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VELOCITY ANALYSIS

The velocity $\mathbf{v}_A$ of a point A on a body is the rate of change of its position $\mathbf{r}_A$.

$$\mathbf{v}_A = \frac{d\mathbf{r}_A}{dt}$$

Different points on a body will have, in general, different velocities. (Recall that velocity is a vector, having direction as well as magnitude.)

The angular velocity, ω, of a body is the rate of change of the angular position, θ, of any line on the body.

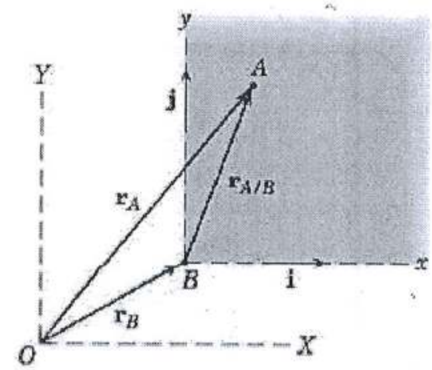
$$\omega = \frac{d\theta}{dt}$$

One body has one angular velocity. For planar motion, we can treat the angular velocity as a scalar with a fixed direction, perpendicular to the plane of motion. Its sense is described as clockwise (cw) or counter clockwise (ccw).

The relative velocity of a point A relative to B is the velocity of A as seen by an observer at B, when B is itself moving.

Consider points A and B, both moving, whose positions measured in a fixed frame of reference (OXY) are given by the vectors $\mathbf{r}_A$ and $\mathbf{r}_B$. These are absolute positions.

Now attach a frame of reference (Bxy) to point B so that it moves with B (but does not rotate). The position of A measured in this new frame is position of A relative to B, written as $\mathbf{r}_{A/B}$.



From the figure,

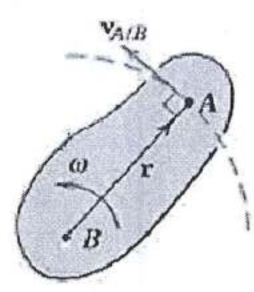
$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$$

Or

$$\mathbf{r}_{A/B} = \mathbf{r}_A - \mathbf{r}_B$$

By taking derivative, we get the relative velocity equation

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$



If points A and B are on the same rigid body rotating with angular velocity ω, then the relative velocity has direction perpendicular to line AB and magnitude $v_{A/B} = \overline{\text{AB}}\omega$.

TYPES OF MOTION

The motion of a rigid body can be divided into three types:

Pure Translation: All points of the body have the same motion. There is no change in the angular position of the body.

Pure Rotation: The body rotates about a fixed point,O. The velocity of any other point, A, on the body has direction perpendicular to the line OA and magnitude $v_A = \overline{\mathrm{OA}}\omega$.

General Motion: The body rotates as well as translates.

We will use the relative velocity equation and the above principles to solve problems of velocity analysis of mechanisms.

The dimensions of the offset slider-crank linkage are given below along with the position solution. If link 2 is rotating with angular velocity $\omega_2 = 2$ rad/s ccw, find the angular velocity of link 3, ω_3 and the velocity of slider, v_c.

$r_1 = OA = 1$ cm

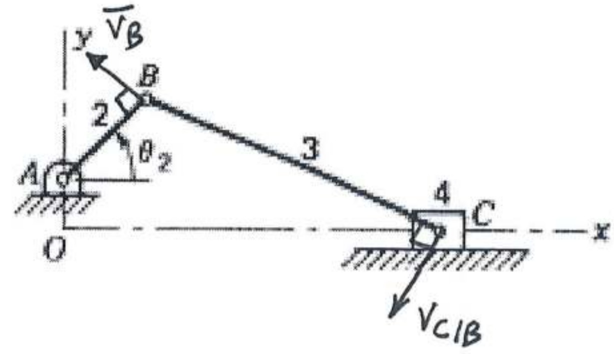
$r_2 = AB = 2.5$ cm

$r_3 = BC = 7$ cm

$\theta_2 = 30^\circ$. $\theta_3 = 18.75^\circ$ (position solution)

$\omega_2 = 2$ rad/s ccw

$\omega_3 = ?$ $V_c = ?$



SOLUTION

Note that

1- Link 2 is in pure rotation about point A, so velocity of point B is $\perp$ to $\overline{AB}$ and its magnitude is

$$V_B = \overline{AB}\ \omega_2 = 2.5 \times 2 = 5 \text{ cm/s}$$

Thus, we know velocity of point B in magnitude and direction

2- Link 4 is in pure translation, so direction of $\overline{V_c}$ is along the x-axis.

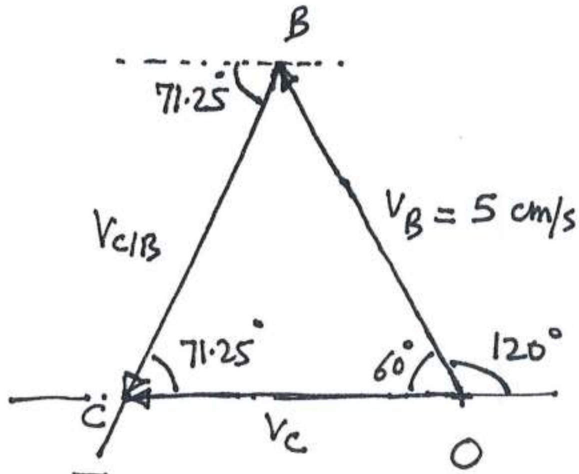
3- Link 3 is in general motion. ~~the~~ Points B and C are on this link as well as on links 2 and 4 respectively. We write the relative velocity equation for points B and C :

$$\overline{V_c} = \overline{V_B} + \overline{V_{C/B}} \quad \text{(or we may use } \overline{V_B} = \overline{V_c} + \overline{V_{B/C}}\text{)}$$

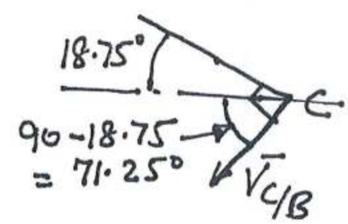
We will solve this equation by drawing the velocity triangle, as follows:

1 - Draw a horizontal line and take any point O on this line. Velocity $\vec{V}_C$ is along this line.

2 - Draw line $OB = V_B = 5$ cm/s (approximate) along angle $30° + 90° = 120°$ to x-axis.



3 - Note that $\vec{V}_{C/B}$ must be $\perp$ to $\overline{BC}$ as points B and C are on the same link. The angle of $\vec{V}_{C/B}$ is calculated to be 71.25°, as shown. Draw line BC from B at 71.25° (approx), thus getting point C.



4 - Complete the triangle, showing directions of vectors according to the velocity equation, $\vec{V}_C = \vec{V}_B + \vec{V}_{C/B}$

5 - Now use Sine law to find V_C and $V_{C/B}$. Angle at B is $180° - 60° - 71.25° = 48.75°$

$$\frac{V_C}{\sin 48.75°} = \frac{5}{\sin 71.25°} \Rightarrow V_C = \underline{\underline{3.97}} \text{ cm/s to the left}$$

$$\frac{V_{C/B}}{\sin 60°} = \frac{5}{\sin 71.25°} \Rightarrow V_{C/B} = 4.57 \text{ cm/s}$$

Now $V_{C/B} = \overline{BC}\,\omega_3 = 7\,\omega_3 \Rightarrow \omega_3 = \frac{4.57}{7} = \underline{\underline{0.653}}$ rad/s CW

The dimensions of the four-bar linkage and the position solution are given below. If link 2 is rotating with angular velocity $\omega_2 = 15$ rad/s ccw find the angular velocities of links 3 and 4, ω_3 and ω_4.

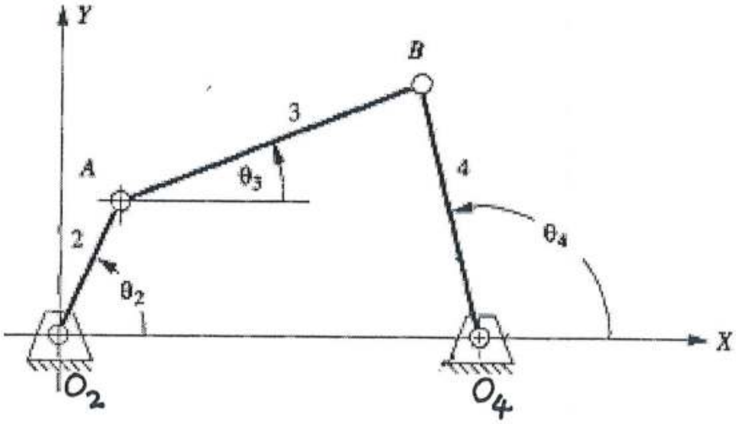
$r_1 = O_2O_4 = 100$ mm

$r_2 = O_2A = 40$ mm

$r_3 = AB = 100$ mm

$r_4 = O_4B = 120$ mm

$\theta_2 = 30°$. $\omega_2 = 15$ rad/s ccw



Position solution : $\theta_3 = 71.9°$. $\theta_4 = 106.6°$

Find ω_3, ω_4

SOLUTION We write the relative velocity equation for points A and B:

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$

Note that

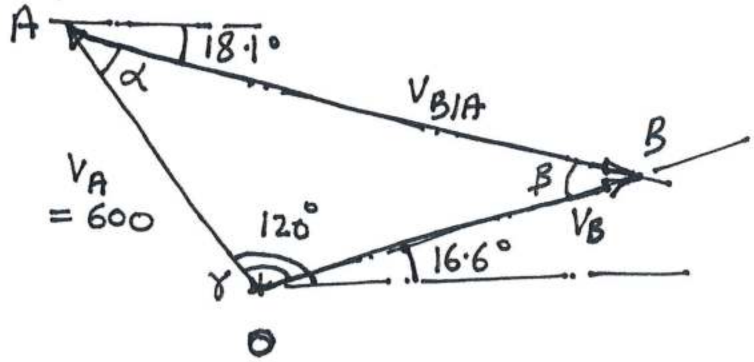
$\vec{V}_B$ is $\perp$ to O_4B, ie, at angle $\theta_4 \pm 90°$

$\vec{V}_A$ is $\perp$ to O_2A, ie, at angle $\theta_2 + 90° = 120°$ from x-axis

$\vec{V}_{B/A}$ is $\perp$ to AB, ie, at angle $\theta_3 \pm 90°$

Also, $V_A = r_2 \omega_2 = 40 \times 15 = 600$ mm/s

Since we know one vector ($\vec{V}_A$) completely and the directions of the two other vectors, we can draw the velocity Triangle. From any point O, draw $V_A = 600$ at angle 120°. Also from O draw a line at $\theta_4 - 90° = 106.6 - 90° = 16.6°$. From A draw line at $\theta_3 - 90° = 71.9 - 90 = -18.1°$. Complete the triangle.



Let the angles at A, B, O be α, β, γ respectively.

From the figure

$$\gamma = 120 - 16.6 = 103.4^\circ$$

$$\beta = 16.6 + 18.1 = 34.7^\circ$$

$$\alpha = 60 - 18.1 = 41.9^\circ$$

(check total : $103.4 + 34.7 + 41.9 = 180^\circ$, OK)

Now use Sine law

$$\frac{V_B}{\sin 41.9^\circ} = \frac{600}{\sin 34.7^\circ} \Rightarrow V_B = 704 \text{ mm/s}$$

$$\frac{V_{B/A}}{\sin 103.4} = \frac{600}{\sin 34.7^\circ} \Rightarrow V_{B/A} = 1025 \text{ mm/s}$$

Then $V_B = r_4 \omega_4$, so $\omega_4 = V_B / r_4 = \frac{704}{120} = \underline{5.87} \text{ rad/s CW}$

$V_{B/A} = r_3 \omega_3$, so $\omega_3 = \frac{V_{B/A}}{r_3} = \frac{1025}{100} = \underline{10.25} \text{ rad/s CW}$

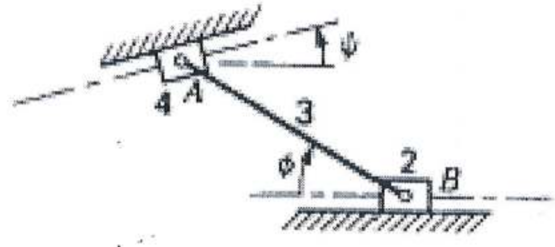
The dimensions of the double-slider linkage and the position solution are given below. If link 2 is moving to the right with velocity $v_2 = 1.5$ m/s, find the angular velocity of link 3, ω_3, and the velocity of link 4, v_4.

$r_3 = AB = 200$ mm $= 0.2$ m

$\psi = 15^\circ$, $\phi = 45^\circ$

$V_B = V_2 = 1.5$ m/s $\longrightarrow$

$V_A = V_4 = ?$, $\omega_3 = ?$



SOLUTION

Write relative velocity equation

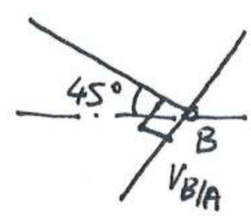
$$\bar{V}_B = \bar{V}_A + \bar{V}_{B/A}$$

We know that

$V_B = 1.5$ along the x-axis to the right

V_A is at $\psi^\circ = 15^\circ$ to the x-axis

$V_{B/A}$ is $\perp$ to AB, so at $45^\circ \pm 90^\circ$

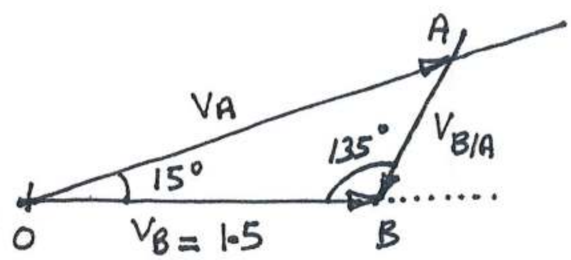


Draw velocity triangle.

Angle at A is $180^\circ - 135^\circ - 15^\circ = 30^\circ$.

Sine law:

$$\frac{V_A}{\sin 135^\circ} = \frac{1.5}{\sin 30^\circ}$$



$\therefore V_A = V_4 = \underline{\underline{2.12}}$ m/s

$$\frac{V_{B/A}}{\sin 15^\circ} = \frac{1.5}{\sin 30^\circ} \Rightarrow V_{B/A} = 0.776 \text{ m/s}$$

But $V_{B/A} = AB\,\omega_3 \Rightarrow \omega_3 = \dfrac{V_{B/A}}{AB} = \dfrac{0.776}{0.2} = \underline{\underline{3.88}}$ rad/s CW