

MECH 350

Measurement and Instrumentation

Chapter (3)

Error and uncertainty

Dr. Eng. Hosam Eldin Elsayed Mohamed

Fahad bin sultan university

Engineering college

Measurement Errors



- What is Error ?
- It is difference between indicated or **measured value and true value.**
- It is impossible to made measurement with perfect accuracy

1.1 Error Calculation Methods

(IN) Accuracy is measured by the absolute and relative errors

$$\text{ABSOLUTE ERROR} = \text{RESULT} - \text{TRUE VALUE}$$

$$\text{RELATIVE ERROR} = \frac{\text{ABSOLUTE ERROR}}{\text{TRUE VALUE}}$$

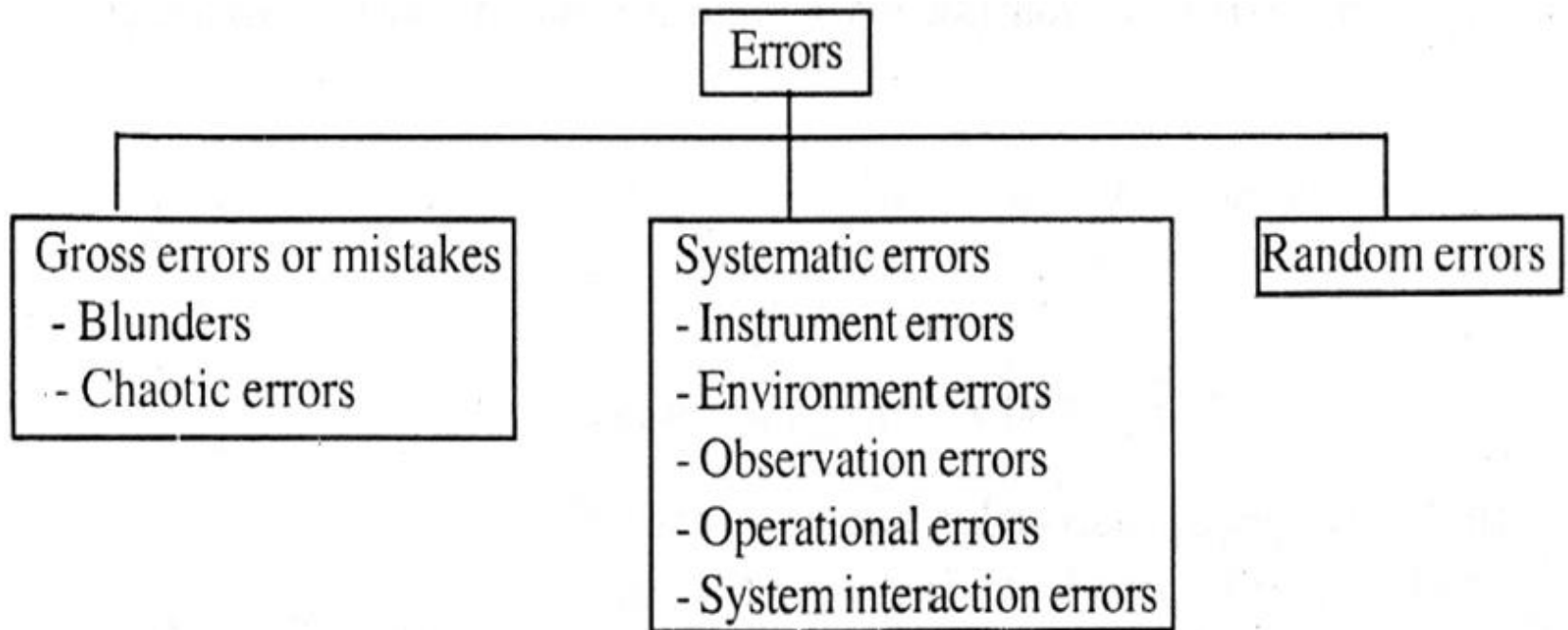
For Example

if the exact value is 50 and the approximation is 49.9, then the absolute error is 0.1, and the relative error is $0.1/50 = 0.002 = 0.2\%$.

Accuracy can be improved by calibration

Measurement Errors

- Types (Classification of Errors)



Measurement Errors

1st type

□ 1. Gross errors

- Human mistakes
- Careless readings, mistake in recordings,
- improper application of instrument
- Can not treated mathematically
- Can be avoided only by taking care in reading and recording

Measurement Errors

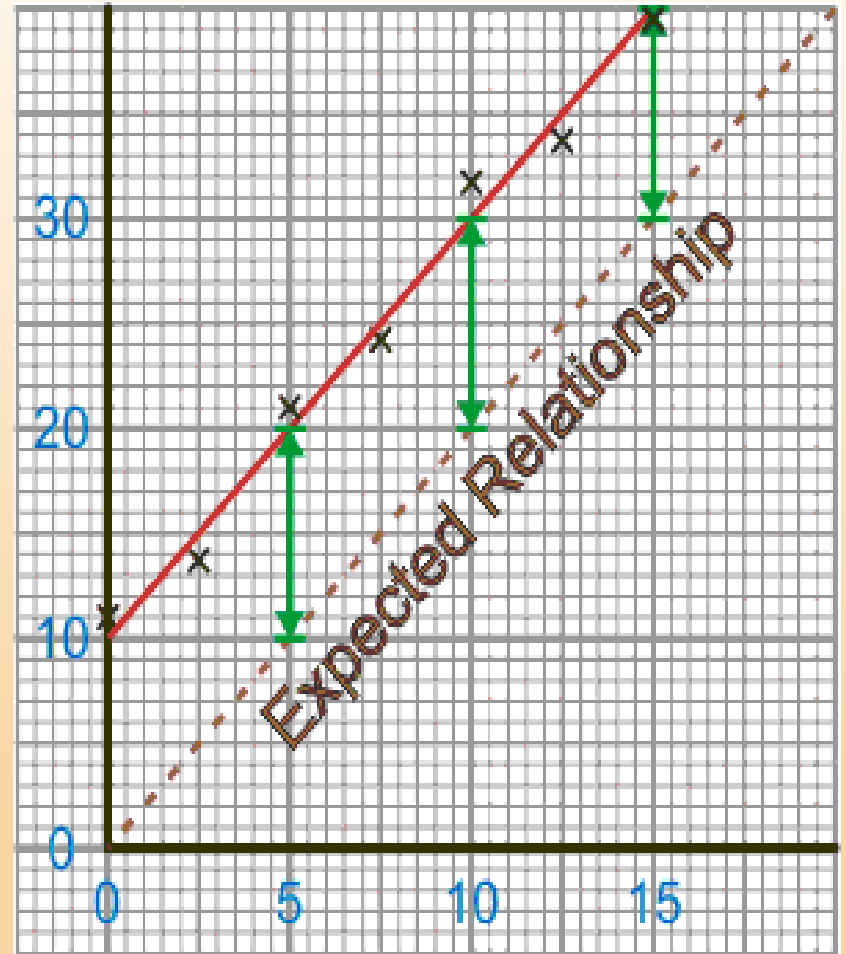
2nd type

2 . Systematic error

- Have definite magnitude and direction.
- Can be repeated consistently with repetition of experiments.
- To locate these errors: repeated measurements under different conditions or with different equipment or possible by an entirely different methods.
- An accurate experiment has a low systematic error

SYSTEMATIC ERROR

- ✘ These cause readings to be spread about some value other than the true value
- ✘ in other words, all the readings are shifted one way or the other way from the true value.



Measurement systematic Errors

Systematic error Types:-

□ 2.1 Instrumental error

- Due to design or construction /assembly of instruments
- Limiting accuracy
- Improper selection of instrument
- Poor maintenance
- For Ex. Errors due to friction, wear, slips, vibration
- Errors due to incorrect fitting of scale at zero, non-uniform division of scale, bent pointer.

Systematic INSTRUMENT error example

Zero errors

Look at this
top-pan balance:

It has a zero error.

There is nothing on it,
but it is not reading zero.

What effect do you think this will have
on all the readings?



Measurement systematic Errors

□ 2.2 Environmental errors

- due to **conditions external** to the measuring instrument, including conditions in the area surrounding the instrument,
- such as effects of change in **temperature**, humidity, **barometric pressure**, or **magnetic or electrostatic fields**.

.

Measurement systematic Errors

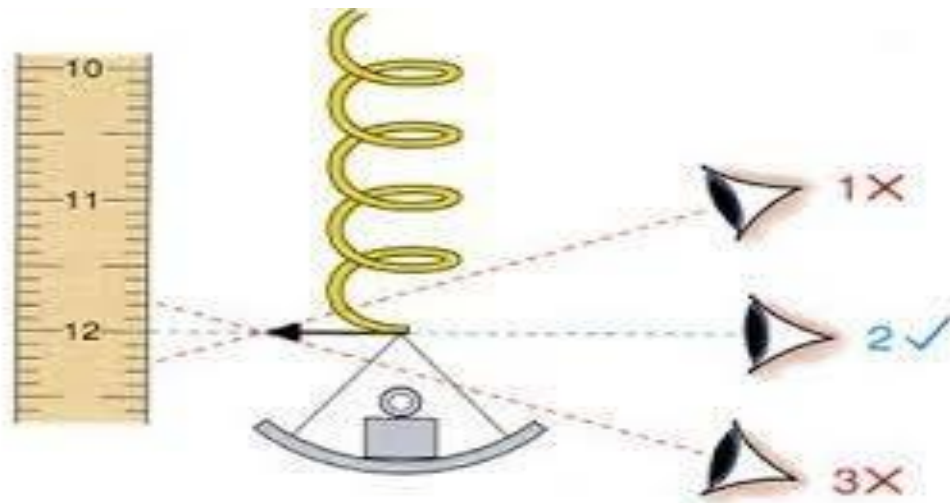
□ 2.2 Environmental errors

- These errors may be avoided by
- (i) Use instrument under **conditions** for which it was **design and calibrated**. This **atmospheric condition** can be **maintain by air conditioning**.
- (ii) Provide **sealing** certain components in the instrument.
- (iii) Make calibration of instrument under the **local atmospheric conditions**

Measurement systematic Errors

□ 2.3 Observation errors :

- Due to poor capabilities and carelessness of operators.
- i. **Parallax** : These errors may arise when the pointer and scale not in same plane or line of vision of observer is not normal to the scale.



Measurement systematic Errors

□ 2.4 Operational error

- Misuse of instrument سوء استخدام
- Poor operational techniques
- For Ex. Errors in flow measurement if flow- meter is placed immediately after a valve or a bend.

Measurement systematic Errors

□ 2.5 System interaction errors

- Interaction between system (to be measured) and instrument body. So it change the condition of the system.
- For Ex. A ruler pressed against a body (system) resulting the deformation of the body.

Measurement Errors

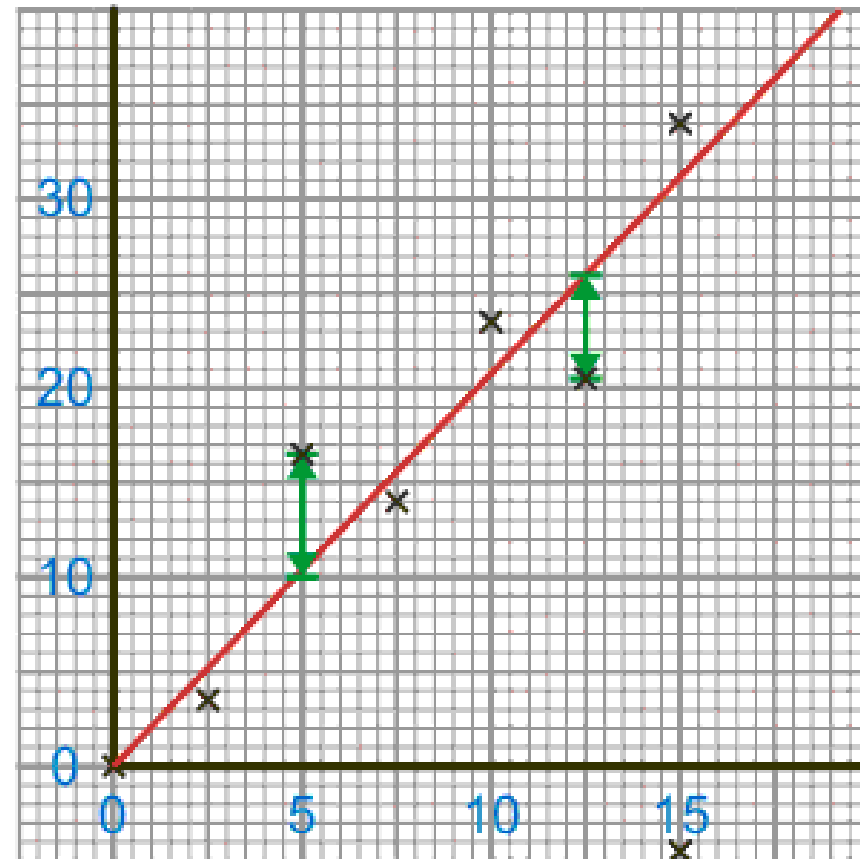
3rd type

□ 3. Random Error

- Accidental in their incidence حدوثها عرضی
- Variable in magnitude and usually follow a certain statistical (probability) law.
- ✓ Friction and stickiness in instrument
- ✓ Vibration in instrument frame or supports
- ✓ Elastic deformation
- ✓ Large dimensional tolerances between the mating parts.
- ✗ Supply power fluctuations
- ✗ These errors cannot be rectified but can be minimized.
- ✓ A precise experiment has a low random error

RANDOM ERROR

- ✘ Random errors are represented on a scatter graph by the distance from the 'Line of Best Fit'.
- ✘ In the adjacent diagram the green lines show the error.
- ✘ It is assumed that the error in measuring the independent variable is low compared to that of the dependent variable - hence the green lines are vertical.
- ✘ If the points are all close to the 'Line of Best Fit' then we can say that our results are good as the error is minimal



Estimate
the uncertainty
of a measured quantity.

2. Determining uncertainty of a measured quantity

Several approaches are used to estimate the uncertainty of a measured quantity.

(a) Instrument Limit of Error (ILE) and Least Count

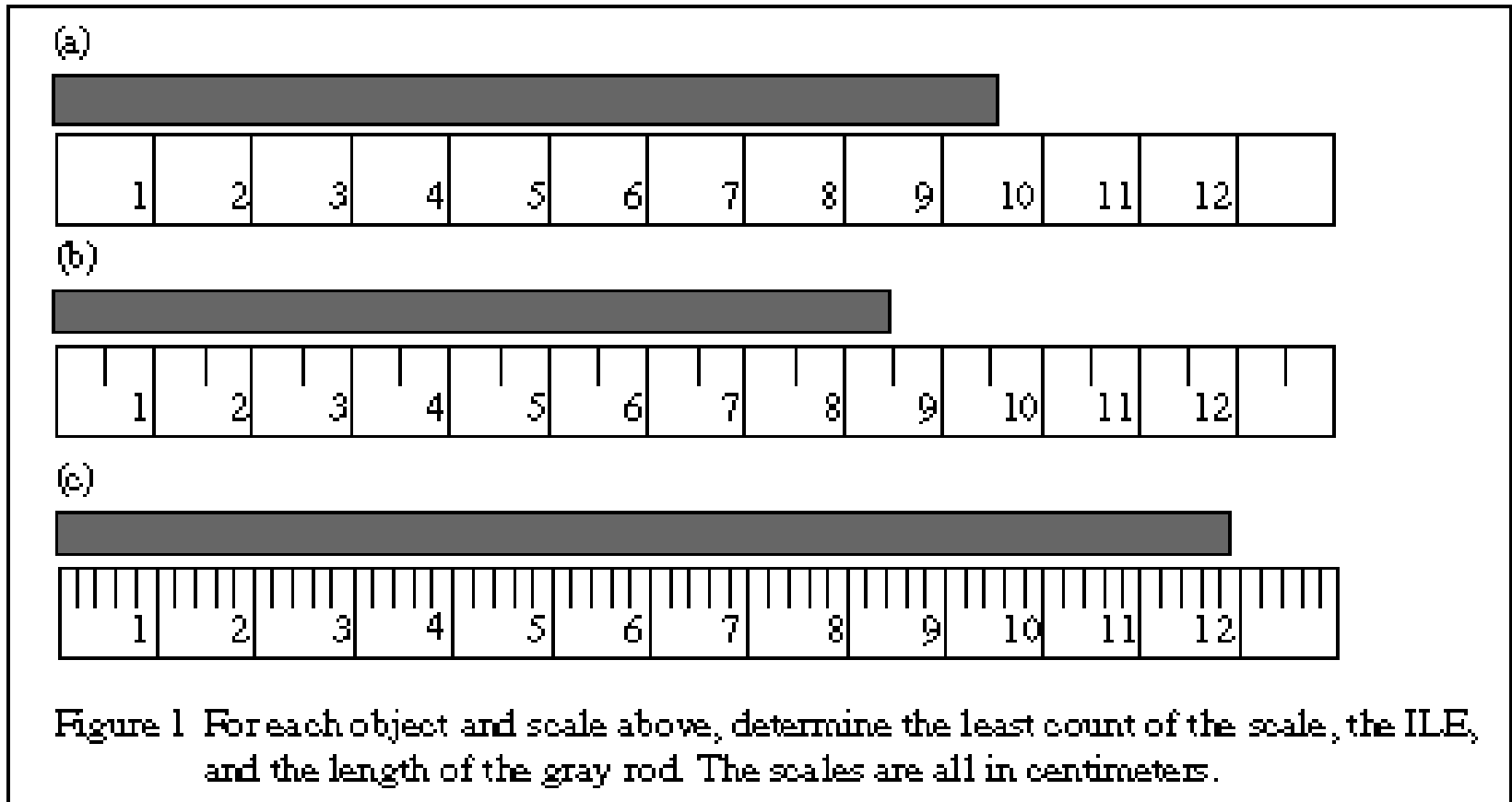
- Least count: the smallest division that is marked on the instrument. For example :-
 - A meter stick will have a least count of 1.0 mm,
 - A digital stop watch might have a least count of 0.01 s.
- Instrument limit of error (ILE):
the precision to which a measuring device can be read, and is always equal to or smaller than the least count.

--

Instrument Limit of Error, ILE

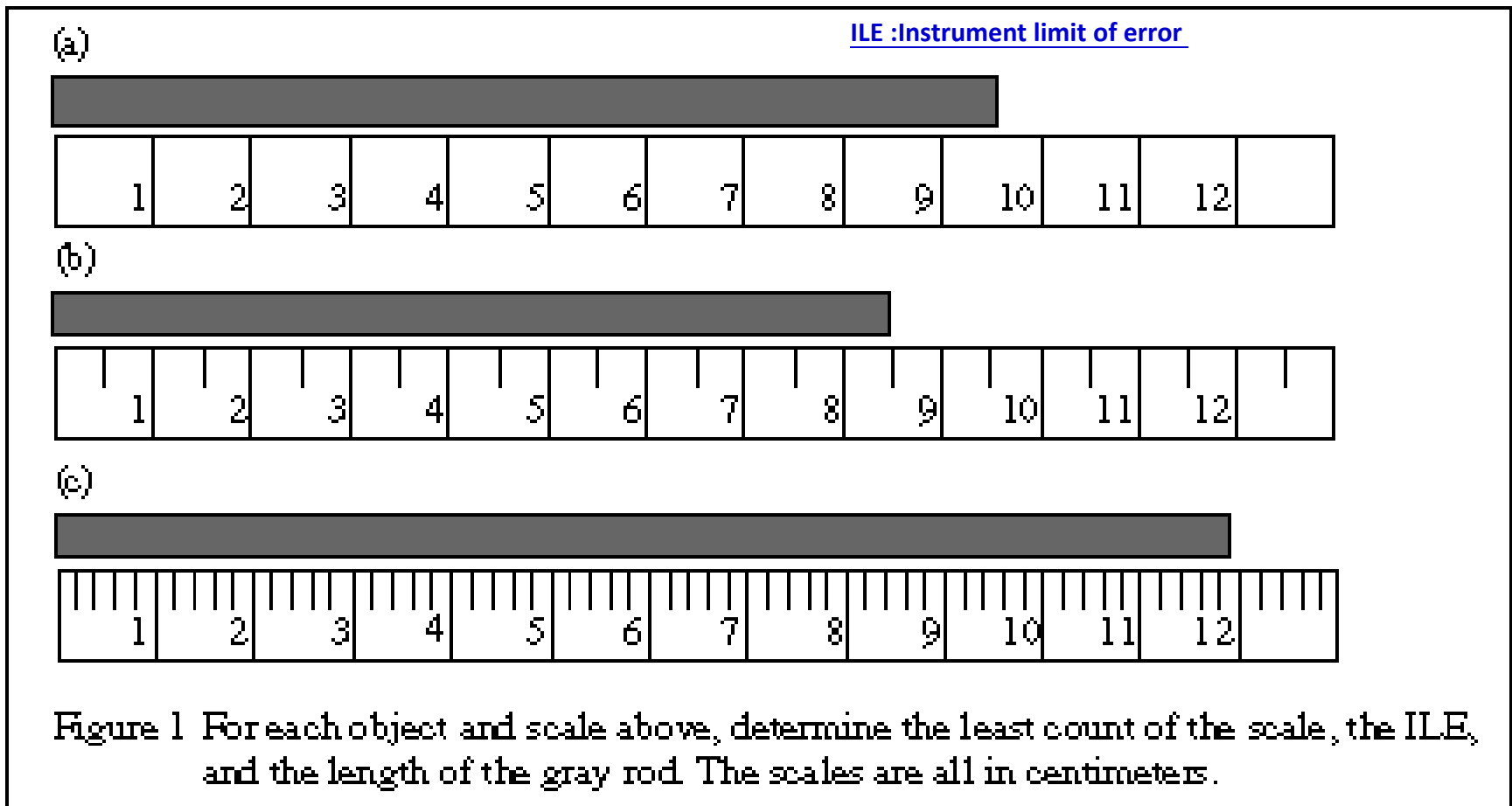
- If the space between the scale divisions **is large**, you may be comfortable in estimating ILE to **1/5** or **1/10** of the **least count**.
- If the scale divisions **are closer together**, you may only be able to estimate ILE to the nearest **1/2** of the **least count**, and
- if the scale divisions are very **close** you may only be able to estimate ILE to the **least count**.

- For some devices the ILE is given as a **tolerance or a percentage**.
- Resistors may be specified as having a tolerance of 5%, meaning that the ILE is 5% of the resistor's value.
- **Example 1:** For each of the following scales (all in centimeters) determine the least count, the ILE, and read the length of the gray rod.



- **Solution** to determine the least count, the ILE, and read the length of the gray rod for each of the following scales (all in centimeters).

	Least Count (cm)	ILE (cm)	Length (cm)
(a)	1	0.2	9.6
(b)	0.5	0.1	8.5
(c)	0.2	0.05	11.90



Estimated Uncertainty methods

تمهيد

١. الانحراف المتوسط (**Average deviation**)
٢. والانحراف المعياري (**Standard deviations**)
هما من مقاييس التشتت في علم الاحصاء ويمكن استعمال اي نوع منهما لدراسة التشتت .
٣. المتوسط الحسابي (**Average**) يعبر عن متوسط مجموع عدة ارقام تتعلق بشيء معين

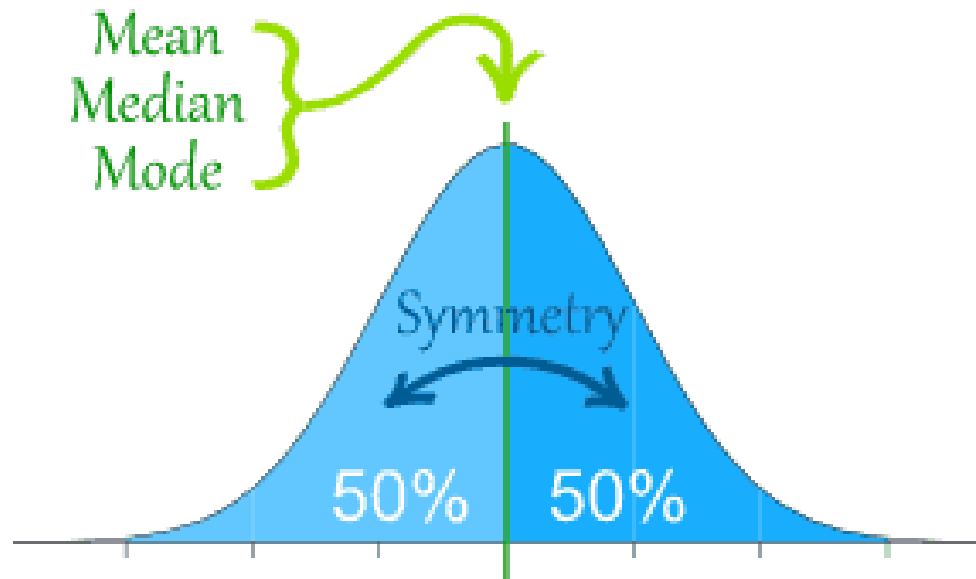
3. Normal(Gaussian) Distribution

What are some examples of things that follow a Normal(Gaussian) Distribution?

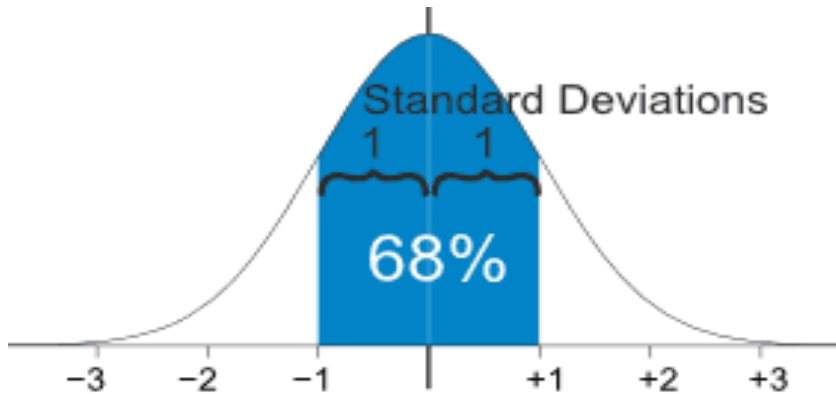
- Heights of people
- Size of things produced by machines
- Errors in measurements
- Blood Pressure
- Test Scores

Normal Distribution Curve

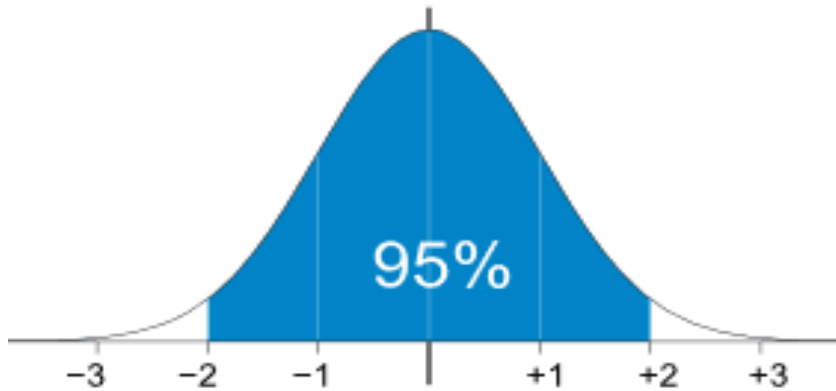
- mean=median=mode
- Symmetry about the center
- 50% of the values less than the mean and 50% greater than the mean



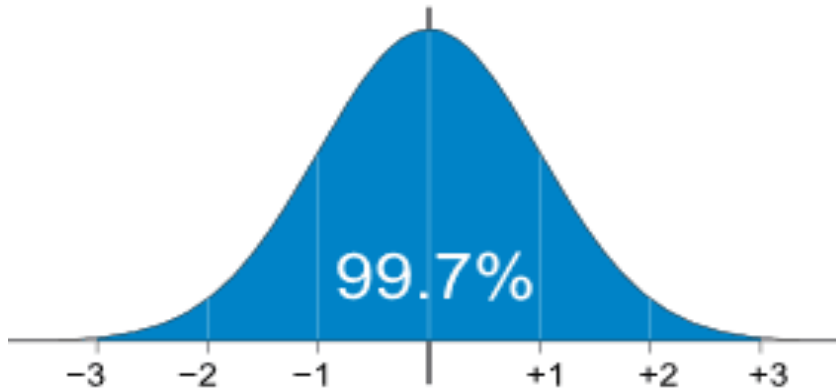
The Standard Deviation :
is a measure of how spread out numbers are.



68% of values are within 1 standard deviation of the mean



95% of values are within 2 standard deviations of the mean



99.7% of values are within 3 standard deviations of the mean

Why do we need to know Standard Deviation?

- Any value is
 - **likely** to be within 1 standard deviation of the mean
من المرجح أن يكون في حدود ١ الانحراف المعياري للمتوسط
 - **very likely** to be within 2 standard deviations
 - **almost certainly** within 3 standard deviations

Estimated Uncertainty

(b) Average and average deviation Method

Example 2:- Find the average, and average deviation for the 5 following data on the length of a pen, L.

Length (cm)	$ L_i - L_{ave} $	$(L_i - L_{ave})^2$
12.2	0.02	0.0004
12.5	0.28	0.0784
11.9	0.32	0.1024
12.3	0.08	0.0064
12.2	0.02	0.0004
Sum 61.1	Sum 0.72	Sum 0.1880
Average $61.1/5 = 12.22$	Average deviation $0.72/5 =$ 0.14	$\sigma = \sqrt{\frac{0.1880}{4}} = 0.22$ Standard deviations

- To get the **average** $\langle L \rangle$:
sum the values and divide by the number of measurements.
 - To get the **average deviation** ΔL ,
 1. Find the absolute values of the **deviations**, $|L - L_{ave}|$
 2. Sum the **absolute deviations**,
 3. Get the average absolute deviation by dividing by the number of measurements
 - To get the **standard deviation**
 1. Find the **deviations and square** of them
 2. Sum the squares
 3. **Divide by (N-1)**, (here it is 4)
 4. Take the square root.
- The pen has a length of
- (12.22 ± 0.14) cm or (12.2 ± 0.1) cm [use average deviations]
- Or (12.22 ± 0.22) cm or (12.2 ± 0.2) cm [use standard deviations].

The proper way to write the answer of uncertainty

Example 3

One make several measurements on the mass of an object.

The balance has an ILE of 0.02 grams.

The average mass is 12.14286 grams,
the average deviation is 0.07313 grams.

What is the correct way to write the mass of the object including its uncertainty? **What is the mistake in each incorrect one?** [Answer](#)

- 12.14286 g
- (12.14 ± 0.02) g
- 12.14286 g ± 0.07313 (lack of unit)
- 12.143 ± 0.073 g
- (12.143 ± 0.073) g
- (12.14 ± 0.07)
- (12.1 ± 0.1) g
- 12.14 g ± 0.07 g

• **The correct answer is (12.14 ± 0.07) g.**

Example 4

- : Find the average and the average deviation of the following measurements of a mass.
- This time there are $N = 6$ measurements, so for the standard deviation we divide by $(N-1) = 5$.

The mass is

(4.342 ± 0.022) g

or **(4.34 ± 0.02) g**

[using average deviations] or

(4.342 ± 0.023) g

or **(4.34 ± 0.02) g**

[using standard deviations].

Mass (grams)	$ m - m_{ave} $	$(m - m_{ave})^2$
4.32	0.0217	0.000471
4.35	0.0083	0.000069
4.31	0.0317	0.001005
4.36	0.0183	0.000335
4.37	0.0283	0.000801
4.34	0.0017	0.000003
Sum 26.05	0.1100	0.002684
Average 4.3417	Average deviation 0.022	Standard deviations $\sigma = 0.023$

Example 5 Finding an average length and an average deviation in length. To round the values, the these values to have an excess of significant figures.

Measured results of the length :

(15.5 ± 0.1) m or (15.47 ± 0.13) m using average dev.

(15.5 ± 0.2) m or (15.47 ± 0.18) m with standard deviation

Length, x, m	$ x - \langle x \rangle , m$	$(x - \langle x \rangle)^2$
15.4	0.06667	0.004445
15.2	0.26667	0.071112
15.6	0.13333	0.017777
15.7	0.23333	0.054443
15.5	0.03333	0.001111
15.4	0.06667	0.004445
Average 15.46667 m	±0.133333 m ±0.13 m (Rounded)	St. dev. ±0.17512 m ±0.18 m (Rounded)

Built-in Functions in *Excel*

- use a spreadsheet such as Excel there are built-in functions that help you to find these quantities.

=SUM(A2:A5)	Find the sum of values in the range of cells A2 to A5.
=COUNT(A2:A5)	Count the number of numbers in the range of cells A2 to A5.
=AVERAGE(A2:A5)	Find the average of the numbers in the range of cells A2 to A5.
=AVEDEV(A2:A5)	Find the average deviation of the numbers in the range of cells A2 to A5.
=STDEV(A2:A5)	Find the sample standard deviation of the numbers in the range of cells A2 to A5.

- To round the uncertainty to one or two significant figures (more on rounding in Section 7), and
- To round the average to the same number of digits relative to the decimal point.
- Thus the average length with average deviation is either (15.47 ± 0.13) m or (15.5 ± 0.1) m.
- If we use standard deviation we report the average length as (15.47 ± 0.18) m or (15.5 ± 0.2) m.
- Follow your instructor's instructions on whether to use average or standard deviation in your reports.

Example 6 (by Excel)

Values showing the determination of **average**, **average deviation**, and **standard deviation** in a measurement of time. Notice that to get a non-zero average deviation we must take the absolute value of the deviation.

Time, t , sec	$(t - \langle t \rangle)$, sec	$ t - \langle t \rangle $, sec	$(t - \langle t \rangle)^2$, sec ²
7.4	-0.2	0.2	0.04
8.1	0.5	0.5	0.25
7.9	0.3	0.3	0.09
7.0	-0.6	0.6	0.36
$\langle t \rangle = 7.6$	$\langle t - \langle t \rangle \rangle =$ 0.0	$\langle t - \langle t \rangle \rangle = 0.4$	$\langle (t - \langle t \rangle)^2 \rangle = 0.247$ Std. dev = 0.50

Problems 7 : Are the following numbers equal within the expected range of values? Answer

(1) (3.42 ± 0.04) m/s and 3.45 m/s?

(2) (13.106 ± 0.014) grams and 13.206 grams?

(3) (2.95 ± 0.03) x m/s and 3.00 x m/s

4. Propagation of Errors, Basic Rules انتشار الأخطاء

- Suppose two measured quantities x and y have uncertainties, Δx and Δy to report $(x \pm \Delta x)$, and $(y \pm \Delta y)$.
- From the measured quantities a new quantity, z , is calculated from x and y . What is the uncertainty, Δz , in z ?
- Use a simplified version of the proper statistical treatment.
- The guiding principle in all cases is to consider the most pessimistic situation.
- Full explanations are covered in statistics courses.
- The examples included in this section also show the proper rounding of answers, which is covered in more detail in Section 6.
- The examples use the propagation of errors using average deviations

(4-1) Addition and Subtraction:

$$z = x + y \quad \text{or} \quad z = x - y$$

- **Derivation:** Assume that the uncertainties are arranged so as to make z as far from its true value as possible.
 - **Average deviations** $\Delta z = |\Delta x| + |\Delta y|$ in both cases.
 - With more than two numbers added or subtracted we continue to add the uncertainties.
1. Using simpler average errors

$$\Delta z = |\Delta x| + |\Delta y| + \dots$$

2. Using standard deviations

$$\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2 + \dots}$$

Example 8

$w = (4.52 \pm 0.02)\text{cm}$, $x = (2.0 \pm 0.2)\text{cm}$, $y = (3.0 \pm 0.6)\text{cm}$.

Find $z = x + y - w$ and its uncertainty.

(1) $z = x + y - w = 2.0 + 3.0 - 4.5 = 0.5 \text{ cm}$

(2) $\Delta z = \Delta x + \Delta y + \Delta w = 0.2 + 0.6 + 0.02 = 0.82 \Rightarrow 0.8 \text{ cm}$

So $z = (0.5 \pm 0.8) \text{ cm}$ with average deviation

(3) Solution with standard deviations, $\Delta z = 0.633 \text{ cm}$,

So $z = (0.5 \pm 0.6) \text{ cm}$

- Notice that we round the uncertainty to one significant figure and round the answer to match.

Example 9

$$x = (2.0 \pm 0.2) \text{ cm}, y = (3.0 \pm 0.6) \text{ cm}.$$

Find $z = x - 2y$ and its uncertainty.

$$(1) \quad z = x - 2y = 2.0 - 2(3.0) = -4.0 \text{ cm}$$

$$(2) \quad \Delta z = \Delta x + 2 \Delta y = 0.2 + 1.2 = 1.4 \text{ cm}$$

So $z = (-4.0 \pm 1.4) \text{ cm}$ (average deviation).

$$z = (-4.0 \pm 0.9) \text{ cm} \text{ (standard deviation)}$$

The 0 after the decimal point in 4.0 is significant and must be written in the answer.

The uncertainty in this case starts with a 1 and is kept to two significant figures. (More on rounding in Section 7.)

(4-2) Multiplication and Division:

$$z = x y \quad \text{or} \quad z = x/y$$

- **Derivation:** derive the relation for multiplication easily. Take the largest values for x and y , that is

$$z \pm \Delta z = (x \pm \Delta x)(y \pm \Delta y) = xy + x \Delta y + y \Delta x + \underline{\Delta x \Delta y}$$

- Usually $\Delta x \ll x$ and $\Delta y \ll y$ \Rightarrow $\Delta x \Delta y \ll 0$

so that the last term $\Delta x \Delta y$ is much smaller than the other terms and can be neglected.

\Rightarrow Since $z = xy$, $\Delta z = y \Delta x + x \Delta y$

- which we write more compactly by forming the relative error, that is the ratio of $\Delta z/z$, namely

$$\frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \dots$$

For multiplication, division, or combinations

The same rule holds, namely **add all the relative errors to get the relative error in the result.**

$$\frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \dots$$

Using average deviation errors

$$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} + \dots$$

Using standard deviations

Example 10

$w = (4.52 \pm 0.02) \text{ cm}$, $x = (2.0 \pm 0.2) \text{ cm}$.

Find $z = w \cdot x$ and its uncertainty.

(1) $z = w \cdot x = (4.52) (2.0) = 9.04 \text{ cm}^2$

(2) Average error:

$$\frac{\Delta z}{9.04 \text{ cm}^2} = \frac{0.02 \text{ cm}}{4.52 \text{ cm}} + \frac{0.2 \text{ cm}}{2.0 \text{ cm}} = 0.1044$$

$\Rightarrow \Delta z = 0.1044 (9.04 \text{ cm}^2) = 0.944 \Rightarrow$ round to 0.9 cm^2 ,

$\Rightarrow z = (9.0 \pm 0.9) \text{ cm}^2$.

(3) Standard deviation: $\Delta z = 0.905 \text{ cm}^2$

$\Rightarrow z = (9.0 \pm 0.9) \text{ cm}^2$

The uncertainty is rounded to one significant figure and the $\langle z \rangle$ is rounded to match. To write 9.0 cm^2 rather than 9 cm^2 since the 0 is significant.

Example 11

$$x = (2.0 \pm 0.2) \text{ cm}, y = (3.0 \pm 0.6) \text{ sec.}$$

Find $z = x/y$ with a dimension of velocity.

(1) $z = 2.0/3.0 = 0.6667 \text{ cm/s.}$

(2) Average error:

$$\Delta z = 0.3 (0.6667 \text{ cm/sec}) = 0.2 \text{ cm/sec}$$

(3) Using average deviation : $z = (0.7 \pm 0.2) \text{ cm/sec}$

(4) Using standard deviation: $z = (0.67 \pm 0.15) \text{ cm/sec}$

- Note that in this case we round off our answer to have no more decimal places than our uncertainty.