KUWAIT UNIVERSITY
DEPARTMENT OF PETROLEUM ENGINEERING

SPRING 2020

PE 437

NUMERICAL METHODS IN PETROLEUM ENGINEERING

PART 2

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2-D EOC for Single Phase, Slight Compressible Fluid Linear Flow

The compressed differential equation for two-dimension, single phase, slight compressible fluid, linear flow **ignoring gravity** is:

$$\Delta_x T_x \Delta_x P + \Delta_y T_y \Delta_y P + (qB)_{ij} = \frac{VR_{ij}}{\Delta t}(\phi C_t)_{ij} \Delta_t P_{ij} \qquad (2\text{-}18)$$

Where,

$$\Delta_x T_x \Delta_x P = 0.001127\left(\frac{k_x}{\mu}\frac{\Delta y \Delta z}{\Delta x}\right)^n_{i+1/2,j}\left(P^{n+1}_{i+1,j} - P^{n+1}_{i,j}\right) - 0.001127\left(\frac{k_x}{\mu}\frac{\Delta y \Delta z}{\Delta x}\right)^n_{i-1/2,j}\left(P^{n+1}_{i,j} - P^{n+1}_{i-1,j}\right)$$

$$\Delta_y T_y \Delta_y P = 0.001127\left(\frac{k_y}{\mu}\frac{\Delta x \Delta z}{\Delta y}\right)^n_{i,j+1/2}\left(P^{n+1}_{i,j+1} - P^{n+1}_{i,j}\right) - 0.001127\left(\frac{k_y}{\mu}\frac{\Delta x \Delta z}{\Delta y}\right)^n_{i,j-1/2}\left(P^{n+1}_{i,j} - P^{n+1}_{i,j\text{-}1}\right)$$

$$\frac{VR_{ij}}{\Delta t}\left(\phi C_t\right)_{ij}\Delta_t P_{ij} = \frac{\left(\Delta x \Delta y \Delta z\right)_{ij}}{5.615\Delta t}\left(\phi C_t\right)_{ij}\left(P^{n+1}_{i,j} - P^{n}_{i,j}\right)$$

$$\text{velocity}\quad v_{x\,i+1/2,j} = -0.001127\frac{k_x}{\mu}\left(\frac{P^{n+1}_{i+1,j} - P^{n+1}_{i,j}}{\Delta_{x\,i+1/2,j}}\right), \text{(function of sign)}$$

Considering a node in the center of a grid with transmissibility to all its sides, equation (2-18) can be expanded:

$$\left[Tx^{n}_{i+1/2,j}\left(P^{n+1}_{i+1,j}-P^{n+1}_{i,j}\right)-Tx^{n}_{i-1/2,j}\left(P^{n+1}_{i,j}-P^{n+1}_{i-1,j}\right)\right]+\left[Ty^{n}_{i,j+1/2}\left(P^{n+1}_{i,j+1}-P^{n+1}_{i,j}\right)-Ty^{n}_{i,j-1/2}\left(P^{n+1}_{i,j}-P^{n+1}_{i,j-1}\right)\right]+q_{i,j}B_{ij}=$$

$$\frac{(\Delta x\Delta y\Delta z)_{i,j}}{\Delta t}\left(\phi C_{t}\right)_{ij}\left(\mathrm{P}^{\mathrm{n+1}}_{\mathrm{i,j}}-\mathrm{P}^{\mathrm{n}}_{\mathrm{i,j}}\right) \qquad (2\text{-}19)$$

Grouping terms, it is possible to write this equation in the form:

$$B_{i,j}P^{n+1}_{i,j-1}+D_{i,j}P^{n+1}_{i-1,j}+E_{i,j}P^{n+1}_{i,j}+F_{i,j}P^{m+1}_{i+1,j}+H_{i,j}P^{n+1}_{i,j+1}=R_{i,j} \qquad (2\text{-}20)$$

Being the coefficients:

$$B_{i,j} = Ty_{i,j-1/2} = 0.001127\left(\frac{ky}{\mu}\right)_{i,j-1/2}\left[\frac{\Delta x \Delta z}{\Delta y_{i,j-1/2}}\right] \quad (2\text{-}21)$$

$$D_{i,j} = Tx_{i-1/2,j} = 0.001127\left(\frac{kx}{\mu}\right)_{i-1/2,j}\left[\frac{\Delta y \Delta z}{\Delta x_{i-1/2,j}}\right] \quad (2\text{-}22)$$

$$F_{i,j} = Tx_{i+1/2,j} = 0.001127\left(\frac{kx}{\mu}\right)_{i+1/2,j}\left[\frac{\Delta y \Delta z}{\Delta x_{i+1/2,j}}\right] \quad (2\text{-}23)$$

$$H_{i,j} = Ty_{i,j+1/2} = 0.001127\left(\frac{ky}{\mu}\right)_{i,j+1/2}\left[\frac{\Delta x \Delta z}{\Delta y_{i,j+1/2}}\right] \quad (2\text{-}24)$$

$$E_{i,j} = -\left[B_{i,j} + H_{i,j} + D_{i,j} + F_{i,j} + \frac{(\Delta x \Delta y \Delta z)_{ij}}{5.615 \Delta t} (\phi C_t)_{ij} \right] \quad (2\text{-}25)$$

$$R_{i,j} = -\left[q_{i,j} B_{ij} + \frac{(\Delta x \Delta y \Delta z)_{ij}}{5.615 \Delta t} (\phi C_t)_{ij} P_{ij}^{n} \right] \quad (2\text{-}26)$$

The compressed differential equation for two-dimension, single phase, slight compressible flow **with gravity** is:

$$\Delta_x T_x (\Delta_x P - \gamma \Delta_x D) + \Delta_y T_y (\Delta_y P - \gamma \Delta_y D) + \left(qB\right)_{ij} = \frac{VR_{ij}}{\Delta t} \left(\phi C_t\right)_{ij} \Delta_t P_{ij}$$

(2-27)

where,

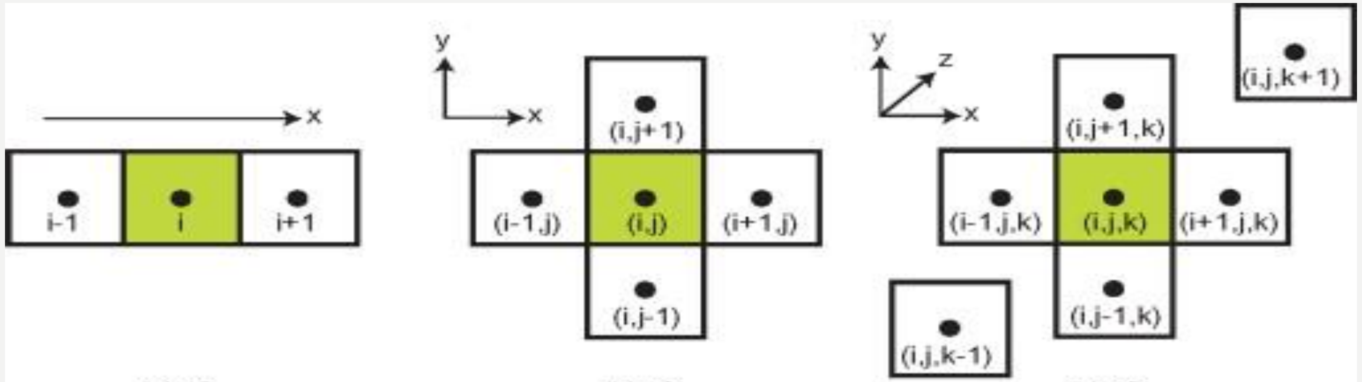
$$\Delta_x T_x (\Delta_x P - \gamma \Delta_x D) = \left[\mathrm{T}_{xi+\frac{1}{2},j} \left(\mathrm{P}^{n+1}_{i+1,j} - \mathrm{P}^{n+1}_{i,j} \right) - \mathrm{T}_{xi-1/2,j} \left(\mathrm{P}^{n+1}_{i,j} - \mathrm{P}^{n+1}_{i-1,j} \right) \right] - \left[\left(\mathrm{T}_x \gamma \right)_{i+\frac{1}{2},j} \left(\mathrm{D}_{i+1,j} - \mathrm{D}_{i,j} \right) - \left(\mathrm{T}_x \gamma \right)_{i-1/2,j} \left(\mathrm{D}_{i,j} - \mathrm{D}_{i-1,j} \right) \right]$$

$$\Delta_y T_y (\Delta_y P - \gamma \Delta_y D) = \left[T_{yi,j+\frac{1}{2}} \left(P^{n+1}_{i,j+1} - P^{n+1}_{i,j} \right) - T_{yi,j-\frac{1}{2}} \left(P^{n+1}_{i,j} - P^{n+1}_{i,j-1} \right) \right] - \left[\left(T_y \gamma \right)_{i,j+\frac{1}{2}} \left(D_{i,j+1} - D_{i,j} \right) - \left(T_y \gamma \right)_{i,j-\frac{1}{2}} \left(D_{i,j} - D_{i,j-1} \right) \right]$$

$$T_{xi+\frac{1}{2},j} = 0.001127 \left(\frac{kx}{\mu} \frac{\Delta y \Delta z}{\Delta x} \right)^n_{i+\frac{1}{2},j}, \quad T_{xi-\frac{1}{2},j} = 0.001127 \left(\frac{kx}{\mu} \frac{\Delta y \Delta z}{\Delta x} \right)^n_{i-\frac{1}{2},j}$$

$$T_{y_{i,j+\frac{1}{2}}} = 0.001127 \left(\frac{ky}{\mu} \frac{\Delta x \Delta z}{\Delta y} \right)^n_{i,j+\frac{1}{2}}, \quad T_{y_{i,j-\frac{1}{2}}} = 0.001127 \left(\frac{ky}{\mu} \frac{\Delta x \Delta z}{\Delta y} \right)^n_{i,j-\frac{1}{2}}$$

$$\frac{VR_{ij}}{\Delta t}(\phi C_t)_{ij}\Delta_t P_{ij} = \frac{(\Delta x \Delta y \Delta z)_{ij}}{5.615\Delta t}(\phi C_t)_{ij}\left(P_{i,j}^{n+1} - P_{i,j}^{n}\right)$$



(a) 1D. (b) 2D. (c) 3D.

CHAPTER 3: 2-D ORDER TECHNIQUES (MATRIX ARRAYS)

A- Standard Order by Column

3	6	9
2	5	8
1	4	7

$i \uparrow$

$j \longrightarrow$

The resultant coefficient matrix (9*9) is as follows:

	1	2	3	4	5	6	7	8	9
1	x	x		x					
2	x	x	x		x				
3		x	x			x			
4	x			x	x		x		
5		x		x	x	x		x	
6			x		x	x			x
7				x			x	x	
8					x		x	x	x
9						x		x	x

There are 3 repeated submatrices of the size 3 by 3 (= number of rows)

B- Standard Order by Raw

9	10	11	12
5	6	7	8
1	2	3	4

i ↑

j →

The resultant coefficient matrix (12*12) is as follows:

	1	2	3	4	5	6	7	8	9	10	11	12
1	x	x			x							
2	x	x	x			x						
3		x	x	x			x					
4			x	x				x				
5	x				x	x			x			
6		x			x	x	x			x		
7			x			x	x	x			x	
8				x			x	x				x
9					x				x	x		
10						x			x	x	x	
11							x			x	x	x
12								x			x	x

There are 3 repeated submatrices of the size 4 by 4 (= number of columns)

C- Standard Order by Column for Irregular Shape

5	10	
4	9	
3	8	13
2	7	12
1	6	11

The resultant coefficient matrix (13*13) is as follows:

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	x	x				x							
2	x	x	x				x						
3		x	x	x				x					
4			x	x	x				x				
5				x	x					x			
6	x					x	x				x		
7		x				x	x	x				x	
8			x				x	x	x				x
9				x				x	x	x			
10					x				x	x			
11						x					x	x	
12							x				x	x	x
13								x				x	x

There are 2 repeated submatrices of the size 5 by 5 and 1 submatrix of the size 3 by 3.

D- Standard Order by rows for Irregular Shape

12	13	
10	11	
7	8	9
4	5	6
1	2	3

The resultant coefficient matrix (13*13) is as follows:

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	x	x		x									
2	x	x	x		x								
3		x	x			x							
4	x			x	x		x						
5		x		x	x	x		x					
6			x		x	x			x				
7				x			x	x		x			
8					x		x	x	x		x		
9						x		x	x				
10							x			x	x	x	
11								x		x	x		x
12										x		x	x
13											x	x	x

There are 3 repeated submatrices of the size 3 by 3 and 2 submatrices of the size 2 by 2.

E- Diagonal Order D-2

7	11	15	18	20
4	8	12	16	19
2	5	9	13	17
1	3	6	10	14

The resultant coefficient matrix (20*20) is as follows:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	x	x	x																	
2	x	x		x	x															
3	x		x		x	x														
4		x		x			x	x												
5		x	x		x			x	x											
6			x			x			x	x										
7				x			x				x									
8				x	x			x			x	x								
9					x	x			x			x	x							
10						x				x			x	x						
11							x	x			x				x					
12								x	x			x			x	x				
13									x	x			x			x	x			
14										x				x			x			
15											x	x			x			x		
16												x	x			x		x	x	
17													x	x			x		x	
18															x	x		x		x
19																x	x		x	x
20																		x	x	x

F- Diagonal Order D-4

13	5	17	9	20
2	14	6	18	10
11	3	15	7	19
1	12	4	16	8

The resultant coefficient matrix (20*20) is as follows:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	x										x	x								
2		x									x		x	x						
3			x								x	x		x	x					
4				x								x			x	x				
5					x								x	x			x			
6						x								x	x		x	x		
7							x								x	x		x	x	
8								x								x			x	
9									x								x	x		x
10										x								x	x	x
11	x	x	x								x									
12	x		x	x								x								
13		x			x								x							
14		x	x		x	x								x						
15			x	x		x	x								x					
16				x			x	x								x				
17					x	x			x								x			
18						x	x		x	x								x		
19							x	x		x									x	
20									x	x										x

G- Cyclic 2-order

12	4	16	8	20
2	14	6	18	10
11	3	15	7	19
1	13	5	17	9

The resultant coefficient matrix (20*20) is as follows:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	x										x		x							
2		x									x	x		x						
3			x								x		x	x	x					
4				x								x		x		x				
5					X								x		x		x			
6						x								x	x	x		x		
7							x								x		x	x	x	
8								x								x		x		x
9									x								x		x	
10										x								x	x	x
11	x	x	x								x									
12		x		x								x								
13	x		x		x								x							
14		x	x	x		x								x						
15			x		x	x	x								x					
16				x		x		x								x				
17					x		x		x								x			
18						x	x	x		x								x		
19							x		x	x									x	
20								x		x										x