Chapter 1

Single phase systems and power measurements

1. Value and representation of a sinusoidal signal

1.1. Periodic, alternating and sinusoidal signal

A voltage *u* is periodic, if its instantaneous value *u (t)* is such that:

$$u(t) = u(t+T); \begin{cases} T \text{ : period} \\ \dfrac{1}{T} \text{ : frequency } f \end{cases}$$

- The signal *u* is called <u>alternative</u> (AC), if it is continually changing from zero up to the positive peak, through zero to the negative peak and back to zero again (sometimes positive and sometimes negative). If *u(t)* vanishes only twice in each period, the positive alternance is the part of the period in which *u* is positive, the negative alternance is that in which *u* is negative.
- The expression of the sinusoidal voltage *u* is given by:

$$u(t) = U_{\max}\cos(\omega t + \varphi), \text{ avec } \begin{cases} U_{\max} = \sqrt{2}U \text{ : peak value} \\ U = U_{RMS} \text{ : effective value} \\ \omega = 2\pi f = \dfrac{2\pi}{T} \text{ : angular velocity (rad/s)} \\ \omega t + \varphi \text{ : instantaneous phase (rad)} \\ \varphi \text{ : initial phase (rad)} \end{cases}$$

1.2. Average value

The mean value of the periodic voltage *u(t)* with period *T* is defined by :

$$U_{mean} = \frac{1}{T}\int_0^T u(t)dt$$

<u>Example</u> :

Let's $u(t) = U_{\max}\cos(\omega t + \varphi)$ a sinusoidal signal defined by

$$U_{mean} = \frac{1}{T}\int_0^T U_{\max}\cos(\omega t + \varphi)dt$$

$$= \frac{U_{\max}}{T}\int_0^T \cos(\omega t + \varphi)dt$$

$$= \frac{U_{\max}}{T} \times \frac{1}{\omega}\left[\sin(\omega t + \varphi)\right]_0^T$$

$$\Rightarrow U_{mean} = \frac{U_{max}}{2\pi}\left[\sin(2\pi+\varphi)-\sin(\varphi)\right]=0$$

1.3. Effective Value: the Root mean Square Value (RMS)

If we are dealing with a periodic voltage signal u such that $u(t)=u(t+T)$, it's effective value U_{eff} is defined by:

$$U_{eff} = \sqrt{\frac{1}{T}\int_0^T u^2(t)dt}$$

1.4. Representations of sinusoidal magnitudes

1.4.1 Vector Representation

We consider a disc with center O rotating with the angular velocity ω. Let's M_1 and M_2 two points of disc showing the points of the vectors, respectively current and voltage (Fig 1.1).

- $\overrightarrow{OM_1} = \vec{I}$ makes an angle θ_1 with the origin (vector rotating at angular velocity ω), $\theta_1 = \omega t + \varphi_1$.
- $\overrightarrow{OM_2} = \vec{U}$ makes an angle θ_2 with the origin (vector rotating at angular velocity ω), $\theta_2 = \omega t + \varphi_2$.
- Instantaneous values of current $i(t)$ and voltage $u(t)$ are the projection of rotating vectors $\vec{I}$ and $\vec{U}$ on a fixed axis.

  $$\begin{cases} i(t) = I_{max}\cos(\omega t+\varphi_1) \\ u(t) = U_{max}\cos(\omega t+\varphi_2) \end{cases}$$

I_{max} and U_{max} are the maximum values of vectors modules $\vec{I}$ and $\vec{U}$.

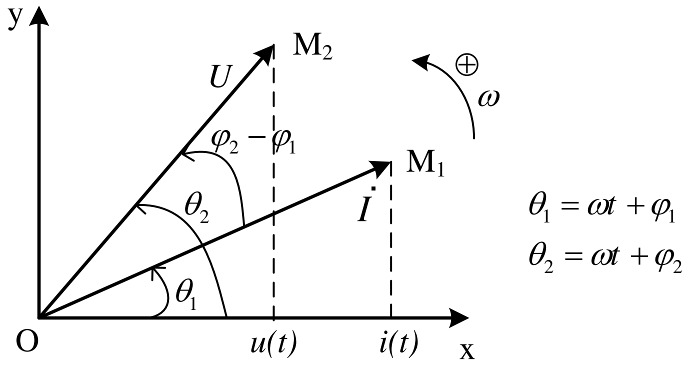


Fig 1.1

The angle deviation φ between $\dot{U}$ and $\bar{I}$ is called <u>angular phase shift</u> ($\varphi = \varphi_2 - \varphi_1 = \varphi_U - \varphi_I$).

- If ($\varphi = 0$), the voltage and the current are *in phase*.
- If ($\varphi > 0$), the voltage leads the current.
- If ($\varphi = \pi/2$), the voltage is 90° *out of phase* with the current.
- If ($\varphi < 0$), the voltage lags the current.
- If ($\varphi = -\pi/2$), the voltage is -90° *out of phase* with the current.
- If ($\varphi = \pi$), the voltage and the current are in phase opposition.

1.4.2. Complex Notation

Consider the instantaneous current i and the voltage u signals defined by:

$$\begin{cases} i(t) = I_{\max}\cos(\omega t) = \sqrt{2}I\cos(\omega t) \\ u(t) = U_{\max}\cos(\omega t + \varphi) = \sqrt{2}U\cos(\omega t + \varphi) \end{cases}$$

These quantities can be characterized by the components of their representative vectors in the complex plane.

- $\bar{I}$ have as components I and 0.
- $\dot{U}$ have as components $U\cos(\varphi)$ and $U\sin(\varphi)$.

The points $M_1(I,\ 0)$ and $M_2(U\cos(\varphi),\ U\sin(\varphi))$ present in the complex plane, the following modules :

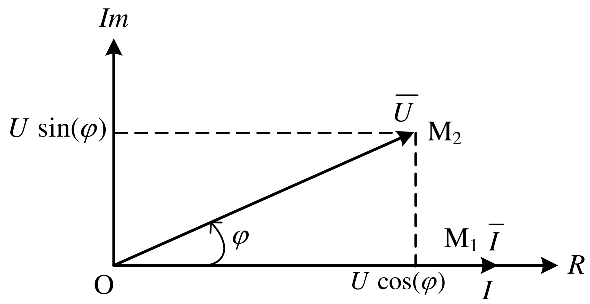


Fig 1.2

$$\begin{cases} Z_1 = I + j0 = Ie^{j0} = \bar{I} \\ Z_2 = U\cos(\varphi) + jU\sin(\varphi) = Ue^{j\varphi} = \bar{U} \end{cases}$$

In complex notation, $i(t)$ and $u(t)$ can be represented by their complex amplitudes $\bar{I}$ and $\bar{U}$ defined by their effective values I and U and their phases in the origin 0 and φ.

1.5. Properties of sinusoidal quantities

1.5.1 Addition - subtraction

we pose :

$$\left.\begin{array}{l} i_1(t)=\sqrt{2}I_1\cos(\omega t+\varphi_1) \\ i_1(t)=\sqrt{2}I_2\cos(\omega t+\varphi_2) \end{array}\right\} i(t)=i_1(t)+i_2(t)=\sqrt{2}I\cos(\omega t+\varphi)$$

$$\Rightarrow \overline{I}=\overline{I}_1+\overline{I}_2$$

$$\Rightarrow I(\cos\varphi+j\sin\varphi)=I_1(\cos\varphi_1+j\sin\varphi_1)+I_2(\cos\varphi_2+j\sin\varphi_2)$$

$$\Rightarrow \begin{cases} I\cos\varphi=I_1\cos\varphi_1+I_2\cos\varphi_2 \\ I\sin\varphi=I_1\sin\varphi_1+I_2\sin\varphi_2 \end{cases}$$

$$\Rightarrow \begin{cases} I=\sqrt{I^2\cos^2\varphi+I^2\sin^2\varphi} \\ \varphi=\operatorname{arctg}\left(\dfrac{I\sin\varphi}{I\cos\varphi}\right) \end{cases}$$

1.1.5.2. Derivation - Integration

$$i(t)=\sqrt{2}I\cos(\omega t+\varphi)$$

$$\Rightarrow \begin{cases} \dfrac{di(t)}{dt}=\omega\sqrt{2}I\cos(\omega t+\varphi+\dfrac{\pi}{2})=-\omega\sqrt{2}I\sin(\omega t+\varphi) \\ \displaystyle\int i(t)dt=\frac{\sqrt{2}I}{\omega}\cos(\omega t+\varphi-\frac{\pi}{2})=\frac{\sqrt{2}I}{\omega}\sin(\omega t+\varphi) \end{cases}$$

$$\Rightarrow \begin{cases} i(t)=\overline{I} \\ \dfrac{di(t)}{dt}\Rightarrow j\omega\overline{I} \\ \displaystyle\int id(t)dt\Rightarrow\frac{1}{j\omega}\overline{I}=-\frac{j}{\omega}\overline{I} \end{cases}$$

2. Impedance

- The impedance Z is defined as the quotient of the effective value of voltage by that of the current : $Z=\dfrac{U}{I}$.
- The complex impedance $\overline{Z}$ is defined as the quotient of the complex quantities representing voltage and current : $\overline{Z}=\dfrac{\overline{U}}{\overline{I}}$.

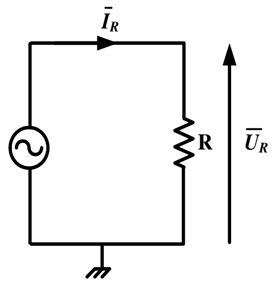
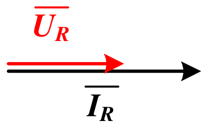
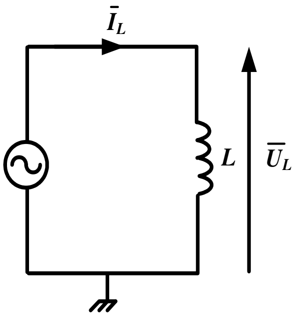
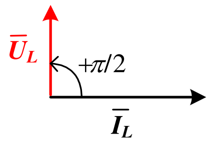
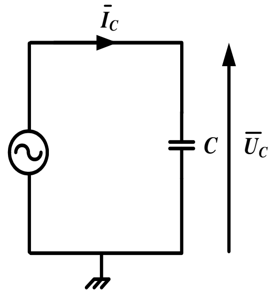
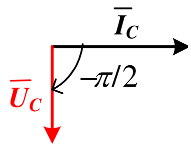
If $\overline{Z}=R+jX$, with $\begin{cases} R: \text{ total resistance of a circuit} \\ X: \text{ total reactance of a circuit} \end{cases}$

$\overline{Z}$ have a module Z such that $\left|\overline{Z}\right|=Z=\sqrt{R^2+X^2}$.

$\overline{Z}$ have an argument φ such that $tg\varphi=\dfrac{X}{R}(\sin\varphi=\dfrac{X}{Z}$ and $\cos\varphi=\dfrac{R}{Z})$.

$$\Rightarrow \overline{Z}=Ze^{j\varphi}$$

- The inverse of the impedance is the admittance $\overline{Y} = \dfrac{1}{\overline{Z}}$

Circuits	Instantaneous Expression	complex Expression	vector Diagram
R Circuit  	$i_R(t) = \sqrt{2} I_R \cos(\omega t)$ $u_R(t) = R i_R(t)$ $u_R(t) = R I_R \sqrt{2} \cos(\omega t)$	$\overline{I}_R = I_R$ $\overline{U}_R = R I_R$ $= R \overline{I}_R$ $= \overline{Z}_R \overline{I}_R$ $\Rightarrow \overline{Z}_R = R = Z e^{j\varphi}$ $\Rightarrow \begin{cases} Z = R \\ \varphi = 0 \end{cases}$	
L Circuit  	$i_L(t) = \sqrt{2} I_L \cos(\omega t)$ $u_L(t) = L \dfrac{di_L(t)}{dt}$ $u_L(t) = L\omega I_L \sqrt{2} \cos(\omega t + \dfrac{\pi}{2})$ $= -L\omega I_L \sqrt{2} \sin(\omega t)$	$\overline{I}_L = I_L$ $\overline{U}_L = jL\omega \overline{I}_L$ $= \overline{Z}_L \overline{I}_L$ $\Rightarrow \overline{Z}_L = jL\omega = Z e^{j\varphi}$ $\Rightarrow \begin{cases} Z = L\omega \\ \varphi = \pi/2 \end{cases}$	
C Circuit  	$i_C(t) = \sqrt{2} I_C \cos(\omega t)$ $u_C(t) = \dfrac{1}{C} \int i_C(t) dt$ $u_C(t) = \dfrac{\sqrt{2} I_C}{C\omega} \cos(\omega t - \dfrac{\pi}{2})$ $= \dfrac{\sqrt{2} I_C}{C\omega} \sin(\omega t)$	$\overline{I}_C = I_C$ $\overline{U}_C = \dfrac{\overline{I}_C}{jC\omega} = -j \dfrac{\overline{I}_C}{C\omega}$ $\overline{U}_C = \overline{Z}_C \overline{I}_C$ $\Rightarrow \overline{Z}_C = \dfrac{1}{jC\omega}$ $= -j \dfrac{1}{C\omega}$ $\Rightarrow \begin{cases} Z = \dfrac{1}{C\omega} \\ \varphi = -\pi/2 \end{cases}$	

3. Power measurements

The instantaneous power *p* supplied by a voltage source *u* providing a current *i* is:

$$p = ui$$

- The active power *P* (or simply power),is defined as the mean value of the instantaneous power:

$$P = \frac{1}{T}\int_0^T p.dt = \frac{1}{T}\int_0^T ui.dt, \; T: \text{ period}$$

In the sinusoidal regime:

$$\begin{cases} u(t) = \sqrt{2}U\cos(\omega t + \varphi) \\ i(t) = \sqrt{2}I\cos(\omega t) \end{cases},$$

We can write:

$$P = \frac{1}{T}\int_0^T p \cdot dt = UI\cos\varphi \text{ (Watt : W)}$$

- The reactive power Q is defined by the following expression :

$Q = UI\sin\varphi$ (VAr : Volt-Ampere reactive)

- The Volt-Amperes (S), a scalar quantity, is the product of the magnitudes the voltage and the current:
  $S = UI$ (en VA : Volt-Ampere)

- We define the complexe power $\overline{S}$ by :

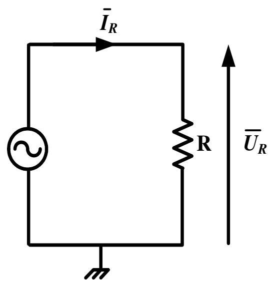
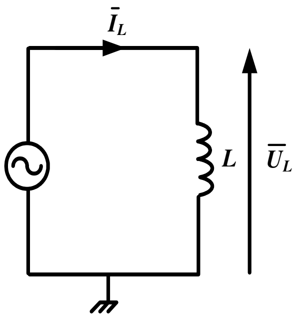
$\overline{S} = \overline{U} \cdot \overline{I}^*$, $\overline{I}^*$: conjugate of $\overline{I}$;

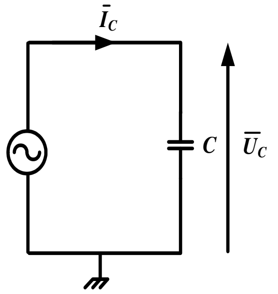
$\Rightarrow \overline{S} = P + jQ$

- We call power factor, the quotient of P by S :

$$F = \frac{P}{S} = \cos\varphi$$

The following table shows the active, reactive and apparent powers of the circuits R,L and C :

Circuits	**Power : $\overline{S}\,/S\,/P\,/Q$**
R Circuit 	$\overline{S}_R = \overline{U}_R \cdot \overline{I}_R^* = R\overline{I}_R \cdot \overline{I}_R^* = RI_R^2$ Or, $\overline{S}_R = P_R + jQ_R$ $\Rightarrow \begin{cases} P_R = RI_R^2 = \frac{U_R^2}{R} \\ Q_R = 0 \end{cases}$
L Circuit 	$\overline{S}_L = \overline{U}_L \cdot \overline{I}_L^* = jL\omega\overline{I}_L \cdot \overline{I}_L^* = jL\omega I_L^2$ Or, $\overline{S}_L = P_L + jQ_L$ $\Rightarrow \begin{cases} P_L = 0 \\ Q_L = L\omega I_L^2 = \frac{U_L^2}{L\omega} \end{cases}$

C Circuit 	$\overline{S}_C = \overline{U}_C \cdot \overline{I}_C^* = \frac{\overline{I}_C}{jC\omega}\cdot\overline{I}_C^* = \frac{I_C^2}{jC\omega} = -j\,\frac{I_C^2}{C\omega}$ Or, $\overline{S}_C = P_C + jQ_C$ $\Rightarrow \begin{cases} P_C = 0 \\ Q_C = -\frac{I_C^2}{C\omega} = -C\omega U_C^2 \end{cases}$

3. Improvement of power factor

We consider a load $\overline{Z}$ that consume an active power P and a reactive power Q with a power factor $\cos\varphi$. We want to improve $\cos\varphi$ $(\cos\varphi \to 1)$ by adding a capacitance in parallel with the load $\overline{Z}$ (Fig 1.3).

The circuit consumes then a reactive power Q' under a new power factor $\cos\varphi'$.

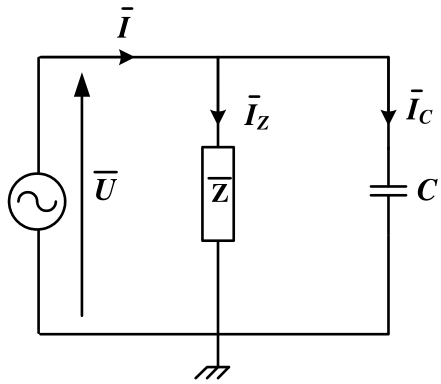


Fig 1.3

Before adding *C* :

We have :

$$\begin{cases} P \\ Q = P \cdot tg\varphi \end{cases}$$

After adding *C* :

The added capacitor produces only reactive power Q_C , so we obtain :

$$\begin{cases} P' = P \\ Q' = Q + Q_C = P \cdot tg\varphi + Q_C = P' \cdot tg\varphi' = P \cdot tg\varphi' \end{cases}$$

$$\left.\begin{array}{l} \Rightarrow Q_C = P\left(tg\varphi' - tg\varphi\right) \\ \text{we have also:} Q_C = -\frac{I_C^2}{C\omega} = -C\omega U_C^2 \end{array}\right\} \Rightarrow C = \frac{-P\left(tg\varphi' - tg\varphi\right)}{\omega U_C^2}$$