Jazan University
Mechanical Engineering Department

CHAPTER 2

KINEMATIC ANALYSIS OF MECHANISMS

EngM271 Theory of Machines

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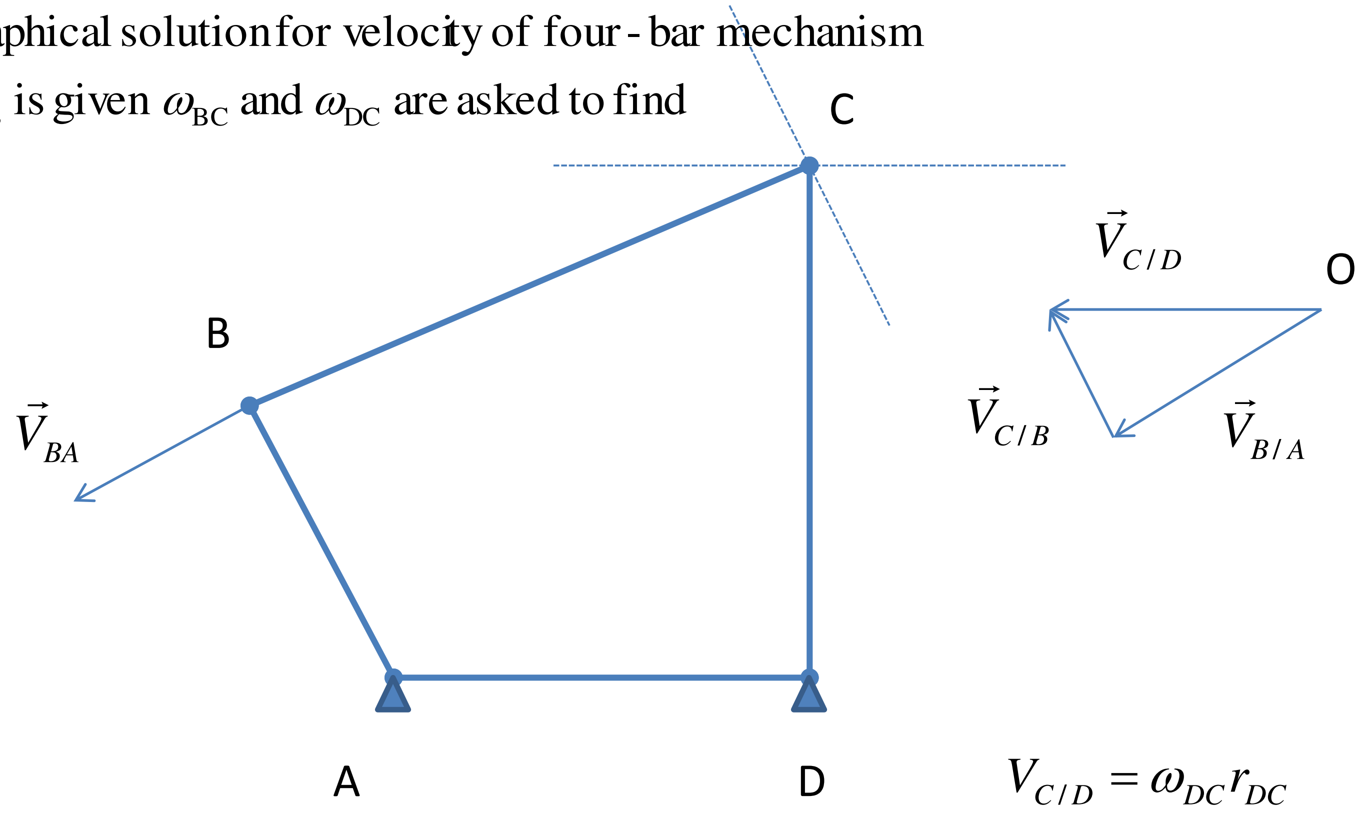
$\text{Graphical solution for velocity of four - bar mechanism}$

ω_{AB} is given ω_{BC} and ω_{DC} are asked to find



$$V_{C/D} = \omega_{DC} r_{DC} \qquad \omega_{DC} = \frac{V_{C/D}}{r_{DC}}$$

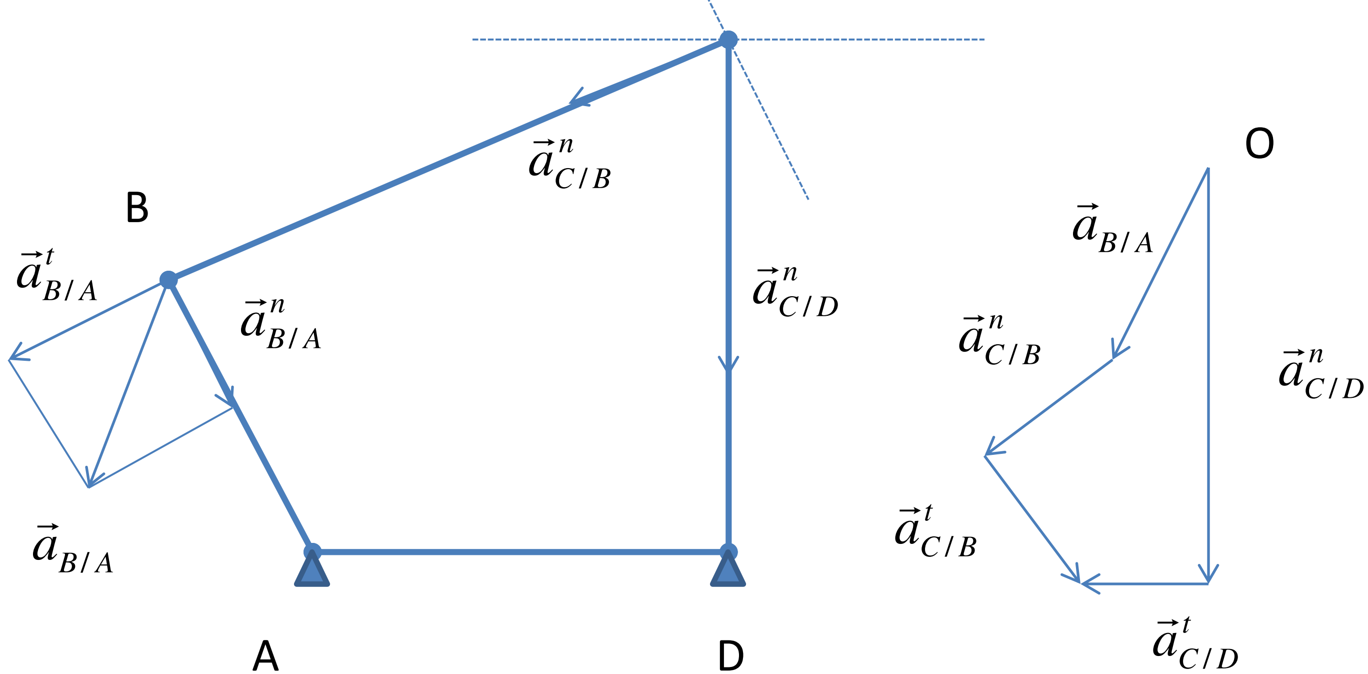
$$V_{C/B} = \omega_{BC} r_{BC} \qquad \omega_{BC} = \frac{V_{C/B}}{r_{BC}}$$

$$\vec{V}_{C/D} = \vec{V}_{C/B} + \vec{V}_{B/A}$$

$$\vec{\omega}_{DC} \times \vec{r}_{DC} = \vec{\omega}_{BC} \times \vec{r}_{BC} + \vec{\omega}_{AB} \times \vec{r}_{AB}$$

Graphical solution for acceleration

α_{AB} is given α_{BC} and α_{DC} are asked to find



$$\vec{a}_{C/D} = \vec{a}_{C/B} + \vec{a}_{B/A}$$

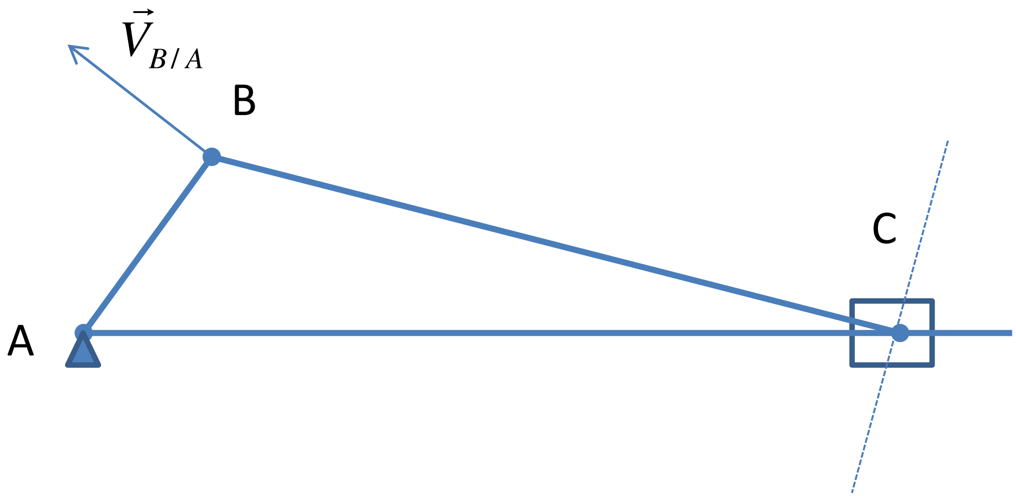
$$\vec{a}^{n}_{C/D} + \vec{a}^{t}_{C/D} = \vec{a}^{n}_{C/B} + \vec{a}^{t}_{C/B} + \vec{a}^{n}_{B/A} + \vec{a}^{t}_{B/A}$$

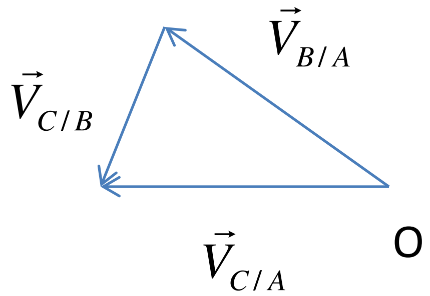
$$\alpha_{BC} = \frac{a^{t}_{C/B}}{r_{BC}} \qquad \alpha_{DC} = \frac{a^{t}_{D/C}}{r_{DC}}$$

$$-\omega^2_{DC}\vec{r}_{DC} + \vec{\alpha}_{DC} \times \vec{r}_{DC} = -\omega^2_{BC}\vec{r}_{BC} + \vec{\alpha}_{BC} \times \vec{r}_{BC} - \omega^2_{AB}\vec{r}_{AB} + \vec{\alpha}_{AB} \times \vec{r}_{AB}$$

Graphical solution for velocity of slider-crank mechanism

ω_{AB} is given ω_{BC} and $V_{C/A}$ are asked to find





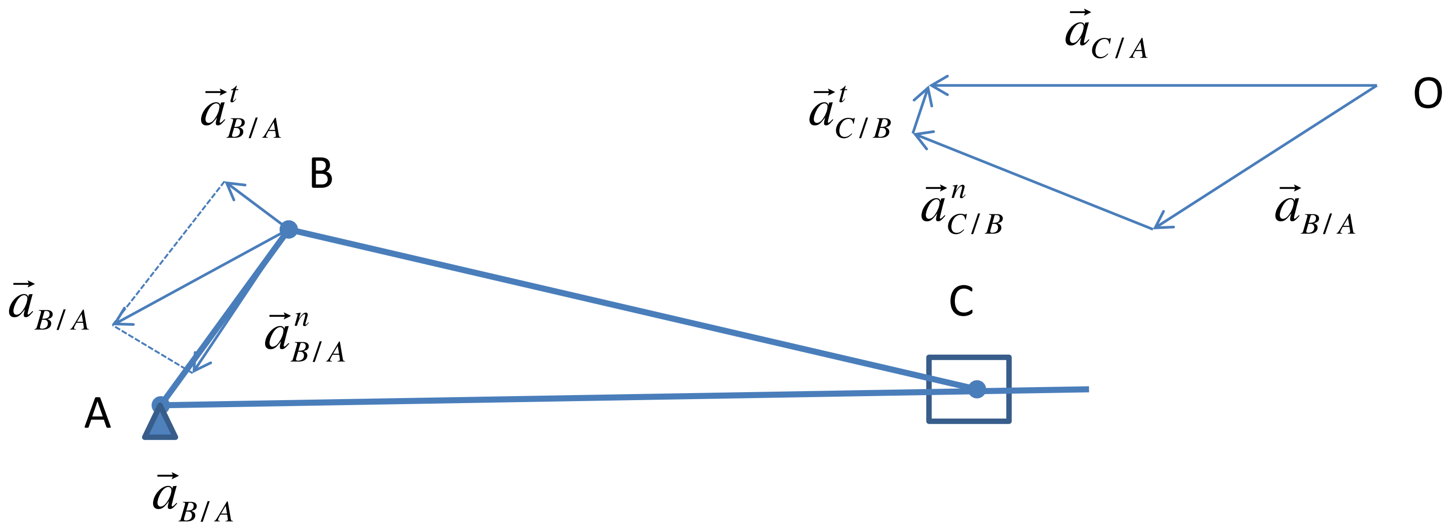
$$\vec{V}_{C/A} = \vec{V}_{C/B} + \vec{V}_{B/A}$$

$$V_{C/B} = \omega_{BC} r_{BC} \qquad \omega_{BC} = \frac{V_{C/B}}{r_{BC}}$$

$$\vec{V}_{C/A} = \vec{\omega}_{BC} \times \vec{r}_{BC} + \vec{\omega}_{AB} \times \vec{r}_{AB}$$

Graphical solution for acceleration of slider-crank mechanism

α_{AB} and ω_{AB} is given α_{BC} and $a_{C/A}$ are asked to find



$$\vec{a}_{C/A} = \vec{a}_{C/B} + \vec{a}_{B/A}$$

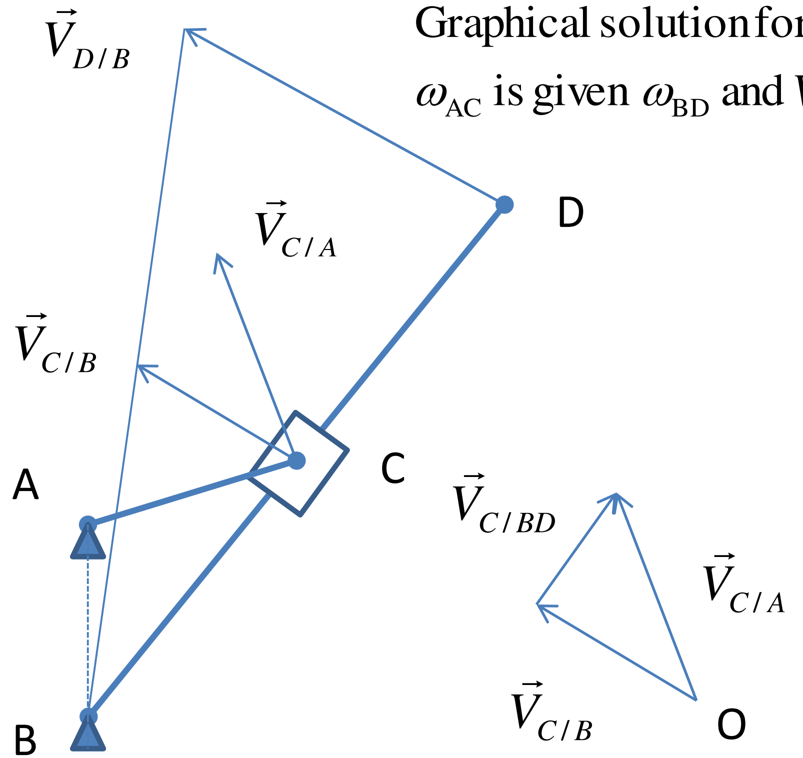
$$\vec{a}_{C/A} = \vec{a}^{n}_{C/B} + \vec{a}^{t}_{C/B} + \vec{a}^{n}_{B/A} + \vec{a}^{t}_{B/A}$$

$$\alpha_{BC} = \frac{a^{t}_{C/B}}{r_{BC}}$$

$$\vec{a}_{C/A} = -\omega^2_{BC}\vec{r}_{BC} + \vec{\alpha}_{BC} \times \vec{r}_{BC} - \omega^2_{AB}\vec{r}_{AB} + \vec{\alpha}_{AB} \times \vec{r}_{AB}$$

Graphical solution for velocity of inverted slider-crank mechanism

ω_{AC} is given ω_{BD} and $V_{C/BD}$ are asked to find



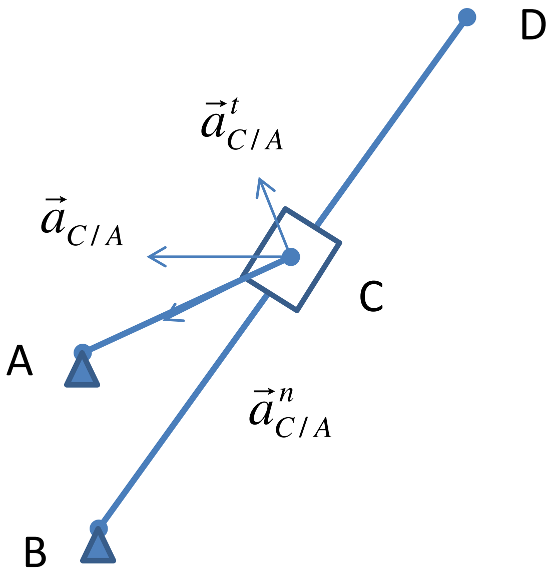
$$\vec{V}_{C/A} = \vec{V}_{C/B} + \vec{V}_{C/BD}$$

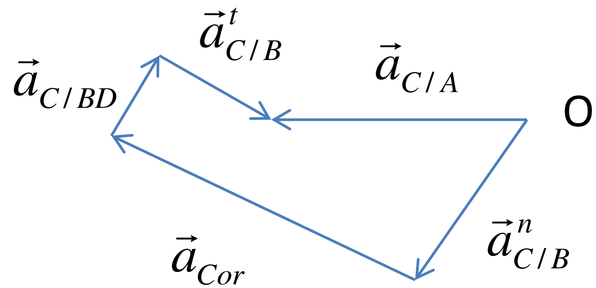
$$\omega_{BD} = \frac{V_{C/B}}{r_{BC}}$$

$$\vec{\omega}_{AC} \times \vec{r}_{AC} = \vec{\omega}_{BD} \times \vec{r}_{BC} + \vec{V}_{C/BD}$$

Graphical solution for acceleration of slider-crank mechanism

α_{AC} and ω_{AC} is given α_{BD} and $a_{C/BD}$ are asked to find





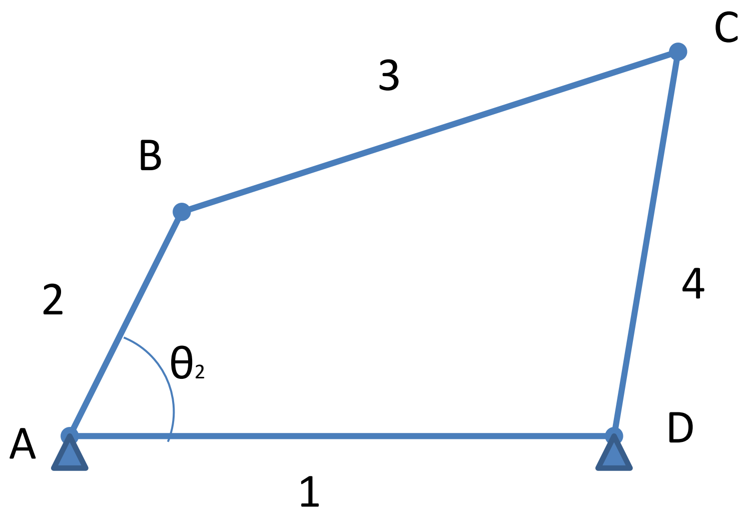
$$\vec{a}_{C/A} = \vec{a}_{C/B} + \vec{a}_{C/BD} + \vec{a}_{Corriolis}$$

$$\vec{a}^n_{C/A} + \vec{a}^t_{C/A} = \vec{a}^n_{C/B} + \vec{a}^t_{C/B} + \vec{a}_{C/BD} + \vec{a}_{Cor}$$

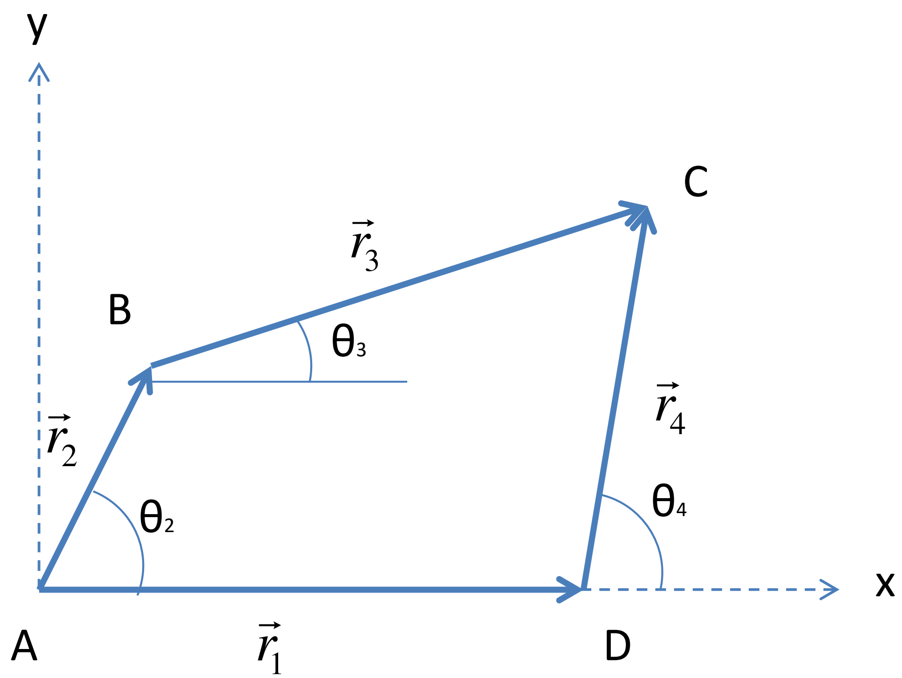
$$\alpha_{BD} = \frac{a^t_{C/B}}{r_{BC}}$$

$$-\omega^2_{AC}\vec{r}_{AC} + \vec{\alpha}_{AC} \times \vec{r}_{AC} = -\omega^2_{BD}\vec{r}_{BC} + \vec{\alpha}_{BD} \times \vec{r}_{BC} + \vec{a}_{C/BD} + 2\vec{\omega}_{BD} \times \vec{V}_{C/BD}$$

Loop-Closure equation for a four-bar linkage



for crank θ_2 , $\dot{\theta}_2$, $\ddot{\theta}_2$ and link lengths r_2, r_3, r_1, r_4 are given,
θ_3 and θ_4 also $\dot{\theta}_3$, $\dot{\theta}_4$, $\ddot{\theta}_3$, $\ddot{\theta}_4$ are asked to find.



$$\vec{r}_2 + \vec{r}_3 = \vec{r}_1 + \vec{r}_4$$

$$\vec{r}_2 + \vec{r}_3 - \vec{r}_1 - \vec{r}_4 = 0 \quad \text{loop-closure equation}$$

$$\vec{r} = re^{j\theta} = r\cos\theta + jr\sin\theta \text{ is called Euler formula}$$

$$r_2 e^{j\theta_2} + r_3 e^{j\theta_3} - r_1 e^{j\theta_1} - r_4 e^{j\theta_4} = 0$$

$$r_2\cos\theta_2 + jr_2\sin\theta_2 + r_3\cos\theta_3 + jr_3\sin\theta_3$$
$$-r_1\cos\theta_1 - jr_1\sin\theta_1 - r_4\cos\theta_4 - jr_4\sin\theta_4 = 0$$

Real and imaginary parts can be written separate as two equations

$$r_2\cos\theta_2 + r_3\cos\theta_3 - r_1\cos\theta_1 - r_4\cos\theta_4 = 0$$

$$r_2\sin\theta_2 + r_3\sin\theta_3 - r_1\sin\theta_1 - r_4\sin\theta_4 = 0$$

Define two functions

$$F_1(\theta_3,\theta_4) = r_2\cos\theta_2 + r_3\cos\theta_3 - r_1\cos\theta_1 - r_4\cos\theta_4 = 0$$

$$F_2(\theta_3,\theta_4) = r_2\sin\theta_2 + r_3\sin\theta_3 - r_1\sin\theta_1 - r_4\sin\theta_4 = 0$$

remember $\theta_1 = 0°$

$$F_1 = r_2\cos\theta_2 + r_3\cos\theta_3 - r_1 - r_4\cos\theta_4 = 0$$

$$F_2 = r_2\sin\theta_2 + r_3\sin\theta_3 - r_4\sin\theta_4 = 0$$

to find velocities take time derivatives of F_1 and F_2

$$\frac{dF_1}{dt} = -r_2 \sin\theta_2\dot{\theta}_2 - r_3 \sin\theta_3\dot{\theta}_3 + r_4 \sin\theta_4\dot{\theta}_4 = 0$$

$$\frac{dF_2}{dt} = r_2 \cos\theta_2\dot{\theta}_2 + r_3 \cos\theta_3\dot{\theta}_3 - r_4 \cos\theta_4\dot{\theta}_4 = 0$$

$$\begin{bmatrix} -r_3 \sin\theta_3 & r_4 \sin\theta_4 \\ r_3 \cos\theta_3 & -r_4 \cos\theta_4 \end{bmatrix} \begin{bmatrix} \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} r_2 \sin\theta_2\dot{\theta}_2 \\ -r_2 \cos\theta_2\dot{\theta}_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} -r_3 \sin\theta_3 & r_4 \sin\theta_4 \\ r_3 \cos\theta_3 & -r_4 \cos\theta_4 \end{bmatrix}^{-1} \begin{bmatrix} r_2\dot{\theta}_2 \sin\theta_2 \\ -r_2\dot{\theta}_2 \cos\theta_2 \end{bmatrix}$$

to find accelerations take time derivatives of $\frac{dF_1}{dt}$ and $\frac{dF_2}{dt}$

$$\frac{d^2F_1}{dt^2} = -r_2 \sin\theta_2\ddot{\theta}_2 - r_2 \cos\theta_2\dot{\theta}_2^2 - r_3 \sin\theta_3\ddot{\theta}_3 - r_3 \cos\theta_3\dot{\theta}_3^2$$

$$+ r_4 \sin\theta_4\ddot{\theta}_4 + r_4 \sin\theta_4\dot{\theta}_4^2 = 0$$

$$\frac{d^2F_2}{dt^2} = -r_2 \cos\theta_2\ddot{\theta}_2 - r_2 \sin\theta_2\dot{\theta}_2^2 - r_3 \cos\theta_3\ddot{\theta}_3 - r_3 \sin\theta_3\dot{\theta}_3^2$$

$$+ r_4 \cos\theta_4\ddot{\theta}_4 + r_4 \sin\theta_4\dot{\theta}_4^2 = 0$$

$$\begin{bmatrix} -r_3 \sin\theta_3 & r_4 \sin\theta_4 \\ r_3 \cos\theta_3 & -r_4 \cos\theta_4 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}$$

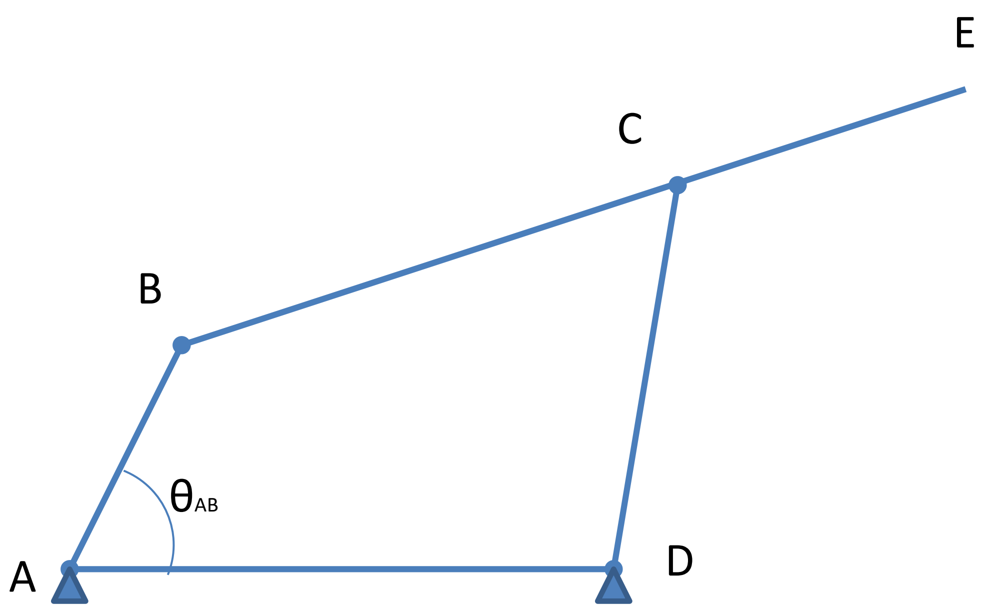
$$\begin{bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_3 \end{bmatrix} = \begin{bmatrix} -r_3 \sin\theta_3 & r_4 \sin\theta_4 \\ r_3 \cos\theta_3 & -r_4 \cos\theta_4 \end{bmatrix}^{-1} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$A = r_2 \sin\theta_2 \ddot{\theta}_2 + r_2 \cos\theta_2 \dot{\theta}_2^2 + r_3 \cos\theta_3 \dot{\theta}_3^2 - r_4 \sin\theta_4 \dot{\theta}_4^2$$

$$B = r_2 \cos\theta_2 \ddot{\theta}_2 + r_2 \sin\theta_2 \dot{\theta}_2^2 + r_3 \sin\theta_3 \dot{\theta}_3^2 - r_4 \sin\theta_4 \dot{\theta}_4^2$$

The following matrix is called Jacobien matrix
which is same for displacement, velocity and acceleration calculations.

$$J = \begin{bmatrix} -r_3 \sin\theta_3 & r_4 \sin\theta_4 \\ r_3 \cos\theta_3 & -r_4 \cos\theta_4 \end{bmatrix}$$



If we want to find velocity or acceleration of any point, write vector of the point and differentiate with respect to time

$$\vec{r}_C = \vec{r}_2 + \vec{r}_3 = r_2 e^{j\theta_2} + r_3 e^{j\theta_3}$$
$$= r_2 \cos\theta_2 + jr_2 \sin\theta_2 + r_3 \cos\theta_3 + jr_3 \sin\theta_3$$
$$= (r_2 \cos\theta_2 + r_3 \cos\theta_3) + j(r_2 \sin\theta_2 + r_3 \sin\theta_3)$$

$$\dot{\vec{r}}_C = (-r_2 \sin\theta_2 \dot{\theta}_2 - r_3 \sin\theta_3 \dot{\theta}_3) + j(r_2 \cos\theta_2 \dot{\theta}_2 + r_3 \cos\theta_3 \dot{\theta}_3)$$

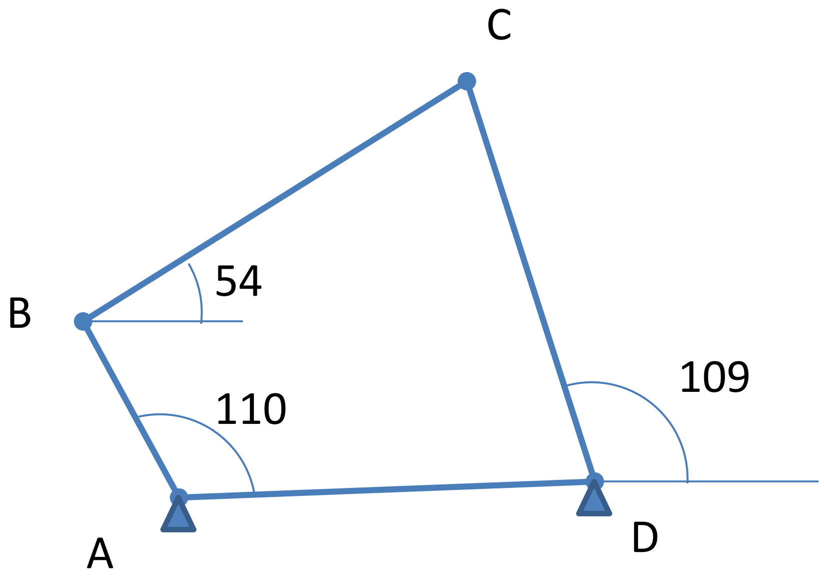
$$\ddot{\vec{r}}_C = (-r_2 \sin\theta_2 \ddot{\theta}_2 - r_2 \cos\theta_2 \dot{\theta}_2^2 - r_3 \sin\theta_3 \ddot{\theta}_3 - r_3 \cos\theta_3 \dot{\theta}_3^2)$$
$$+ j(r_2 \cos\theta_2 \ddot{\theta}_2 + r_2 \sin\theta_2 \dot{\theta}_2^2 + r_3 \cos\theta_3 \ddot{\theta}_3 + r_3 \sin\theta_3 \dot{\theta}_3^2)$$

or we can use $\vec{r}_C = \vec{r}_1 + \vec{r}_4 = r_1 e^{j\theta_1} + r_4 e^{j\theta_4}$

for point E $\vec{r}_E = \vec{r}_1 + \vec{r}_4 + \vec{r}_{CE} = r_1 e^{j\theta_1} + r_4 e^{j\theta_4} + r_{CE} e^{j\theta_3}$ or

$\vec{r}_E = \vec{r}_2 + \vec{r}_{BE} = r_2 e^{j\theta_2} + r_{BE} e^{j\theta_3}$ can be used

Example



$$AB = 3\ \text{cm},\ BC = 7\ \text{cm},\ AD = 6\ \text{cm},\ DC = 9\ \text{cm}$$

$$\omega_2 = 10\ \text{rad/s (constant, or uniform, and CCW)}$$

$$\omega_3 = ?\ \ \omega_4 = ?\ \alpha_3 = ?\ \ \alpha_4 = ?$$

$$\begin{bmatrix}\dot{\theta}_3\\ \dot{\theta}_4\end{bmatrix}=\begin{bmatrix}-r_3\sin\theta_3 & r_4\sin\theta_4\\ r_3\cos\theta_3 & -r_4\cos\theta_4\end{bmatrix}^{-1}\begin{bmatrix}r_2\dot{\theta}_2\sin\theta_2\\ -r_2\dot{\theta}_2\cos\theta_2\end{bmatrix}$$

$$\begin{bmatrix}\dot{\theta}_3\\ \dot{\theta}_4\end{bmatrix}=\begin{bmatrix}-7\sin 54 & 9\sin 109\\ 7\cos 54 & -9\cos 109\end{bmatrix}^{-1}\begin{bmatrix}3(10)\sin 110\\ 3(10)\cos 110\end{bmatrix}$$

remember

$$A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix} \qquad A^{-1}=\begin{bmatrix}\dfrac{a_{22}}{\Delta} & \dfrac{-a_{12}}{\Delta}\\ \dfrac{-a_{21}}{\Delta} & \dfrac{a_{11}}{\Delta}\end{bmatrix}$$

determinant

$$\Delta=a_{11}a_{22}-a_{21}a_{12}$$

$$\begin{bmatrix}\dot{\theta}_3\\ \dot{\theta}_4\end{bmatrix}=\begin{bmatrix}-5.66 & 8.51\\ 4.11 & 2.93\end{bmatrix}^{-1}\begin{bmatrix}28.19\\ 10.26\end{bmatrix}$$

$$\text{determinant } \Delta=(-5.66)(2.93)-(4.11)(8.51)=-51.56$$

$$\begin{bmatrix} \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} \dfrac{2.93}{-51.56} & \dfrac{-8.51}{-51.56} \\ \dfrac{-4.11}{-51.56} & \dfrac{-5.66}{-51.56} \end{bmatrix} \begin{bmatrix} 28.19 \\ 10.26 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} -0.057 & 0.16 \\ 0.08 & 0.11 \end{bmatrix} \begin{bmatrix} 28.19 \\ 10.26 \end{bmatrix}$$

$$\dot{\theta}_3 = \omega_3 = (-0.057)(28.19) + (0.16)(10.26) = 0.035\,\text{rad/s}$$

$$\dot{\theta}_4 = \omega_4 = (0.08)(28.19) + (0.11)(10.26) = 3.38\,\text{rad/s}$$

$$\begin{bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{bmatrix} = \begin{bmatrix} -r_3 \sin\theta_3 & r_4 \sin\theta_4 \\ r_3 \cos\theta_3 & -r_4 \cos\theta_4 \end{bmatrix}^{-1} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{bmatrix} = \begin{bmatrix} -0.057 & 0.16 \\ 0.08 & 0.11 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$A = r_2\ddot{\theta}_2 \sin\theta_2 + r_2\dot{\theta}_2^2 \cos\theta_2 + r_3\dot{\theta}_3^2 \cos\theta_3 - r_4\dot{\theta}_4^2 \sin\theta_4$$

$$A = 3(0)\sin 110 + 3(10)^2 \cos 110 + 7(0.035)^2 \cos 54 - 9(3.38)^2 \sin 109 = -69.12$$

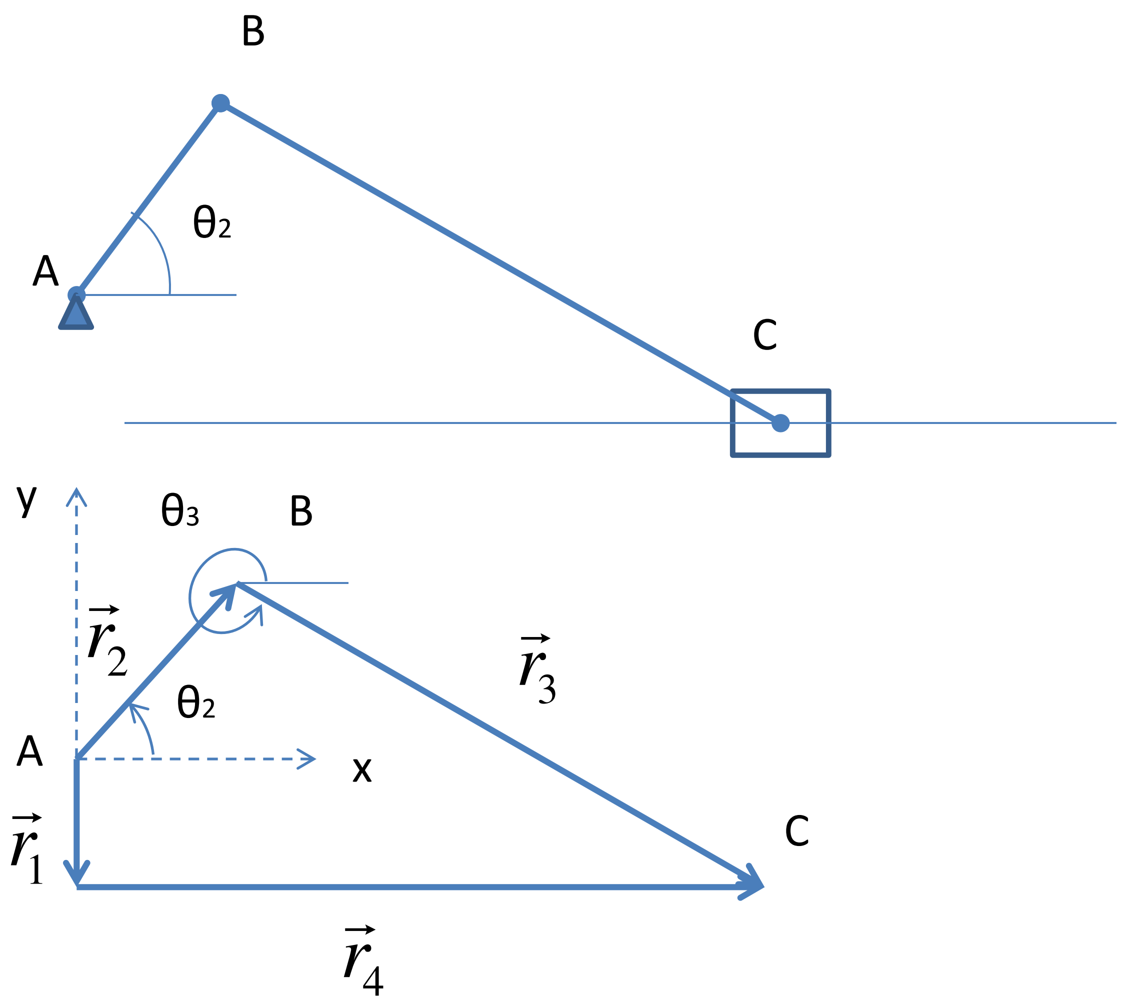
$$B = r_2\ddot{\theta}_2 \cos\theta_2 + r_2\dot{\theta}_2^2 \sin\theta_2 + r_3\dot{\theta}_3^2 \sin\theta_3 - r_4\dot{\theta}_4^2 \sin\theta_4$$

$$B = 3(0)\cos 110 + 3(10)^2 \sin 110 + 7(0.035)^2 \sin 54 - 9(3.38)^2 \sin 109 = 184.69$$

$$\begin{bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{bmatrix} = \begin{bmatrix} -0.057 & 0.16 \\ 0.08 & 0.11 \end{bmatrix} \begin{bmatrix} -69.12 \\ 184.69 \end{bmatrix}$$

$$\ddot{\theta}_3 = \alpha_3 = (-0.057)(-69.12) + (0.16)(184.69) = 33.49\,\text{rad/s}^2$$
$$\ddot{\theta}_4 = \alpha_4 = (0.08)(-69.12) + (0.11)(184.69) = 14.78\,\text{rad/s}^2$$

Offset slider-crank mechanism loop-closure equation



$$\vec{r}_1 + \vec{r}_4 = \vec{r}_2 + \vec{r}_3$$

$$\vec{r}_1 + \vec{r}_4 - \vec{r}_2 - \vec{r}_3 = 0 \quad \text{loop-closure equation}$$

$$r_1 e^{j\theta_1} + r_4 e^{j\theta_4} - r_2 e^{j\theta_2} - r_3 e^{j\theta_3} = 0$$

$$r_1 \cos\theta_1 + j r_1 \sin\theta_1 + r_4 \cos\theta_4 + j r_4 \sin\theta_4$$
$$- r_2 \cos\theta_2 - j r_2 \sin\theta_2 - r_3 \cos\theta_3 - j r_3 \sin\theta_3 = 0$$

Real and imaginary parts can be written separate as two equations

$$r_1 \cos\theta_1 + r_4 \cos\theta_4 - r_2 \cos\theta_2 - r_3 \cos\theta_3 = 0$$

$$r_1 \sin\theta_1 + r_4 \sin\theta_4 - r_2 \sin\theta_2 - r_3 \sin\theta_3 = 0$$

Define two functions

$$F_1 = r_1 \cos\theta_1 + r_4 \cos\theta_4 - r_2 \cos\theta_2 - r_3 \cos\theta_3 = 0$$

$$F_2 = r_1 \sin\theta_1 + r_4 \sin\theta_4 - r_2 \sin\theta_2 - r_3 \sin\theta_3 = 0$$

$$\text{remember} \quad \theta_1 = 270^\circ \text{ and } \theta_4 = 0^\circ$$

$$F_1 = r_4 - r_2 \cos\theta_2 - r_3 \cos\theta_3 = 0$$

$$F_2 = -r_1 - r_2 \sin\theta_2 - r_3 \sin\theta_3 = 0$$

to find velocities take time derivative of F_1 and F_2

$$\frac{dF_1}{dt} = \dot{r}_4 + r_2\dot{\theta}_2 \sin\theta_2 + r_3\dot{\theta}_3 \sin\theta_3 = 0$$

$$\frac{dF_2}{dt} = -r_2\dot{\theta}_2 \cos\theta_2 - r_3\dot{\theta}_3 \cos\theta_3 = 0$$

$$\begin{bmatrix} r_3 \sin\theta_3 & 1 \\ -r_3 \cos\theta_3 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_3 \\ \dot{r}_4 \end{bmatrix} = \begin{bmatrix} -r_2\dot{\theta}_2 \sin\theta_2 \\ r_2\dot{\theta}_2 \cos\theta_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_3 \\ \dot{r}_4 \end{bmatrix} = \begin{bmatrix} r_3 \sin\theta_3 & 1 \\ -r_3 \cos\theta_3 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -r_2 \dot{\theta}_2 \sin\theta_2 \\ r_2 \dot{\theta}_2 \cos\theta_2 \end{bmatrix}$$

to find accelerations take time derivative of $\frac{dF_1}{dt}$ and $\frac{dF_2}{dt}$

$$\frac{d^2 F_1}{dt^2} = \ddot{r}_4 + r_2 \ddot{\theta}_2 \sin\theta_2 + r_2 \dot{\theta}_2^2 \cos\theta_2 + r_3 \ddot{\theta}_3 \sin\theta_3 + r_3 \dot{\theta}_3^2 \cos\theta_3 = 0$$

$$\frac{d^2 F_2}{dt^2} = -r_2 \ddot{\theta}_2 \cos\theta_2 + r_2 \dot{\theta}_2^2 \sin\theta_2 - r_3 \ddot{\theta}_3 \cos\theta_3 + r_3 \dot{\theta}_3^2 \sin\theta_3 = 0$$

$$\begin{bmatrix} r_3 \sin\theta_3 & 1 \\ -r_3 \cos\theta_3 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_3 \\ \ddot{r}_4 \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} \ddot{\theta}_3 \\ \ddot{r}_4 \end{bmatrix} = \begin{bmatrix} r_3 \sin\theta_3 & 1 \\ -r_3 \cos\theta_3 & 0 \end{bmatrix}^{-1} \begin{bmatrix} A \\ B \end{bmatrix}$$

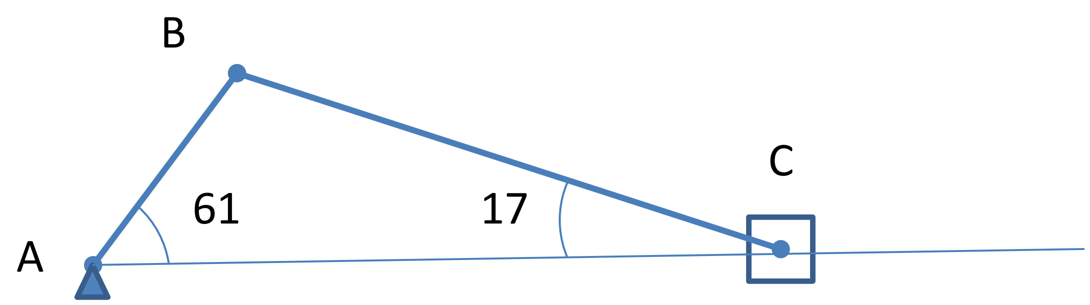
$$A = -r_2\ddot{\theta}_2 \sin\theta_2 - r_2\dot{\theta}_2^2 \cos\theta_2 - r_3\dot{\theta}_3^2 \cos\theta_3$$

$$B = r_2\ddot{\theta}_2 \cos\theta_2 - r_2\dot{\theta}_2^2 \sin\theta_2 - r_3\dot{\theta}_3^2 \sin\theta_3$$

The following matrix is the Jacobean matrix of the mechanism

$$J = \begin{bmatrix} r_3 \sin\theta_3 & 1 \\ -r_3 \cos\theta_3 & 0 \end{bmatrix}$$

Example



$$AB = 2\ \text{cm}, \quad BC = 6\ \text{cm}$$

$$\omega_{AB} = 10\ \text{rad/s (constant, or uniform, and CCW)}$$

$$\omega_{BC} = ?\ \ V_C = ?\ \ \alpha_{BC} = ?\ \ a_c = ?$$

$$\begin{bmatrix} \dot{\theta}_3 \\ \dot{r}_4 \end{bmatrix} = \begin{bmatrix} r_3 \sin\theta_3 & 1 \\ -r_3 \cos\theta_3 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -r_2\dot{\theta}_2 \sin\theta_2 \\ r_2\dot{\theta}_2 \cos\theta_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_3 \\ \dot{r}_4 \end{bmatrix} = \begin{bmatrix} 6\sin 343 & 1 \\ -6\cos 343 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -2(10)\sin 61 \\ 2(10)\cos 61 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_3 \\ \dot{r}_4 \end{bmatrix} = \begin{bmatrix} -1.7542 & 1 \\ -5.7378 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -17.4924 \\ 9.6962 \end{bmatrix}$$

$$\text{determinant } \Delta = (1.7542)(0) - (-5.7378)(1) = 5.7378$$

$$\begin{bmatrix} \dot{\theta}_3 \\ \dot{r}_4 \end{bmatrix} = \frac{1}{5.7378} \begin{bmatrix} 0 & -1 \\ 5.7378 & -1.7542 \end{bmatrix} \begin{bmatrix} -17.4924 \\ 9.6962 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_3 \\ \dot{r}_4 \end{bmatrix} = \begin{bmatrix} 0 & -0.1743 \\ 1 & -0.3057 \end{bmatrix} \begin{bmatrix} -17.4924 \\ 9.6962 \end{bmatrix}$$

$$\dot{\theta}_3 = \omega_3 = (0)(-17.4924) + (-0.1743)(9.6962) = -1.69\,\text{rad/s}$$

$$\dot{r}_4 = V_C = (1)(-17.4924) + (-0.3057)(9.6962) = -20.4565\,\text{cm/s}$$

$$\begin{bmatrix} \ddot{\theta}_3 \\ \ddot{r}_4 \end{bmatrix} = \begin{bmatrix} r_3 \sin\theta_3 & 1 \\ -r_3 \cos\theta_3 & 0 \end{bmatrix}^{-1} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_3 \\ \dot{r}_4 \end{bmatrix} = \begin{bmatrix} 0 & -0.1743 \\ 1 & -0.3057 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$A = -r_2\ddot{\theta}_2 \sin\theta_2 - r_2\dot{\theta}_2^2 \cos\theta_2 - r_3\dot{\theta}_3^2 \cos\theta_3$$

$$A = -2(0)\sin 61 - 2(10)^2 \cos 61 - 6(-1.69)^2 \cos 343 = -113.3497$$

$$B = r_2\ddot{\theta}_2 \cos\theta_2 + r_2\dot{\theta}_2^2 \sin\theta_2 - r_3\dot{\theta}_3^2 \sin\theta_3$$

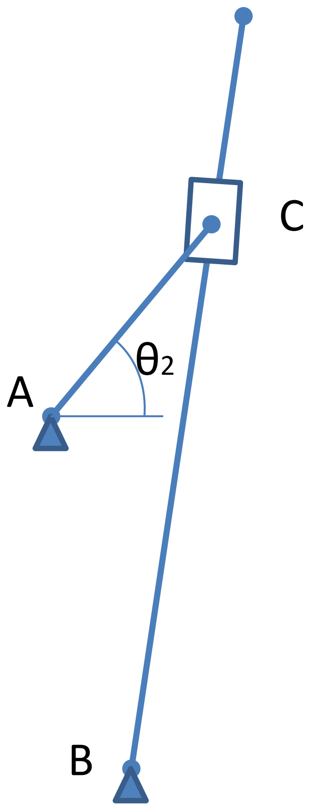
$$B = 2(0)\cos 61 + 2(10)^2 \sin 61 - 6(-1.69)^2 \sin 343 = -169.9137$$

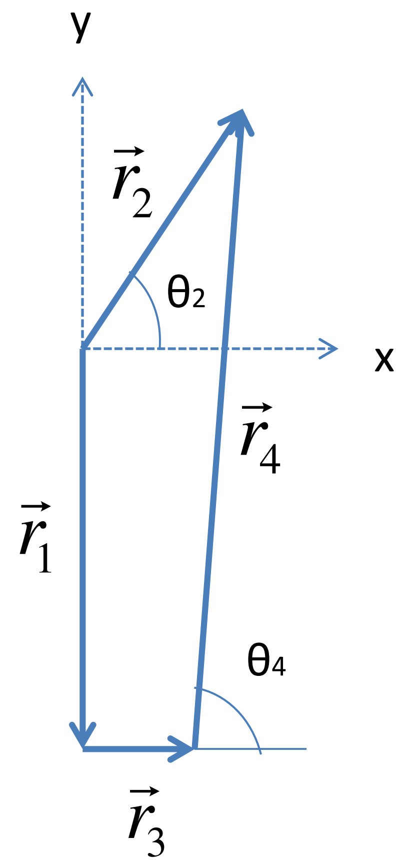
$$\begin{bmatrix} \dot{\theta}_3 \\ \dot{r}_4 \end{bmatrix} = \begin{bmatrix} 0 & -0.1743 \\ 1 & -0.3057 \end{bmatrix} \begin{bmatrix} -113.3497 \\ -169.9137 \end{bmatrix}$$

$$\ddot{\theta}_3 = \alpha_3 = (0)(-113.3497) + (-0.1743)(-169.9137) = 29.6159\,\text{rad/s}^2$$

$$\ddot{r}_4 = a_C = (1)(-113.3497) + (-0.3057)(-169.9137) = -61.4070\,\text{cm/s}^2$$

Inverted slider-crank mechanism loop-closure equation





Our variables are θ_4 and r_4

$$\vec{r}_1 + \vec{r}_3 + \vec{r}_4 = \vec{r}_2$$

$$\vec{r}_1 + \vec{r}_3 + \vec{r}_4 - \vec{r}_2 = 0 \quad \text{loop-closure equation}$$

$$r_1 e^{j\theta_1} + r_3 e^{j\theta_3} + r_4 e^{j\theta_4} - r_2 e^{j\theta_2} = 0$$

$$r_1 \cos\theta_1 + j r_1 \sin\theta_1 + r_3 \cos\theta_3 + j r_3 \sin\theta_3$$
$$+ r_4 \cos\theta_4 + j r_4 \sin\theta_4 - r_2 \cos\theta_2 - j r_2 \sin\theta_2 = 0$$

Real and imaginary parts can be written separate as two equations

$$r_1 \cos\theta_1 + r_3 \cos\theta_3 + r_4 \cos\theta_4 - r_2 \cos\theta_2 = 0$$

$$r_1 \sin\theta_1 + r_3 \sin\theta_3 + r_4 \sin\theta_4 - r_2 \sin\theta_2 = 0$$

Define two functions

$$F_1 = r_1 \cos\theta_1 + r_3 \cos\theta_3 + r_4 \cos\theta_4 - r_2 \cos\theta_2 = 0$$

$$F_2 = r_1 \sin\theta_1 + r_3 \sin\theta_3 + r_4 \sin\theta_4 - r_2 \sin\theta_2 = 0$$

$$\text{remember} \quad \theta_1 = 270^\circ \text{ and } \theta_3 = 0^\circ$$

$$F_1 = r_3 + r_4 \cos\theta_4 - r_2 \cos\theta_2 = 0$$

$$F_2 = -r_1 + r_4 \sin\theta_4 - r_2 \sin\theta_2 = 0$$

to find velocities take time derivative of F_1 and F_2

$$\frac{dF_1}{dt} = \dot{r}_4 \cos\theta_4 - r_4\dot{\theta}_4 \sin\theta_4 + r_2\dot{\theta}_2 \sin\theta_2 = 0$$

$$\frac{dF_2}{dt} = \dot{r}_4 \sin\theta_4 + r_4\dot{\theta}_4 \cos\theta_4 - r_2\dot{\theta}_2 \cos\theta_2 = 0$$

$$\begin{bmatrix} -r_4 \sin\theta_4 & \cos\theta_4 \\ r_4 \cos\theta_4 & \sin\theta_4 \end{bmatrix} \begin{bmatrix} \dot{\theta}_4 \\ \dot{r}_4 \end{bmatrix} = \begin{bmatrix} -r_2\dot{\theta}_2 \sin\theta_2 \\ r_2\dot{\theta}_2 \cos\theta_2 \end{bmatrix}$$

$$\begin{bmatrix}\dot{\theta}_4\\ \dot{r}_4\end{bmatrix}=\begin{bmatrix}-r_4\sin\theta_4 & \cos\theta_4\\ r_4\cos\theta_4 & \sin\theta_4\end{bmatrix}^{-1}\begin{bmatrix}-r_2\dot{\theta}_2\sin\theta_2\\ r_2\dot{\theta}_2\cos\theta_2\end{bmatrix}$$

to find accelerations take time derivative of $\dfrac{dF_1}{dt}$ and $\dfrac{dF_2}{dt}$

$$\frac{d^2F_1}{dt^2}=\ddot{r}_4\cos\theta_4-r_4\dot{\theta}_4\sin\theta_4-r_4\dot{\theta}_4\sin\theta_4-r_4\ddot{\theta}_4\sin\theta_4$$
$$-r_4\dot{\theta}_4^2\cos\theta_4+r_2\ddot{\theta}_2\sin\theta_2+r_2\dot{\theta}_2^2\cos\theta_2=0$$

$$\frac{d^2F_2}{dt^2}=\ddot{r}_4\sin\theta_4+\dot{r}_4\dot{\theta}_4\cos\theta_4+\dot{r}_4\dot{\theta}_4\cos\theta_4+r_4\ddot{\theta}_4\cos\theta_4$$
$$-r_4\dot{\theta}_4^2\sin\theta_4+r_2\ddot{\theta}_2\cos\theta_2-r_2\dot{\theta}_2^2\sin\theta_2=0$$

$$\begin{bmatrix} -r_4 \sin\theta_4 & \cos\theta_4 \\ r_4 \cos\theta_4 & \sin\theta_4 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_4 \\ \ddot{r}_4 \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} \ddot{\theta}_4 \\ \ddot{r}_4 \end{bmatrix} = \begin{bmatrix} -r_4 \sin\theta_4 & \cos\theta_4 \\ r_4 \cos\theta_4 & \sin\theta_4 \end{bmatrix}^{-1} \begin{bmatrix} A \\ B \end{bmatrix}$$

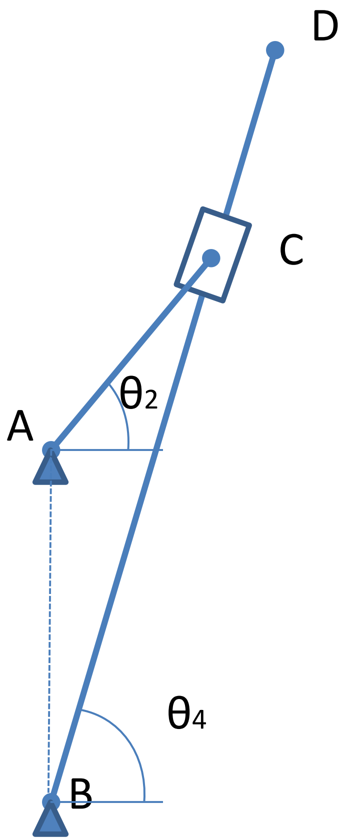
$$A = 2\dot{r}_4\dot{\theta}_4 \sin\theta_4 + r_4\dot{\theta}_4^2 \cos\theta_4 - r_2\ddot{\theta}_2 \sin\theta_2 - r_2\dot{\theta}_2^2 \cos\theta_2$$

$$B = -2\dot{r}_4\dot{\theta}_4 \cos\theta_4 + r_4\dot{\theta}_4^2 \sin\theta_4 - r_2\ddot{\theta}_2 \cos\theta_2 + r_2\dot{\theta}_2^2 \sin\theta_2$$

The following matrix is called Jacobien matrix
which is same for displacement, velocity and acceleration calculations

$$J = \begin{bmatrix} -r_4 \sin\theta_4 & \cos\theta_4 \\ r_4 \cos\theta_4 & \sin\theta_4 \end{bmatrix}$$

Example



AC= 2 cm

AB= 5 cm

BC= 6.14 cm

$$\theta_2 = 116^\circ \quad \theta_4 = 163^\circ$$

$$\dot{\theta}_4 = ? \quad \ddot{\theta}_4 = ? \quad V_{C/BD} = ? \quad a_{C/BD} = ?$$

$$\omega_{AC} = 10 \text{ rad/s (constant, or uniform, and CCW)}$$

$$\begin{bmatrix} \dot{\theta}_4 \\ \dot{r}_4 \end{bmatrix} = \begin{bmatrix} -r_4 \sin\theta_4 & \cos\theta_4 \\ r_4 \cos\theta_4 & \sin\theta_4 \end{bmatrix}^{-1} \begin{bmatrix} -r_2\dot{\theta}_2 \sin\theta_2 \\ r_2\dot{\theta}_2 \cos\theta_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_4 \\ \dot{r}_4 \end{bmatrix} = \begin{bmatrix} -(6.14)\sin 163 & \cos 163 \\ (6.14)\cos 163 & \sin 163 \end{bmatrix}^{-1} \begin{bmatrix} -2(10)\sin 116 \\ 2(10)\cos 116 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_4 \\ \dot{r}_4 \end{bmatrix} = \begin{bmatrix} -1.7952 & -0.9563 \\ -5.8717 & 0.2924 \end{bmatrix}^{-1} \begin{bmatrix} -17.9759 \\ -8.7674 \end{bmatrix}$$

$$\text{determinant } \Delta = (-1.7952)(0.2924) - (-5.8717)(-0.9563) = -6.14$$

$$\begin{bmatrix} \dot{\theta}_4 \\ \dot{r}_4 \end{bmatrix} = \frac{1}{-6.14} \begin{bmatrix} 0.2924 & 0.9563 \\ 5.8717 & -1.7952 \end{bmatrix} \begin{bmatrix} -17.9759 \\ -8.7674 \end{bmatrix}$$

$$\begin{bmatrix}\dot{\theta}_4\\ \dot{r}_4\end{bmatrix}=\begin{bmatrix}-0.0476 & -0.1557\\ -0.9563 & 0.2924\end{bmatrix}\begin{bmatrix}-17.9759\\ -8.7674\end{bmatrix}$$

$$\dot{\theta}_4=\omega_4=(-0.0476)(-17.9759)+(-0.1557)(-8.7674)=2.22\,\text{rad/s}$$

$$\dot{r}_4=V_{C/BD}=(-0.9563)(-17.9759)+(0.2924)(-8.7674)=14.62\,\text{cm/s}$$

$$\begin{bmatrix}\ddot{\theta}_4\\ \ddot{r}_4\end{bmatrix}=\begin{bmatrix}r_4\sin\theta_4 & \cos\theta_4\\ -r_4\cos\theta_4 & \sin\theta_4\end{bmatrix}^{-1}\begin{bmatrix}A\\ B\end{bmatrix}$$

$$A=2\dot{r}_4\dot{\theta}_4\sin\theta_4+r_4\dot{\theta}_4^2\cos\theta_4-r_2\ddot{\theta}_2\sin\theta_2-r_2\dot{\theta}_2^2\cos\theta_2$$

$$B=-2\dot{r}_4\dot{\theta}_4\cos\theta_4+r_4\dot{\theta}_4^2\sin\theta_4-r_2\ddot{\theta}_2\cos\theta_2+r_2\dot{\theta}_2^2\sin\theta_2$$

$$A=2(14.62)(2.22)\sin 163+2(2.22)^2\cos 163-2(0)\sin 116-2(10)^2\cos 116=77.7147$$

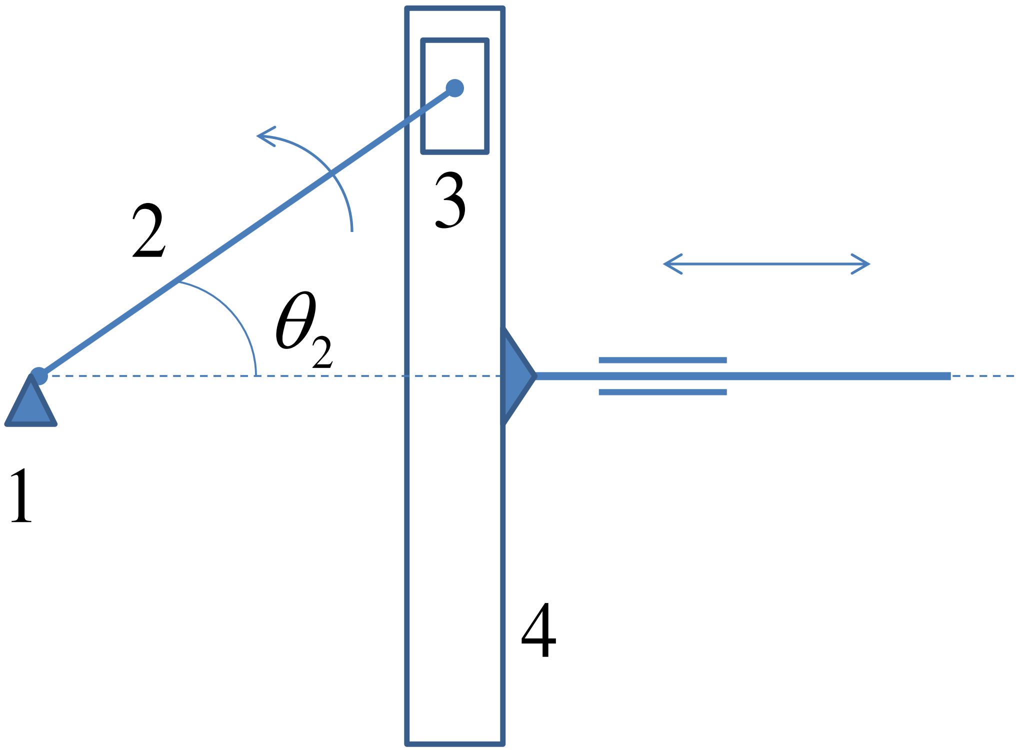
$$B=-2(14.62)(2.22)\cos 163+2(2.22)^2\sin 163-2(0)\cos 116+2(10)^2\sin 116=-108.8351$$

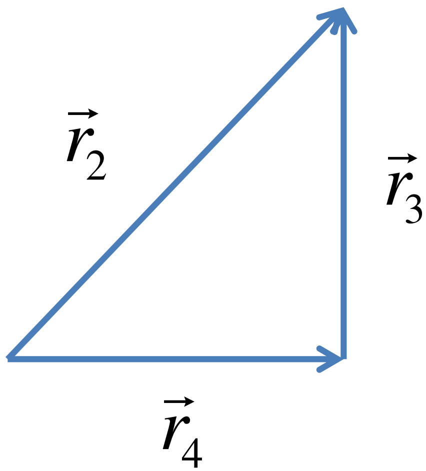
$$\begin{bmatrix} \ddot{\theta}_4 \\ \ddot{r}_4 \end{bmatrix} = \begin{bmatrix} -0.0476 & -0.1557 \\ -0.9563 & 0.2924 \end{bmatrix} \begin{bmatrix} 77.7147 \\ -108.8351 \end{bmatrix}$$

$$\ddot{\theta}_4 = \alpha_4 = (-0.0476)(77.7147) + (-0.1557)(-108.8351) = 13.27\ \mathrm{rad/s^2}$$

$$\ddot{r}_4 = a_{C/BD} = (-0.9563)(77.7147) + (0.2924)(-108.8351) = -105.66\ \mathrm{cm/s^2}$$

Scotch-Yoke Mechanism





$$\vec{r}_2 = \vec{r}_3 + \vec{r}_4$$

$$\vec{r}_2 - \vec{r}_3 - \vec{r}_4 = 0 \quad \text{loop-closure equation}$$

$$r_2 e^{j\theta_2} - r_3 e^{j\theta_3} - r_4 e^{j\theta_4} = 0$$

$$r_2 \cos\theta_2 + j r_2 \sin\theta_2 - r_3 \cos\theta_3 - j r_3 \sin\theta_3 - r_4 \cos\theta_4 - j r_4 \sin\theta_4 = 0$$

Real and imaginary parts can be written separate as two equations

$$r_2 \cos\theta_2 - r_3 \cos\theta_3 - r_4 \cos\theta_4 = 0$$

$$r_2 \sin\theta_2 - r_3 \sin\theta_3 - r_4 \sin\theta_4 = 0$$

Define two functions

$$F_1 = r_2 \cos\theta_2 - r_3 \cos\theta_3 - r_4 \cos\theta_4 = 0$$

$$F_2 = r_2 \sin\theta_2 - r_3 \sin\theta_3 - r_4 \sin\theta_4 = 0$$

remember $\theta_4 = 0°$ and $\theta_3 = 90°$

$$F_1 = r_2 \cos\theta_2 - r_4 = 0$$

$$F_2 = r_2 \sin\theta_2 - r_3 = 0$$

variables are r_4 and r_3

to find velocities take time derivative of F_1 and F_2

$$\frac{dF_1}{dt} = -r_2\dot{\theta}_2 \sin\theta_2 - \dot{r}_4 = 0$$

$$\frac{dF_2}{dt} = r_2\dot{\theta}_2 \cos\theta_2 - \dot{r}_3 = 0$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \dot{r}_3 \\ \dot{r}_4 \end{bmatrix} = \begin{bmatrix} -r_2\dot{\theta}_2 \cos\theta_2 \\ r_2\dot{\theta}_2 \sin\theta_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{r}_3 \\ \dot{r}_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}^{-1} \begin{bmatrix} -r_2\dot{\theta}_2 \cos\theta_2 \\ r_2\dot{\theta}_2 \sin\theta_2 \end{bmatrix}$$

to find accelerations take time derivative of $\frac{dF_1}{dt}$ and $\frac{dF_2}{dt}$

$$\frac{d^2F_1}{dt^2} = r_2\ddot{\theta}_2 \sin\theta_2 + r_2\dot{\theta}_2^2 \cos\theta_2 - \ddot{r}_4 = 0$$

$$\frac{d^2F_2}{dt^2} = r_2\ddot{\theta}_2 \cos\theta_2 - r_2\dot{\theta}_2^2 \sin\theta_2 - \ddot{r}_3 = 0$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \ddot{r}_3 \\ \ddot{r}_4 \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} \ddot{r}_3 \\ \ddot{r}_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}^{-1} \begin{bmatrix} A \\ B \end{bmatrix}$$

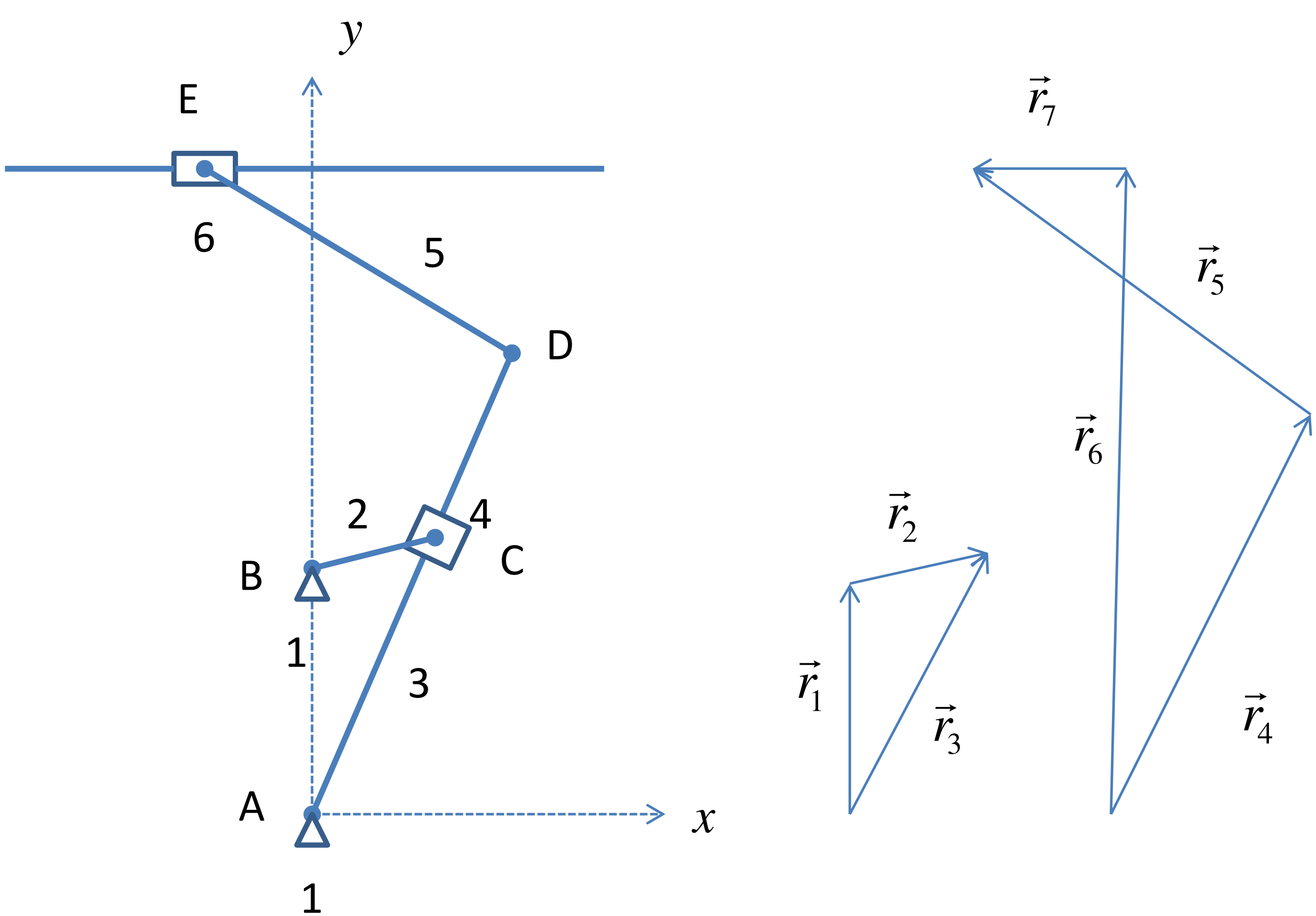
$$A = -r_2\ddot{\theta}_2 \cos\theta_2 + r_2\dot{\theta}_2^2 \sin\theta_2$$

$$B = -r_2\ddot{\theta}_2 \sin\theta_2 - r_2\dot{\theta}_2^2 \cos\theta_2$$

The following matrix is called Jacobien matrix
which is same for displacement, velocity and acceleration calculations

$$J = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Quick return mechanism



$$\vec{r}_1 + \vec{r}_2 = \vec{r}_3$$

$$\vec{r}_4 + \vec{r}_5 = \vec{r}_6 + \vec{r}_7$$

$$\vec{r}_1 + \vec{r}_2 - \vec{r}_3 = 0$$

$$\vec{r}_4 + \vec{r}_5 - \vec{r}_6 - \vec{r}_7 = 0$$

$$r_1 e^{j\theta_1} + r_2 e^{j\theta_2} - r_3 e^{j\theta_3} = 0$$

$$r_4 e^{j\theta_4} + r_5 e^{j\theta_5} - r_6 e^{j\theta_6} - r_7 e^{j\theta_7} = 0$$

$$r_1 \cos\theta_1 + r_2 \cos\theta_2 - r_3 \cos\theta_3 = 0$$

$$r_1 \sin\theta_1 + r_2 \sin\theta_2 - r_3 \sin\theta_3 = 0$$

$$r_4 \cos\theta_4 + r_5 \cos\theta_5 - r_6 \cos\theta_6 - r_7 \cos\theta_7 = 0$$

$$r_4 \sin\theta_4 + r_5 \sin\theta_5 - r_6 \sin\theta_6 - r_7 \sin\theta_7 = 0$$

$$\theta_1 = 90°, \quad \theta_6 = 90°, \theta_7 = 180°, \theta_3 = \theta_4,$$

$$r_1 \cos 90 + r_2 \cos\theta_2 - r_3 \cos\theta_3 = 0$$

$$r_1 \sin 90 + r_2 \sin\theta_2 - r_3 \sin\theta_3 = 0$$

$$r_4 \cos\theta_3 + r_5 \cos\theta_5 - r_6 \cos 90 - r_7 \cos 180 = 0$$

$$r_4 \sin\theta_3 + r_5 \sin\theta_5 - r_6 \sin 90 - r_7 \sin 180 = 0$$

$$F_1 = r_2 \cos\theta_2 - r_3 \cos\theta_3 = 0$$

$$F_2 = r_1 + r_2 \sin\theta_2 - r_3 \sin\theta_3 = 0$$

$$F_3 = r_4 \cos\theta_3 + r_5 \cos\theta_5 + r_7 = 0$$

$$F_4 = r_4 \sin\theta_3 + r_5 \sin\theta_5 - r_6 = 0$$

variables are; $r_3,\ \theta_3,\ \theta_5,\ r_7$

$$F_1^0 + \frac{\partial F_1}{\partial r_3}\delta r_3 + \frac{\partial F_1}{\partial \theta_3}\delta \theta_3 + \frac{\partial F_1}{\partial \theta_5}\delta \theta_5 + \frac{\partial F_1}{\partial r_7}\delta r_7 = 0$$

$$F_2^0 + \frac{\partial F_2}{\partial r_3}\delta r_3 + \frac{\partial F_2}{\partial \theta_3}\delta \theta_3 + \frac{\partial F_2}{\partial \theta_5}\delta \theta_5 + \frac{\partial F_2}{\partial r_7}\delta r_7 = 0$$

$$F_3^0 + \frac{\partial F_3}{\partial r_3}\delta r_3 + \frac{\partial F_3}{\partial \theta_3}\delta \theta_3 + \frac{\partial F_3}{\partial \theta_5}\delta \theta_5 + \frac{\partial F_3}{\partial r_7}\delta r_7 = 0$$

$$F_4^0 + \frac{\partial F_4}{\partial r_3}\delta r_3 + \frac{\partial F_4}{\partial \theta_3}\delta \theta_3 + \frac{\partial F_4}{\partial \theta_5}\delta \theta_5 + \frac{\partial F_4}{\partial r_7}\delta r_7 = 0$$

$$\begin{bmatrix} \frac{\partial F_1}{\partial r_3} & \frac{\partial F_1}{\partial \theta_3} & \frac{\partial F_1}{\partial \theta_5} & \frac{\partial F_1}{\partial r_7} \\ \frac{\partial F_2}{\partial r_3} & \frac{\partial F_2}{\partial \theta_3} & \frac{\partial F_2}{\partial \theta_5} & \frac{\partial F_2}{\partial r_7} \\ \frac{\partial F_3}{\partial r_3} & \frac{\partial F_3}{\partial \theta_3} & \frac{\partial F_3}{\partial \theta_5} & \frac{\partial F_3}{\partial r_7} \\ \frac{\partial F_4}{\partial r_3} & \frac{\partial F_4}{\partial \theta_3} & \frac{\partial F_4}{\partial \theta_5} & \frac{\partial F_4}{\partial r_7} \end{bmatrix} \begin{bmatrix} \delta r_3 \\ \delta \theta_3 \\ \delta \theta_5 \\ \delta r_7 \end{bmatrix} = \begin{bmatrix} -F_1^0 \\ -F_2^0 \\ -F_3^0 \\ -F_4^0 \end{bmatrix}$$

$$\begin{bmatrix} \delta r_3 \\ \delta\theta_3 \\ \delta\theta_5 \\ \delta r_7 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial F_1}{\partial r_3} & \dfrac{\partial F_1}{\partial \theta_3} & \dfrac{\partial F_1}{\partial \theta_5} & \dfrac{\partial F_1}{\partial r_7} \\ \dfrac{\partial F_2}{\partial r_3} & \dfrac{\partial F_2}{\partial \theta_3} & \dfrac{\partial F_2}{\partial \theta_5} & \dfrac{\partial F_2}{\partial r_7} \\ \dfrac{\partial F_3}{\partial r_3} & \dfrac{\partial F_3}{\partial \theta_3} & \dfrac{\partial F_3}{\partial \theta_5} & \dfrac{\partial F_3}{\partial r_7} \\ \dfrac{\partial F_4}{\partial r_3} & \dfrac{\partial F_4}{\partial \theta_3} & \dfrac{\partial F_4}{\partial \theta_5} & \dfrac{\partial F_4}{\partial r_7} \end{bmatrix}^{-1} \begin{bmatrix} -F_1^0 \\ -F_2^0 \\ -F_3^0 \\ -F_4^0 \end{bmatrix}$$

$$\frac{\partial F_1}{\partial r_3} = -\cos\theta_3, \quad \frac{\partial F_1}{\partial \theta_3} = r_3 \sin\theta_3, \quad \frac{\partial F_1}{\partial \theta_5} = 0, \quad \frac{\partial F_1}{\partial r_7} = 0$$

$$\frac{\partial F_2}{\partial r_3} = -\sin\theta_3, \quad \frac{\partial F_2}{\partial \theta_3} = -r_3 \cos\theta_3, \quad \frac{\partial F_2}{\partial \theta_5} = 0\,, \quad \frac{\partial F_2}{\partial r_7} = 0$$

$$\frac{\partial F_3}{\partial r_3} = 0, \quad \frac{\partial F_3}{\partial \theta_3} = -r_4 \sin\theta_3, \quad \frac{\partial F_3}{\partial \theta_5} = -r_5 \sin\theta_5, \quad \frac{\partial F_3}{\partial r_7} = 1$$

$$\frac{\partial F_4}{\partial r_3} = 0, \quad \frac{\partial F_4}{\partial \theta_3} = r_4 \cos\theta_3, \quad \frac{\partial F_4}{\partial \theta_5} = r_5 \cos\theta_5, \quad \frac{\partial F_4}{\partial r_7} = 0$$

$$\begin{bmatrix} \delta r_3 \\ \delta\theta_3 \\ \delta\theta_5 \\ \delta r_7 \end{bmatrix} = \begin{bmatrix} -\cos\theta_3 & r_3\sin\theta_3 & 0 & 0 \\ -\sin\theta_3 & -r_3\cos\theta_3 & 0 & 0 \\ 0 & -r_4\sin\theta_3 & -r_5\sin\theta_5 & 1 \\ 0 & r_4\cos\theta_3 & r_5\cos\theta_5 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -F_1^0 \\ -F_2^0 \\ -F_3^0 \\ -F_4^0 \end{bmatrix}$$

if conditions $\left|\mathrm{F}_1^0\right| \le \varepsilon, \left|\mathrm{F}_2^0\right| \le \varepsilon, \left|\mathrm{F}_3^0\right| \le \varepsilon, \left|\mathrm{F}_4^0\right| \le \varepsilon$ are not satisfied, new guesses

$r_3^1 = r_3^0 + \delta r_3$

$\theta_3^1 = \theta_3^0 + \delta\theta_3$

$\theta_5^1 = \theta_5^0 + \delta\theta_5$

$r_7^1 = r_7^0 + \delta r_7$

are used and calculations are repeated until conditons are satisfied.

ε is a Numerical Tolerance like 10^{-5}.

These calculations will be repeated for $0° < \theta_2 < 360°$

To find velocities $\dot{r}_3,\ \dot{\theta}_3,\ \dot{\theta}_5,\ \dot{r}_7$ derive Fs with respect totime

$$\dot{F}_1 = -r_2\dot{\theta}_2 \sin\theta_2 - \dot{r}_3 \cos\theta_3 + r_3\dot{\theta}_3 \sin\theta_3 = 0$$

$$\dot{F}_2 = r_2\dot{\theta}_2 \cos\theta_2 - \dot{r}_3 \sin\theta_3 - r_3\dot{\theta}_3 \cos\theta_3 = 0$$

$$\dot{F}_3 = -r_4\dot{\theta}_3 \sin\theta_3 - r_5\dot{\theta}_5 \sin\theta_5 + \dot{r}_7 = 0$$

$$\dot{F}_4 = r_4\dot{\theta}_3 \cos\theta_3 + r_5\dot{\theta}_5 \cos\theta_5 = 0$$

$$-\dot{r}_3 \cos\theta_3 + r_3 \sin\theta_3\dot{\theta}_3 = r_2\dot{\theta}_2 \sin\theta_2$$

$$-\dot{r}_3 \sin\theta_3 - r_3 \cos\theta_3\dot{\theta}_3 = -r_2\dot{\theta}_2 \cos\theta_2$$

$$-r_4 \sin\theta_3\dot{\theta}_3 - r_5 \sin\theta_5\dot{\theta}_5 + \dot{r}_7 = 0$$

$$r_4 \cos\theta_3\dot{\theta}_3 + r_5 \cos\theta_5\dot{\theta}_5 = 0$$

$$\begin{bmatrix} -\cos\theta_3 & r_3\sin\theta_3 & 0 & 0 \\ -\sin\theta_3 & -r_3\cos\theta_3 & 0 & 0 \\ 0 & -r_4\sin\theta_3 & -r_5\sin\theta_5 & 1 \\ 0 & r_4\cos\theta_3 & r_5\cos\theta_5 & 0 \end{bmatrix} \begin{bmatrix} \dot{r}_3 \\ \dot{\theta}_3 \\ \dot{\theta}_5 \\ \dot{r}_7 \end{bmatrix} = \begin{bmatrix} r_2\dot{\theta}_2\sin\theta_2 \\ -r_2\dot{\theta}_2\cos\theta_2 \\ 0 \\ 0 \end{bmatrix}$$

To find accelerations $\ddot{r}_3,\ \ddot{\theta}_3,\ \ddot{\theta}_5,\ \ddot{r}_7$ derive $\dot{\text{F}}$s with respect to time

$$\ddot{F}_1 = -r_2\ddot{\theta}_2\sin\theta_2 - r_2\theta_2^2\cos\theta_2 - \ddot{r}_3\cos\theta_3 + \dot{r}_3\dot{\theta}_3\sin\theta_3 + \dot{r}_3\dot{\theta}_3\sin\theta_3 + r_3\ddot{\theta}_3\sin\theta_3 + \dot{r}_3\dot{\theta}_3^2\cos\theta_3 = 0$$

$$\ddot{F}_2 = r_2\ddot{\theta}_2\cos\theta_2 - r_2\theta_2^2\sin\theta_2 - \ddot{r}_3\sin\theta_3 - \dot{r}_3\dot{\theta}_3\cos\theta_3 - \dot{r}_3\dot{\theta}_3\cos\theta_3 - r_3\ddot{\theta}_3\cos\theta_3 + \dot{r}_3\dot{\theta}_3^2\sin\theta_3 = 0$$

$$\ddot{F}_3 = -r_4\ddot{\theta}_3\sin\theta_3 - r_4\dot{\theta}_3^2\cos\theta_3 - r_5\ddot{\theta}_5\sin\theta_5 - r_5\dot{\theta}_5^2\cos\theta_5 + \ddot{r}_7 = 0$$

$$\ddot{F}_3 = r_4\ddot{\theta}_3\cos\theta_3 + r_4\dot{\theta}_3^2\sin\theta_3 + r_5\ddot{\theta}_5\cos\theta_5 - r_5\dot{\theta}_5^2\sin\theta_5 = 0$$

$$-\cos\theta_3\ddot{r}_3 + r_3\sin\theta_3\ddot{\theta}_3 = r_2\ddot{\theta}_2\sin\theta_2 + r_2\theta_2^2\cos\theta_2 - 2\dot{r}_3\dot{\theta}_3\sin\theta_3 - \dot{r}_3\dot{\theta}_3^2\cos\theta_3$$

$$-\sin\theta_3\ddot{r}_3 - r_3\cos\theta_3\ddot{\theta}_3 = -r_2\ddot{\theta}_2\cos\theta_2 + r_2\theta_2^2\sin\theta_2 + 2\dot{r}_3\dot{\theta}_3\cos\theta_3 - \dot{r}_3\dot{\theta}_3^2\sin\theta_3$$

$$-r_4\sin\theta_3\ddot{\theta}_3 - r_5\sin\theta_5\ddot{\theta}_5 + \ddot{r}_7 = +r_4\dot{\theta}_3^2\cos\theta_3 + r_5\dot{\theta}_5^2\cos\theta_5$$

$$r_4\cos\theta_3\ddot{\theta}_3 + r_5\cos\theta_5\ddot{\theta}_5 = -r_4\dot{\theta}_3^2\sin\theta_3 + r_5\dot{\theta}_5^2\sin\theta_5$$

$$\begin{bmatrix} -\cos\theta_3 & r_3\sin\theta_3 & 0 & 0 \\ -\sin\theta_3 & -r_3\cos\theta_3 & 0 & 0 \\ 0 & -r_4\sin\theta_3 & -r_5\sin\theta_5 & 1 \\ 0 & r_4\cos\theta_3 & r_5\cos\theta_5 & 0 \end{bmatrix} \begin{bmatrix} \ddot{r}_3 \\ \ddot{\theta}_3 \\ \ddot{\theta}_5 \\ \ddot{r}_7 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}$$

$$A = r_2\ddot{\theta}_2 \sin\theta_2 + r_2\theta_2^2 \cos\theta_2 - 2\dot{r}_3\dot{\theta}_3 \sin\theta_3 - \dot{r}_3\dot{\theta}_3^2 \cos\theta_3$$

$$B = -r_2\ddot{\theta}_2 \cos\theta_2 + r_2\theta_2^2 \sin\theta_2 + 2\dot{r}_3\dot{\theta}_3 \cos\theta_3 - \dot{r}_3\dot{\theta}_3^2 \sin\theta_3$$

$$C = r_4\dot{\theta}_3^2 \cos\theta_3 + r_5\dot{\theta}_5^2 \cos\theta_5$$

$$D = -r_4\dot{\theta}_3^2 \sin\theta_3 + r_5\dot{\theta}_5^2 \sin\theta_5$$

Jacobian matrix of the problemis

$$J = \begin{bmatrix} -\cos\theta_3 & r_3\sin\theta_3 & 0 & 0 \\ -\sin\theta_3 & -r_3\cos\theta_3 & 0 & 0 \\ 0 & -r_4\sin\theta_3 & -r_5\sin\theta_5 & 1 \\ 0 & r_4\cos\theta_3 & r_5\cos\theta_5 & 0 \end{bmatrix}$$