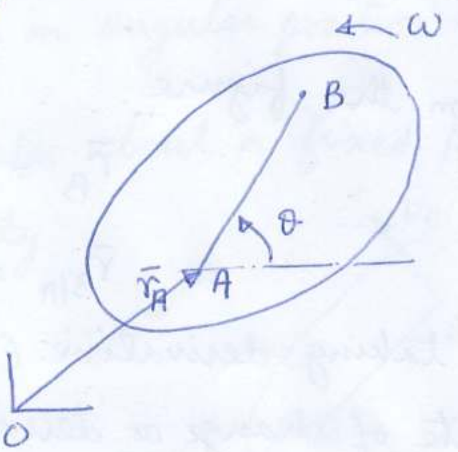
VELOCITY ANALYSIS

Velocity $\bar{V}_A$ of a point A on a body is the rate of change of its position, $\bar{r}_A$

$$\bar{V}_A = \frac{d\bar{r}_A}{dt}$$

Different points will have, in general, different velocity.



Velocity is a vector, having direction as well as magnitude.

Angular velocity ω of a body is the rate of change of angular position θ of any line on the body.

$$\omega = \frac{d\theta}{dt}$$

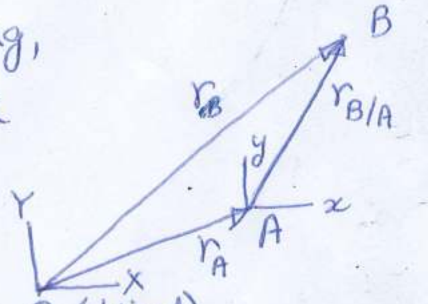
One body has one angular velocity.

Angular velocity for planar motion is a scalar with fixed direction ⊥ to plane of motion. It is often described as clockwise or counterclockwise.

Relative Velocity of a point B relative to A is the velocity of B as seen from A when A is itself moving.

Consider points A and B, both moving, whose positions measured from a fixed point O are given by vectors

$\bar{r}_A$ and $\bar{r}_B$



These are absolute positions.

However, position of B measured from A (or relative to A) is given by the vector

$$\bar{r}_{B/A}$$

From the figure

$$\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$$

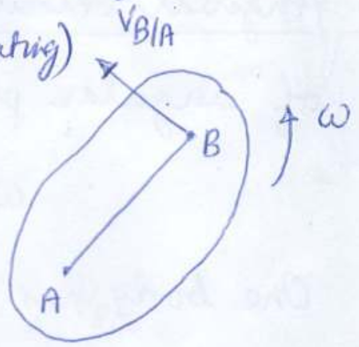
$$\text{or} \quad \bar{r}_{B/A} = \bar{r}_B - \bar{r}_A$$

By taking derivative (and remembering that velocity is rate of change or derivative of position), we get

$$\bar{V}_{B/A} = \bar{V}_B - \bar{V}_A$$

which is the equation of relative velocity.

If points A and B are on the same (rotating) body, then



$$\bar{V}_{B/A} \text{ is } \perp \text{ to line } \bar{AB}$$

and the magnitude of $\bar{V}_{B/A}$ is

$$V_{B/A} = \bar{AB}\,\omega$$

Finally,

$$\bar{V}_{A/B} = -\bar{V}_{B/A}$$

$$= \bar{V}_A - \bar{V}_B$$

Types of Motion

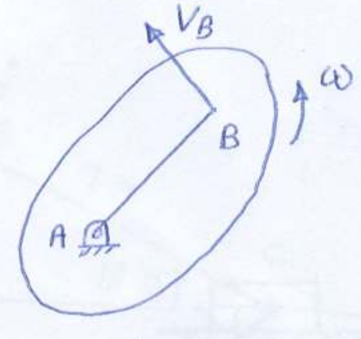
1- Pure Translation: All points of the body have same motion. There is no change in angular position of body.

2- Pure Rotation: Body rotates about a fixed point.

Any point B of the body has velocity

$\bar{V}_B \perp$ to line $\bar{AB}$

$V_B = \bar{AB}\,\omega$



3- General Motion: Body rotates as well as translates.

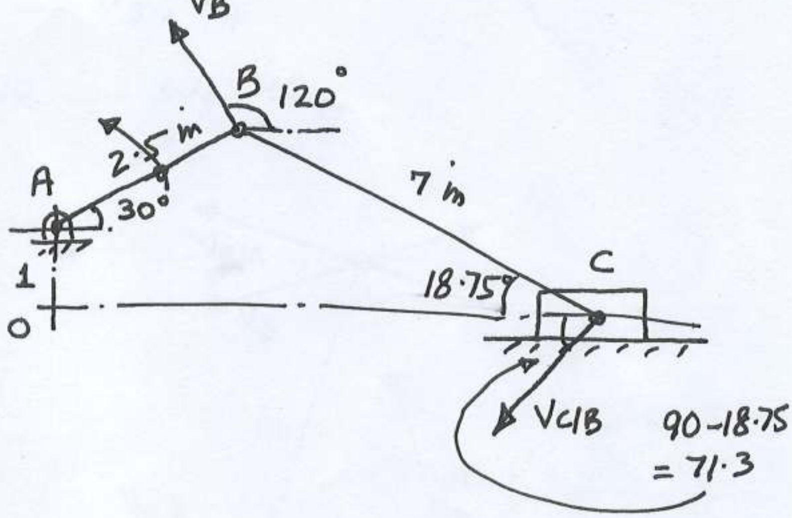
Velocity Analysis of slider-crank

Example

Offset slider-crank with dimensions shown. At $\theta_2 = 30^\circ$, $\theta_3 = 18.75^\circ$.

When $\omega_2 = 2$ rad/s ccw

find ω_3, V_C.



SOLUTION

$$\overline{V_C} = \overline{V_B} + \overline{V_{C/B}}$$

We know

$\overline{V_B}$ is $\perp$ to AB (because $\overline{AB}$ rotates about A)

$V_B = 2.5\,\omega_2 = 2.5 \times 2 = 5$ in/s

$\overline{V_{C/B}}$ is $\perp$ to BC (because B, C are on the same link $\overline{BC}$)

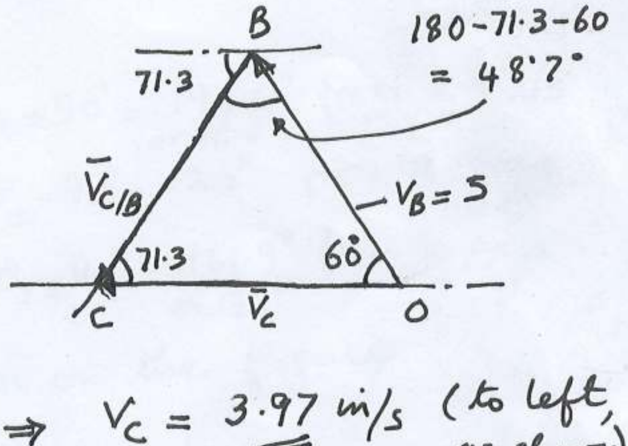
$\overline{V_C}$ is parallel to line $\overline{OC}$ (slider link)

Draw velocity triangle.

From O draw OB = ~~2~~ 5 in/s at 60°

From O draw line at 0° for V_C

From B draw line at 71.3° for $V_{C/B}$, cutting horizontal line at C.



Calculate angles.

Sine law gives $\dfrac{V_C}{\sin 48.7^\circ} = \dfrac{5}{\sin 71.3^\circ} \Rightarrow V_C = \underline{\underline{3.97}}$ in/s (to left, as shown)

$\dfrac{V_{C/B}}{\sin 60^\circ} = \dfrac{5}{\sin 71.3^\circ} \Rightarrow V_{C/B} = 4.57$. $\omega_3 = \dfrac{V_{C/B}}{\overline{CB}} = \dfrac{4.57}{7} = \underline{\underline{0.653}}$ rad/s

From fig, this is cw.

Four-Bar Example

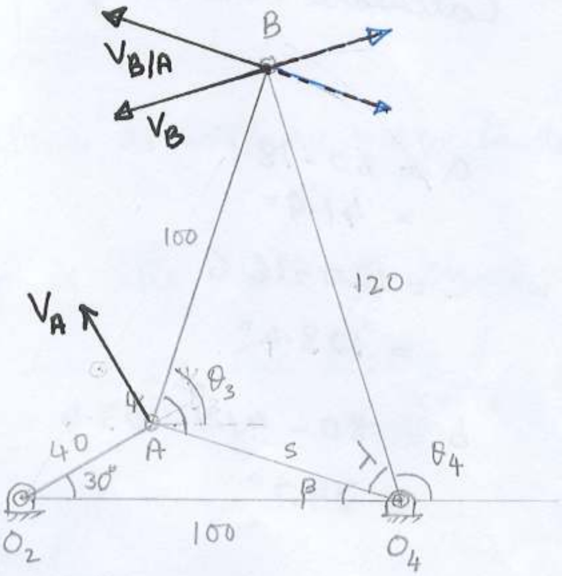
Given: $R_1 = 100$ mm, $R_2 = 40$, $R_3 = 100$, $R_4 = 120$

$\theta_2 = \cancel{60}\ 30^\circ$, $\omega_2 = 15$ rad/s ccw

Find: (a) Position solution — θ_3, θ_4

(b) Velocity solution — ω_3, ω_4

SOLUTION



(a) Steps of position analysis:

$$s = \sqrt{40^2 + 100^2 - 2 \times 40 \times 100 \times \cos 30^\circ}$$
$$= 68.4$$

$$\beta = \sin^{-1}\left[\frac{40 \sin 30^\circ}{68.4}\right] = 17.0^\circ$$

$$\psi = \cos^{-1}\left[\frac{100^2 + 68.4^2 - 120^2}{2 \times 100 \times 68.4}\right]$$
$$= 88.9^\circ$$

$$\lambda = \sin^{-1}\left[\frac{100 \sin 88.9^\circ}{120}\right] = 56.4^\circ$$

$\therefore\ \theta_3 = \psi - \beta = 71.9^\circ$, $\theta_4 = 180 - \lambda - \beta = 106.6^\circ$

(b) Velocity Solution

$$\overline{V_B} = \overline{V_A} + \overline{V_{B/A}}$$

$\overline{V_B}$ is $\perp$ to $\overline{O_4B}$, ie at angle $\theta_4 \pm 90^\circ = 196.6^\circ$ or 16.6° from x-axis

$\overline{V_A}$ is $\perp$ to $\overline{O_2A}$, ie at angle $\theta_2 + 90^\circ = 120^\circ$ from x-axis

$\overline{V_{B/A}}$ is $\perp$ to $\overline{AB}$, ie at angle $\theta_3 \pm 90^\circ = 161.9^\circ$ or -18.1° from x-axis

All these directions are shown on the figure.

Also, $V_A = R_2 \omega_2 = 40 \times 15 = 600$ mm/s

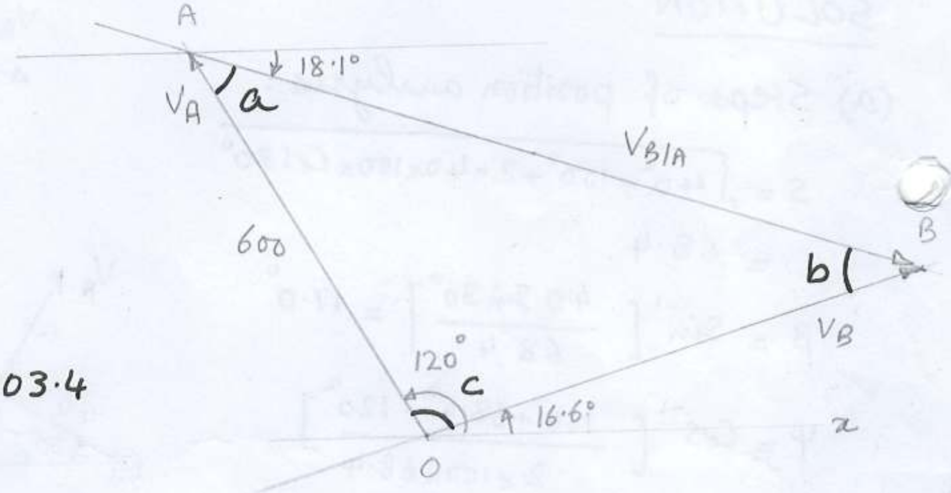
Draw velocity Triangle.

From O, draw $V_A = 600$ at 120° to x-axis, going up to A

From O, draw line at ~~196.6~~ 16.6° to x-axis (for V_B, length unknown)

From A, draw line at ~~161.9°~~ -18.1° to x-axis, cutting above line at B.

Calculate inner angles of $\triangle ABO$



$a = 60 - 18.1$
$= 41.9^\circ$

$c = 120 - 16.6$
$= 103.4^\circ$

$b = 180 - 41.9 - 103.4$
$= 34.7^\circ$

We can now use sine law to find V_B, $V_{B/A}$

$$\frac{V_B}{\sin 41.9^\circ} = \frac{600}{\sin 34.7^\circ} \Rightarrow V_B = 704 \text{ mm/s}$$

$$\frac{V_{B/A}}{\sin 103.4} = \frac{600}{\sin 34.7^\circ} \Rightarrow V_{B/A} = 1025 \text{ mm/s}$$

Finally, $V_B = R_4 \omega_4 \Rightarrow \omega_4 = \frac{704}{120} = \underline{\underline{5.87}}$ rad/s CW

$V_{B/A} = R_3 \omega_3 \Rightarrow \omega_3 = \frac{1025}{100} = \underline{\underline{10.25}}$ rad/s CW

Correct velocity directions are as shown here →