



Introduction to Refrigeration & Air Conditioning

Applied Thermodynamics & Heat Engines

S.Y. B. Tech.

ME0223 SEM - IV

Production Engineering



Outline

- **Applications of Refrigeration.**
- **Bell – Coleman Cycle.**
- **COP and Power Calculations**
- **Vapour – Compression Refrigeration System.**
- **Presentation on T-S and P-h diagram.**
- **Vapour – Absorption System.**



Refrigeration

REFRIGERATION – *Science of producing and maintaining temperature below that of surrounding / atmosphere.*

REFRIGERATION – *Cooling of or removal of heat from a system.*

Refrigerating System – *Equipment employed to maintain the system at a low temperature.*

Refrigerated System – *System which is kept at lower temperature.*

- Refrigeration** – 1) By *melting of a solid*,
- 2) By *sublimation of a solid*,
- 3) By *evaporation of a liquid*.

Most of the commercial refrigeration production : **Evaporation of liquid**.

This liquid is known as *Refrigerant*.



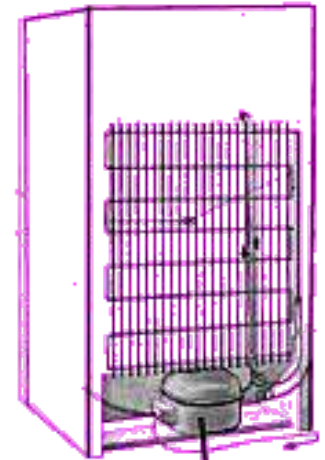
Refrigeration Circuit



Evaporator



Compressor



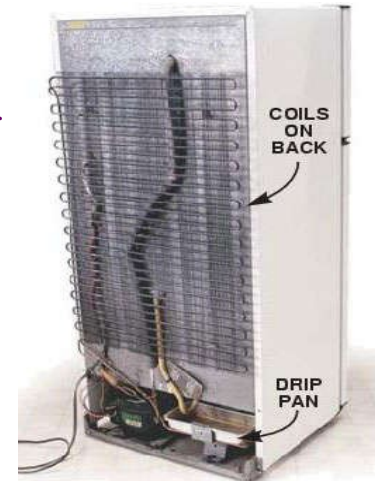
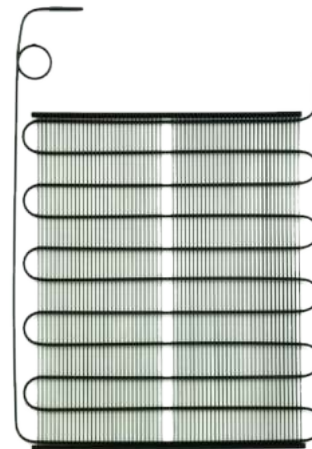
Compressor



Expansion Valve

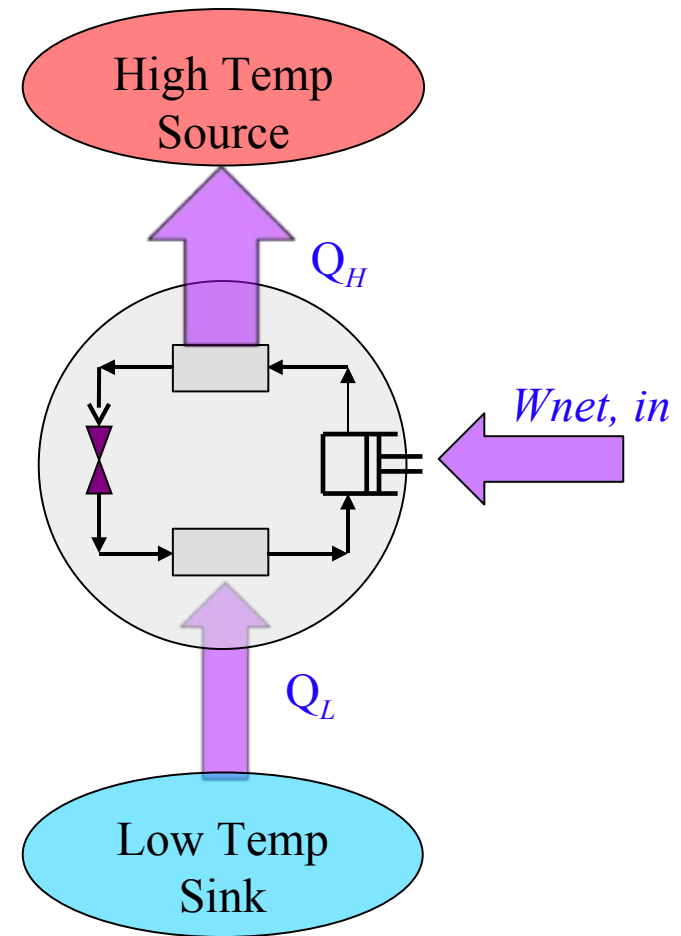
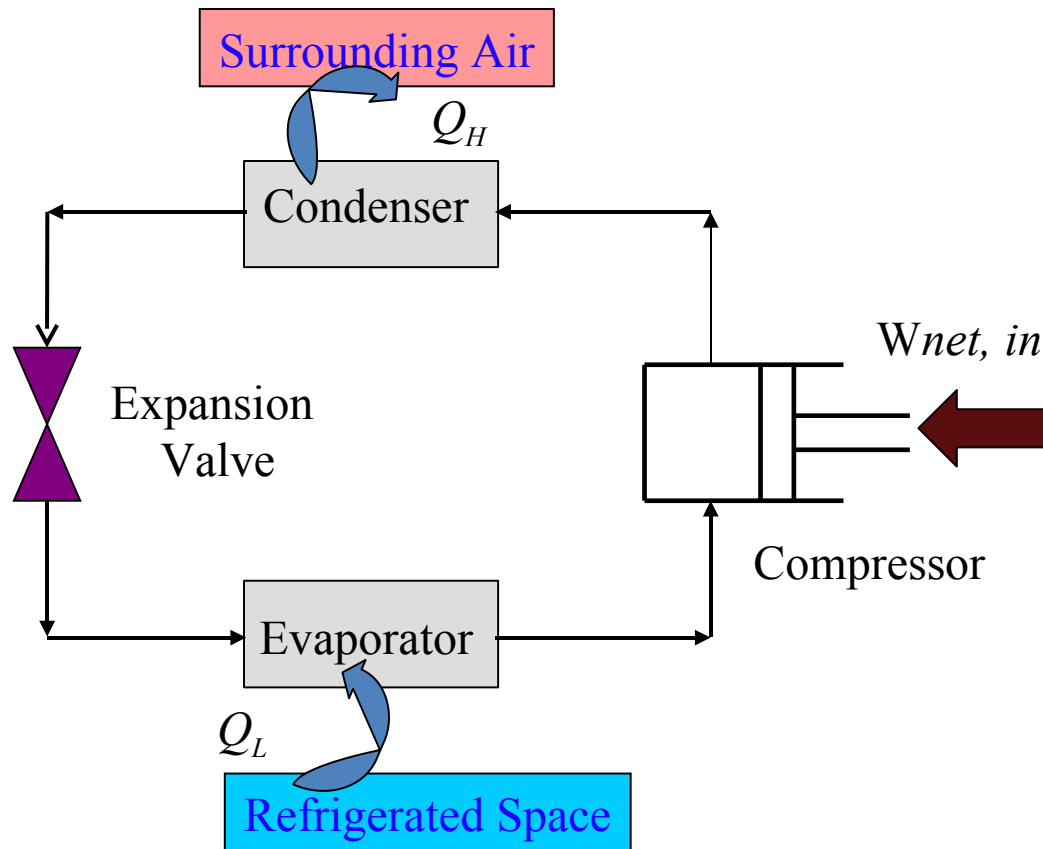


Condenser





Refrigeration - Elements





Refrigeration - Applications

Applications :

1. Ice making.
2. Transportation of food items above and below freezing.
2. Industrial Air – Conditioning.
4. Comfort Air – Conditioning.
5. Chemical and related industries.
6. Medical and Surgical instruments.
7. Processing food products and beverages.
8. Oil Refining.
9. Synthetic Rubber Manufacturing.
10. Manufacture and treatment of metals.
11. Freezing food products.
12. Manufacturing Solid Carbon Dioxide.
13. Production of extremely low temperatures (Cryogenics)
14. Plumbing.
15. Building Construction.



Refrigeration Systems

Refrigeration Systems :

1. Ice Refrigeration System.
2. Air Refrigeration System.
2. Vapour Compression Refrigeration System.
4. Vapour Absorption Refrigeration System.
5. Adsorption Refrigeration System.
6. Cascade Refrigeration System.
7. Mixed Refrigeration System.
8. Thermoelectric Refrigeration System.
9. Steam Jet Refrigeration System.
10. Vortex Tube Refrigeration System.



Performance - COP

Performance of Refrigeration System :

- Measured in terms of **COP (Coefficient of Performance)**.

COP – *Ratio of Heat absorbed by the Refrigerant while passing through the Evaporator to the Work Input required to compress the Refrigerant in the Compressor.*

If; R_n = Net Refrigerating Effect. W = Work required by the machine.

Then;
$$COP = \frac{R_n}{W}$$

$$\text{Relative COP} = \frac{\text{Actual COP}}{\text{Theoretical COP}}$$

Actual COP = Ratio of R_n and W actually measured.

Theoretical COP = Ratio of *Theoretical values* of R_n and W obtained by applying *Laws of Thermodynamics* to the Refrigerating Cycle.



Performance - Rating

Rating of Refrigeration System :

- Refrigeration Effect / Amount of Heat extracted from a body in a given time.

Definition :

- Refrigeration Effect produced by melting 1 tonne of ice from and at 0 °C in 24 hours.

Unit :

- Standard commercial Tonne of Refrigeration / TR Capacity

Latent Heat of ice = 336 kJ/kg.



Air Refrigeration System

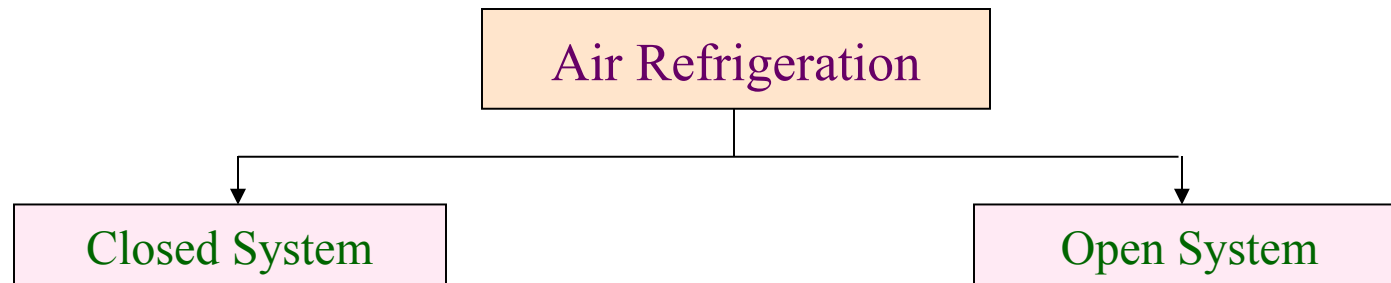
One of the earliest method.

Obsolete due to **low COP** and **high operating cost**.

Preferred in **Aircraft Refrigeration** due to its low weight.

Characteristic :

- Throughout the cycle, Refrigerant remains in *gaseous* state.



- Air refrigerant contained within piping or components of system.
- Pressures above atm. Pr.

- Refrigerator space is actual room to be cooled.
- Air expansion to atm. Pr. And then compressed to cooler pressure.
- Pressures limited to near atm. Pr. levels..



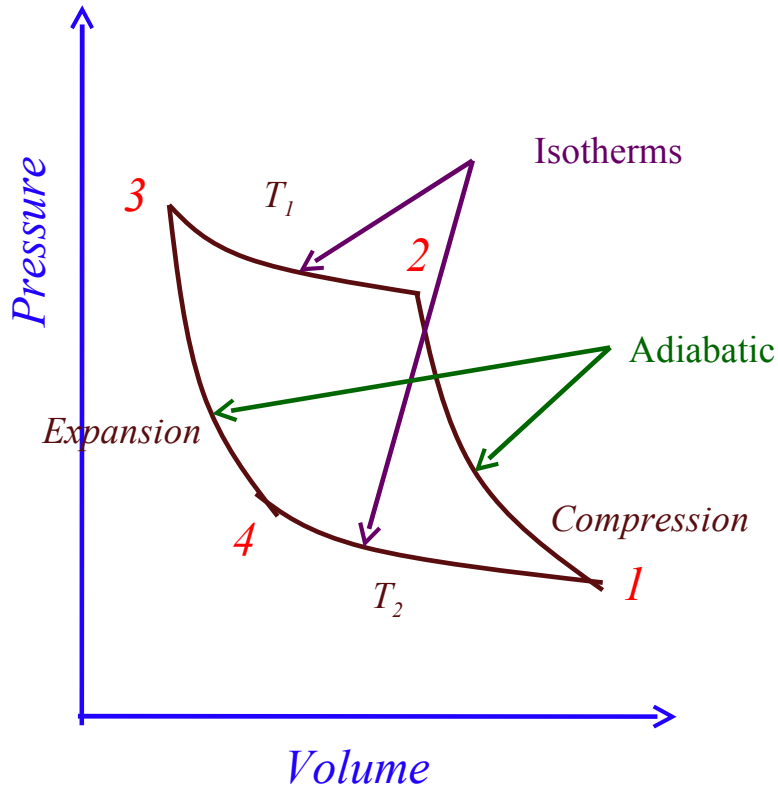
Air Refrigeration System

Closed System Vs. Open System :

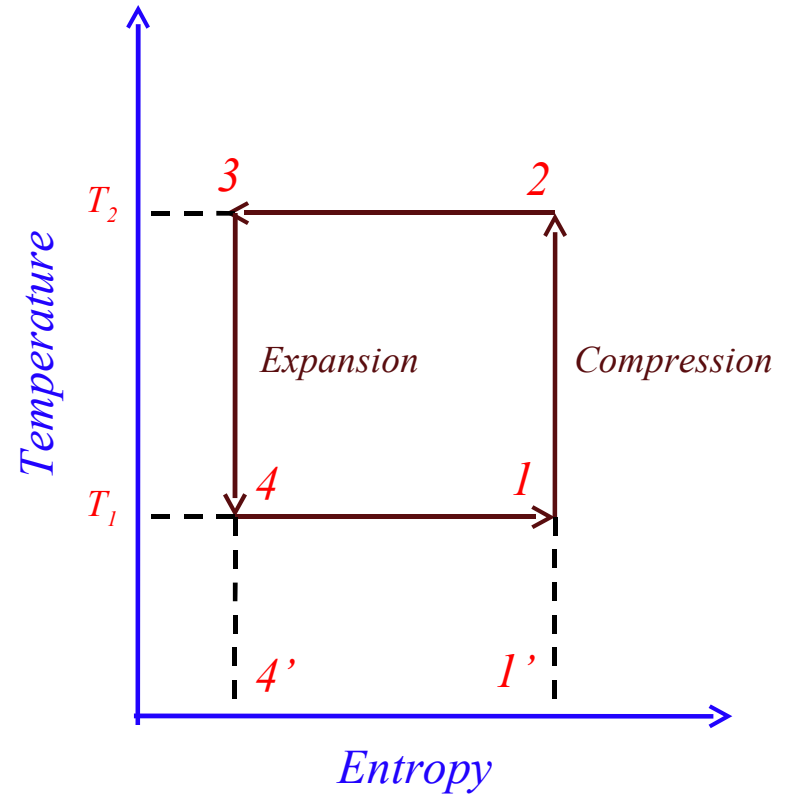
1. Suction to compressor in **Closed System** may be at high pressures. Hence, the size of Expander and Compressor can be kept small.
2. In **Open Systems**, air picks up the moisture from refrigeration chamber. This moisture freezes and chokes the valves.
3. Expansion in **Open System** is limited to atm. Pr. Level only. No such restriction to **Closed System**.



Reverse Carnot Cycle



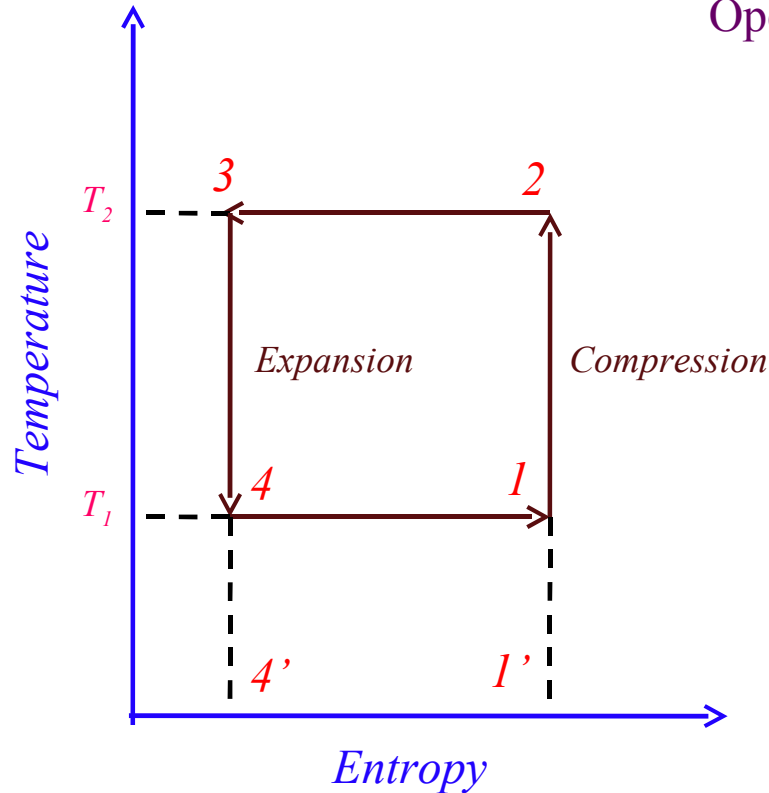
P - V Diagram



T - s Diagram



Reverse Carnot Cycle



Operation :

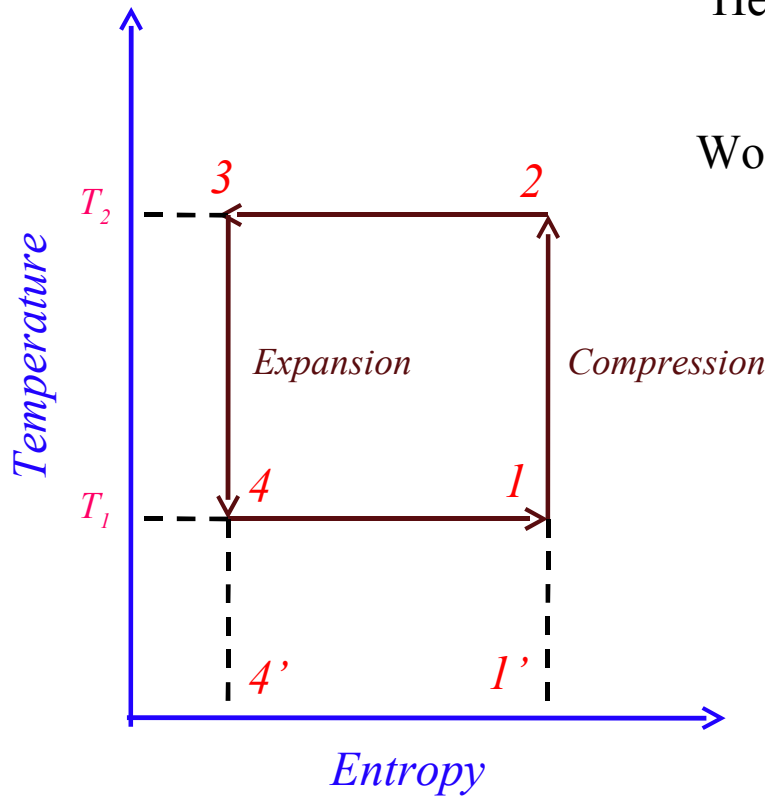
- 1 – 2 : Adiabatic Compression.**
Requires external power.
Temp. rises from T_1 to T_2 .
Cylinder in contact with Hot Body at T_2
- 2 – 3 : Isothermal Compression.**
Heat Rejection to Hot Body.
- 3 – 4 : Adiabatic Expansion.**
Temp. falls from T_2 to T_1 .
Cylinder in contact with Cold Body at T_1 .
- 4 – 1 : Isothermal Expansion.**
Heat Extraction from Cold Body.



Reverse Carnot Cycle

Heat extracted from cold Body : $Area\ 1-1'-4'-4$
 $= T_1 \times 1-4$

Work done per cycle : $Area\ 1-2-3-4$
 $= (T_2 - T_1) \times 1-4$



$$COP = \frac{\text{Heat Extracted}}{\text{Work Done}}$$

$$= \frac{Area\ 1-1'-4'-4}{Area\ 1-2-3-4}$$

$$= \frac{T_1 \times (1-4)}{(T_2 - T_1) \times (1-4)}$$

$$= \frac{T_1}{T_2 - T_1}$$



Example 1

A Carnot Refrigerator requires 1.3 kW per tonne of refrigeration to maintain a region at low temperature of -38 °C. Determine:

ii) COP of Carnot Refrigerator.

iii) Higher temperature of the cycle.

iv) Heat delivered and COP, if the same device is used Heat Pump.

$$COP_{refrig} = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{1 \text{ tonne}}{1.3 \text{ kW}} = \frac{14,000 \text{ kJ / hr}}{(1.3 \text{ kW}) (3600 \text{ sec / hr})} = 2.99 \dots \text{ANS}$$

$$COP_{refrig} = \frac{T_1}{T_2 - T_1} \Rightarrow 2.99 = \frac{235 \text{ K}}{T_2 - 235 \text{ K}} \Rightarrow T_1 = 313.6 \text{ K} \dots \text{ANS}$$

Heat Delivered as Heat Pump ;

$$= \text{Heat absorbed} + \text{Work done}$$

$$= 1 \text{ tonne} + 1.3 \text{ kW} = \frac{14,000 \text{ kJ / hr}}{3600} + 1.3 = 5.189 \text{ kJ / sec} \dots \text{ANS}$$

$$COP_{HP} = \frac{\text{Heat delivered}}{\text{Work done}} = \frac{5.189 \text{ kJ / sec}}{1.3 \text{ kW}} = 3.99 \dots \text{ANS}$$



Example 2

A refrigerating system works on reverse Carnot cycle. The higher temperature in the system is 35 °C and the lower temperature is -15 °C. The capacity is to be 12 tonnes. Determine :

ii) COP of Carnot Refrigerator.

iii) Heat rejected from the system per hour.

iv) Power required.

$$COP_{refrig} = \frac{T_1}{T_2 - T_1} = \frac{258 K}{308 K - 258 K} = 5.18 \dots \text{ANS}$$

$$COP_{refrig} = \frac{\text{Refrig. Effect}}{\text{Work Input}} \Rightarrow 5.16 = \frac{12 \text{ tonne}}{\text{Work Input}} = \frac{12 \times 14,000 \text{ kJ / hr}}{\text{Work Input}}$$

$$\Rightarrow \text{Work Input} = 32558 \text{ kJ / hr}$$

$$\text{Heat Rejected / hr} = \text{Refrig. Effect / hr} + \text{Work Input / hr}$$

$$= 12 \times 14,000 \text{ (kJ/hr)} + 32,558 \text{ (kJ/hr)} = 2,00,558 \text{ kJ/hr.} \dots \text{ANS}$$

$$\text{Power} = \frac{\text{Work Input / hr}}{3600} = \frac{32558 \text{ kJ / hr}}{3600} = 9.04 \text{ kW} \dots \text{ANS}$$



Example 3

Ice is formed at 0 °C from water at 20 °C. The temperature of the brine is -8 °C. Find out the kg of ice per kWh. Assume that the system operates on reversed Carnot cycle. Take latent heat of ice as 335 kJ/kg.

$$COP_{refrig} = \frac{T_1}{T_2 - T_1} = \frac{265 \text{ K}}{293 \text{ K} - 265 \text{ K}} = 9.46$$

Heat to be extracted per kg of water (to from ice at 0 °C)

$$\begin{aligned} R_n &= 1 \text{ (kg)} \times C_{pw} \text{ (kJ/kg.K)} \times (293 - 273) \text{ (K)} + \text{Latent Heat (kJ/kg) of ice} \\ &= 1 \text{ (kg)} \times 4.18 \text{ (kJ/kg.K)} \times 20 \text{ (K)} + 335 \text{ (kJ/kg)} \\ &= 418.6 \text{ kJ/kg.} \end{aligned}$$

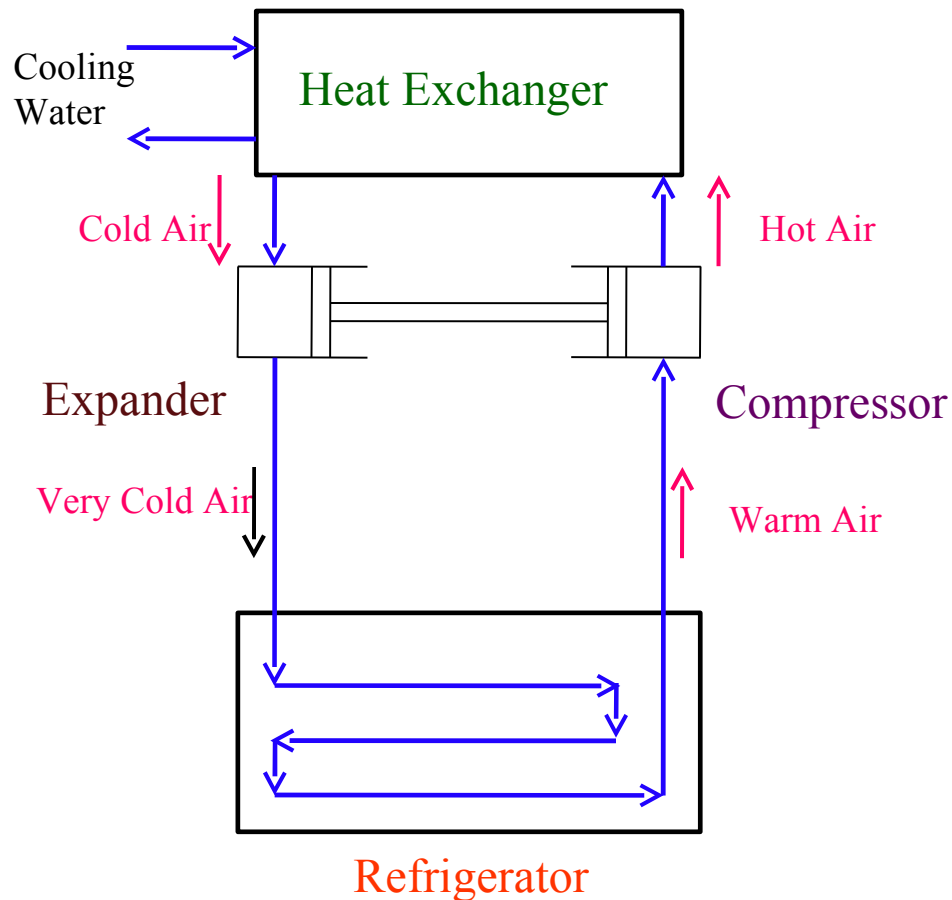
Also, $1 \text{ kWh} = 1 \text{ (kJ)} \times 3600 \text{ (sec/hr)} = 3600 \text{ kJ.}$

$$COP_{refrig} = \frac{R_n}{W} = \frac{\text{Re frig. Effect (kJ)}}{\text{Work done (kJ)}}$$

$$\Rightarrow 9.46 = \frac{m_{ice} \text{ (kg)} \times 418.6 \text{ (kJ / kg)}}{3600 \text{ kJ}} \Rightarrow m_{ice} = 81.35 \text{ kg...ANS}$$



Bell – Coleman / Reverse Bryaton Cycle



Elements of this system :

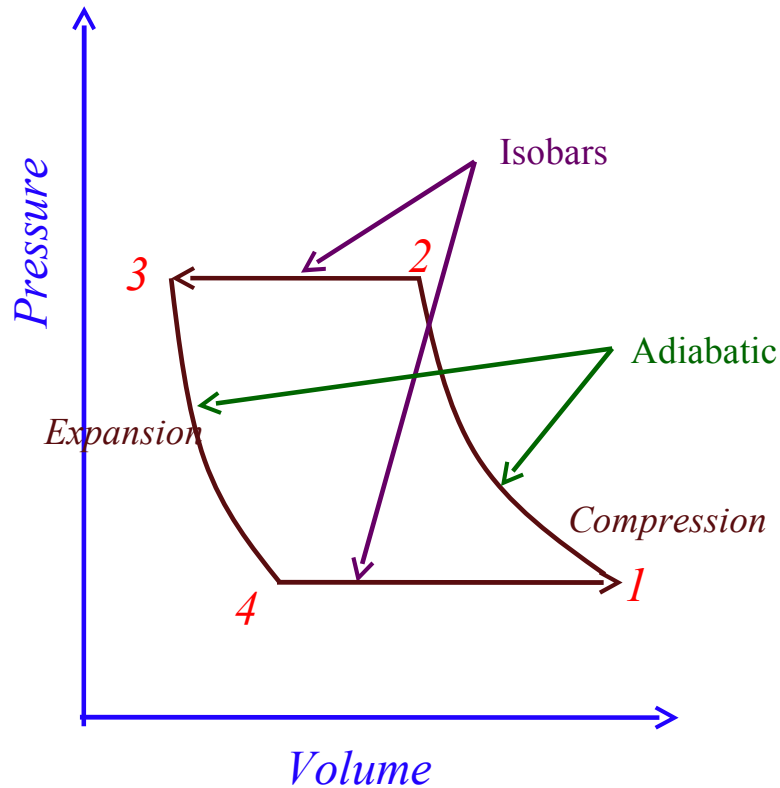
- Compressor.
- Heat Exchanger.
- Expander.
- Refrigerator.

Work gained from *Expander* is used to drive *Compressor*.

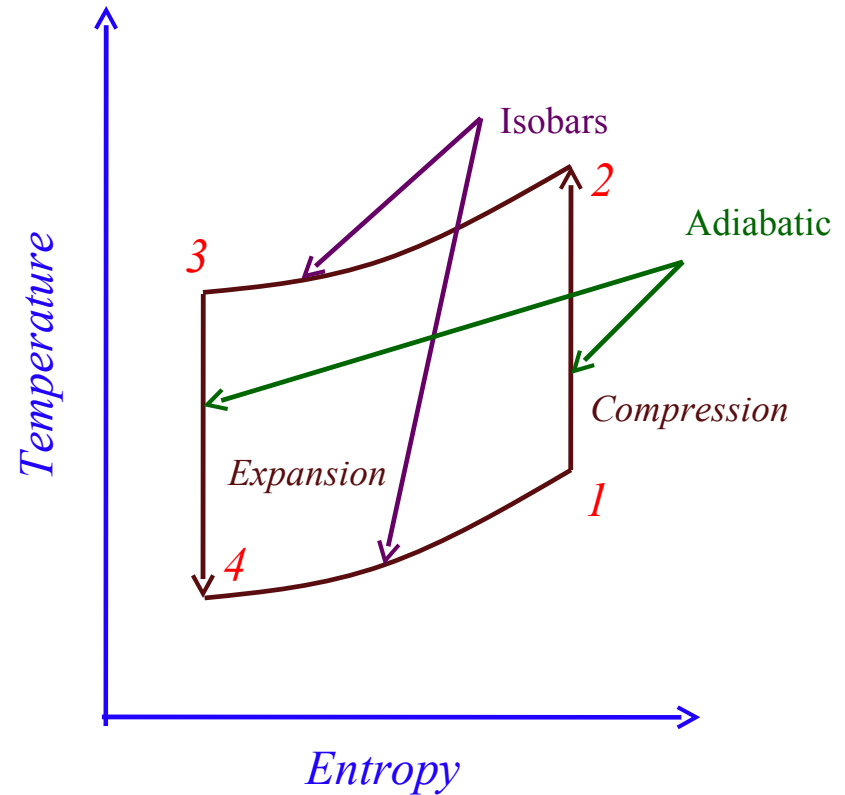
Hence, less external work is required.



Bell – Coleman / Reverse Bryaton Cycle



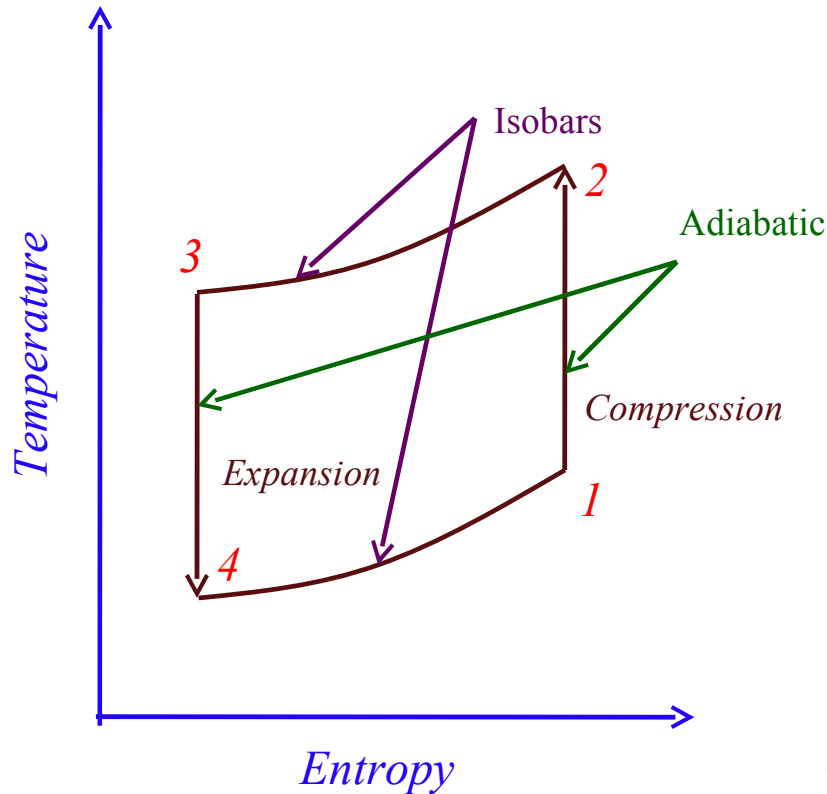
P – V Diagram



T – s Diagram



Bell – Coleman / Reverse Bryaton Cycle



Heat Absorbed in Refrigerator :

$$Q_{added} = m C_P (T_1 - T_4)$$

Heat Rejected in Heat Exchanger :

$$Q_{rejected} = m C_P (T_2 - T_3)$$

If process changes from *Adiabatic* to *Polytropic*;

$$Q_{comp} = \frac{n}{n-1} (P_2 V_2 - P_1 V_1)$$

$$Q_{expn} = \frac{n}{n-1} (P_3 V_3 - P_4 V_4)$$

We know,

$$R = C_P \left(\frac{\gamma - 1}{\gamma} \right)$$



Bell – Coleman / Reverse Bryaton Cycle

Net Work Done :

$$\begin{aligned} W &= W_{comp} - W_{expn} \\ &= \frac{n}{n-1} (P_2 V_2 - P_1 V_1 - P_3 V_3 + P_4 V_4) \\ &= \frac{n}{n-1} m R (T_2 - T_1 - T_3 + T_4) \\ &= \frac{n}{n-1} \left(\frac{\gamma-1}{\gamma} \right) m C_P (T_4 - T_3 + T_2 - T_1) \end{aligned}$$

For Isentropic Process :

$$\begin{aligned} W &= W_{comp} - W_{expn} \\ &= m C_P (T_4 - T_3 + T_2 - T_1) \end{aligned}$$



Bell – Coleman / Reverse Bryaton Cycle

COP :

$$\begin{aligned} COP &= \frac{\text{Work Added}}{Q_{\text{rejected}} - Q_{\text{added}}} = \frac{Q_{\text{added}}}{W_{\text{net}}} \\ &= \frac{m C_P (T_1 - T_4)}{\left(\frac{n}{n-1}\right) \left(\frac{\gamma-1}{\gamma}\right) m C_P (T_4 - T_3 + T_2 - T_1)} \\ COP &= \frac{(T_1 - T_4)}{\left(\frac{n}{n-1}\right) \left(\frac{\gamma-1}{\gamma}\right) (T_4 - T_3 + T_2 - T_1)} \end{aligned}$$



Air Refrigeration Cycle - Merits / Demerits

Merits :

1. No risk of fire (as in case of NH_3); as air is non – flammable.
2. Cheaper (than other systems); as air is easily available.
3. Weight per tonne of refrigeration is quite low (compared to other systems).

Demerits :

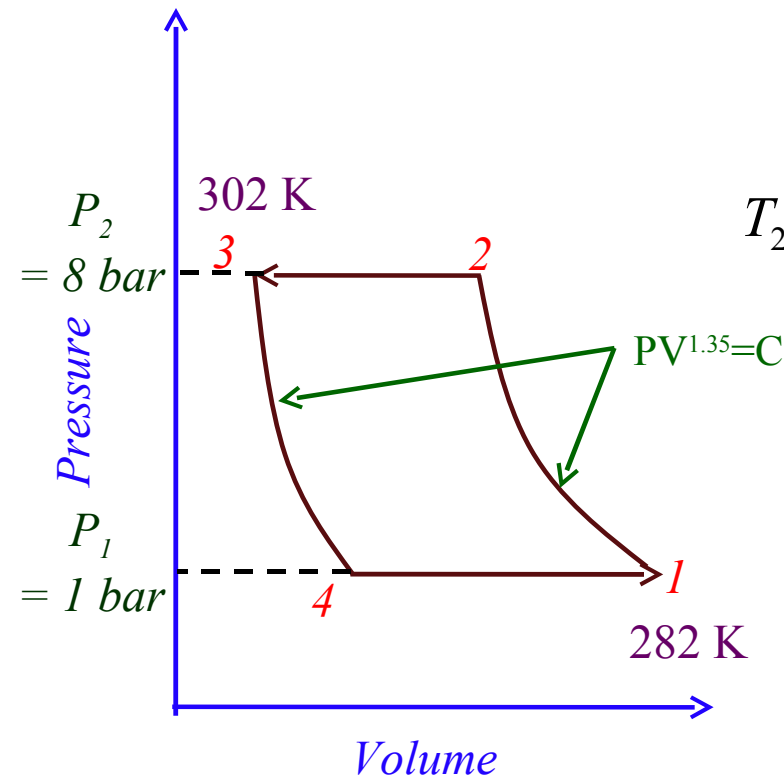
1. Low COP (compared with other systems).
2. Weight of air (as Refrigerant) is more (compared to other systems).



Example 4

A Bell – Coleman refrigerator operates between pressure limits of 1 bar and 8 bar. Air is drawn from the cold chamber at 9 °C, compressed and then cooled to 29 °C before entering the expansion cylinder. Expansion and compression follow the law $PV^{1.35} = \text{Const}$. Calculate the theoretical COP.

For air, take $\gamma = 1.4$ and $C_p = 1.003 \text{ kJ/kg}$.



Polytropic Compression 1-2 :

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = (282 \text{ K}) \left(\frac{8 \text{ bar}}{1 \text{ bar}} \right)^{\frac{1.35-1}{1.35}} = 482.2 \text{ K}$$

Polytropic Expansion 3-4 :

$$T_3 = T_4 \left(\frac{P_3}{P_4} \right)^{\frac{n-1}{n}} \Rightarrow (302 \text{ K}) = T_4 \left(\frac{8 \text{ bar}}{1 \text{ bar}} \right)^{\frac{1.35-1}{1.35}}$$

$$\Rightarrow T_4 = 176.6 \text{ K}$$



Example 4...contd

Heat Extracted from Cold Chamber :

$$= C_p (T_1 - T_4) = 1.003 (kJ / kg) \times (282 K - 176.6 K) = 105.7 kJ / kg$$

Heat Rejected to Heat Exchanger :

$$= C_p (T_2 - T_3) = 1.003 (kJ / kg) \times (482.2 K - 302 K) = 180.7 kJ / kg$$

Net Work Done :

$$W_{net} = \frac{n}{n-1} \left(\frac{\gamma-1}{\gamma} \right) m C_p (T_4 - T_3 + T_2 - T_1)$$

$$W_{net} = \frac{1.35}{1.35-1} \left(\frac{1.4-1}{1.4} \right) (1.003 kJ / kg) (176.6 K - 302 K + 482.2 K - 282 K)$$

$$W_{net} = 82.8 kJ / kg$$

$$COP_{refrig} = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{105.7 kJ / kg}{82.8 kJ / kg} = 1.27 \dots \text{ANS}$$

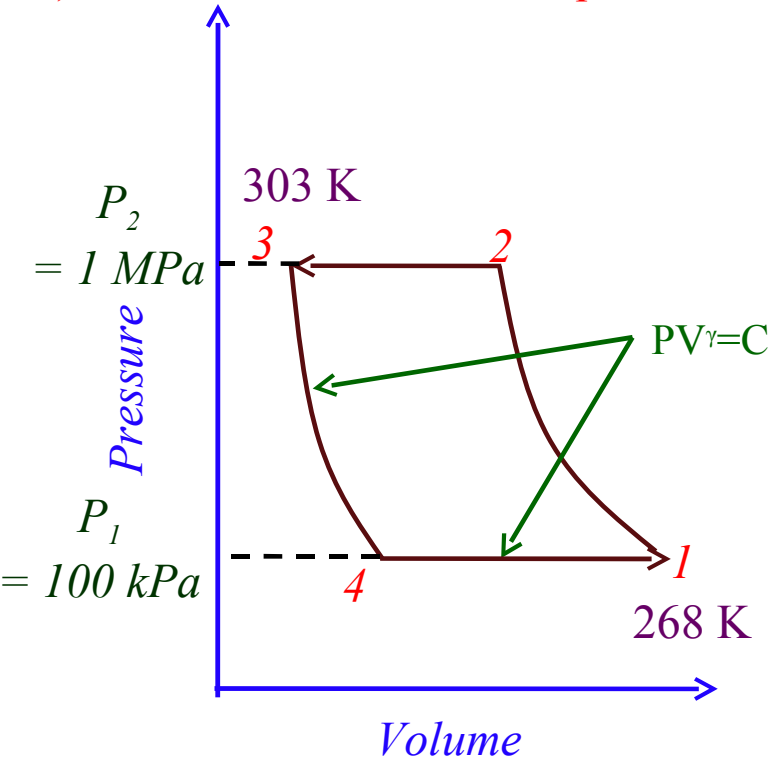


Example 5

An air refrigeration open system operating between 1 MPa and 100 kPa is required to produce a cooling effect of 2000 kJ/min. temperature of the air leaving the cold chamber is -5 °C, and at leaving the cooler is 30 °C. Neglect losses and clearance in the compressor and expander.

Determine :

- ii) Mass of air circulated per min. ii) Compressor Work, Expander Work, Cycle Work.
- iii) COP and Power in kW required.



Polytropic Expansion 3-4 :

$$T_3 = T_4 \left(\frac{P_3}{P_4} \right)^{\frac{\gamma-1}{\gamma}} \Rightarrow (302 \text{ K}) = T_4 \left(\frac{1 \text{ MPa}}{0.1 \text{ MPa}} \right)^{\frac{1.4-1}{1.4}}$$

$$\Rightarrow T_4 = 156.9 \text{ K}$$

Refrig. Effect per kg :

$$= C_p (T_1 - T_4)$$

$$= 1.003 (kJ / kg) \times (268 \text{ K} - 156.9 \text{ K})$$

$$= 111.66 \text{ kJ / kg}$$



Example 5...contd

Mass of air circulated per min :

$$= \frac{\text{Re frig. Effect}}{\text{Re frig. Effect per kg}} = \frac{2000 \text{ kJ / min}}{111.66 \text{ kJ / kg}} = 17.91 \text{ kg / min} \dots \text{ANS}$$

Polytropic Compression 1-2 : $T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = (268 \text{ K}) \left(\frac{1000 \text{ kPa}}{100 \text{ kPa}} \right)^{\frac{1.4-1}{1.4}} = 517.4 \text{ K} \dots \text{ANS}$

Compressor Work :

$$W_{comp} = \left(\frac{\gamma}{\gamma-1} \right) m R (T_2 - T_1)$$

$$W_{comp} = \left(\frac{1.4}{1.4-1} \right) (17.91 \text{ kg / min}) (0.287 \text{ kJ / kg}) (517.4 \text{ K} - 268 \text{ K})$$

$$W_{comp} = 4486.85 \text{ kJ / min} \dots \text{ANS}$$



Example 5...contd

Expander Work :

$$W_{\text{exp}} = \left(\frac{\gamma}{\gamma - 1} \right) m R (T_3 - T_4)$$

$$W_{\text{comp}} = \left(\frac{1.4}{1.4 - 1} \right) (17.91 \text{ kg} / \text{min}) (0.287 \text{ kJ} / \text{kg}) (303 \text{ K} - 156.9 \text{ K})$$

$$W_{\text{comp}} = 2628.42 \text{ kJ} / \text{min} \quad \dots\text{ANS}$$

$$\text{Cycle Work} = W_{\text{cycle}} = W_{\text{comp}} - W_{\text{exp}}$$

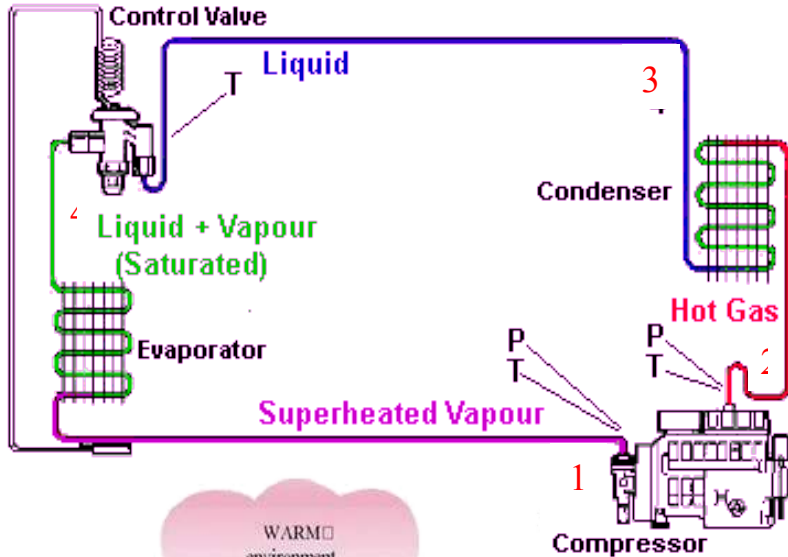
$$= 4486.85 \text{ kJ} / \text{min} - 2628.42 \text{ kJ} / \text{min} = 1858.43 \text{ kJ} / \text{min} \dots\text{ANS}$$

$$COP_{\text{refrig}} = \frac{\text{Re frig. Effect}}{\text{Work required}} = \frac{2000 \text{ kJ} / \text{min}}{1858.43 \text{ kJ} / \text{min}} = 1.076 \dots\text{ANS}$$

$$\text{Power required : } P = \frac{W_{\text{cycle}}}{\text{time}} = \frac{1858.43 \text{ kJ} / \text{min}}{60 \text{ sec} / \text{min}} = 30.97 \text{ kW} \dots\text{ANS}$$



Vapour Compression System



Elements of this system :

- Compressor.
- Condenser.
- Expansion Valve.
- Evaporator.

Vapour @ \downarrow Pr. and \downarrow Temp. (State 1)

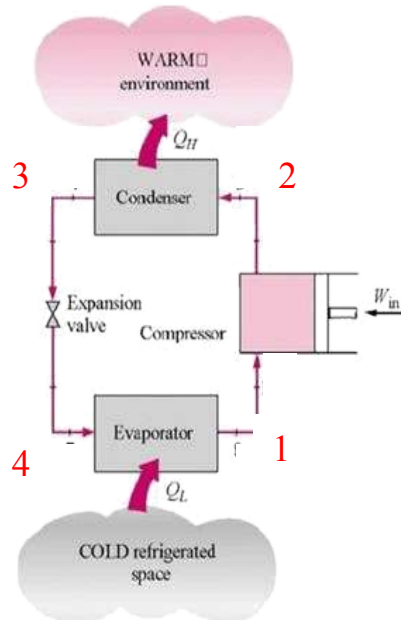
Isentropic Compression :

\uparrow Pr. and \uparrow Temp. (State 2)

Condenser : \uparrow Pr. Liquid (State 3)

Throttling : \downarrow Pr. \downarrow Temp. (State 4)

Evaporator : Heat Extraction from surrounding;
 \downarrow Pr. vapour (State 1).





Vapour Compression System

Merits :

1. High COP; as very close to Reverse Carnot Cycle.
2. Running Cost is $1/5^{\text{th}}$ of that of Air Refrigeration Cycle.
3. Size of Evaporator is small; for same Refrigeration Effect.
4. Evaporator temperature adjustment is simple; by adjusting Throttle Valve.

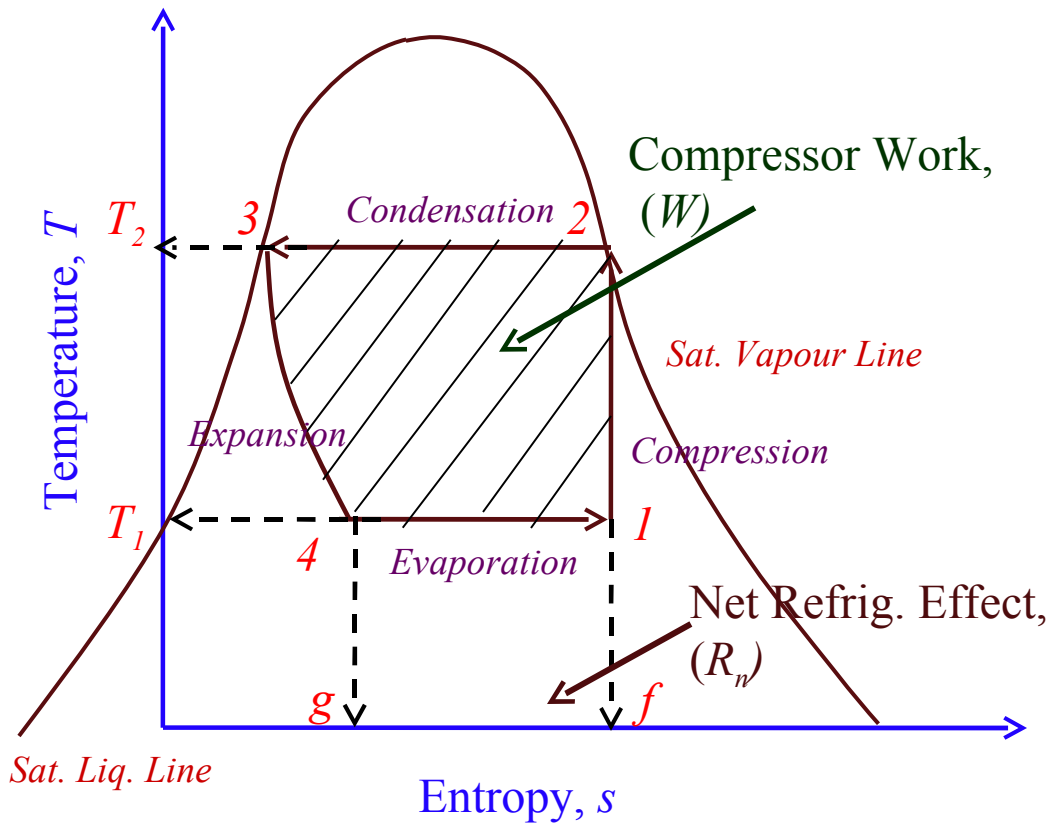
Demerits :

1. Initial cost is high.
2. Inflammability.
3. Leakage.
4. Toxicity.



Vapour Compression System : T-s Diagram

Case A. *Dry and Saturated Vapour after Compression :*



Work done by Compressor

$$= W = \text{Area } 1-2-3-4-1$$

Heat Absorbed

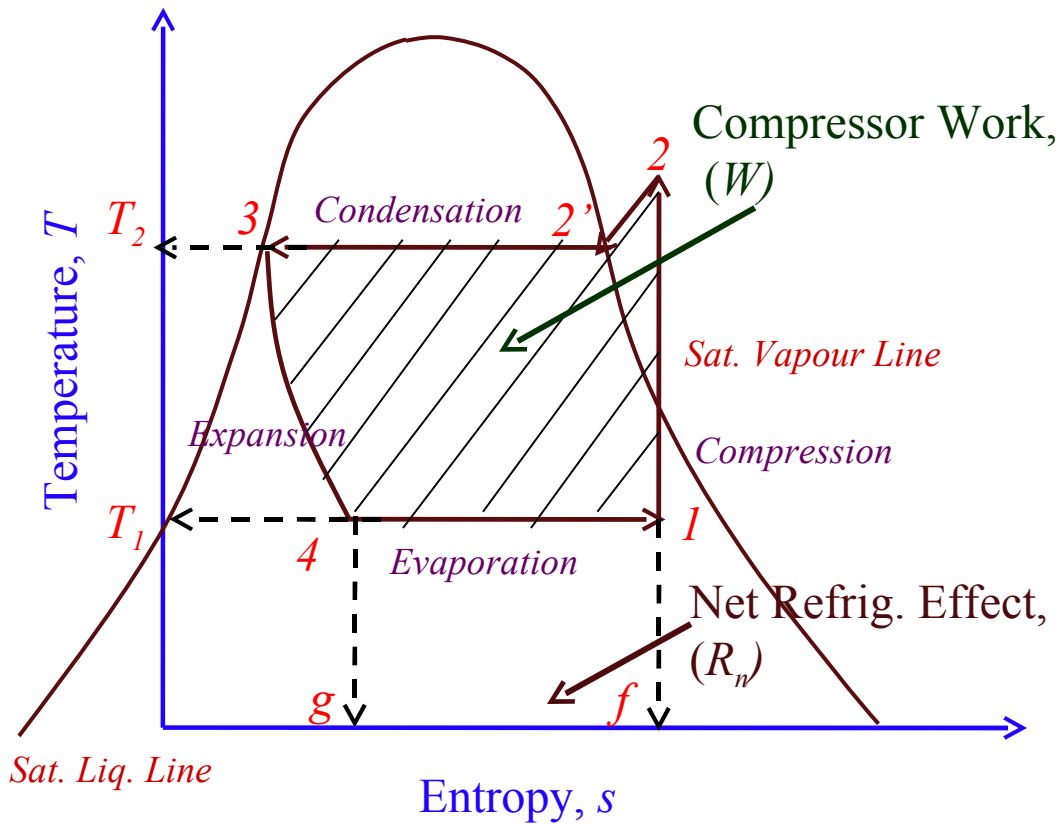
$$= W = \text{Area } 1-4-g-f-1$$

$$\begin{aligned} COP &= \frac{\text{Heat Absorbed}}{\text{Work Done}} \\ &= \frac{\text{Area } 1-4-g-f-1}{\text{Area } 1-2-3-4-1} \\ &= \frac{h_1 - h_4}{h_2 - h_1} \end{aligned}$$



Vapour Compression System : T-s Diagram

Case B. *Superheated Vapour after Compression :*



Work done by Compressor

$$= W = \text{Area } 1-2-2'-3-4-1$$

Heat Absorbed

$$= W = \text{Area } 1-4-g-f-1$$

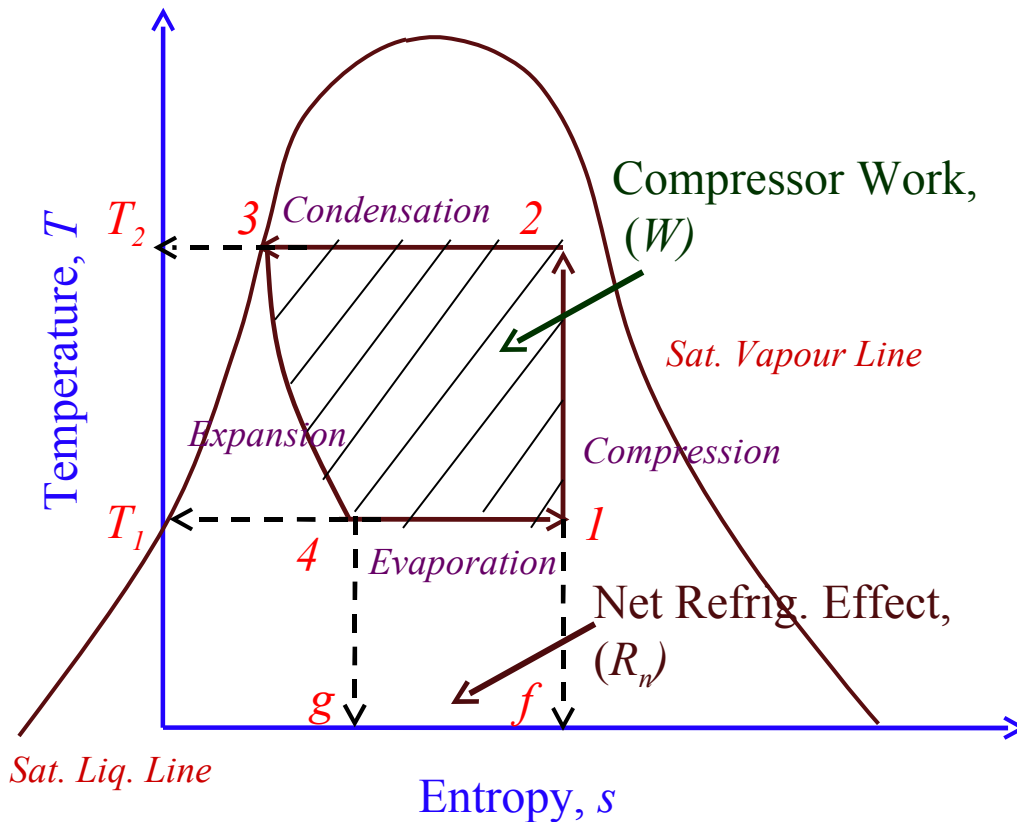
$$\begin{aligned} COP &= \frac{\text{Heat Absorbed}}{\text{Work Done}} \\ &= \frac{\text{Area } 1-4-g-f-1}{\text{Area } 1-2-2'-3-4-1} \\ &= \frac{h_1 - h_4}{h_2 - h_1} \end{aligned}$$

NOTE : $h_2 = h_{2'} + C_p (T_{\text{sup}} - T_{\text{sat}})$



Vapour Compression System : T-s Diagram

Case C. *Wet Vapour after Compression :*



Work done by Compressor

$$= W = \text{Area } 1-2-3-4-1$$

Heat Absorbed

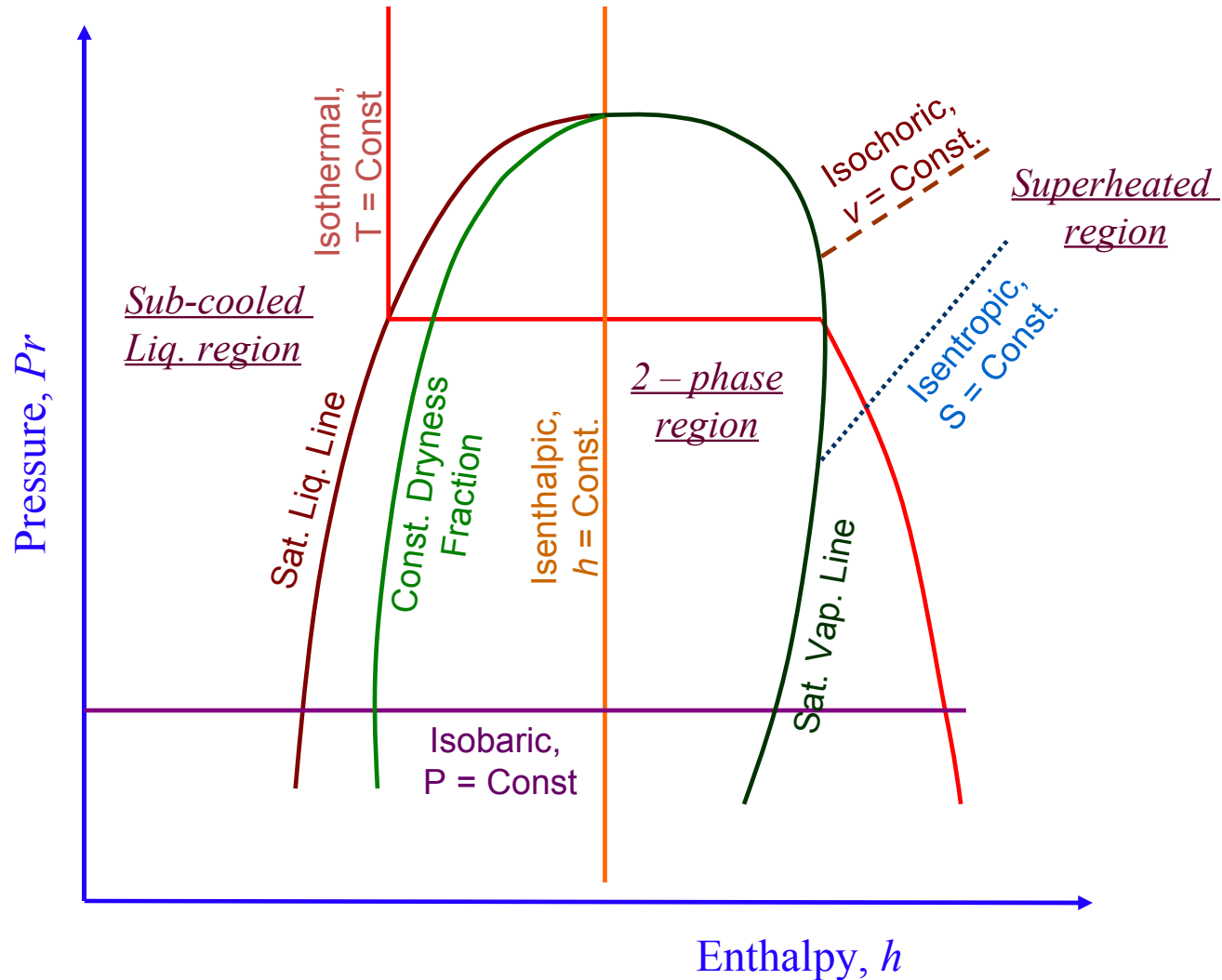
$$= W = \text{Area } 1-4-g-f-1$$

$$\begin{aligned} COP &= \frac{\text{Heat Absorbed}}{\text{Work Done}} \\ &= \frac{\text{Area } 1-4-g-f-1}{\text{Area } 1-2-3-4-1} \\ &= \frac{h_1 - h_4}{h_2 - h_1} \end{aligned}$$

NOTE : $h_2 = (h_f + x \cdot h_{fg})_2$

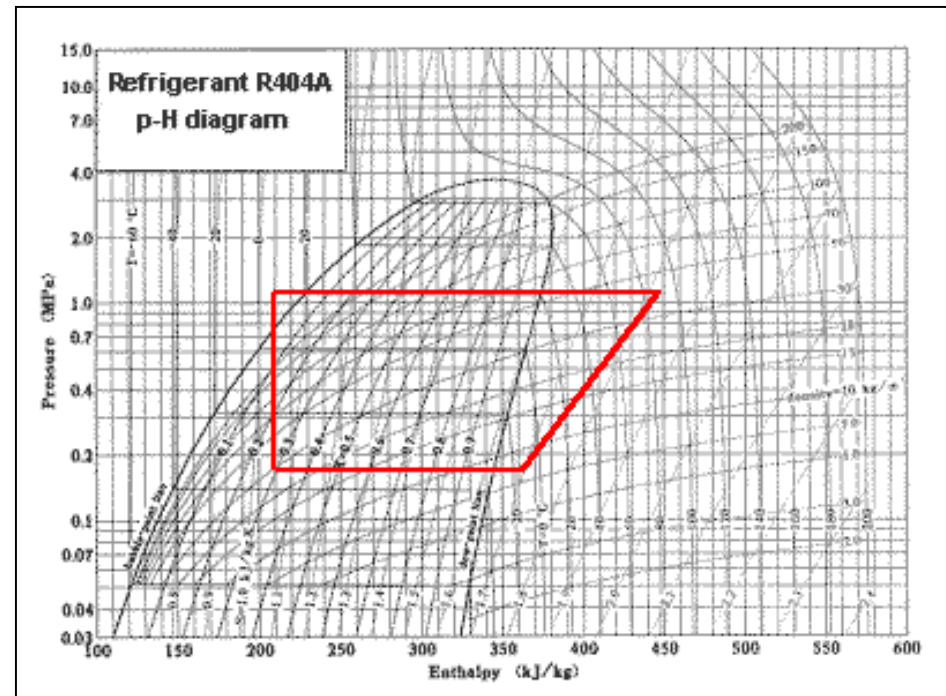
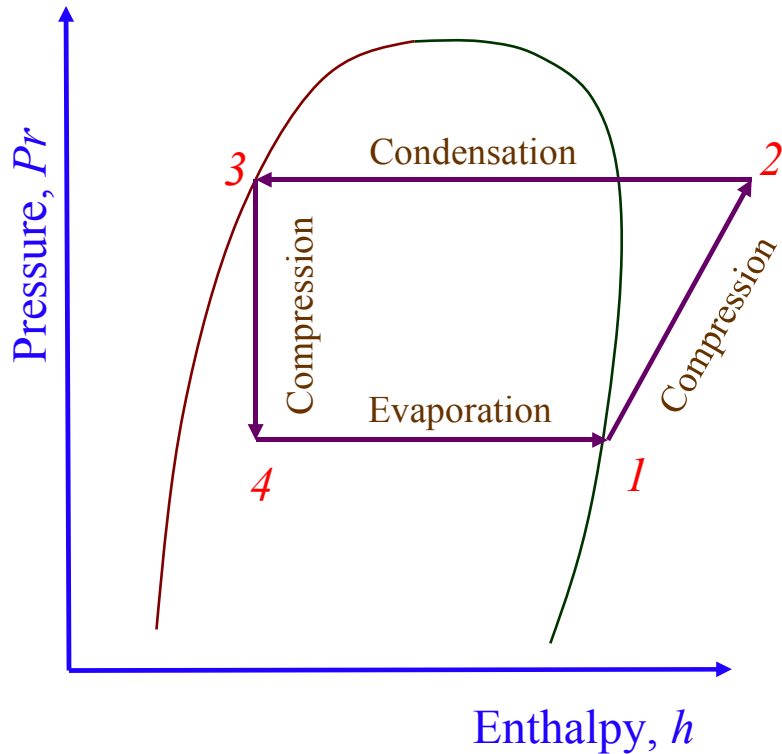


Vapour Compression System : P-h Diagram





Vapour Compression System : P-h Diagram

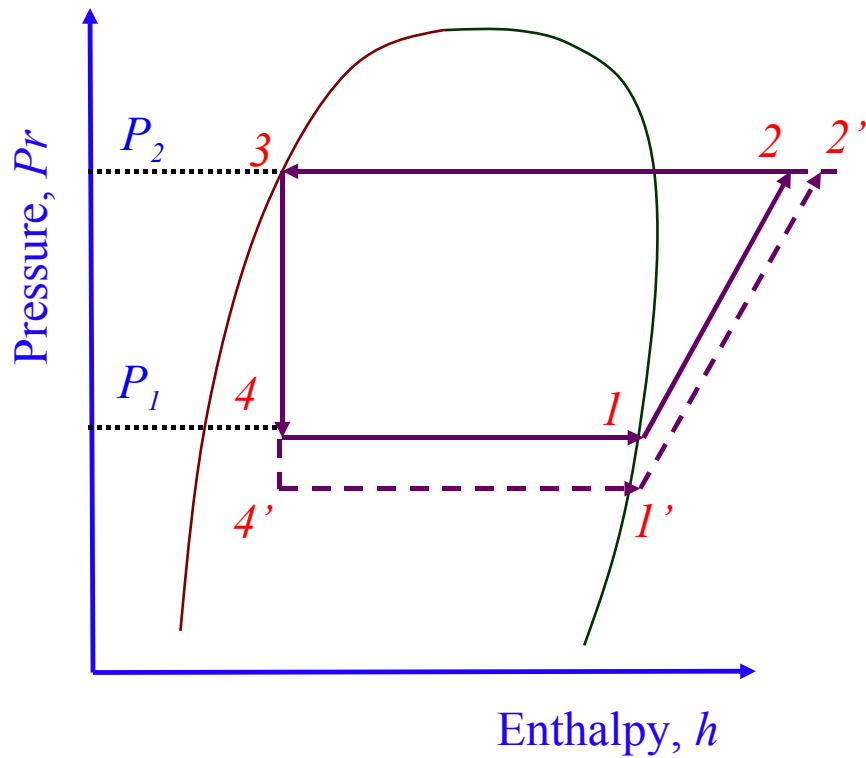


$$\left. \begin{aligned} R_n &= h_1 - h_4 \\ W &= h_2 - h_1 \end{aligned} \right\} COP = \frac{R_n}{W} = \frac{h_1 - h_4}{h_2 - h_1}$$



Factors Affecting Vapour Compression System

A. Effect of Suction Pressure :



COP of Original Cycle :

$$COP = \frac{R_n}{W} = \frac{h_1 - h_4}{h_2 - h_1}$$

COP when Suction Pr. decreased :

$$\begin{aligned} COP &= \frac{R_n}{W} = \frac{h_{1'} - h_{4'}}{h_{2'} - h_{1'}} \\ &= \frac{(h_1 - h_4) - (h_1 - h_{1'})}{(h_2 - h_1) + (h_1 - h_{1'}) + (h_{2'} - h_2)} \end{aligned}$$

Thus,

Refrig. Effect ↓

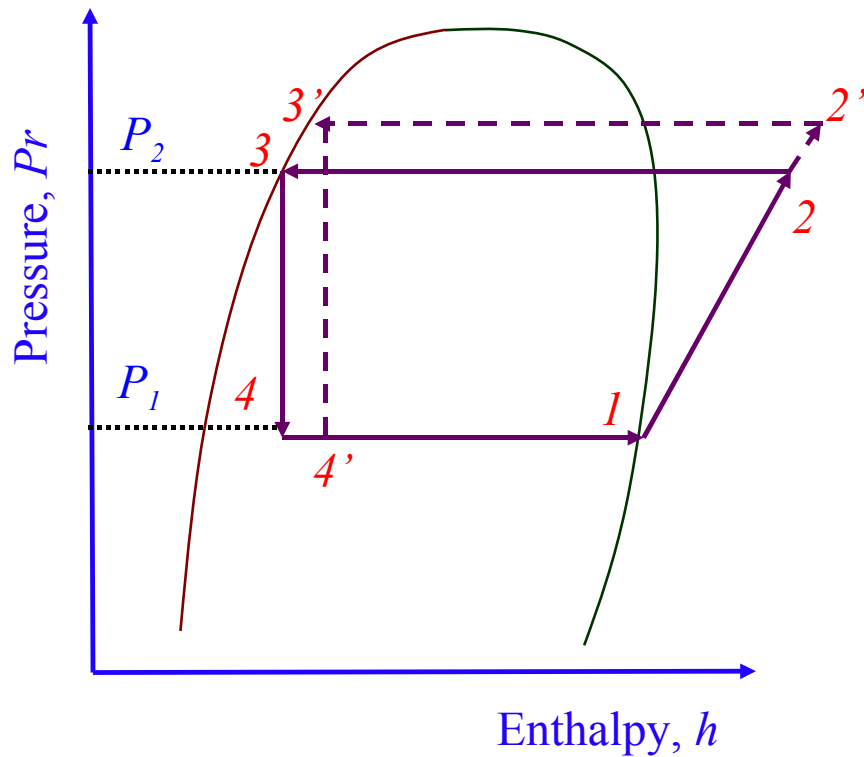
Work Input ↑

⇒ COP ↓



Factors Affecting Vapour Compression System

B. Effect of Delivery Pressure :



COP of Original Cycle :

$$COP = \frac{R_n}{W} = \frac{h_1 - h_4}{h_2 - h_1}$$

COP when Delivery Pr. increased :

$$\begin{aligned} COP &= \frac{R_n}{W} = \frac{h_1 - h_{4'}}{h_{2'} - h_1} \\ &= \frac{(h_1 - h_4) - (h_{4'} - h_4)}{(h_2 - h_1) + (h_{2'} - h_2)} \end{aligned}$$

Thus,

Refrig. Effect ↓

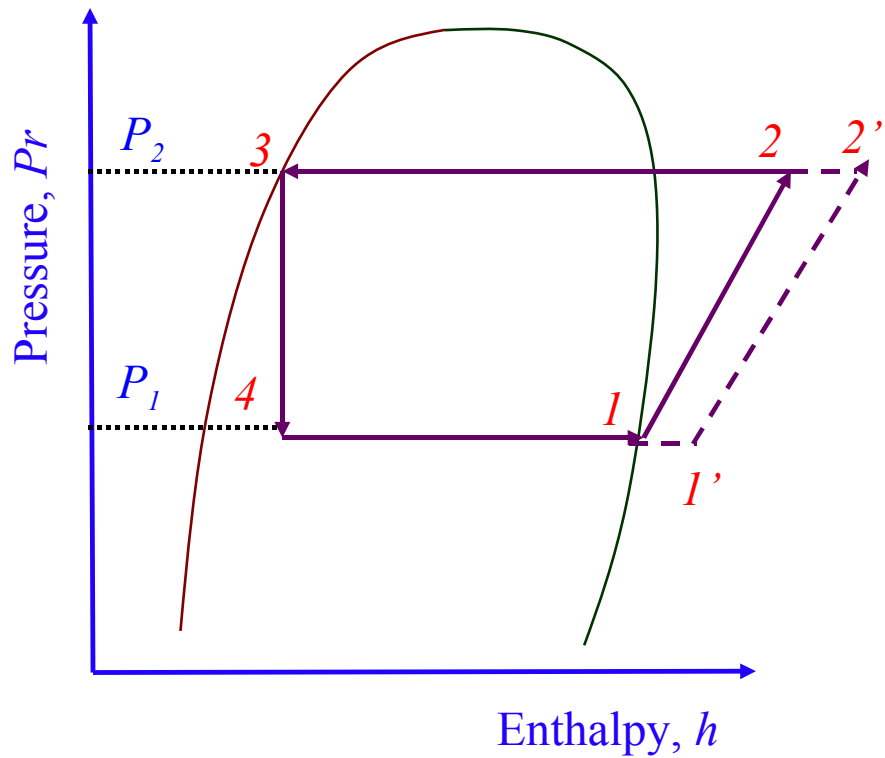
Work Input ↑

⇒ COP ↓



Factors Affecting Vapour Compression System

C. Effect of Superheating :



COP of Original Cycle :

$$COP = \frac{R_n}{W} = \frac{h_1 - h_4}{h_2 - h_1}$$

COP when Delivery Pr. increased :

$$COP = \frac{R_n}{W} = \frac{h_{1'} - h_4}{h_{2'} - h_{1'}} = \frac{(h_1 - h_4) + (h_{1'} - h_1)}{(h_2 - h_1) + (h_{2'} - h_2) + (h_{1'} - h_1)}$$

Thus,

Refrig. Effect ↑

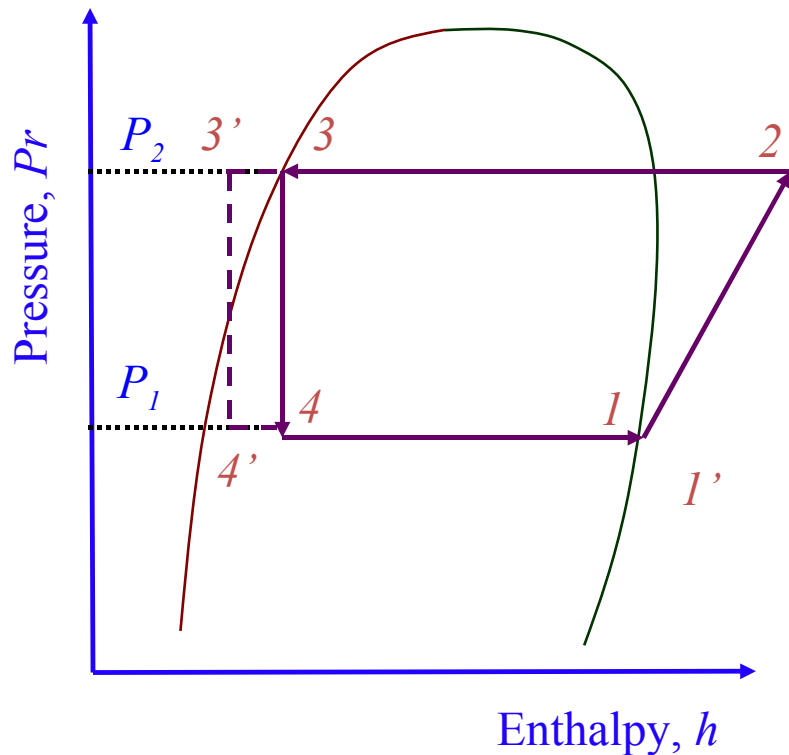
Work Input ↑ or ↓

⇒ COP ↓ or ↑



Factors Affecting Vapour Compression System

D. Effect of Sub-cooling :



COP of Original Cycle :

$$COP = \frac{R_n}{W} = \frac{h_1 - h_4}{h_2 - h_1}$$

COP when Delivery Pr. increased :

$$\begin{aligned} COP &= \frac{R_n}{W} = \frac{h_1 - h_{4'}}{h_2 - h_1} \\ &= \frac{(h_1 - h_4) + (h_4 - h_{4'})}{(h_2 - h_1)} \end{aligned}$$

Thus,

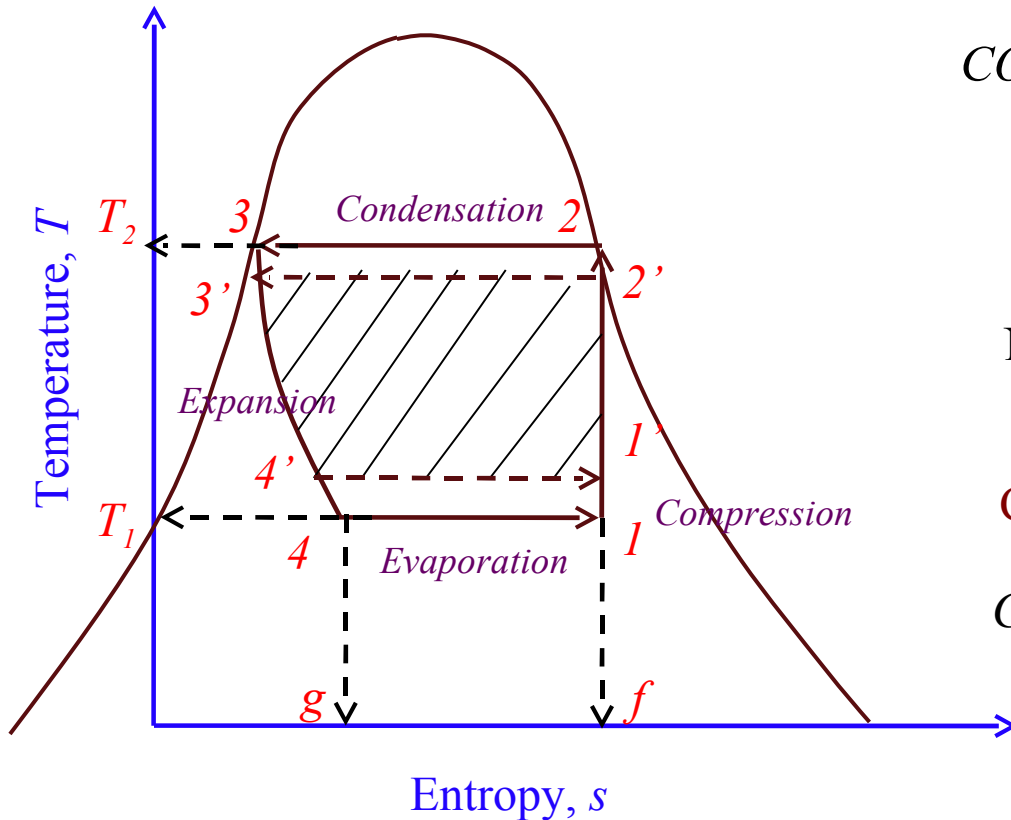
Refrig. Effect ↑
Work Input : SAME

⇒ COP ↑



Factors Affecting Vapour Compression System

E. Effect of Suction & Condenser Temperatures :



COP of Original Cycle :

$$COP = \frac{\text{Heat Absorbed}}{\text{Work Done}}$$

$$= \frac{\text{Area } 1-4-g-f-1}{\text{Area } 1-2-3-4-1} = \frac{h_1 - h_4}{h_2 - h_1}$$

Now, Condenser Temp. ↓
Evaporator Temp. ↑

COP of Modified Cycle :

$$COP = \frac{\text{Heat Absorbed } (\uparrow)}{\text{Work Done } (\downarrow)}$$

$$= \frac{\text{Area } 1-1'-4-4'-g-f-1}{\text{Area } 1'-2'-3'-4'-1'} > \frac{h_1 - h_4}{h_2 - h_1}$$

⇒ COP ↑



Vapour Compression System – Mathematical Analysis

A. Refrigerating Effect :

= Amount of Heat absorbed in Evaporator.

$$Q_{evap} = (h_1 - h_4) + \text{Latent Heat} + \text{Superheated Heat} \quad (\text{kJ} / \text{kg})$$

B. Mass of Refrigerant :

= Amount of Heat absorbed / Refrigerating Effect.

$$m = \frac{14,000}{3600 (h_1 - h_4)} \quad (\text{kg} / \text{sec} - \text{tonne})$$

1 TR = 14,000 kJ/hr

C. Theoretical Piston Displacement :

= Mass of Refrigerant X Sp. Vol. of Refrigerant Gas $(v_g)_1$.

$$\text{Th. Piston Displ.} = \frac{14,000}{3600 (h_1 - h_4)} * (v_g)_1 \quad (\text{m}^3 / \text{sec} - \text{tonne})$$



Vapour Compression System – Mathematical Analysis

D. Theoretical Power Required :

a) Isentropic Compression :

$$W_{comp} = (h_2 - h_1) \quad (kJ / kg)$$

$$P_{theor} = m (h_2 - h_1) \quad (kW)$$

a) Polytropic Compression :

$$W_{comp} = \frac{n}{n-1} (P_2 V_2 - P_1 V_1) \quad (kJ / kg)$$

$$P_{theor} = m \frac{n}{n-1} (P_2 V_2 - P_1 V_1) \quad (kW)$$

E. Heat removed through Condenser :

$$Q_{cond} = m (h_2 - h_3) \quad (kJ / kg)$$



Example 6

A refrigeration machine is required to produce ice at 0° C from water at 20 °C. The machine has a condenser temperature of 298 K while the evaporator temperature is 268 K. The relative efficiency of the machine is 50 % and 6 kg of Freon-12 refrigerant is circulated through the system per minute. The refrigerant enters the compressor with a dryness fraction of 0.6. Specific heat of water is 4.187 kJ/kg.K and the latent heat of ice is 335 kJ/kg. Calculate the amount of ice produced on 24 hours. The table of properties if Freon-12 is given below:

Temperature (K)	Liquid Heat (kJ/kg)	Latent Heat (kJ/kg)	Entropy of Liquid (kJ/kg)
298	59.7	138.0	0.2232
268	31.4	154.0	0.1251

Given :

$$m = 6 \text{ kg/min}$$

$$\eta_{\text{rel}} = 50 \%$$

$$x_2 = 0.6$$

$$C_{\text{pw}} = 4.187 \text{ kJ/kg.K}$$

$$\text{Latent Heat of ice} = 335.7 \text{ kJ/kg}$$

$$h_{f1} = 31.4 \text{ kJ/kg}$$

$$h_{fg1} = 154.0 \text{ kJ/kg}$$

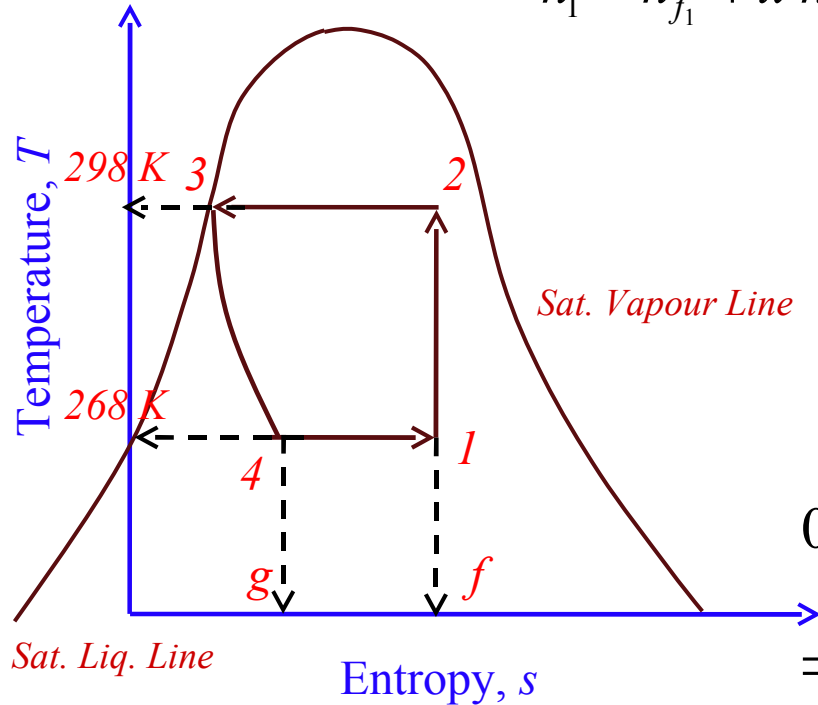
$$h_{f2} = 59.7 \text{ kJ/kg}$$

$$h_{fg2} = 138.0 \text{ kJ/kg}$$

$$h_{f3} = h_4 = 59.7 \text{ kJ/kg}$$



Example 6...contd



$$h_1 = h_{f_1} + x h_{fg_1} = 31.4 + (0.6)154.0 = 123.8 \text{ kJ/kg}$$

Isentropic Compression : 1-2

$$s_2 = s_1$$

$$s_{f_2} + x_2 * s_{fg_2} = s_{f_1} + x_1 * s_{fg_1}$$

$$s_{f_2} + x_2 \left(\frac{h_{fg_2}}{T_2} \right) = s_{f_1} + x_1 \left(\frac{h_{fg_1}}{T_1} \right)$$

$$0.2232 + x_2 \left(\frac{138.0}{298} \right) = 0.1251 + (0.6) \left(\frac{154.0}{268} \right)$$

$$\Rightarrow x_2 = 0.5325$$

$$h_2 = h_{f_2} + x_2 h_{fg_2} = 59.7 + (0.5325)138.0 = 133.2 \text{ kJ/kg}$$

$$h_4 = h_{f_3} = 59.7 \text{ kJ/kg}$$

COP of Original Cycle :
$$COP = \frac{R_n}{W} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{(123.8 - 59.7) \text{ kJ/kg}}{(133.2 - 123.8) \text{ kJ/kg}} = 6.82$$

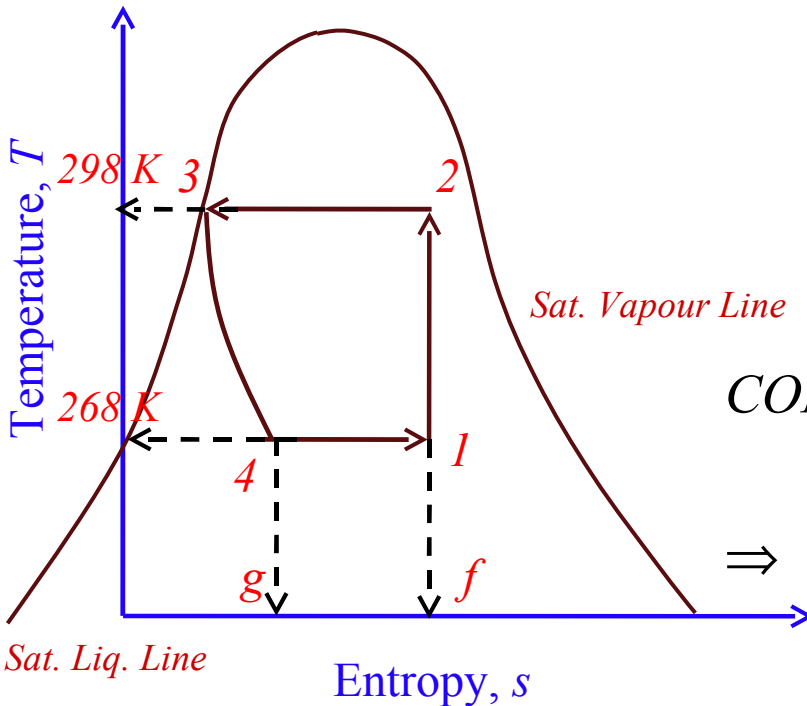


Example 6...contd

$$\text{Actual COP} = \eta_{\text{rel}} \times \text{COP}_{\text{theor}} = 0.5 \times 6.82 = 3.41$$

Heat extracted from 1 kg of water at 20 °C to form 1 kg of ice at 0 °C :

$$\begin{aligned} &= 1 \text{ (kg)} \times 4.187 \text{ (kJ / kg.K)} \times (20 - 0) \text{ (}^\circ\text{C)} \\ &\quad + 335 \text{ (kJ / kg)} \\ &= 418.74 \text{ kJ / kg} \end{aligned}$$



Now;

$$\begin{aligned} \text{COP}_{\text{actual}} &= 3.41 = \frac{R_{n(\text{actual})}}{W} = \frac{m_{\text{ice}} \times 418.74}{m (h_2 - h_1)} \\ \Rightarrow m_{\text{ice}} &= \frac{6 \text{ (kg)} \times (133.2 - 123.8) \text{ (kJ / kg)}}{418.74 \text{ kJ / kg}} * 3.41 \\ &= 0.459 \text{ kg / min} \\ &= \frac{0.459 \times 60 \times 24}{1000} = 0.661 \text{ tonne in 24 hrs} \\ &\quad \dots\text{ANS} \end{aligned}$$



Example 7

28 tonnes of ice from and at 0 °C is produced per day in an ammonia refrigerator. The temperature range in the compressor is from 25 °C to -15°C. The vapour is dry and saturated at the end of compression and an expansion valve is used. Assuming a co-efficient of performance of 62% of the theoretical, calculate the power required to drive the compressor. Take latent heat of ice = 335 kJ/kg.

Temp (°C)	Enthalpy (kJ/kg)		Entropy of Liquid (kJ/kg.K)	Entropy of Vapour (kJ/kg.K)
	Liquid	Vapour		
25	100.04	1319.22	0.3473	4.4852
-15	-54.56	1304.99	-2.1338	5.0585

} Given :

$$T_{\text{cond}} = 25 \text{ }^\circ\text{C}$$

$$T_{\text{evap}} = -15 \text{ }^\circ\text{C}$$

$$x_2 = 1 \dots \text{dry saturated vapour}$$

$$\text{COP}_{\text{actual}} = 0.62 (\text{COP}_{\text{theor}})$$

$$\text{Latent Heat of ice} = 335.7 \text{ kJ/kg}$$

$$h_{f1} = -54.56 \text{ kJ/kg}$$

$$h_{g1} = 1304.99 \text{ kJ/kg}$$

$$h_{f2} = 100.04 \text{ kJ/kg}$$

$$h_{g2} = 1319.22 \text{ kJ/kg}$$

$$h_{f3} = h_4 = 100.04 \text{ kJ/kg}$$



Example 7...contd

$$h_2 = h_{g2} = 1319.22 \text{ kJ / kg}$$

$$h_3 = h_4 = 100.04 \text{ kJ / kg} \dots \text{Isenthalpic process}$$

Isentropic Compression : 1-2

$$s_2 = s_1$$

$$s_{g2} = s_{f1} + x_1 * s_{fg1}$$

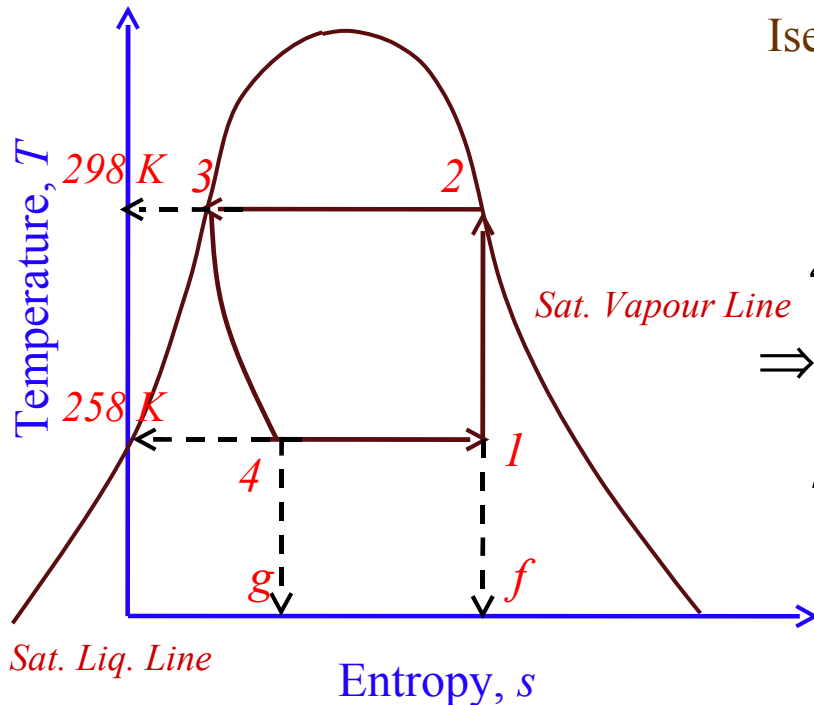
$$4.4852 = (-2.1338) + (x_1)[5.0585 - (-2.1338)]$$

$$\Rightarrow x_2 = 0.92$$

$$h_1 = h_{f1} + x_1(h_{fg1})$$

$$= (-54.56) + (0.92)[1304.99 - (-54.56)]$$

$$= 1196.23 \text{ kJ / kg}$$



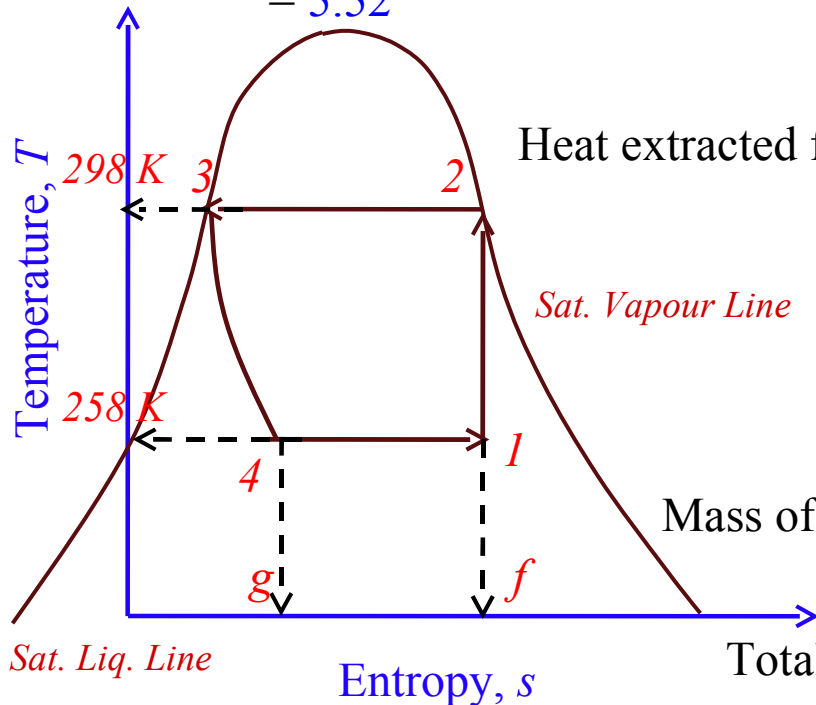
$$\text{COP of the Cycle : } COP_{\text{theoretical}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{(1196.23 - 100.04)}{(1319.22 - 1196.23)} = 8.91$$



Example 7....contd

$$\begin{aligned} \text{Actual COP} &= \eta_{\text{rel}} \times \text{COP}_{\text{theor}} \\ &= 0.62 \times 8.91 \\ &= 5.52 \end{aligned}$$

$$\begin{aligned} \text{Actual } R_n &= \text{COP}_{\text{actual}} \times \text{Work done} \\ &= 5.52 \times (h_2 - h_1) \\ &= 5.52 \times (1319.22 - 1196.23) \\ &= 678.9 \text{ kJ/kg} \end{aligned}$$



Heat extracted from 28 tonnes of water at 0 °C to form ice at 0 °C :

$$\begin{aligned} &= \frac{28 \text{ (kg)} \times 1000 \text{ (kg / tonne)} \times 335 \text{ (kJ / kg)}}{24 \text{ (hr)} \times 3600 \text{ (sec / hr)}} \\ &= 108.56 \text{ kJ / sec (kW)} \end{aligned}$$

$$\text{Mass of refrigerant : } = \frac{108.56 \text{ (kJ / sec)}}{678.9 \text{ (kJ / kg)}} = 0.1599 \text{ kg}$$

Total Work done by Compressor :

$$\begin{aligned} &= m_{\text{refrig}} \times (h_2 - h_1) = 0.1599 \text{ (kg)} \times (1319.22 - 1196.23) \text{ kJ / kg} \\ &= 19.67 \text{ kJ / sec (kW)ANS} \end{aligned}$$



Example 8

In a standard vapour compression refrigeration cycle, operating between an evaporator temperature of $-10\text{ }^{\circ}\text{C}$ and a condenser temperature of $40\text{ }^{\circ}\text{C}$, the enthalpy of the refrigerant, Freon-12, at the end of compression is 220 kJ/kg . Show the cycle diagram on T-s plane. Calculate:

1. The C.O.P. of the cycle.
2. The refrigerating capacity and the compressor power assuming a refrigerant flow rate of 1 kg/min .

You may use the extract of Freon-12 property table given below:

Temp ($^{\circ}\text{C}$)	Pr (MPa)	h_f (kJ/kg)	h_g (kJ/kg)
-10	0.2191	26.85	183.1
40	0.9607	74.53	203.1

Given :

$$T_{\text{cond}} = 40\text{ }^{\circ}\text{C}$$

$$T_{\text{evap}} = -10\text{ }^{\circ}\text{C}$$

$$x_1 = 1 \dots \text{dry saturated vapour}$$

$$h_2 = 220\text{ kJ/kg}$$

$$h_{f1} = 26.85\text{ kJ/kg}$$

$$h_{g1} = h_1 = 183.1\text{ kJ/kg}$$

$$h_{f2} = 74.53\text{ kJ/kg}$$

$$h_{g2} = 203.1\text{ kJ/kg}$$

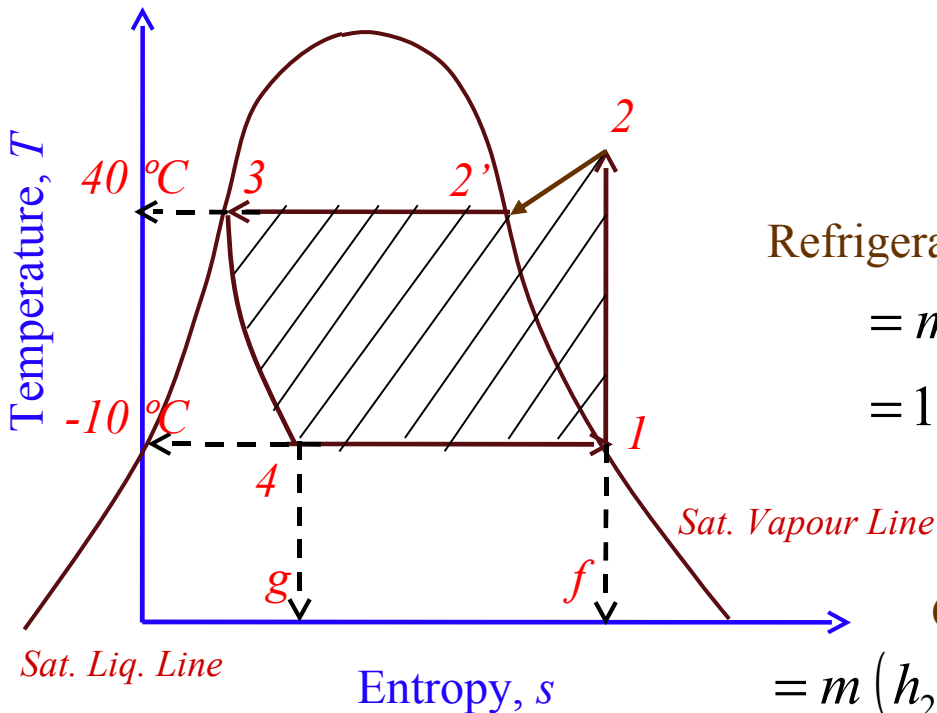
$$h_{f3} = h_4 = 74.53\text{ kJ/kg}$$



Example 8...contd

COP of Original Cycle :
$$COP = \frac{R_n}{W} = \frac{h_1 - h_4}{h_2 - h_1}$$

$$= \frac{(183.1 - 74.53) \text{ kJ / kg}}{(220.0 - 183.1) \text{ kJ / kg}} = 2.94 \text{ANS}$$



Refrigerating Capacity :

$$= m (h_1 - h_4) = 1 \text{ (kg)} \times (183.1 - 74.53) \text{ kJ / kg}$$

$$= 108.57 \text{ kJ / min} \text{ANS}$$

Compressor Power :

$$= m (h_2 - h_1) = 1 \text{ (kg)} \times (220.0 - 183.1) \text{ kJ / kg}$$

$$= 36.9 \text{ kJ / min}$$

$$= 0.615 \text{ kW} \text{ANS}$$



Example 9

A Freon-12 refrigerator producing a cooling effect of 20 kJ/sec operates on a simple cycle with pressure limits of 1.509 bar and 9.607 bar. The vapour leaves the evaporator dry saturated and there is no undercooling. Determine the power required by the machine. If the compressor operates at 300 rpm and has a clearance volume of 3% of stroke volume, determine the piston displacement of the compressor. For compressor assume that the expansion following the law $PV^{1.3} = \text{Constant}$.

Temp (°C)	P _s (bar)	v _g (m ³ /kg)	Enthalpy h _f (kJ/kg)	Enthalpy h _g (kJ/kg)	Entropy s _f (kJ/kg)	Entropy s _g (kJ/kg)	Specific heat (kJ/kg.K)
-20	1.509	0.1088	17.8	178.61	0.073	0.7082	---
40	9.607	---	74.53	203.05	0.2716	0.682	0.747

Given :

$$T_{\text{cond}} = 40 \text{ }^\circ\text{C}$$

$$T_{\text{evap}} = -20 \text{ }^\circ\text{C}$$

$x_1 = 1$dry saturated vapour

$$h_2 = 220 \text{ kJ/kg}$$

$$h_{f1} = 17.8 \text{ kJ/kg}$$

$$h_{g1} = h_1 = 178.61 \text{ kJ/kg}$$

$$h_{f2} = 74.53 \text{ kJ/kg}$$

$$h_{g2} = 203.05 \text{ kJ/kg}$$

$$h_{f3} = h_4 = 74.53 \text{ kJ/kg}$$



Example 9...contd

Refrigerating Capacity : $= \dot{m} (h_1 - h_4) \Rightarrow 20 \text{ kW} = \dot{m} \times (178.61 - 74.53) \text{ kJ / kg}$

$\Rightarrow \dot{m} = 0.192 \text{ kg / sec}$

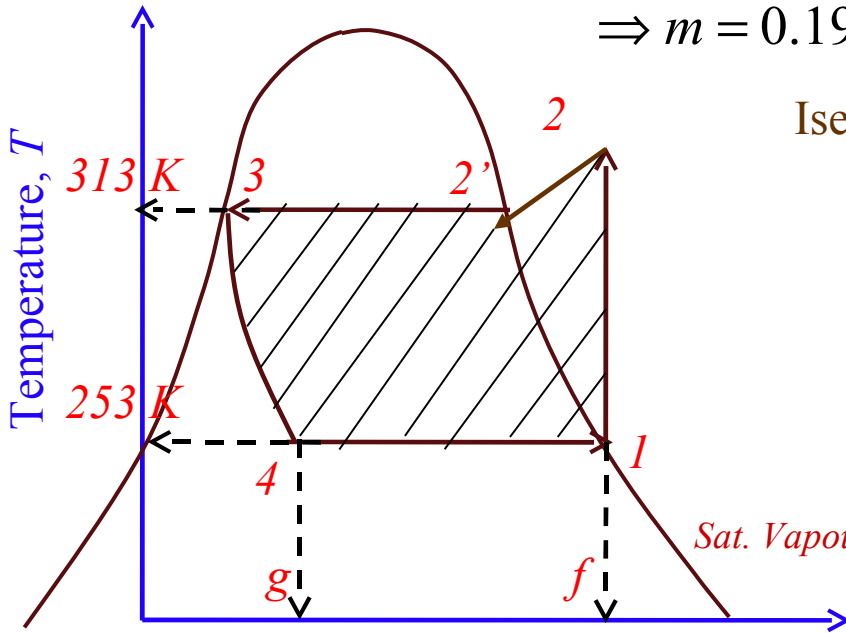
Isentropic Compression : 1-2

$s_1 = s_2$

$s_1 = s_{2'} + C_p \ln \left(\frac{T_2}{T_{2'}} \right)$

$0.7082 = 0.682 + (0.747) \ln \left(\frac{T_2}{313} \right)$

$\Rightarrow T_2 = 324.2 \text{ K}$



Sat. Liq. Line

Entropy, s

$h_2 = h_{2'} + C_p (T_2 - T_{2'})$

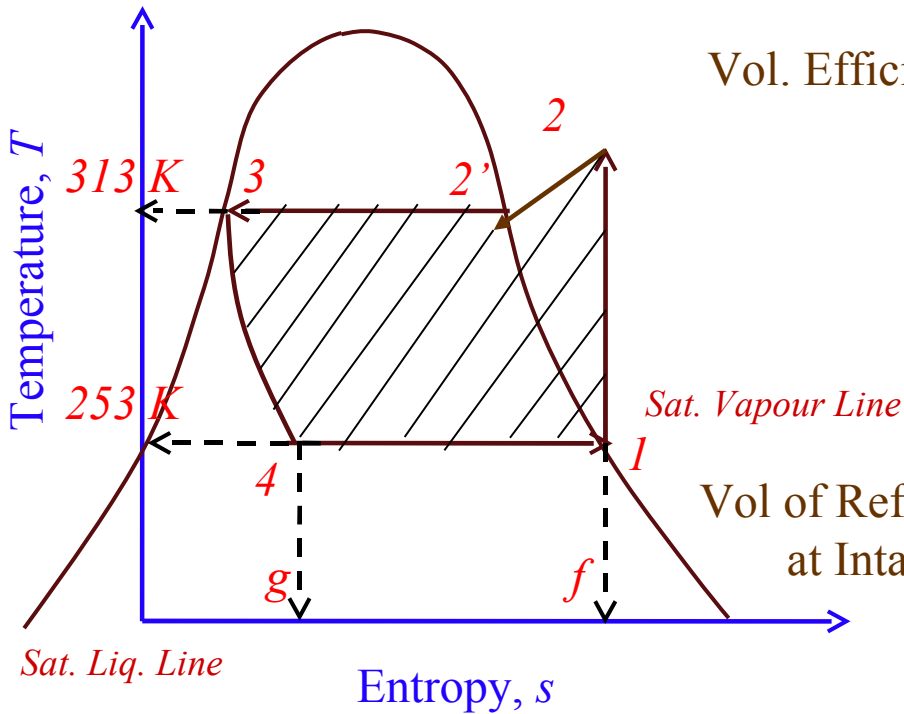
$= 203.05 \text{ (kJ / kg)} + (0.747 \text{ kJ / kg.K}) (324.2 - 313.0) \text{ K}$

$= 211.4 \text{ kJ / kg}$



Example 9

Power Required : $\dot{m} (h_2 - h_1) = 0.192 \text{ (kg / sec)} \times (211.4 - 178.61) \text{ kJ / kg}$
 $= 6.29 \text{ kW} \dots \text{ANS}$



Vol. Efficiency : $\eta_{vol} = 1 + k - k \left(\frac{P_d}{P_s} \right)^{1/n}$

$$= 1 + 0.03 - 0.03 \left(\frac{9.607 \text{ bar}}{1.509 \text{ bar}} \right)^{1/1.13}$$

$$= 87.6 \%$$

Vol of Refrigerant at Intake : $\dot{m} * v_g$

$$= 0.192 \text{ (kg / sec)} \times 0.1088 \text{ (m}^3 \text{ / kg)}$$

$$= 0.02089 \text{ m}^3 \text{ / sec}$$

Piston Displ. Vol. $\dot{V} = \frac{\text{Actual Vol.}}{\eta_{vol} * (\text{rpm})} = \frac{0.02089 \text{ (m}^3 \text{ / sec)} * 60 \text{ (sec / min)}}{0.876 * 300 \text{ (rpm)}} = 0.00477 \text{ m}^3$

....ANS



Example 10

A food storage locker requires a refrigeration capacity of 50 kW. It works between a condenser temperature of 35 °C and an evaporator temperature of -10 °C. The refrigerator is ammonia. It is sub-cooled by 5 °C before entering the expansion valve. By the dry saturated vapour leaving the evaporator. Assuming a single-cylinder, single-acting compressor operating at 1000 rpm with stroke equal to 1.2 times the bore, determine :

6. The power required.
7. The cylinder dimensions.

Properties of ammonia are :

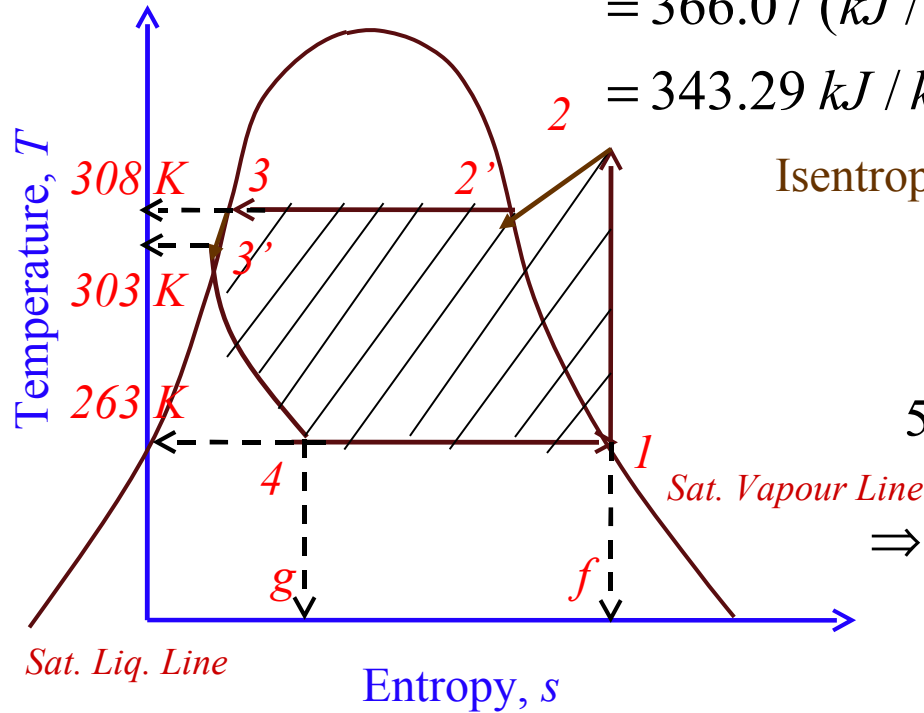
Sat. Temp. (°C)	Pr. (bar)	Enthalpy (kJ/kg)		Entropy (kJ/kg)		Sp. Vol. (m ³ /kg)		Sp. Heat (kJ/kg.K)	
		Liquid	Vapour	Liquid	Vapour	Liquid	Vapour	Liquid	Vapour
-10	2.9157	154.056	1450.22	0.82965	5.7550	---	0.417477	---	2.492
35	13.522	366.072	1488.57	1.56605	5.2086	1.7023	0.095629	4.556	2.903

}	}	Given :	$T_{\text{cond}} = 35 \text{ }^\circ\text{C}$	$h_1 = 1450.22 \text{ kJ/kg}$
			$T_{\text{evap}} = -10 \text{ }^\circ\text{C}$	$h_{2'} = 1488.57 \text{ kJ/kg}$
			$x_1 = 1 \dots \text{dry saturated vapour}$	$h_{f3} = 366.072 \text{ kJ/kg}$
			$\text{State 3} = \text{Sub-cooled by } 5 \text{ }^\circ\text{C}$	



Example 10...contd

$$\begin{aligned}
 h_{3'} &= h_4 = h_{f3} - C_{Pliq}(T_{sat} - T_{subcool}) \\
 &= 366.07 \text{ (kJ / kg)} - 405.56 (308 - 303) \text{ (kJ / kg)} \\
 &= 343.29 \text{ kJ / kg}
 \end{aligned}$$



Isentropic Compression : 1-2

$$s_1 = s_2 \Rightarrow s_1 = s_{2'} + C_P \ln\left(\frac{T_2}{T_{2'}}\right)$$

$$5.755 = 5.2086 + (2.903) \ln\left(\frac{T_2}{308}\right)$$

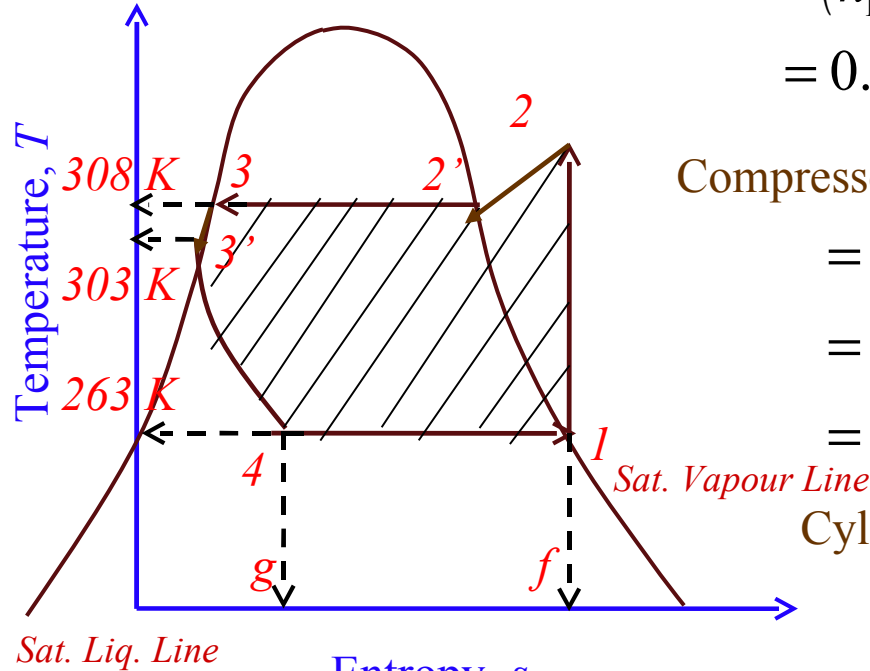
$$\Rightarrow T_2 = 371.8 \text{ K}$$

$$\begin{aligned}
 h_2 &= h_{2'} + C_P(T_2 - T_{2'}) \\
 &= 1488.57 \text{ (kJ / kg)} + (2.903 \text{ kJ / kg.K}) (371.8 - 308.0) \text{ K} \\
 &= 1673.8 \text{ kJ / kg}
 \end{aligned}$$



Example 10...contd

Mass of Refrigerant : $\dot{m} = \frac{50 \text{ (kW)}}{(h_1 - h_4) \text{ kJ / kg}} = \frac{50 \text{ (kW)}}{(1450.22 - 343.29) \text{ kJ / kg}}$
 $= 0.04517 \text{ kg / sec}$



Compressor Power :
 $= \dot{m} (h_2 - h_1)$
 $= 0.04517 \text{ (kg)} \times (1673.8 - 1450.22) \text{ kJ / kg}$
 $= 10.1 \text{ kW} \dots \text{ANS}$

Cylinder Dimensions :

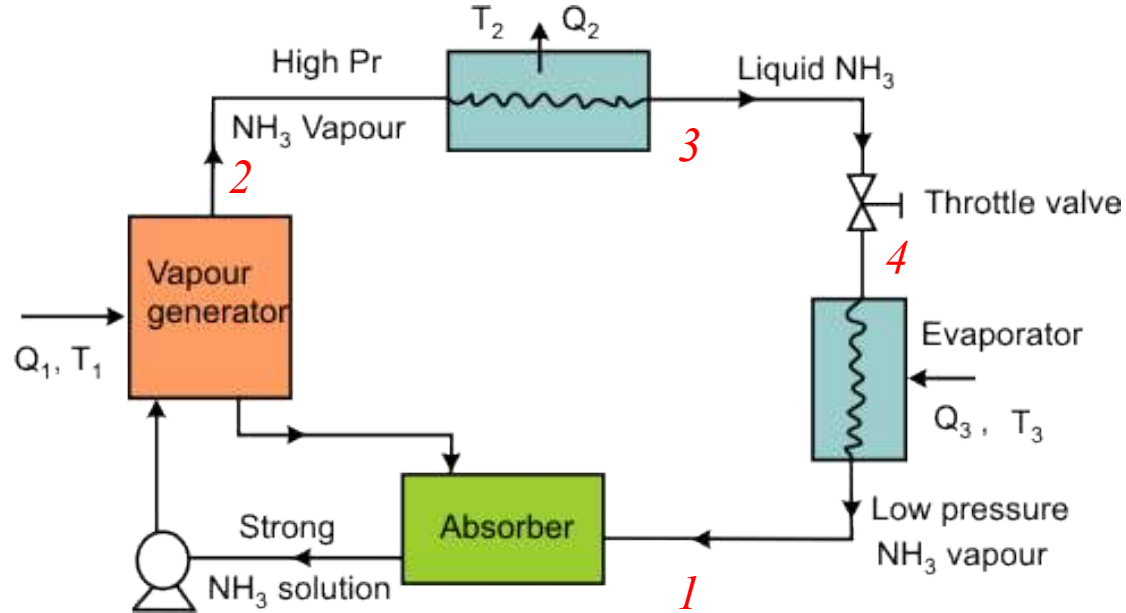
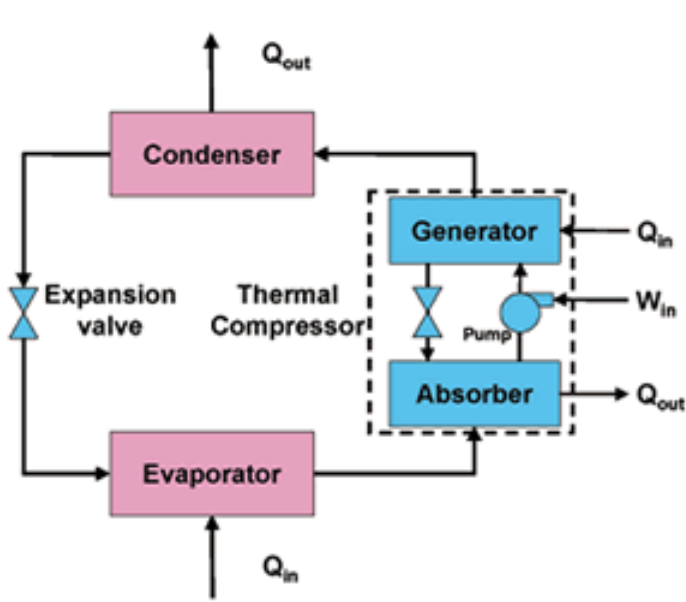
Entropy, s
 $m = 0.04517 \text{ (kg / sec)} = \frac{\left(\frac{\pi}{4} D^2 L \frac{N}{60} \right)}{v_g} = \frac{\left(\frac{\pi}{4} D^2 (1.2 D) \frac{1000 \text{ (rpm)}}{60} \right)}{0.417477 \text{ (m}^3 \text{ / kg)}}$

$\Rightarrow D = 0.19 \text{ m} \dots \text{ANS}$

$\Rightarrow L = 1.2 * (0.19 \text{ m}) = 0.228 \text{ m} \dots \text{ANS}$



Vapour Absorption System



Solubility of NH_3 in water @ \downarrow Temp and Pr. is **MORE** than that @ \uparrow Temp. and Pr.

NH_3 vapour from **Evaporator** (State 1) is readily absorbed in **Absorber**. \Rightarrow **Heat Rejection**

This solution is then pumped to \uparrow Temp. and Pr. @ **Generator**.

Reduction in stability of solution \Rightarrow **Vapour removed from Solution**.

Vapour passes to **Condenser**.

Weak Solution returns to **Absorber**.



Vapour Absorption System

COP :

$$COP = \frac{\text{Heat extracted in Evaporator}}{\text{Heat supplied in Generator} + \text{Work done on Liquid Pump}}$$

Merits :

1. Pumping work is much less than work for Compressing vapour.
2. Work done on Compression is LESS.

Demerits :

1. Heat input to the Generator is required.
2. Low COP.



Vapour Compression Vs. Vapour Absorption

Sr. No.	Particulars	Vapour Compression Systems	Vapour Absorption Systems
1.	Type of Energy Supplied	Mechanical – High Grade	Heat – Low Grade
2.	Energy Supply Rate	Low	High
3.	Wear & Tear	More	Less
4.	Performance of Part Load	Poor	Not affected at Part Load
5.	Suitability	Used where High Grade Mechanical Energy is available	Can be used at Remote Places, as can be used with simple Kerosene lamp
6.	Charging of Refrigerant	Simple	Difficult
7.	Leakage	More chances	No chances, as no Compressor or Reciprocating
8.	Damage	Liquid traces in Suction Line may damage Compressor	No danger



Thank You !