

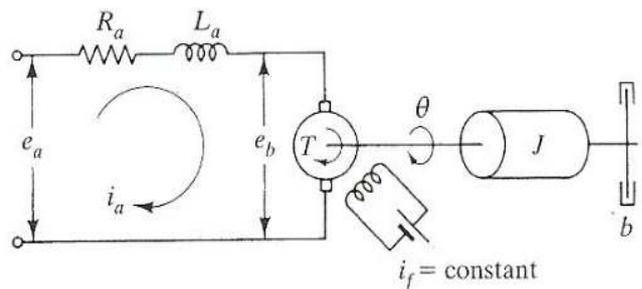


Course #: 804465 – Automatic Control Lab #4

MODELING A DC MOTOR (ELECTROMECHANICAL SYSTEM)

Objective To derive the transfer function of a DC motor (electromechanical system) and simulate its response to a unit step input.

Introduction



(a) (b)

Figure 1: (a) Cut-away view of a DC motor; (b) schematic diagram of a DC motor

The majority of mechanical systems are driven by an electric motor. A DC motor is an electromechanical system since it consists of both electrical and mechanical components. Fig.1(b) shows the schematic diagram of a dc motor with a constant field current (constant magnetic flux), where,

R_a = armature resistance (4 ohm)

L_a = armature inductance (2.75E-6 H)

i_a = armature current and i_f = field current (Amp)

e_a = input or applied voltage (V)

e_b = back electromotive force, emf (V);

Faraday's law of electromagnetic induction states that $e_b = K_b \dot{\theta}$

K_b = emf constant and $\dot{\theta}$ = angular velocity of the motor shaft (radian/sec)

T = torque developed by the motor (Nm), $T = K i_a$, K is the motor-torque constant

$K = K_b = 0.0274$ Nm/Amp

J = mass moment of inertia of the rotor (3.2284E-6 kgm²)

b = damping coefficient of the motor (3.5077E-6 Nm-s/rad)

Theory

The speed of the dc motor is controlled by the armature current i_a , which depends on e_a , the input to the system. The angular rotation of the rotor θ is the output. Applying Kirchoff's voltage law to the electrical circuit loop,

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a, \text{ but } e_b = K_b \dot{\theta}$$

$$L_a \frac{di_a}{dt} + R_a i_a + K_b \dot{\theta} = e_a$$

(1)



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The armature current produces torque that rotates the rotor hence the equation of motion for the mechanical components is given as:

$$J \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} = T = Ki_a$$

(2)

**Lab
Assignments**

- a. Take Laplace transforms of Equations (1) and (2) and derive the transfer function $\frac{\theta(s)}{E_a(s)}$ of the electromechanical system (a dc motor).
- b. Write MATLAB m-file to compare the responses of the dc motor due to a unit step input using the transfer function obtained in (a) above and the transfer function when $L_a = 0$. Use the values given in the Introduction Section.
- c. Draw BLOCK diagram of the system and model this system in SIMULINK using block diagram.

Hints:

```
clear all  
close all
```

```
%Parameters
```

```
J = 3.2284E-6; % kg.m^2  
b = 3.5077E-6; % Nms/rad  
K = 0.0274; Kb = K; % Nm/Amp  
Ra = 4; %ohm  
La = 2.75E-6; % H
```

```
num = [ ]; % write this in terms of the parameters (K, La, etc)  
% with La considered  
den_La = [ ]; % write this in terms of the parameters (K, La,  
etc)  
[y1 x1 t1] = step(num,den_La);  
disp('TF with inductance')  
tf(num,den_La)
```

```
% without La  
La = 0;  
den = [ ]; % the same as den_La
```

```
[y2 x2 t2] = step(num,den);  
disp('TF without inductance')  
tf(num,den)
```

```
%Unit step input responses,  
plot(t1,y1,'r',t2,y2,'b--')  
title('Unit response of a dc motor')  
legend('with inductance', 'without inductance')  
xlabel('time (s)')  
ylabel('Theta')
```



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Course #: 804465 – Automatic Control Lab #4 - Answer Sheet

Student's name: _____ **ID #:** _____ **Group #:** _____