

Jazan University  
Mechanical Engineering Department

CHAPTER 1

# LINKAGES

EngM271 Theory of Machines

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- Reuleaux (1829-1905, German kinematician)  
Defines a **machine** as a “combination of resistant bodies so arranged that by their means the mechanical forces of nature can be compelled to do work accompanied by certain determinate motions.”

- He also defines a **mechanism** as an “assemblage of resistant bodies connected by moveable joints to form a closed kinematic chain with one link fixed and having the purpose of transforming motion.

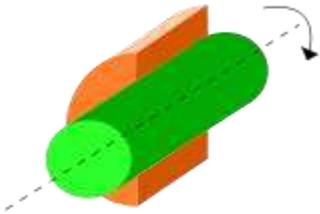
- There is no clear-cut between “machine” and “mechanism”
- ***Mechanism***: A system of elements arranged to ***transmit motion*** in a predetermined fashion
- ***Machine***: A system of elements arranged to ***transmit motion and energy*** in a predetermined fashion (Independently functioning structure → includes a power source)

- **Link (or rigid body)** is a machine part or a component of a mechanism.
- A link is assumed to be completely rigid.
- The links of a mechanism must be connected together in some manner in order to transmit motion from the **driver/input** link to the **follower/output** link.

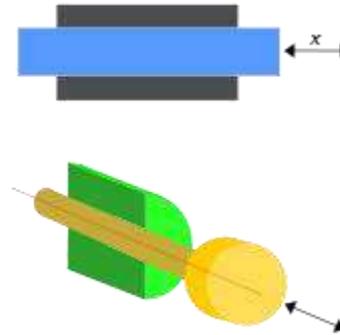
- A **kinematic pair (or joint)** is the general name for two rigid bodies that can move with respect to each other via a mechanical constraint.
- kinematic pairs can be classified in two groups:

- **Lower pairs:** the constraint is of the *surface* type. The following joint types exist: revolute joint ("pin" or "hinge"), prismatic joint ("slider"), cylindrical joint, screw joint, planar joint, and ball joint or ball and socket joint.
- **Higher pairs:** the constraint is of the curve or point type. For example: **cams** or **gears**.

- Kinematic pairs are the building blocks of most kinematic chains and mechanical linkages, e.g., gimbals, robots, and car suspensions.
- A kinematic pair must satisfy two conditions
  1. physical contact
  2. complete constrained motion



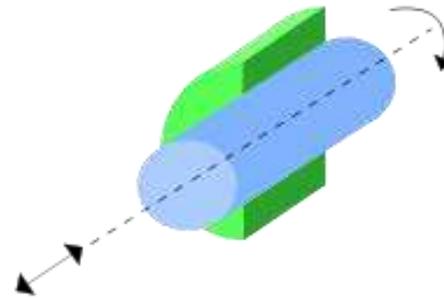
Revolute joint (only rotation)  
(lower pair)



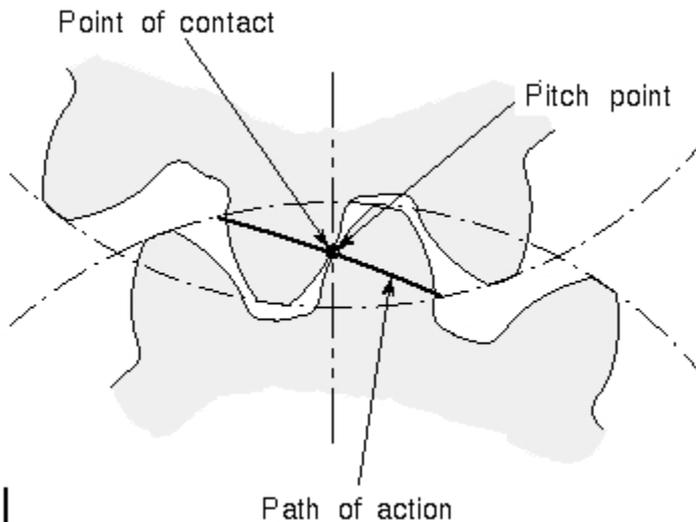
Prismatic joint (only translation)  
(lower pair)



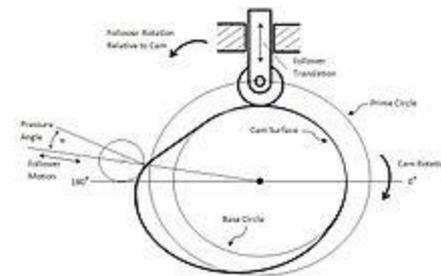
Ball joint (only rotation)  
(lower pair)



Cylindrical joint (rotation and translation)  
(lower pair)



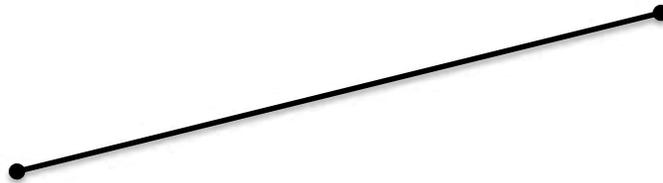
Gear (higher pair)



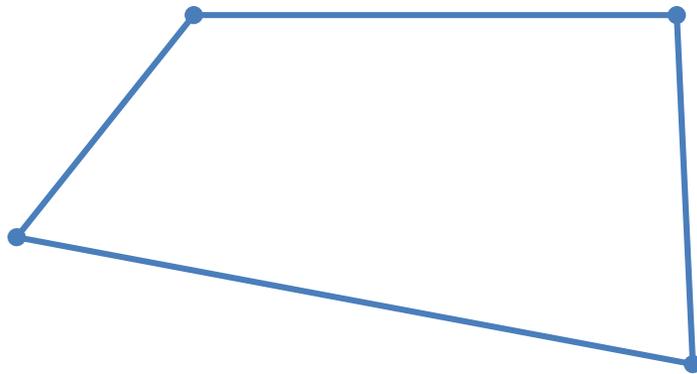
Cam (higher pair)

- We can also define a link as the rigid connection between two or more elements of different kinematic pairs.
- When several links are movably connected together by joints, they are said to form a **kinematic chain**.

- Links containing only two pair-element connections are called binary links.

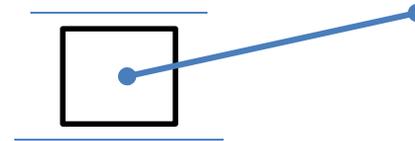
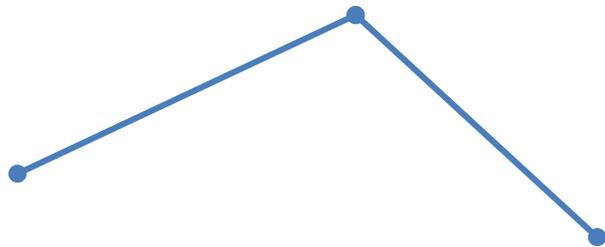


- If every link in the chain is connected to at least two other links, the chain forms one or more closed loops and is called a **closed kinematic chain**.



- For a mechanism it is necessary to have a closed kinematic chain with one link fixed. When we say one link is fixed, we mean that it is chosen as a frame of reference for all other links and is called the **frame** or **base link**.

- The degree of freedom of a joint is defined as the number of independent parameters required to determine the relative position of one link connected by the joint with respect to the other link.



## MOBILITY

One of the first concerns in either the design or the analysis of a mechanism is the number of degrees of freedom, also called the mobility of the device.

- The **mobility** of the mechanism is the number of input parameters (usually pair variables) which must be independently controlled in order to bring the device into a particular position.

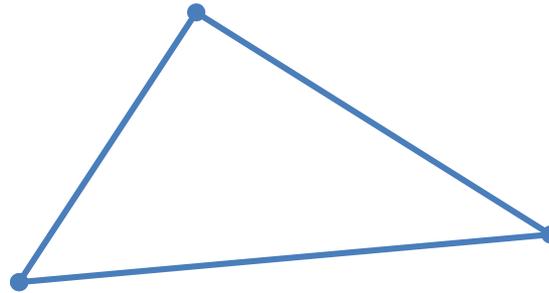
- Mobility is positive  $\rightarrow$  mechanism (links can have relative motion)
- Mobility is zero  $\rightarrow$  structure (no motion is possible)
- Mobility is negative  $\rightarrow$  preloaded structure (no motion is possible and assemblage may induce stress)

$$m = 3(n-1) - 2(j_1) - (j_2)$$

- m: mobility
- n: number of links of a mechanism
- j<sub>1</sub>: number of single degree of freedom pair (lower pair)
- j<sub>2</sub>: number of two degree of freedom pair (higher pair)

- Revolute pair and prismatic pair are single degree of freedom pairs or **lower pairs**.
- Cams and gears are two degree of freedom pairs or **higher pairs**.

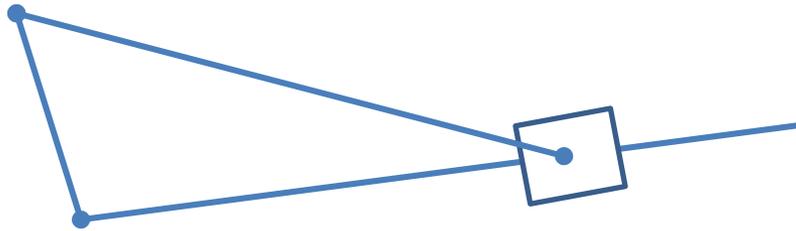
- Example



- $n=3, j_1=3, j_2=0$

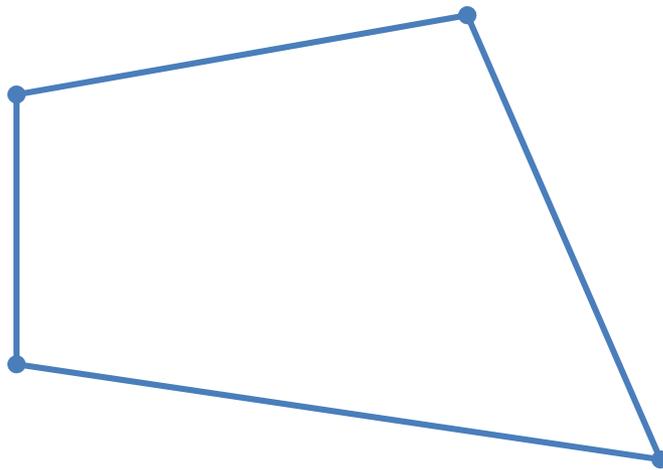
- $m=3(3-1)-2 \times 3-0=0$  degrees of freedom

- Example



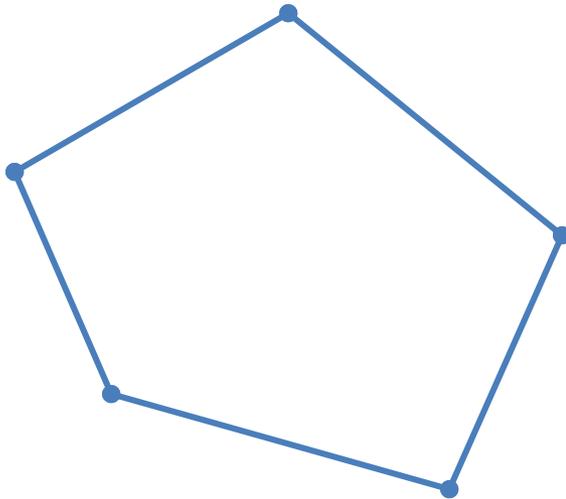
- $n=4, j1=4, j2=0$
- $m=3(4-1)-2 \times 4-0=1$  DOF

- Example



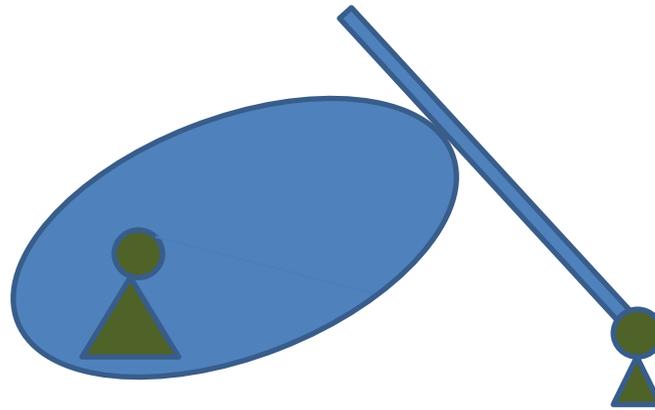
- $n=4, j_1=4, j_2=0$        $m=3(4-1)-2 \times 4-0=1$  DOF

- Example



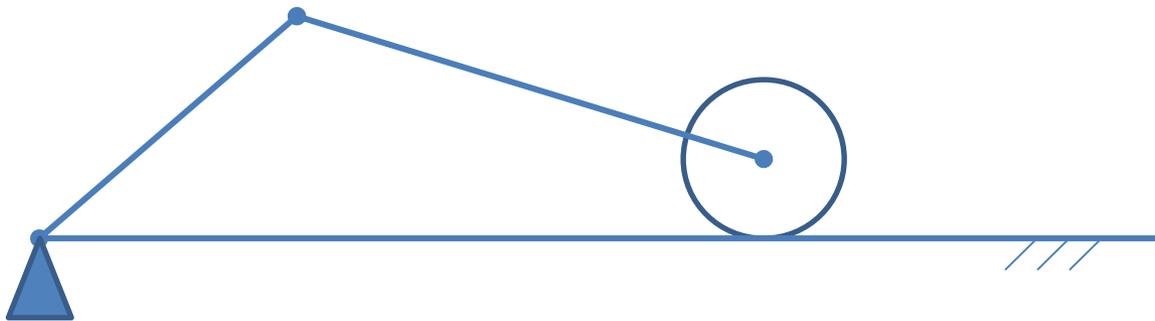
- $n=5, j_1=5, j_2=0, m=3(5-1)-2 \times 5-0=2$  DOF

- Example



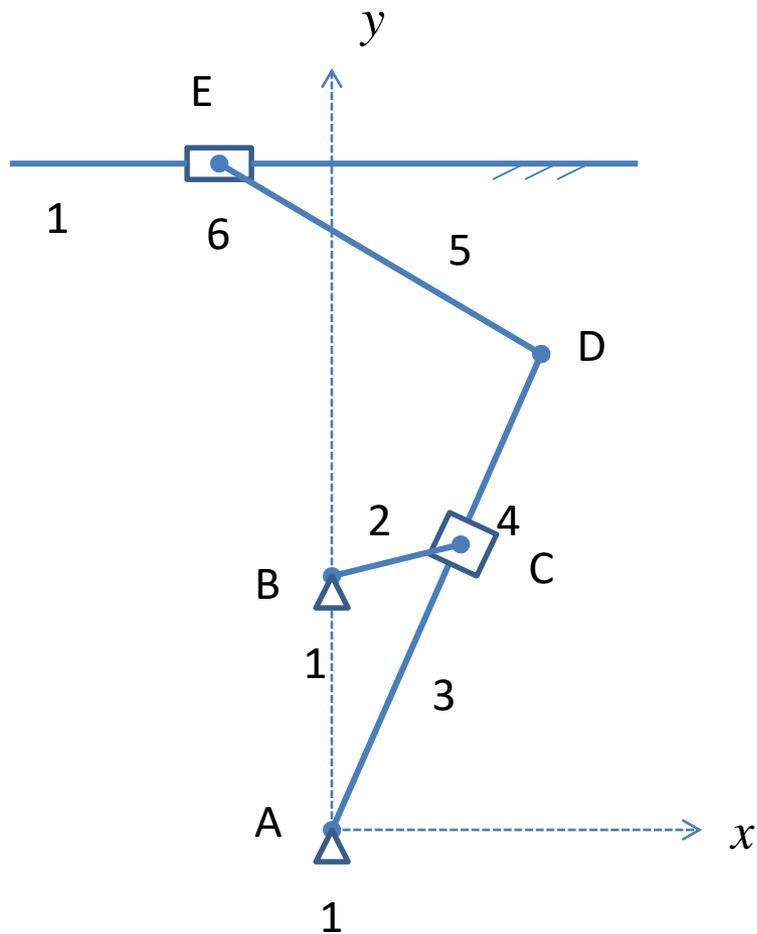
- $n=3, j_1=2, j_2=1, m=3(3-1)-2 \times 2-1=1$  DOF

- Example



- $n=4, j_1=3, j_2=1, m=3(4-1)-2 \times 3-1=2$  DOF

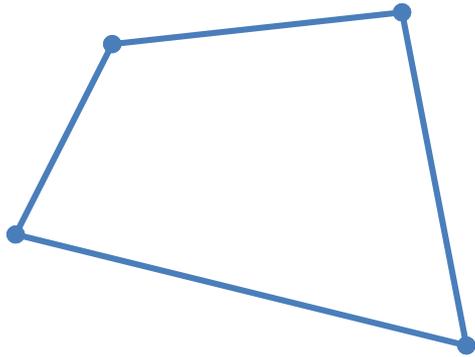
# Example



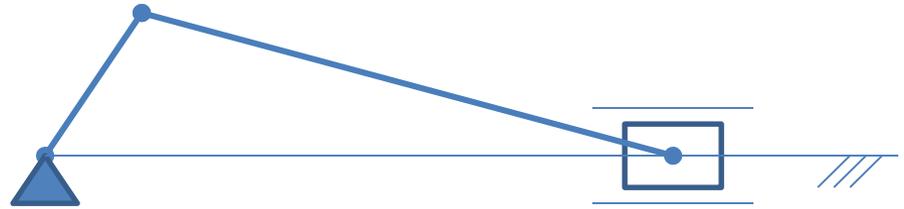
$$n=6, \quad j_1=7, \quad j_2=0, \quad m=3(6-1)-2 \times 7-0=1 \text{ DOF}$$



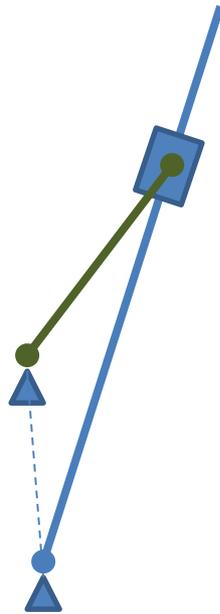
- For single degree of freedom mechanism having lower pairs only
- $m=1$   $j_2=0$
- $1=3(n-1)-2xj_1$
- $3n-2xj_1-4=0$  GRUBLER'S CRITERION
- Minimum possible joint and link number is 4
- $n=4, j_1=4$



Four bar linkage  
 $n=4, j_1=4$



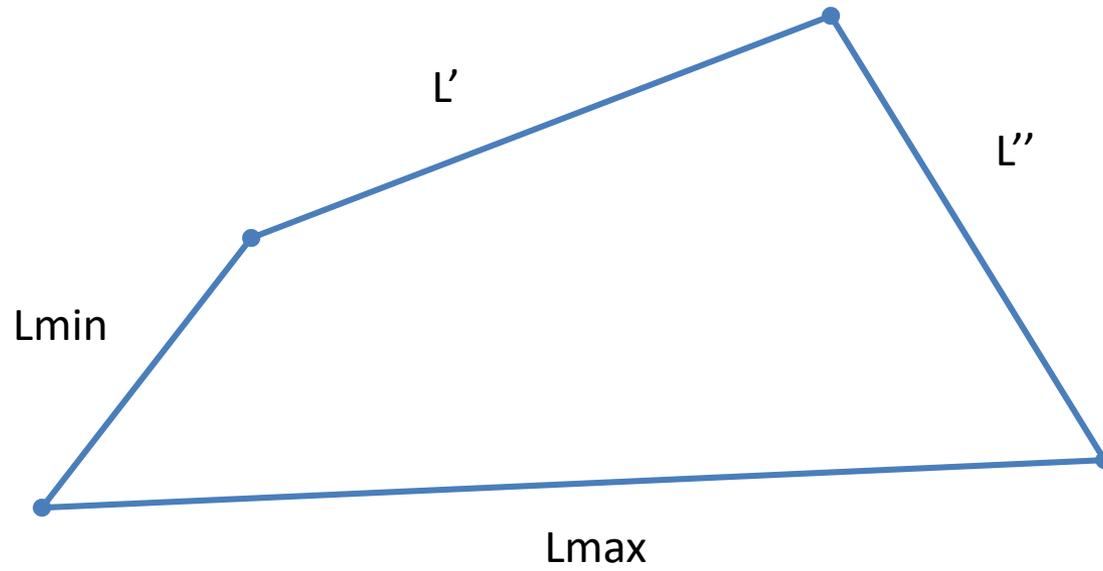
Slider-crank mechanism  
 $n=4, j_1=4$



Inverted slider crank mechanism  
 $n=4$   $j_1=4$

## GRASHOF'S LAW

For a planar four bar linkage, the sum of the shortest and longest links cannot be greater than the sum of the remaining links if there is to be continuous relative rotation between two members.

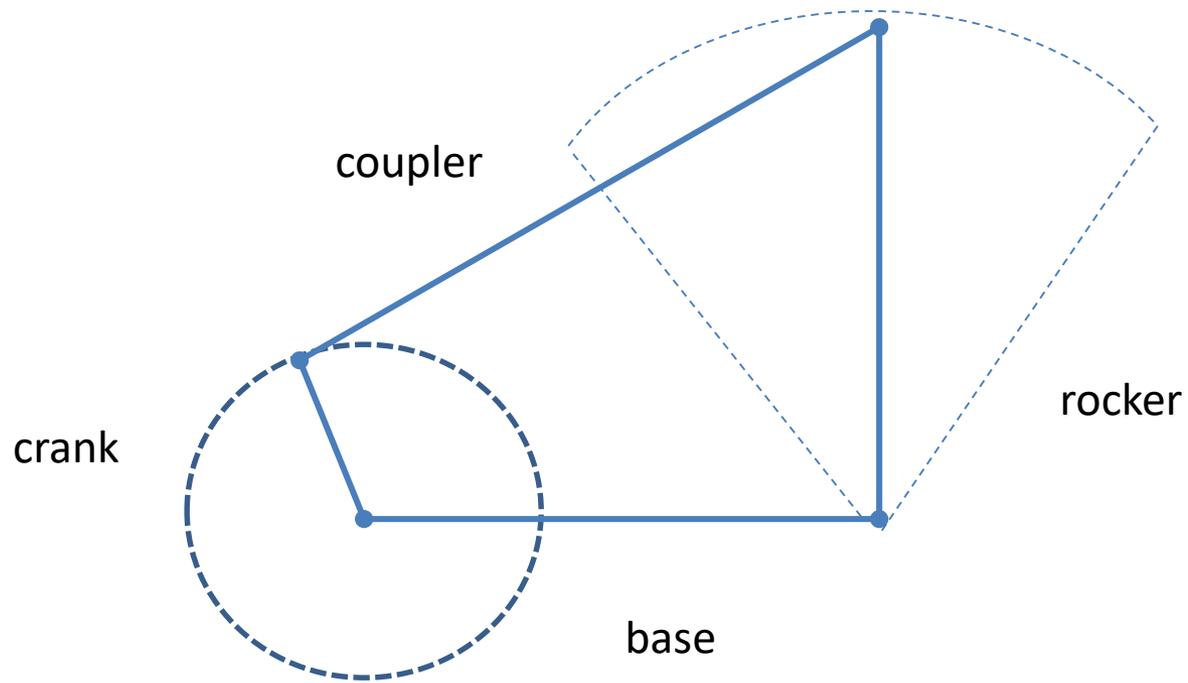


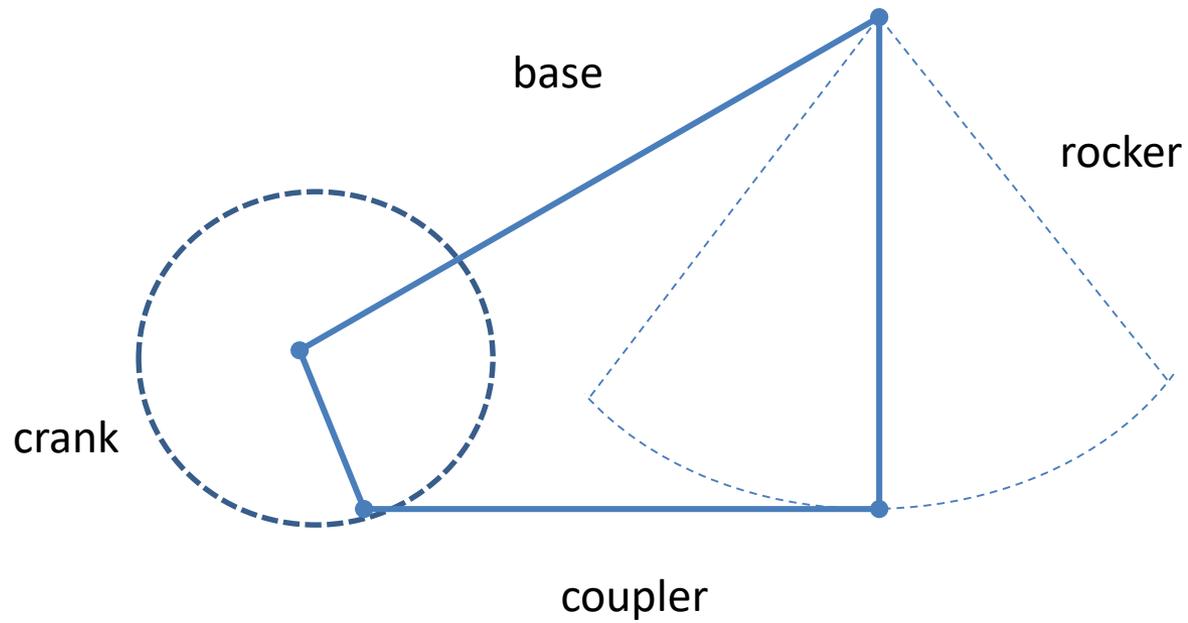
$$L_{min} + L_{max} < L' + L''$$

# Inversion

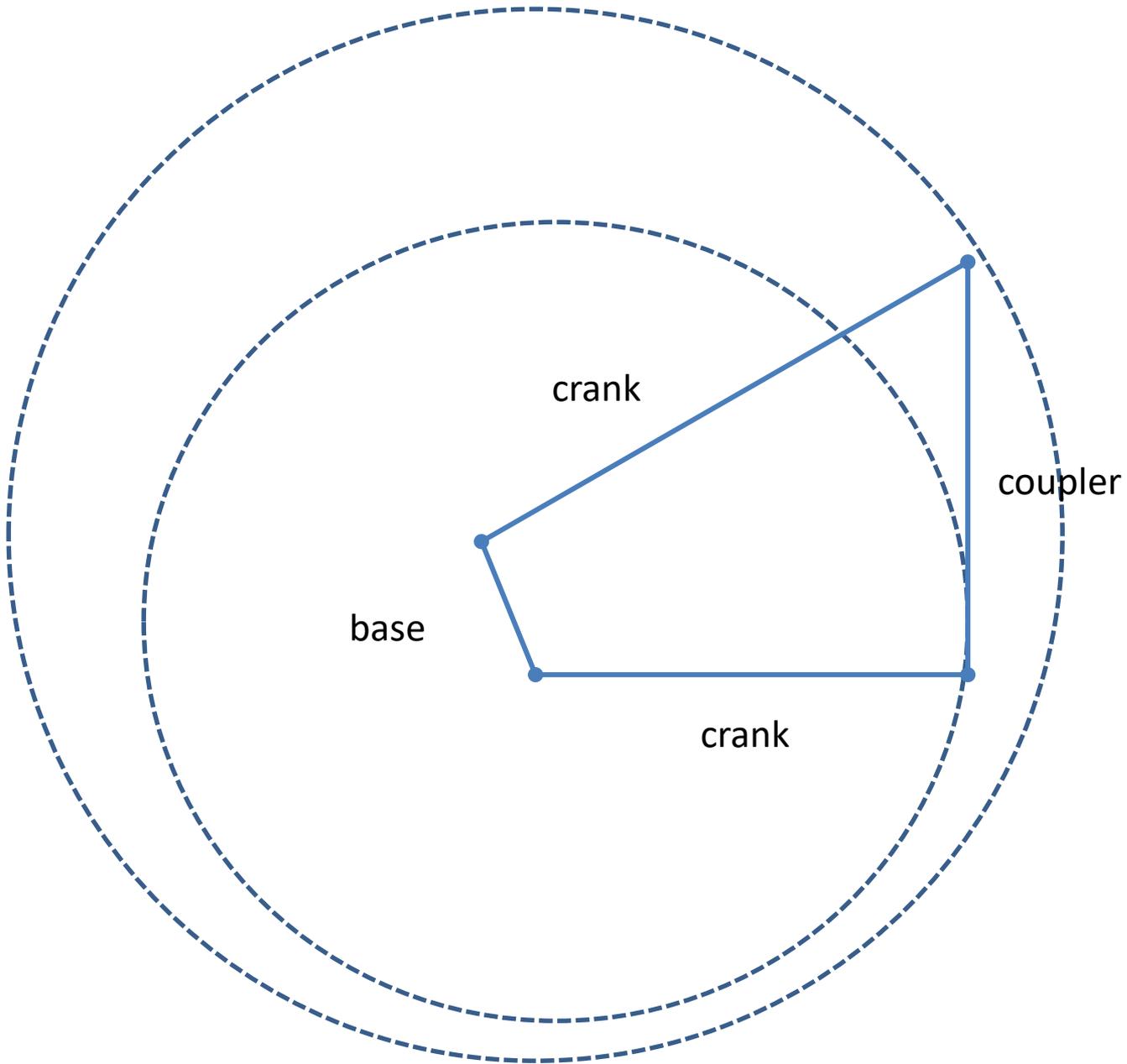
- An **inversion** is created by grounding a different link in the kinematic chain

- If the shortest link  $L_{\min}$  is the crank (input) and either of the adjacent members  $L'$  or  $L_{\max}$  is fixed a **CRANK-ROCKER** mechanism results.  $L_{\min}$  can make complete revolution,  $L''$  can only rock back and forth.

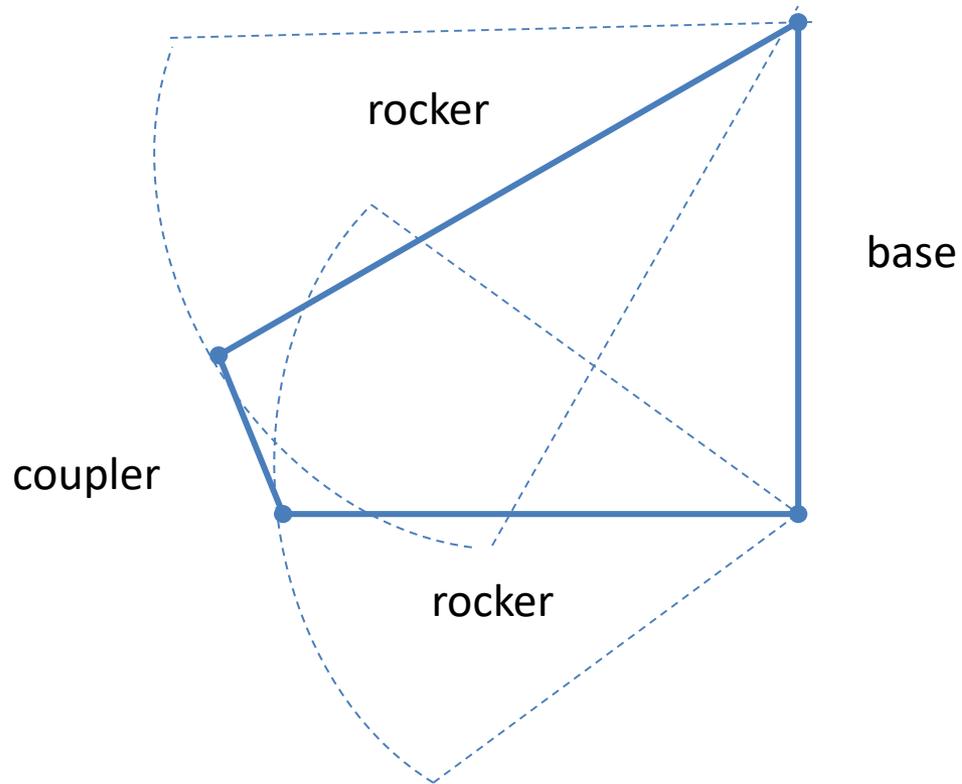




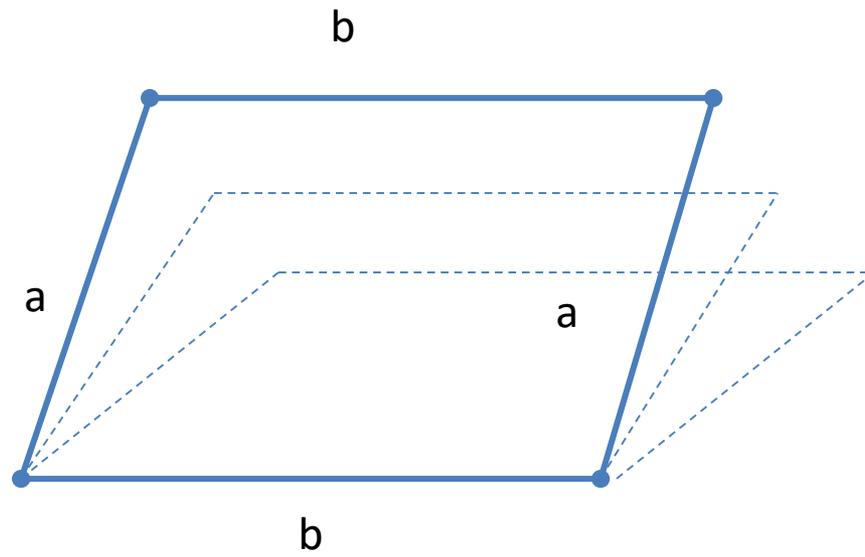
- When shortest link  $L_{\min}$  is fixed  
**DOUBLE\_CRANK** linkage will result. Both link adjacent to  $L_{\min}$  can rotate continuously



By fixing the link opposite to  $L_{min}$ , we obtain **DOUBLE-ROCKER** mechanism

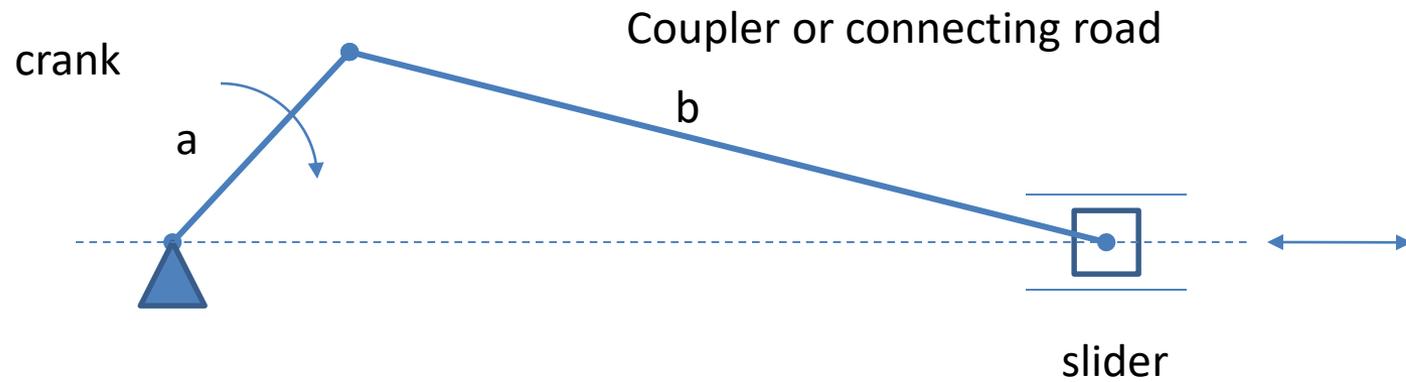


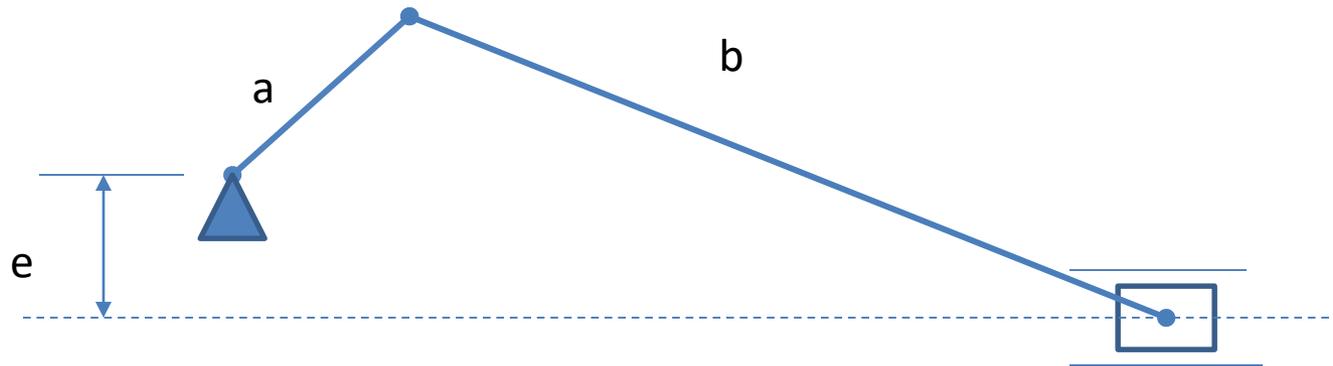
- If  $L_{\min} + L_{\max} > L' + L''$  then a **DOUBLE-ROCKER** mechanism will result no matter which link is fixed.



Parallel linkage mechanism

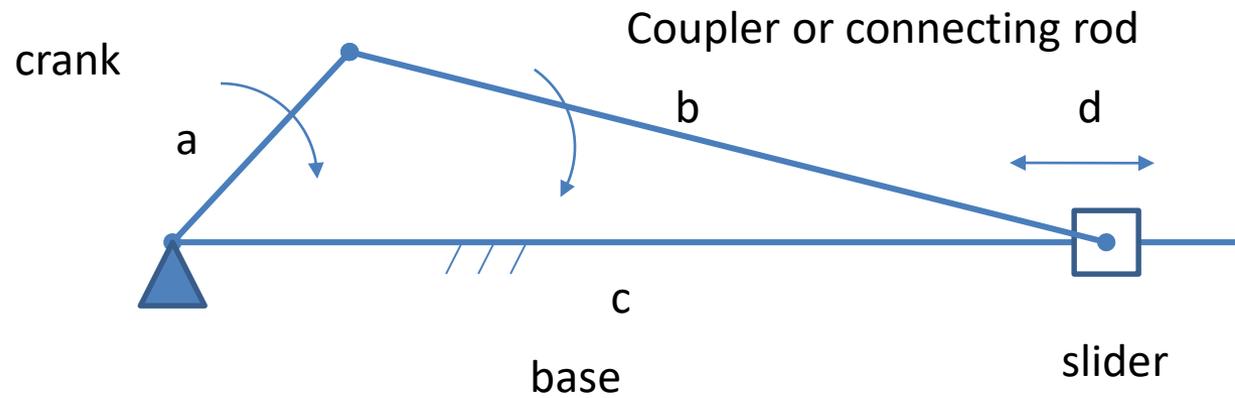
- Slider-Crank mechanism





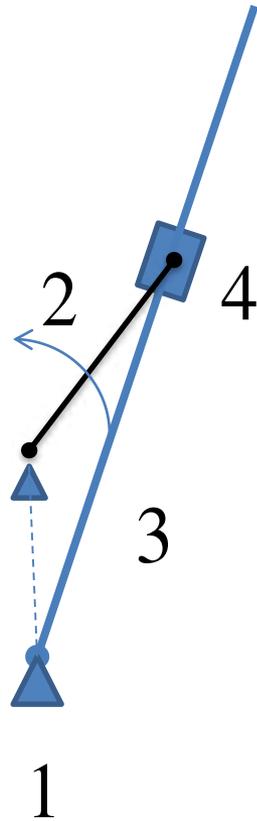
- “ $e$ ” is **offset** or **eccentricity**
- Complete rotation of “ $a$ ” is possible if  $e < (b-a)$

- Slider-Crank mechanism



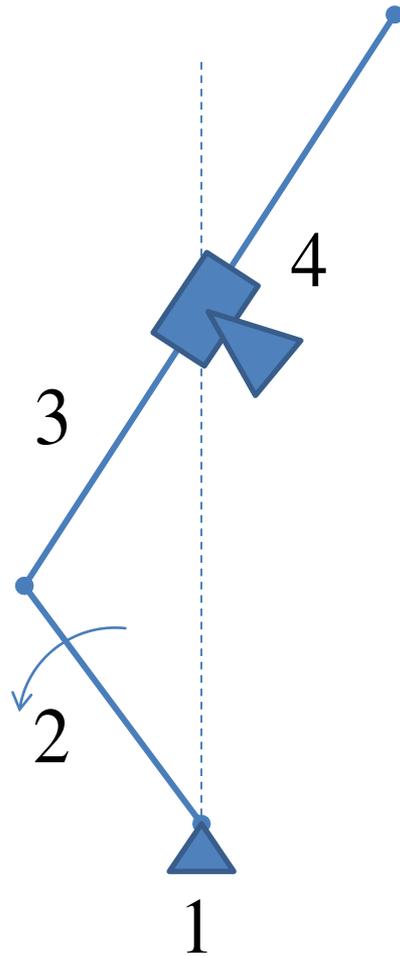
$c$  is fixed

# Inverted Slider-Crank mechanism



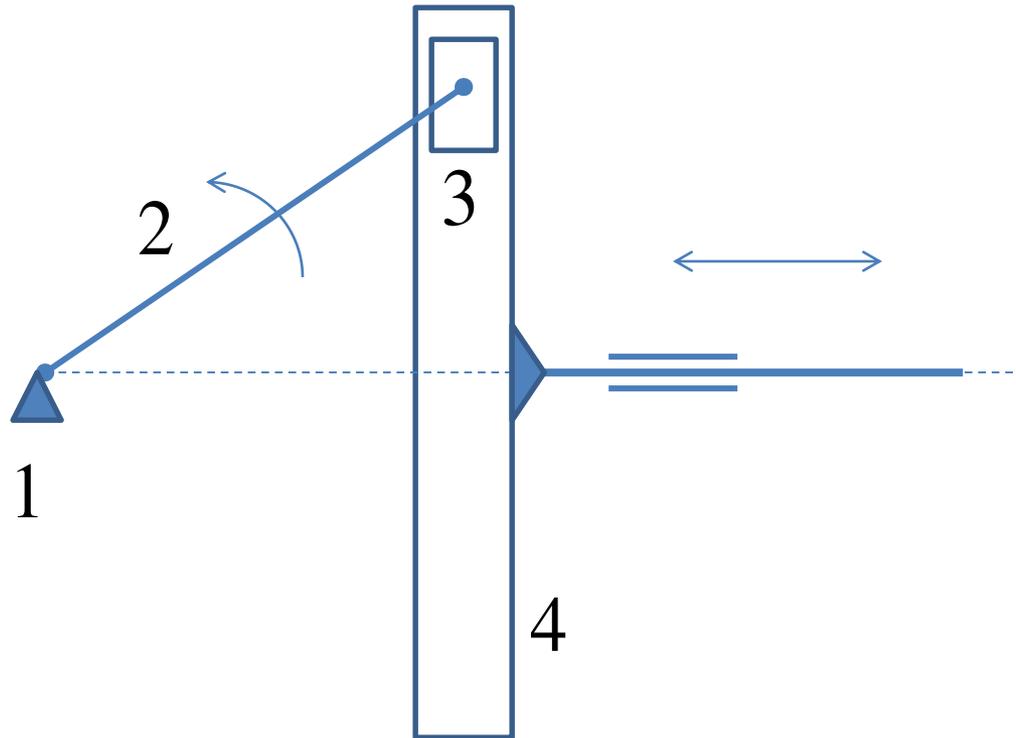
- 1: fixed
- 2: crank
- 3: rocker
- 4: slider

# Inverted Slider-Crank mechanism



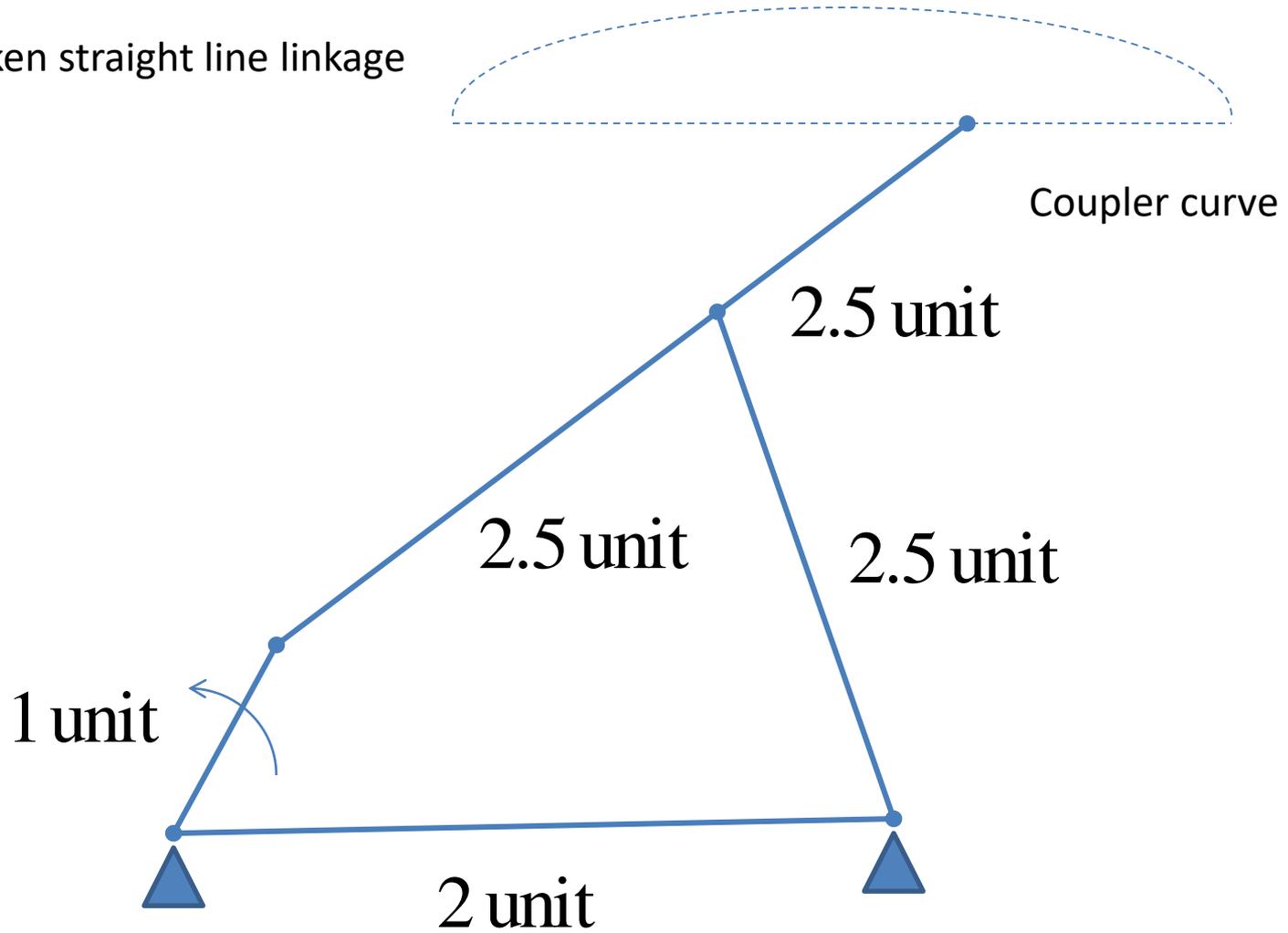
- 1: fixed
- 2: crank
- 3: rocker
- 4: slider

## Scotch-Yoke Mechanism



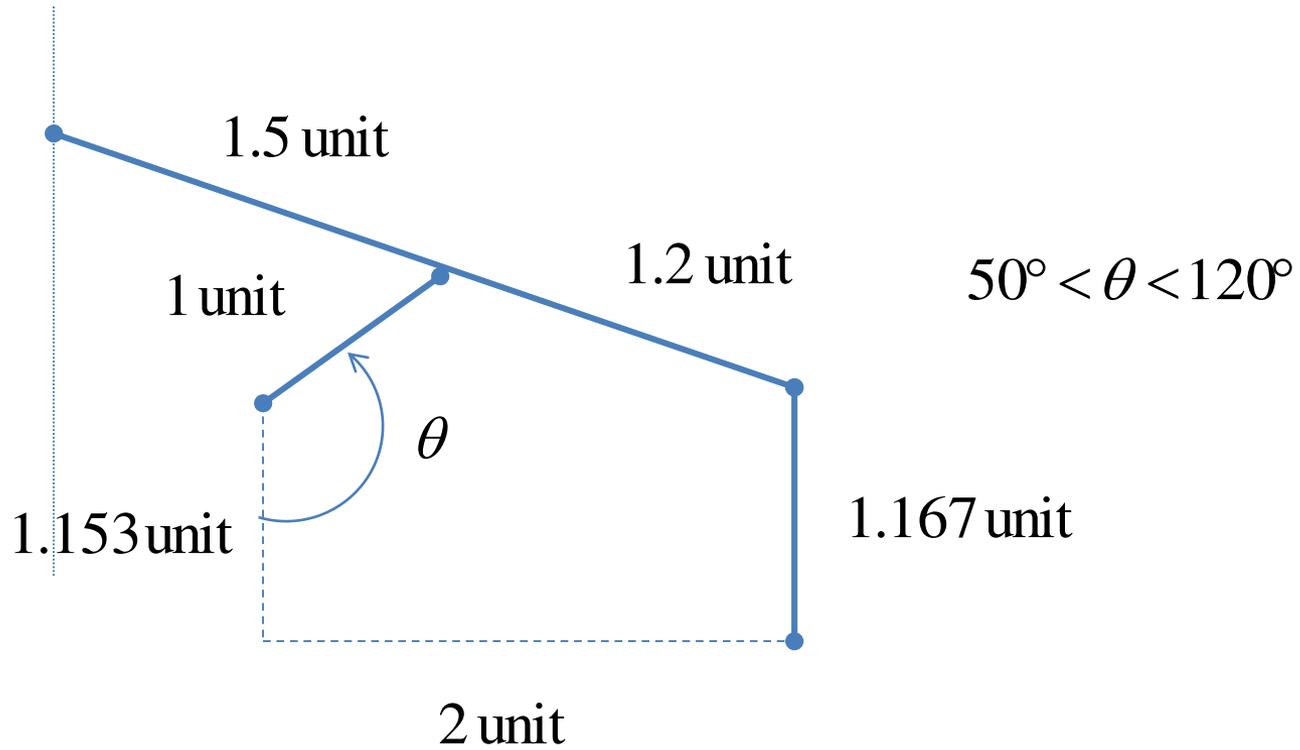
$$n=4, \quad j_1=4, \quad j_2=0, \quad m=3(4-1)-2 \times 4-0=1 \text{ DOF}$$

Hoeken straight line linkage



# Evans straight line linkage

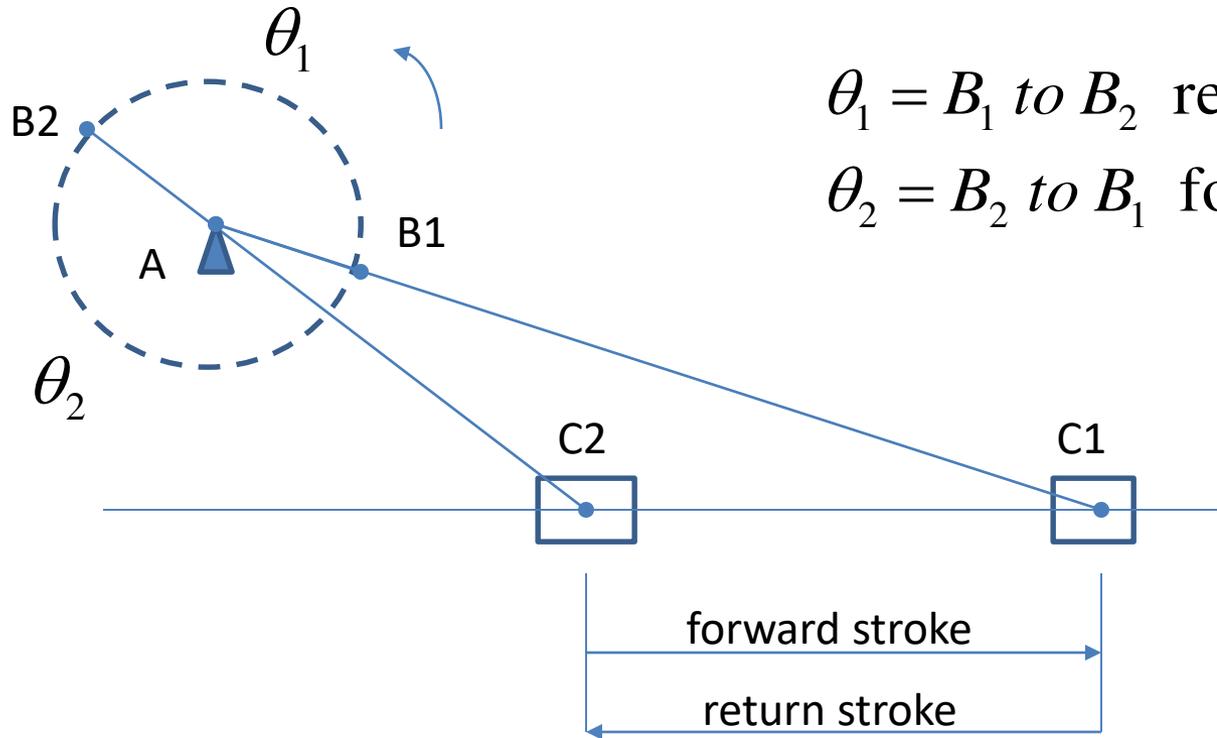
Coupler curve



## TIME RATIO

$$\text{time ratio} = \frac{\text{time of forward stroke}}{\text{time of return stroke}}$$

## Time ratio for slider-crank mechanism



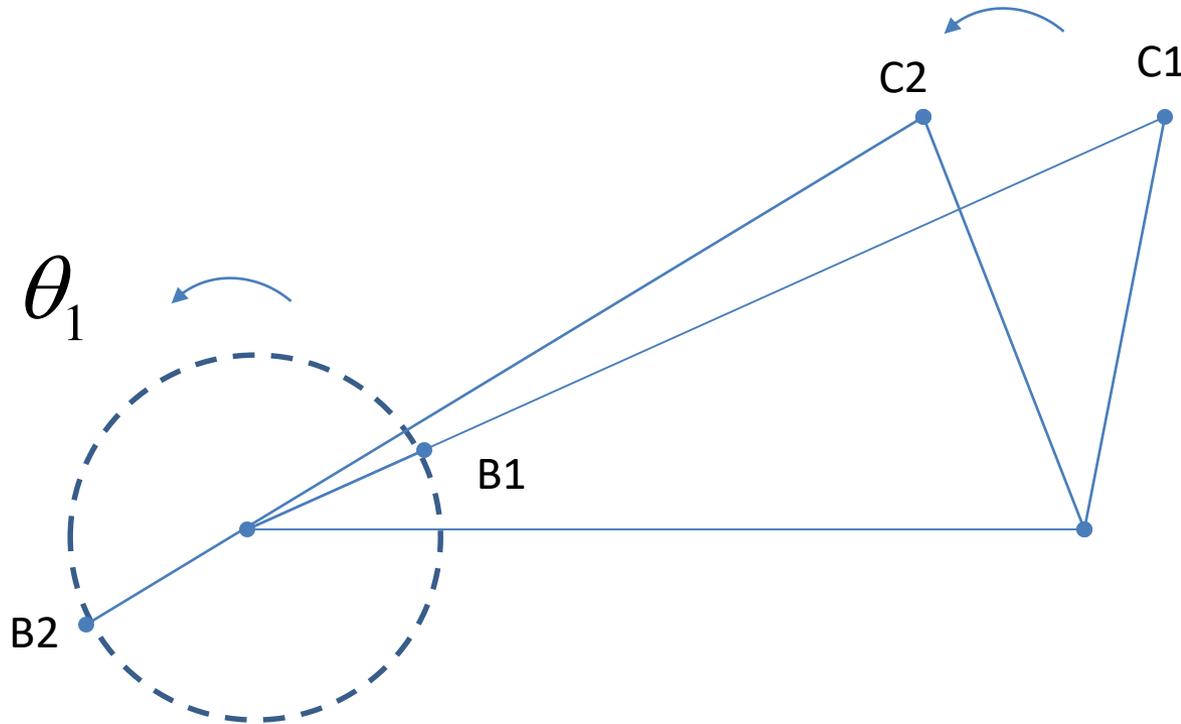
$$tr = \frac{\theta_2 / \omega}{\theta_1 / \omega} = \frac{\theta_2}{\theta_1} = \frac{\theta_2}{360 - \theta_2} = \frac{360 - \theta_1}{\theta_1}$$

$\omega$  is constant speed of crank in rad/s

If crank is rotating with constant speed, time ratio can be calculated from angles.

- If rotation is reversed time ratio will also be reversed
- If time ratio is greater than one mechanism is called **quick-return** mechanism.
- If offset is zero time ratio is always one

## Time ratio for four-bar mechanism



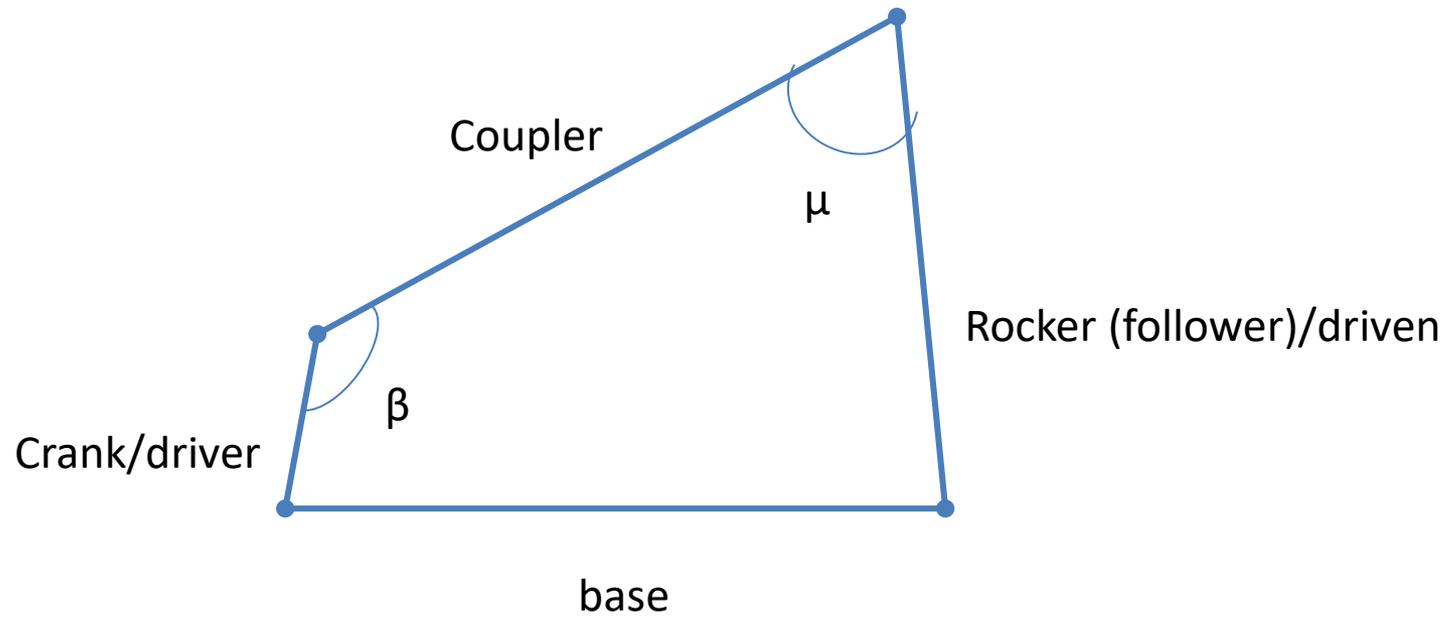
$$tr = \frac{\angle B_2 B_1}{\angle B_1 B_2} = \frac{\theta_2}{\theta_1} = \frac{\text{forward angle}}{\text{return angle}}$$

- Mechanical Advantage
- Mechanical advantage of a linkage is the ratio of the output torque exerted by the driven link to the necessary input torque required at the driver.

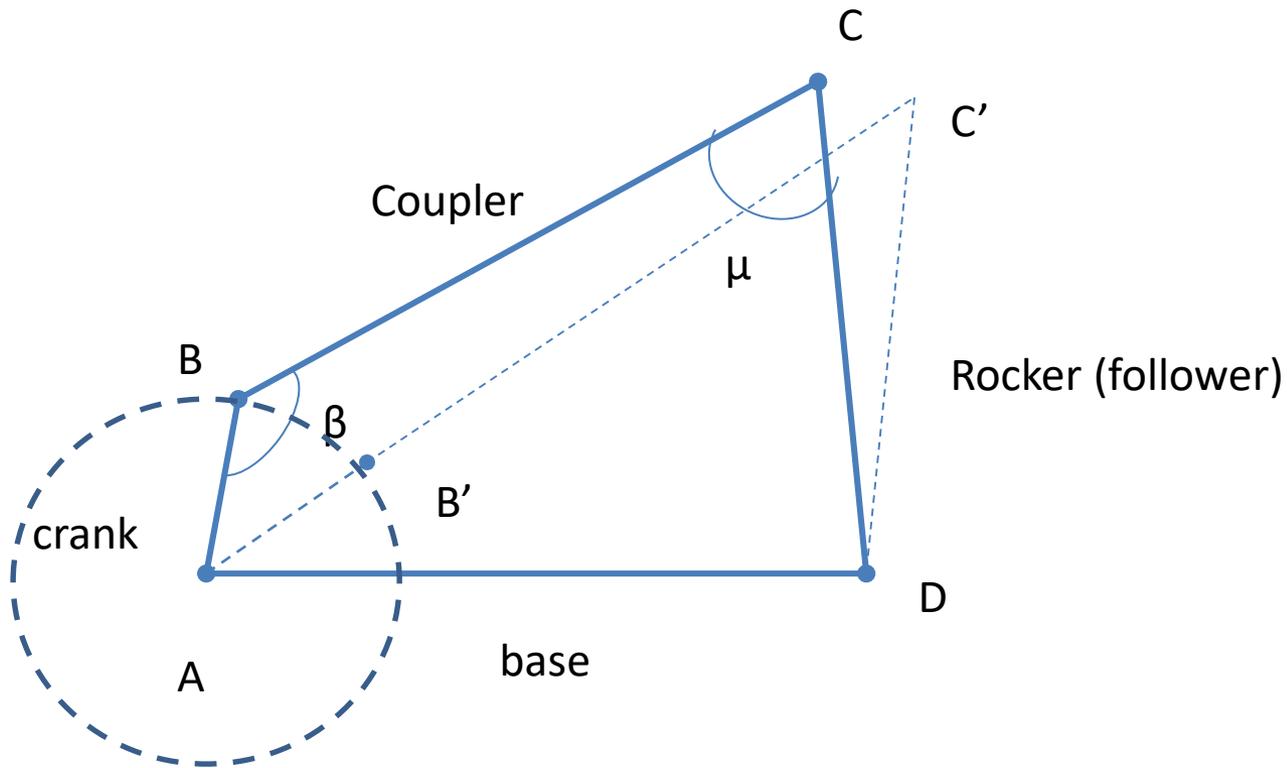
$$\text{mechanical advantage} = \frac{T_{\text{output}}}{T_{\text{input}}}$$

- **Mechanical advantage** of the four-bar linkage is directly proportional to the sine of the angle  $\mu$  between the coupler and the follower/rocker (driven link) and inversely proportional to the sine of the angle  $\beta$  between the coupler and the driver link. As the linkage moves mechanical advantage will continuously change.

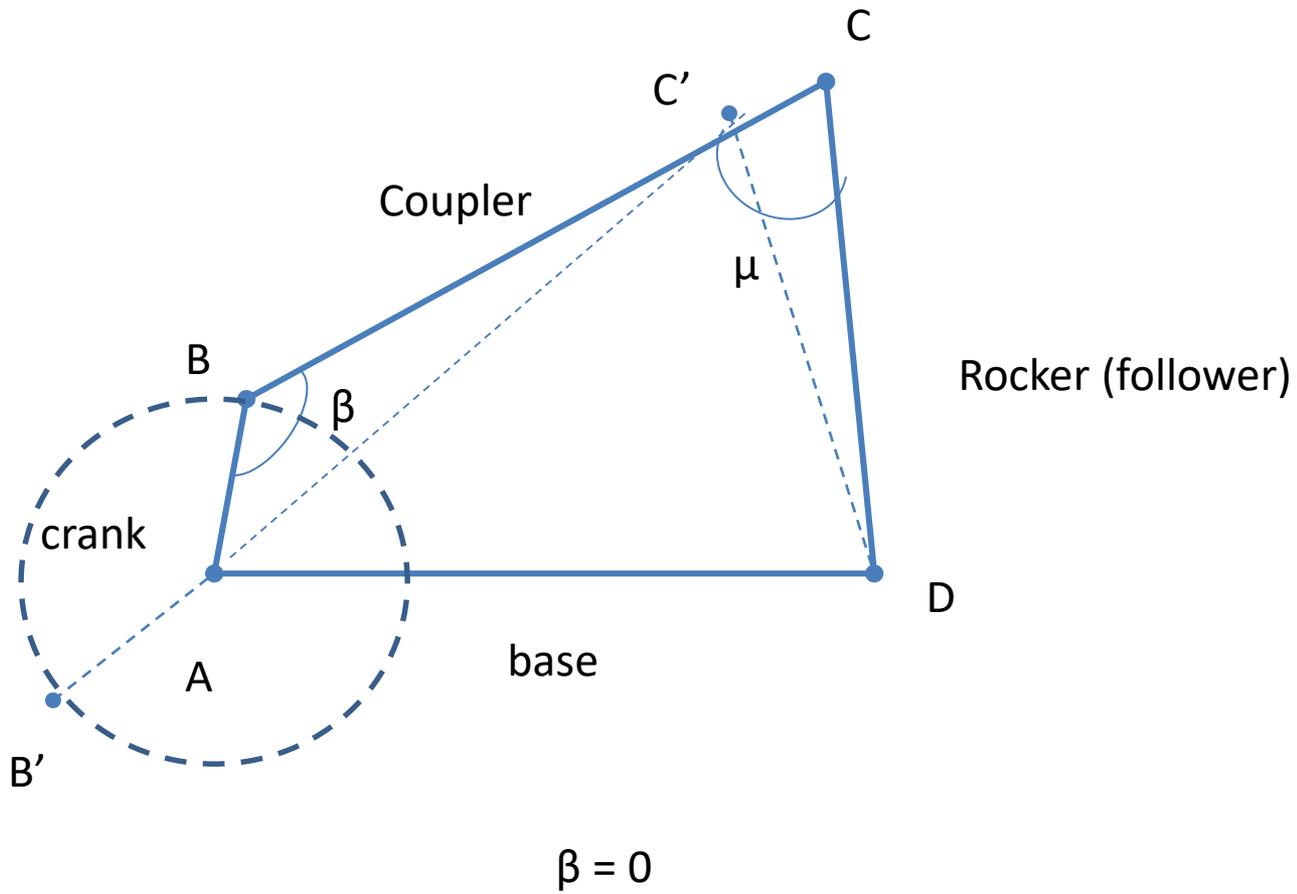
$$\text{mechanical advantage} \propto \frac{\sin\mu}{\sin\beta}$$



- When  $\sin\beta$  becomes zero ( $\beta=0$  or  $\pi$ ), the mechanical advantage becomes infinite; thus at such position, only a small input torque is necessary to overcome a large output torque.



$$\beta = 180$$

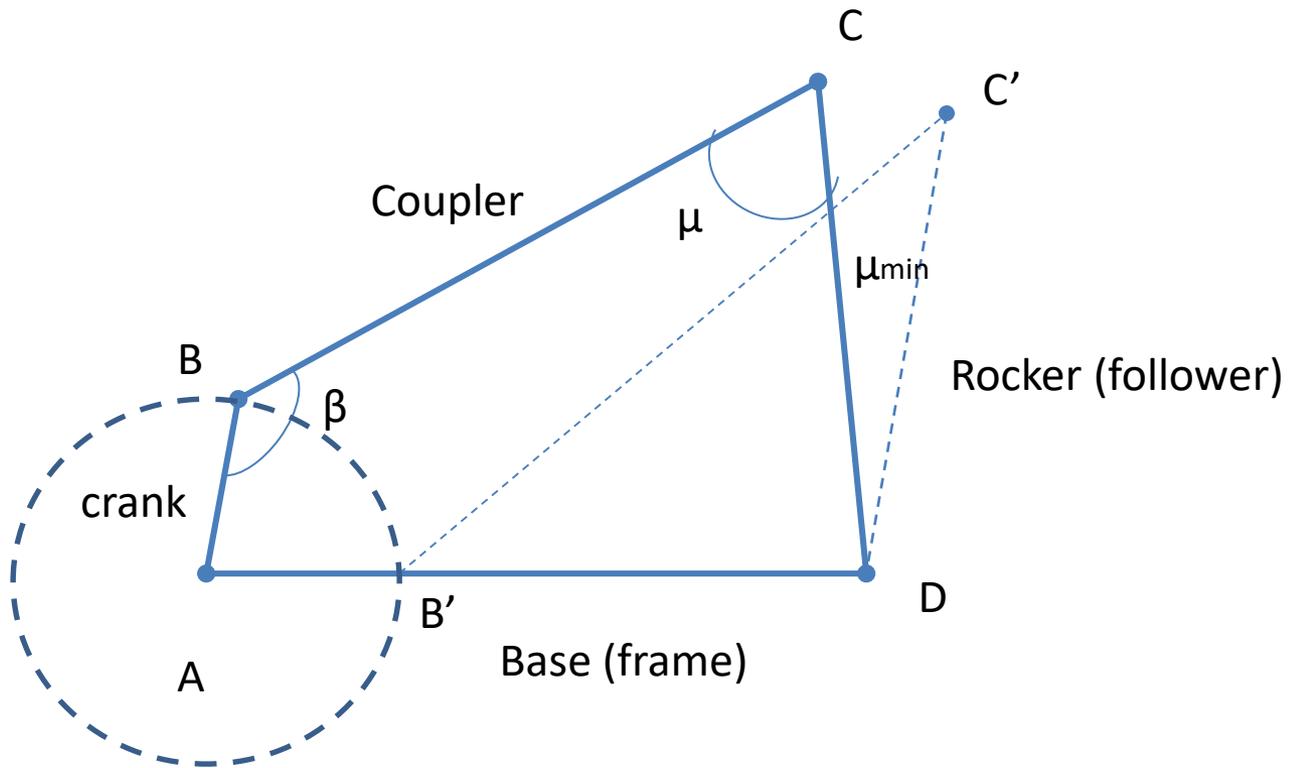


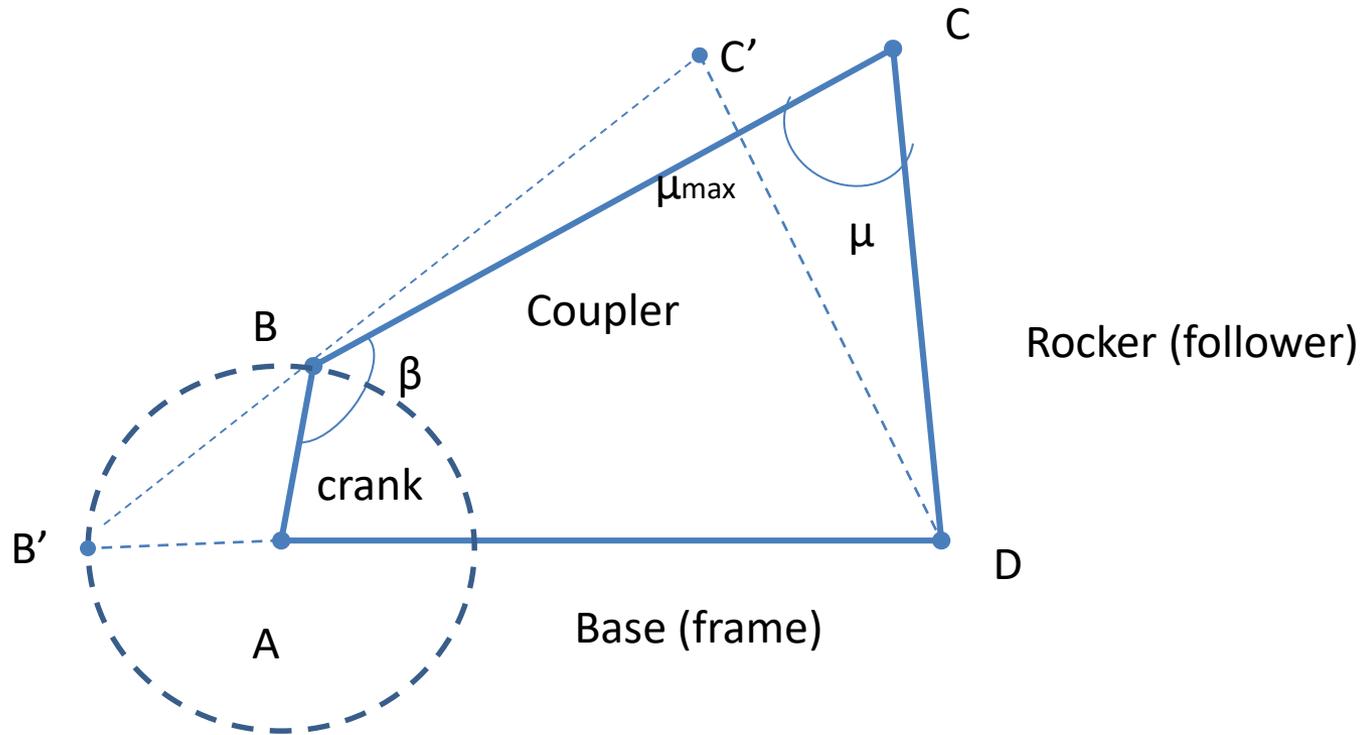
- C' positions are extreme positions of rocker DC. In these positions, the mechanical advantage is infinite and the linkage is said to be in a **toggle** position.

- The angle  $\mu$  between the coupler and the follower (driven link) is called the **transmission angle**. As this angle becomes small, the mechanical advantage decreases and even small amount of friction will cause the mechanism to lock or jam.

- A common rule of thumb is that a four-bar linkage should not be used in the region where the transmission angle is less than,  $45^\circ$  or  $50^\circ$ .

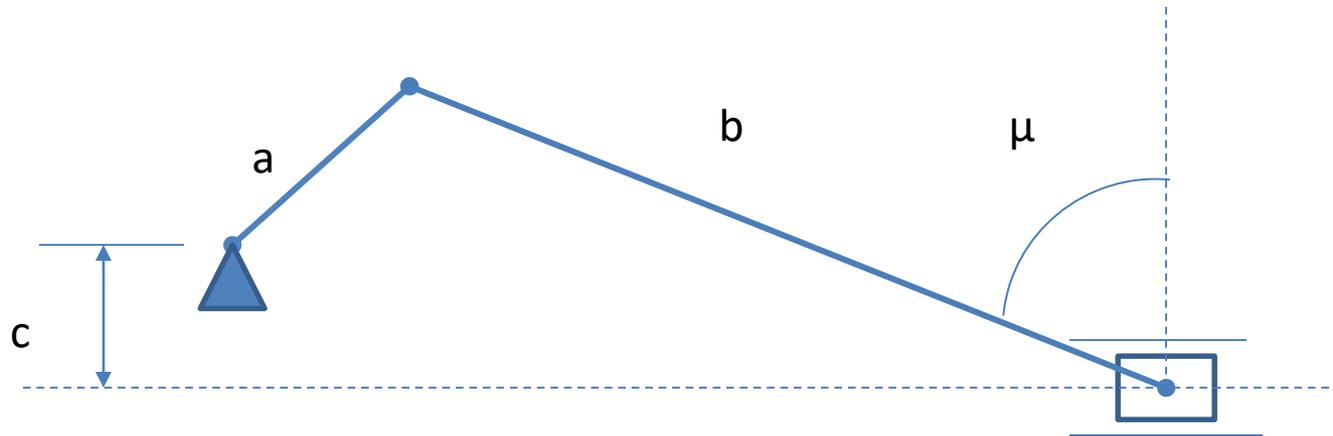
- In the four-bar linkage, extreme values (max or min) of the transmission angle occur when the crank AB lies along the line of the frame (base) AD.





- Because of the ease with which it can be visually inspected, the **transmission angle** has become a commonly accepted measure of the quality of the design of a four-bar linkage.

- Definitions of mechanical advantage, toggle position, and transmission angle depend on the choice of the driver and driven links. If follower is the driver and crank is the driven link, the roles of  $\beta$  and  $\mu$  will be reversed.



Slider crank mechanism transmission angle

In the slider-crank mechanism  $\mu_{\min}$  and  $\mu_{\max}$  occurs when the crank is making 90 degrees and 270 degrees with the horizontal line.

