جامعة بيشة
UNIVERSITY OF BISHA

بسم الله الرحمن الرحيم

«قال ياقوم أرأيتم إن كنت على بينة من ربى ورزقنى منه رزقا حسنا
وما أريد أن أخالفكم إلى ما أنهاكم عنه إن أريد إلا الإصلاح ما أستطعت
وما توفيقي إلا بالله عليه توكلت وإليه أنيب»

صدق الله العظيم (٨٨ هود)

Prof. Ashraf A. Ali ا.د. أشرف عبدالفتاح على حسانين
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تصميم اجزاء ماكينات (١) – همك ٤١١
Design of Machine Elements (1) – ME 411

Prof. Ashraf A. Ali

PhD Materials Engineering Department, Drexel University, USA 2002

Egyptian Fulbright Awarded Grantee and Visiting Scientist, ISN, MIT, USA 2007
Assistant professor, KSU, Saudi Arabia (2007-2010)
Associate Professor, PSAU, Saudi Arabia (2010-2012)

Tenure Track Professor, Mechanical Design and Production Engineering Department, Faculty of Engineering, Zagazig University,
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Professor

Vice Dean for Development and Quality, College of Engineering 2017-2020
Part Time Consultant at Vice Presidency for Graduate Studies and Scientific Research
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1st Term AY 1441-1442/2020-2021

الاسبوع الرابع – المحاضرة الرابعة

التعليم عن بعد

الانحناء فى الكمرات

Distant Learning

Beam Deflection

Design Concepts

- ✓ **Design based on Stresses**
- ✓ **Design based on Stiffness**

Chapter 12

If the curvature of this pole is measured, it is then possible to determine the bending stress developed within it.

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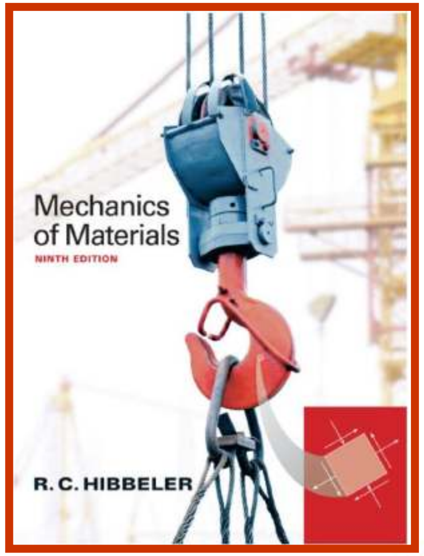
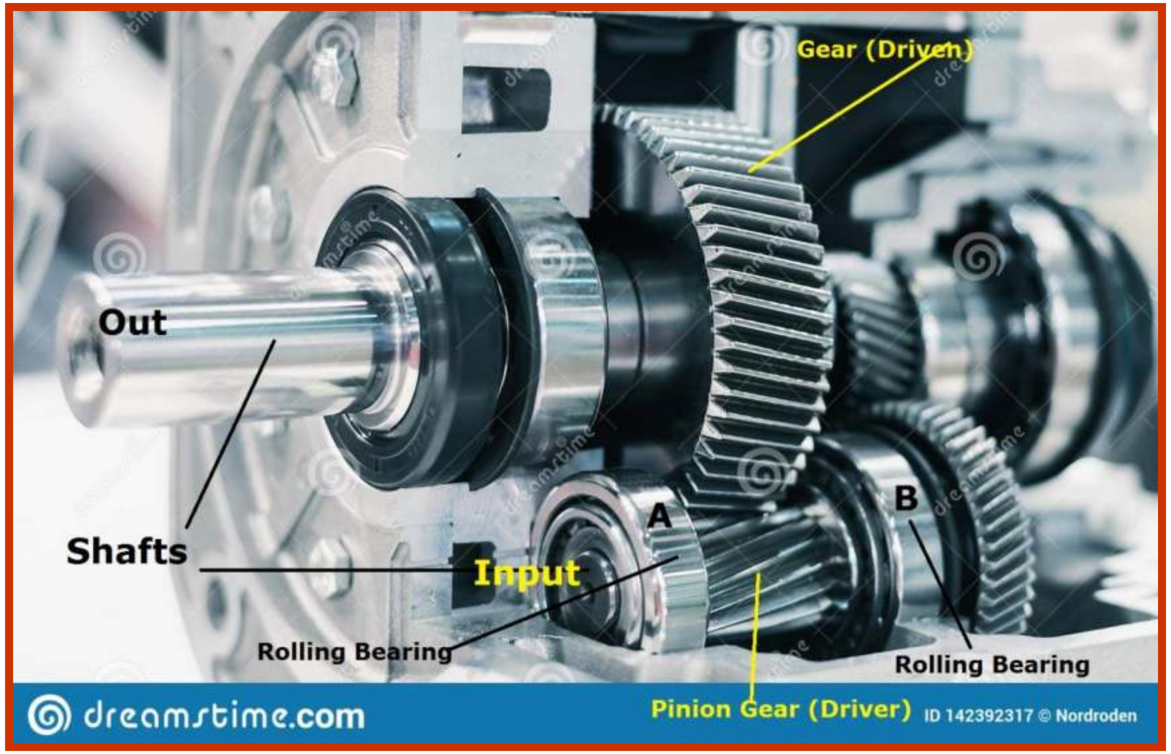


Design Concepts

✓ **Design based on Stresses**
✓ **Design based on Stiffness**



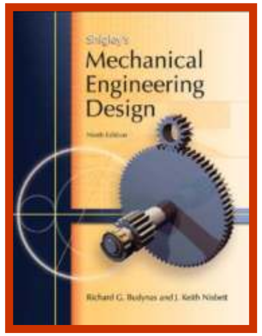


Table 7–2

Typical Maximum Ranges for Slopes and Transverse Deflections

Slopes	
Tapered roller	0.0005–0.0012 rad
Cylindrical roller	0.0008–0.0012 rad
Deep-groove ball	0.001–0.003 rad
Spherical ball	0.026–0.052 rad
Self-align ball	0.026–0.052 rad
Uncrowned spur gear	< 0.0005 rad
Transverse Deflections	
Spur gears with $P < 10$ teeth/in	0.010 in
Spur gears with $11 < P < 19$	0.005 in
Spur gears with $20 < P < 50$	0.003 in

After: https://www.dreamstime.com/mechanical-gearbox-cross-section-helical-gear-close-up-image142392317

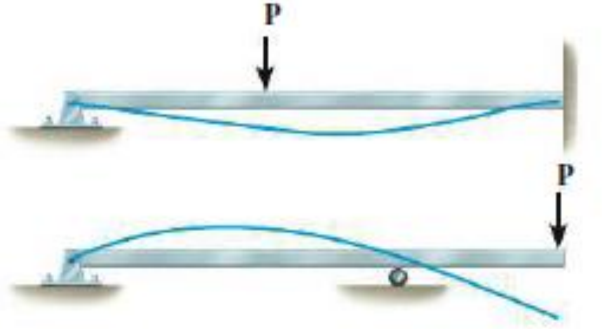
Slope and Deflection vs. $M_{(x)}$

Elastic Curve

where ρ is the radius of curvature. From studies in mathematics we also learn that the curvature of a plane curve is given by the equation

$$\frac{1}{\rho} = \frac{d^2y/dx^2}{[1+(dy/dx)^2]^{3/2}}$$

$$\boxed{\frac{1}{\rho} = \frac{M}{EI}}$$



where the interpretation here is that y is the lateral deflection of the centroidal axis of the beam at any point x along its length. The slope of the beam at any point x is

$$\boxed{\theta = \frac{dy}{dx}}$$

For many problems in bending, the slope is very small,

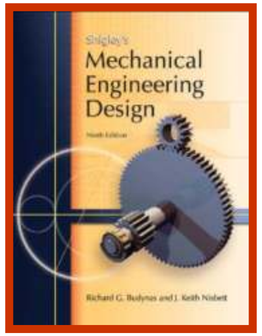
$$\boxed{\frac{M}{EI} = \frac{d^2y}{dx^2}}$$

$$\boxed{EI\frac{dy}{dx} = \int M\,dx}$$

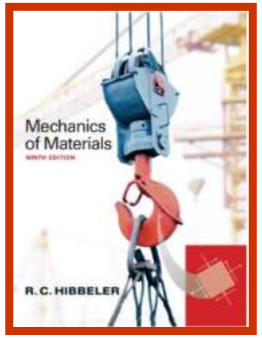
Slope = θ

$$\boxed{EIy = \iint M\,dx}$$

Deflection = y = δ



Ch.4 p. 147



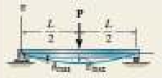
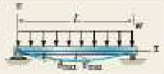
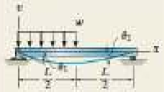
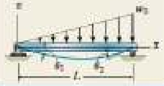
Ch. 12 p.573

Deflection and Slope Correlations in Different Types of Supported and Loaded Beams

APPENDIX C

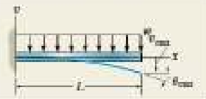
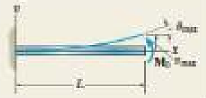
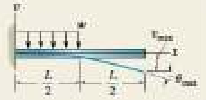
Slopes and Deflections of Beams

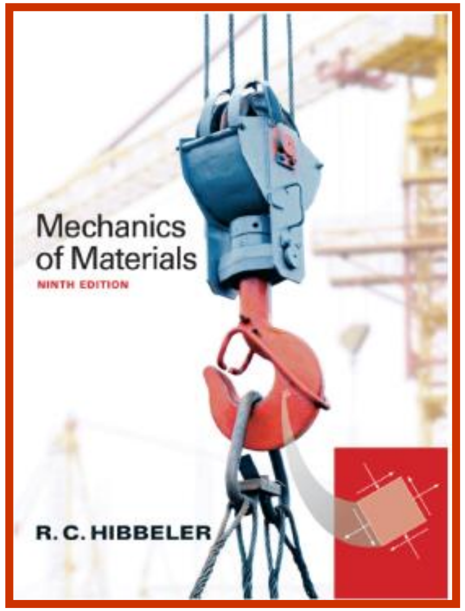
Simply Supported Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \frac{-PL^2}{16EI}$	$v_{max} = \frac{-PL^3}{48EI}$	$v = \frac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \frac{-Pab(L+b)}{6EIL}$ $\theta_2 = \frac{Pab(L+a)}{6EIL}$	$v\Big\vert_{x=a} = \frac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \frac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$\theta_1 = \frac{-M_0L}{6EI}$ $\theta_2 = \frac{M_0L}{3EI}$	$v_{max} = \frac{-M_0L^2}{9\sqrt{3}EI}$ at $x = 0.5774L$	$v = \frac{-M_0x}{6EIL}(L^2 - x^2)$
	$\theta_{max} = \frac{-wL^3}{24EI}$	$v_{max} = \frac{-5wL^4}{384EI}$	$v = \frac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$
	$\theta_1 = \frac{-3wL^3}{128EI}$ $\theta_2 = \frac{7wL^3}{384EI}$	$v\Big\vert_{x=L/2} = \frac{-5wL^4}{768EI}$ $v_{max} = -0.006563\frac{wL^4}{EI}$ at $x = 0.4598L$	$v = \frac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \frac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x < L$
	$\theta_1 = \frac{-7w_0L^3}{360EI}$ $\theta_2 = \frac{w_0L^3}{45EI}$	$v_{max} = -0.00652\frac{w_0L^4}{EI}$ at $x = 0.5193L$	$v = \frac{-w_0x}{360EIL}(3x^4 - 10L^2x^2 + 7L^4)$

812

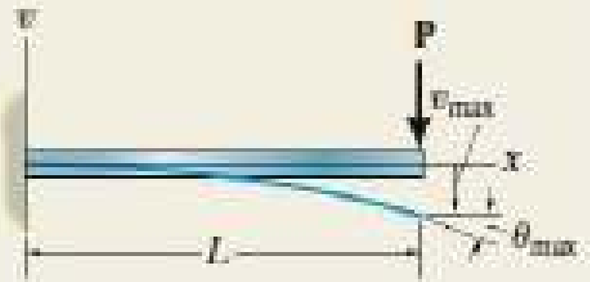
Cantilevered Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \frac{-PL^2}{2EI}$	$v_{max} = \frac{-PL^3}{3EI}$	$v = \frac{-Px^2}{6EI}(3L - x)$
	$\theta_{max} = \frac{-PL^2}{8EI}$	$v_{max} = \frac{-5PL^3}{48EI}$	$v = \frac{-Px^2}{12EI}(3L - 2x) \quad 0 \le x \le L/2$ $v = \frac{-PL^2}{48EI}(6x - L) \quad L/2 \le x \le L$
	$\theta_{max} = \frac{-wL^3}{6EI}$	$v_{max} = \frac{-wL^4}{8EI}$	$v = \frac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$
	$\theta_{max} = \frac{M_0L}{EI}$	$v_{max} = \frac{M_0L^2}{2EI}$	$v = \frac{M_0x^2}{2EI}$
	$\theta_{max} = \frac{-wL^3}{48EI}$	$v_{max} = \frac{-7wL^4}{384EI}$	$v = \frac{-wx^2}{24EI}\left(x^2 - 2Lx + \frac{3}{2}L^2\right)$ $0 \le x \le L/2$ $v = \frac{-wL^3}{384EI}(8x - L)$ $L/2 \le x \le L$
	$\theta_{max} = \frac{-w_0L^3}{24EI}$	$v_{max} = \frac{-w_0L^4}{30EI}$	$v = \frac{-w_0x^2}{120EIL}(10L^3 - 10L^2x + 5Lx^2 - x^3)$



Beam Deflection Derivation

Case No. (1) Tabulated

Cantilevered Beam Slopes and Deflections			
Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \frac{-PL^2}{2EI}$	$v_{max} = \frac{-PL^3}{3EI}$	$v = \frac{-Px^2}{6EI}(3L - x)$

δ

Delta

$$EI(\theta) = \frac{Px^2}{2} - PLx$$

$$EI(\delta) = \frac{Px^3}{6} - \frac{PLx^2}{2}$$

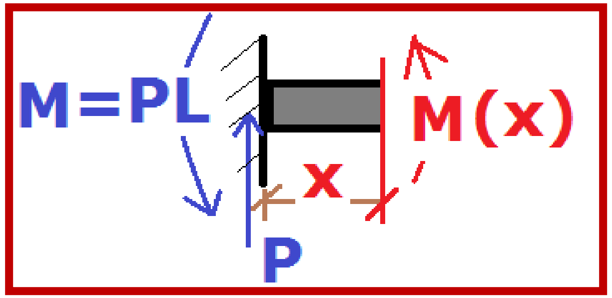
Beam Deflection Derivation

Case No. (1)

Derived by Integration Method

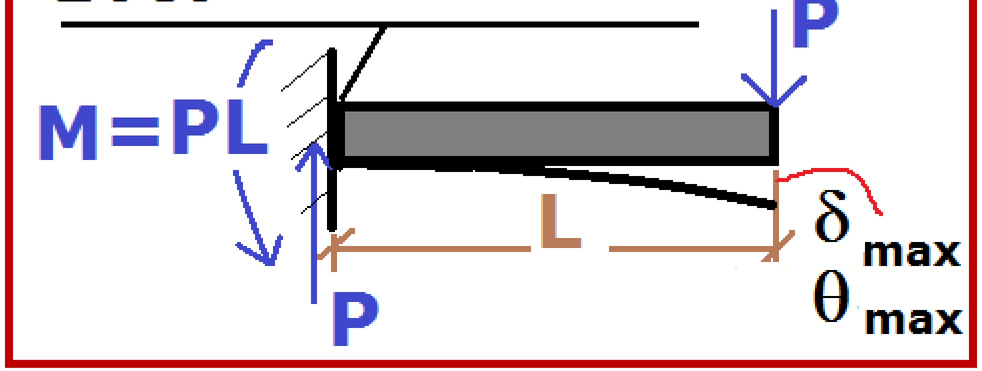
$$E\,I\,(\theta) = \int M(x)dx$$

$$E\,I\,(\delta) = \iint M(x)dx$$



$$M(x) = Px - PL$$

B.C. $\theta = 0$ & $\delta = 0$



$$\boxed{E\,I\,(\theta) =} \int (Px - PL)\,dx = \boxed{\frac{Px^2}{2} - PLx} + C1$$ ➔ @ $x=0;\ \theta=0$ gives $C1=0$

$$\boxed{E\,I(\delta) =} \iint M(x)dx = \boxed{\frac{Px^3}{6} - \frac{PLx^2}{2}} + C1x + C2$$ ➔ @ $x=0;\ \delta=0$ gives $C2=0$

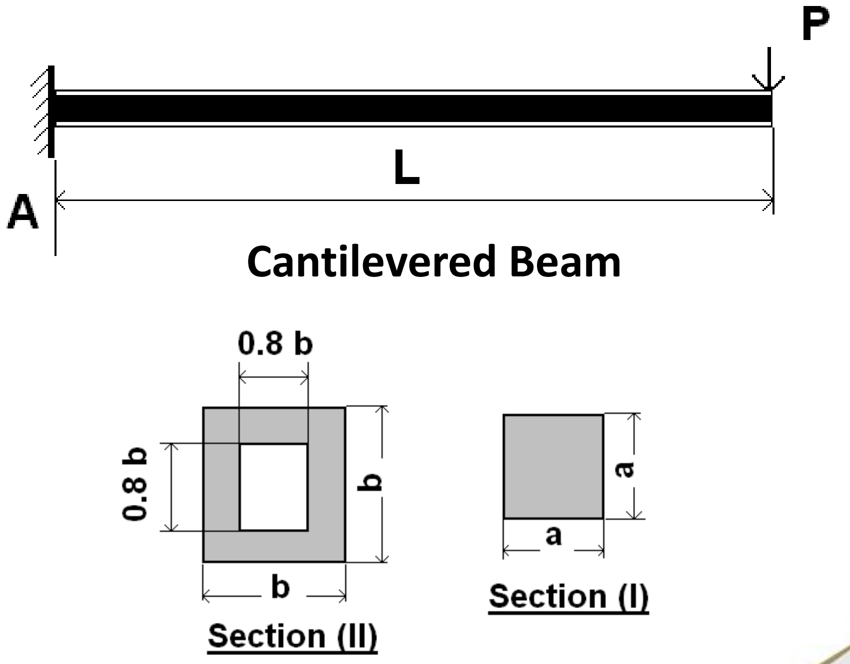
Beam Design on Both Stresses and Deflection

Solved Problem (Practice No.1)

1. Design standard two cross-sections: Solid square (section I), and Box square (Section II) (Use static flexural stress formula only).
2. Calculate the maximum deflection in (mm) and slope in (rad) of the loaded beam for each cross-section and identify its position.
3. Comment on the previous results and state one conclusion.

Given:

Young's modulus = 210 GPa, yield strength = 390 MPa and Design factor = 3, P = 10 kN, L = 1 m



Two cross-sections

Beam Design on Both Stresses and Deflection
Solved Problem (Practice No.1)

$M_{(max)}$ = PL = 10x1= 10 (kN.m) at the support as $[M_{(max)}/ Z] = [\sigma_y/n_d]$
Z = (10x1000X1000) N.mm/130 (N/mm^2) = 76.9231x1000 (mm^3)
a = 42.53 mm, for standard **use $a_{st.}$ = 45 mm**
b = 98.16 mm, for standard **use $b_{st.}$ = 100 mm**

I (moment of Inertia) for solid square = $a_{st}^4/12$ = 341718.75 mm^4
I (moment of Inertia) for Box square = $(b_{st}^4-(0.8b)^4)/12$ = 4920000 mm4

Maximum Deflection at free end = PL³/3EI =
{[(10000(1000)^3 N.mm^3)]/[3(210x1000 N/mm^2)(I)]} = **{15873015.87/(I)} (mm)**

Maximum Slope at free end = PL²/2EI =
{[(10000(1000)^{2}N.mm^2)]/[2(210x1000 N/mm^2)(I)]} = **{500000/21(I)} (rad.)**

Beam Design on Both Stresses and Deflection

Solved Problem (Practice No.1)

For **solid square cross-section** (Maximum Deflection) **= 47 mm**
For **solid square cross-section** (Maximum Slope) **= 0.0697 rad. = 4 degrees**

For **Box square cross-section** (Maximum Deflection) **= 3.2 mm**
For **Box square cross-section** (Maximum Slope) **= 0.00048 rad. = 0.3 degree**

Comments

1- Deflection and Slope of the loaded beam showed **lower values** by using box structure cross section instead of solid square cross section.

2- Beams with **box structure cross-section** not only lighter in weight but also behave more rigid (stiff) in its elastic curve (deflection and slope) than solid square cross-section

Conclusion: Cross-section **profile and orientation** is very important parameter in designing of beams; not only to save weight (money) but also to make **firm, stiff and rigid beam** in its applications.

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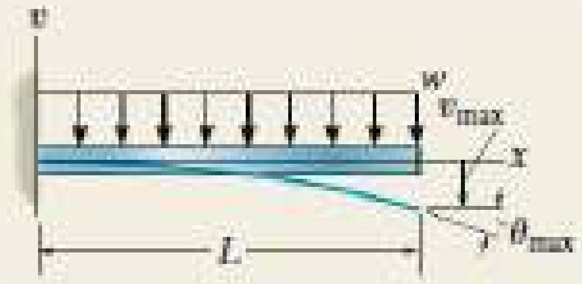
Beam Deflection Derivation
Case No. (2) Tabulated

Cantilevered Beam Slopes and Deflections

Beam	Slope √	Deflection √	Elastic Curve
	$\theta_{max} = \dfrac{-wL^3}{6EI}$	$v_{max} = \dfrac{-wL^4}{8EI}$	$v = \dfrac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$

δ

Delta

Beam Deflection Derivation

Case No. (2)

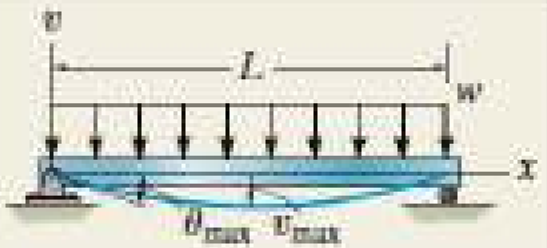
Derived by Integration Method

Two Pones points for all students in tomorrow practice (Competition)
And in case of no student contribution
T.B.S and I in Practical Lecture No. 4 (Tomorrow)

Beam Deflection Derivation

Case No. (3) Tabulated

Simply Supported Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \dfrac{-wL^3}{24EI}$	$v_{max} = \dfrac{-5wL^4}{384EI}$	$v = \dfrac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$

√ √

δ

Delta

Beam Deflection Derivation

Case No. (3)

Derived by Integration Method

EXAMPLE 4–1 For the beam in Fig. 4–2, the bending moment equation, for $0 \le x \le l$, is

$$M = \frac{wl}{2}x - \frac{w}{2}x^2$$

Using Eq. (4–12), determine the equations for the slope and deflection of the beam, the slopes at the ends, and the maximum deflection.

Solution Integrating Eq. (4–12) as an indefinite integral we have

$$EI\frac{dy}{dx} = \int M\,dx = \frac{wl}{4}x^2 - \frac{w}{6}x^3 + C_1 \quad (1)$$

where C_1 is a constant of integration that is evaluated from geometric boundary conditions. We could impose that the slope is zero at the midspan of the beam, since the beam and loading are symmetric relative to the midspan. However, we will use the given boundary conditions of the problem and verify that the slope is zero at the midspan. Integrating Eq. (1) gives

$$EIy = \iint M\,dx = \frac{wl}{12}x^3 - \frac{w}{24}x^4 + C_1x + C_2 \quad (2)$$

The boundary conditions for the simply supported beam are $y = 0$ at $x = 0$ and l. Applying the first condition, $y = 0$ at $x = 0$, to Eq. (2) results in $C_2 = 0$. Applying the second condition to Eq. (2) with $C_2 = 0$,

$$EIy(l) = \frac{wl}{12}l^3 - \frac{w}{24}l^4 + C_1l = 0$$

Solving for C_1 yields $C_1 = -wl^3/24$. Substituting the constants back into Eqs. (1) and (2) and solving for the deflection and slope results in

$$y = \frac{wx}{24EI}(2lx^2 - x^3 - l^3) \quad (3)$$

$$\theta = \frac{dy}{dx} = \frac{w}{24EI}(6lx^2 - 4x^3 - l^3) \quad (4)$$

Comparing Eq. (3) with that given in Table A–9, beam 7, we see complete agreement.

For the slope at the left end, substituting $x = 0$ into Eq. (4) yields

$$\theta|_{x=0} = -\frac{wl^3}{24EI}$$

and at $x = l$,

$$\theta|_{x=l} = \frac{wl^3}{24EI}$$

At the midspan, substituting $x = l/2$ gives $dy/dx = 0$, as earlier suspected.

The maximum deflection occurs where $dy/dx = 0$. Substituting $x = l/2$ into Eq. (3) yields

$$y_{max} = -\frac{5wl^4}{384EI}$$

الاسبوع الخامس – المحاضرة الخامسة

التعليم عن بعد

تصميم الكمرات على الاجهاد والانحناء – مسائل محلولة

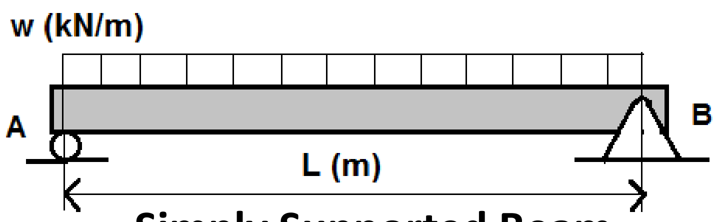
Distant Learning

Beam Design on both stresses and Deflection
Solved Problems

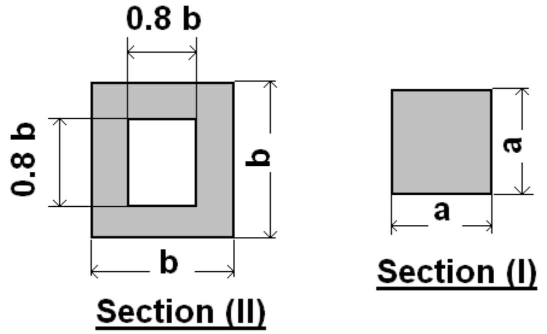
Beam Design on Both Stresses and Deflection

Solved Problem (Drill No. 1)

1. Deduce the deflection and slope equations for the shown simply supported beam shown in Figure (1).
2. Design standard two cross-sections: Solid square (section I), and Box square (Section II) (Use static flexural stress formula only).
3. Get the factor of safety for each standard cross-section to the corrected nearest 5 mm of each side length.
4. Calculate the weight of the beam for each cross-section in (kg) and identify the lighter beam weight?
5. Calculate the maximum deflection in (mm) and slope in (rad) of the loaded beam for each cross-section and identify its position.
6. Comment on the previous results (parts 4 and 5) and state one conclusion.



Simply Supported Beam



Two cross-sections

Assume the loaded beam has the following data
Young's modulus = 210 GPa, yield strength = 390 MPa and Design factor = 3
w = 4 kN/m, L = 1 m and material's density = 7.8 g/cm^3

بسم الله الرحمن الرحيم

«يوم تجد كل نفس ما عملت من خير محضرا وما عملت من سوء تود لو أن بينها وبينه أمدا بعيدا ويحذركم الله نفسه والله رءوف بالعباد» صدق الله العظيم

٣٠ آل عمران

Prof. Ashraf A. Ali ا.د. أشرف عبدالفتاح على حسانين
ahassanein@ub.edu.sa