

ME 8043114 Automatic Control

Lecture 11: Frequency domain lead-lag compensator design Frequency response shaping in Matlab

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Lecture Outline

- Frequency domain lead-lag compensator design
- Frequency response shaping in Matlab



Course roadmap

Modeling

- ✓ Laplace transform
- ✓ Transfer function
- ✓ Models for systems
 - ✓ electrical
 - ✓ mechanical
 - ✓ electromechanical
- ✓ Linearization, delay

Analysis

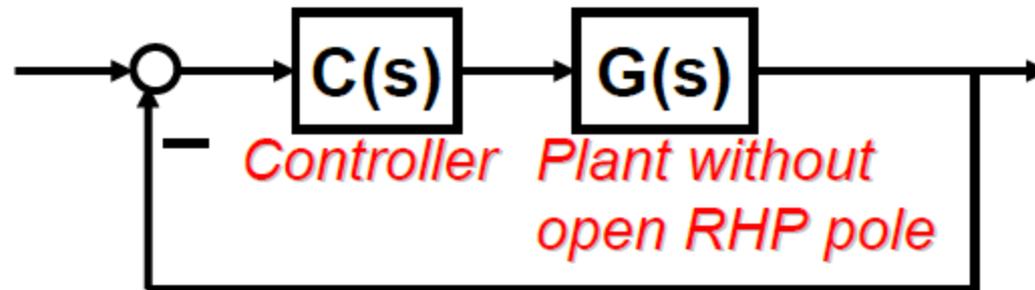
- ✓ Time response
 - ✓ Transient
 - ✓ Steady state
- ⇒ ✓ Frequency response
 - ✓ Bode plot
- ✓ Stability
 - ✓ Routh-Hurwitz
 - ✓ Nyquist

Design

- ✓ Design specs
- ✓ Root locus
- ⇒ ✓ Frequency domain
- ✓ PID & Lead-lag
- Design examples



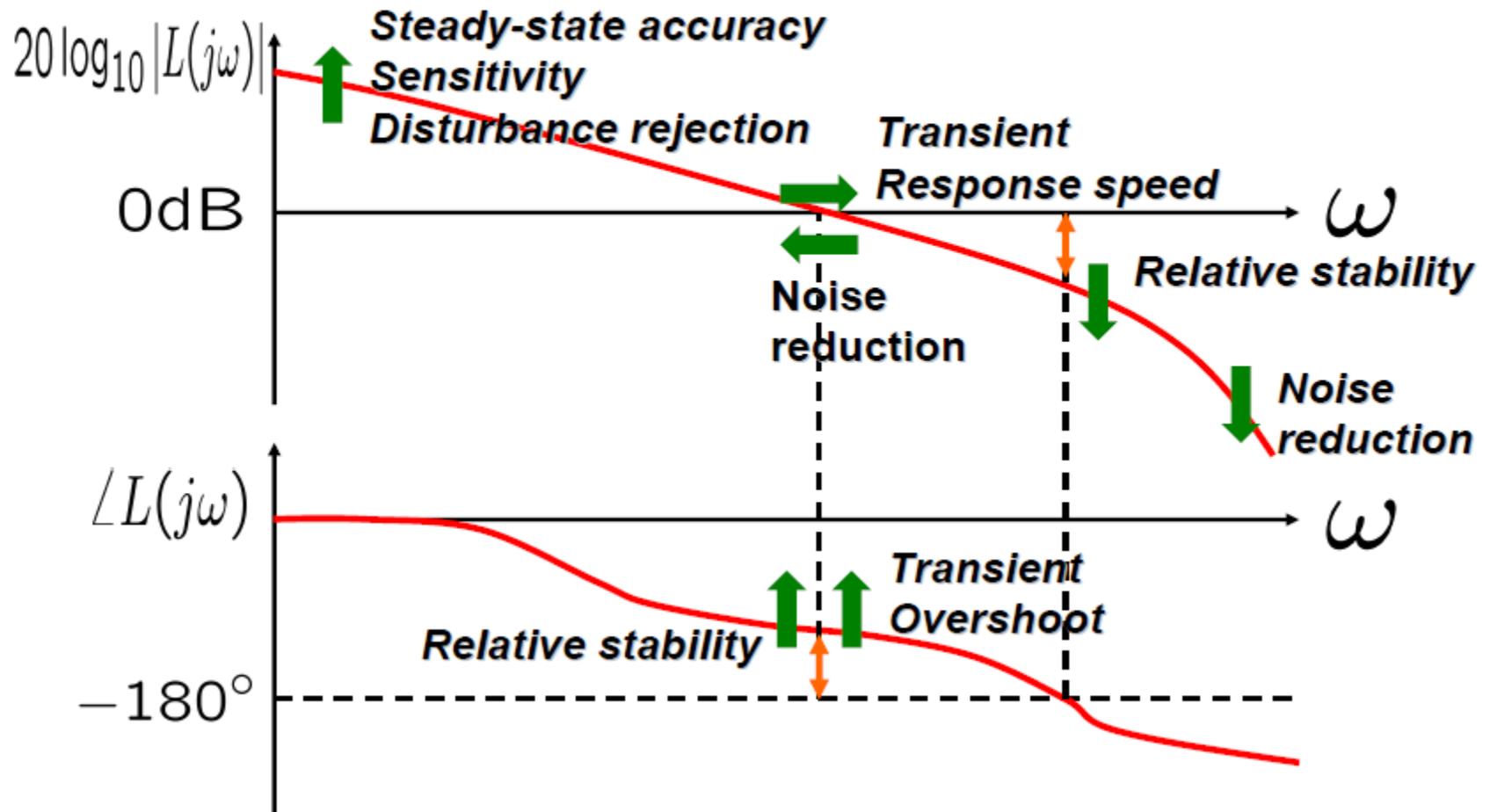
Frequency shaping (loop shaping)



- Design $C(s)$ so that $L(j\omega) := G(j\omega)C(j\omega)$ has a desired shape.
- We study the design of simple compensators:
 - Gain compensator
 - Lag compensator
 - Lead compensator



Typical shaping goal (review)

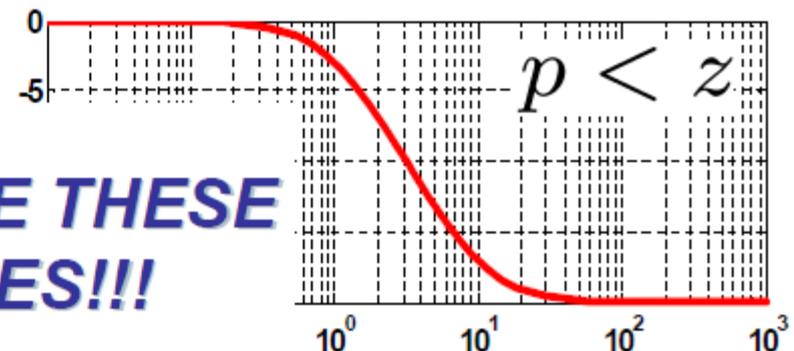
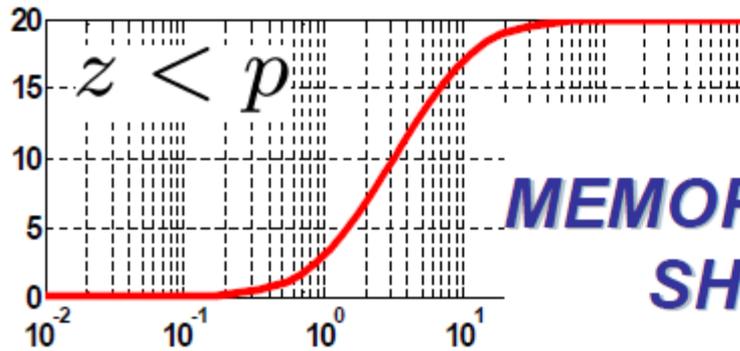


Bode plots of lead/lag C(s) (review)

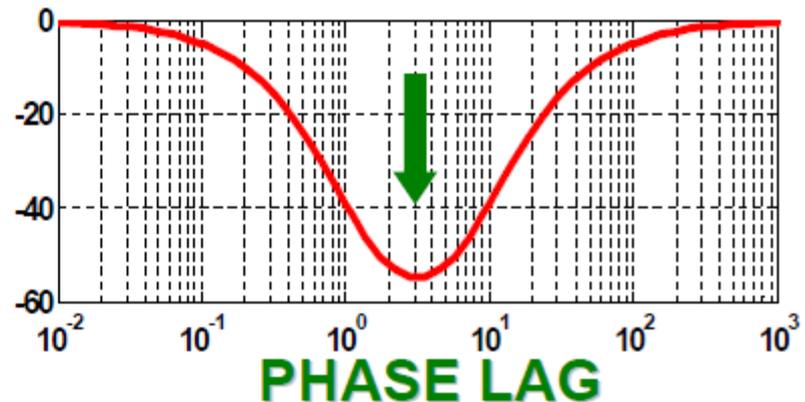
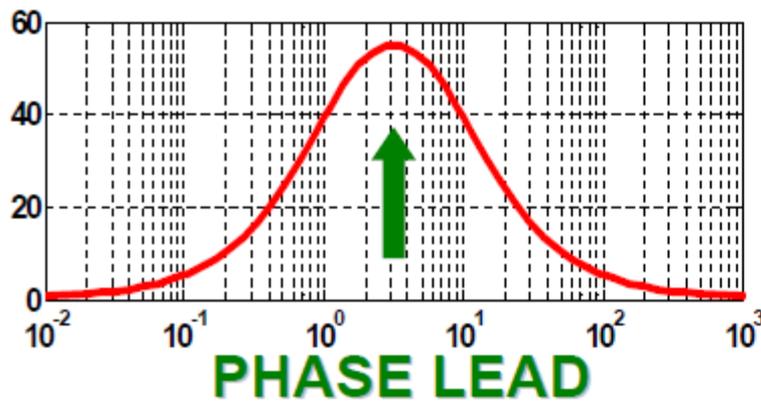
$$C(s) = \frac{\frac{s}{z} + 1}{\frac{s}{p} + 1}$$

Lead compensator

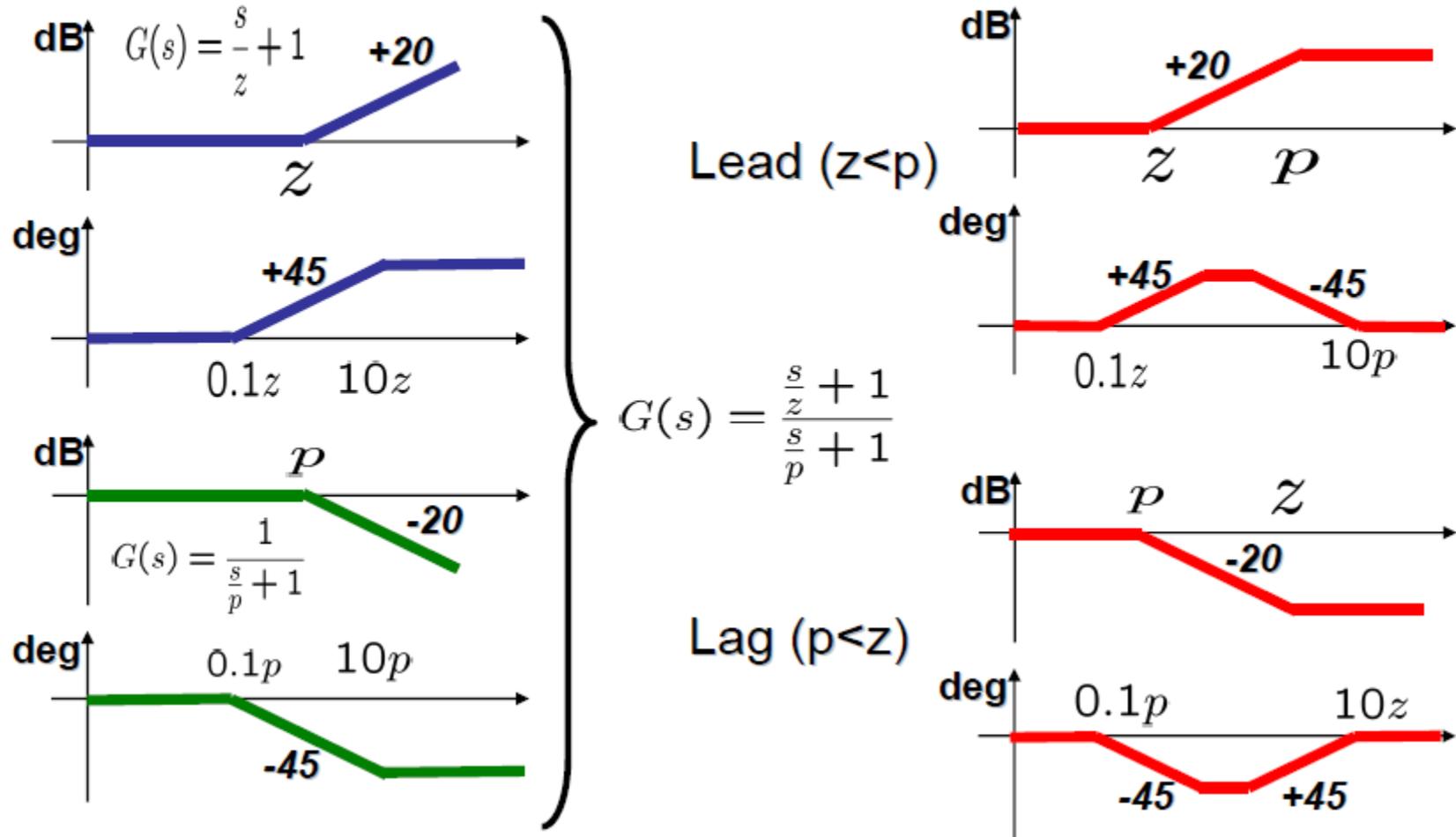
Lag compensator



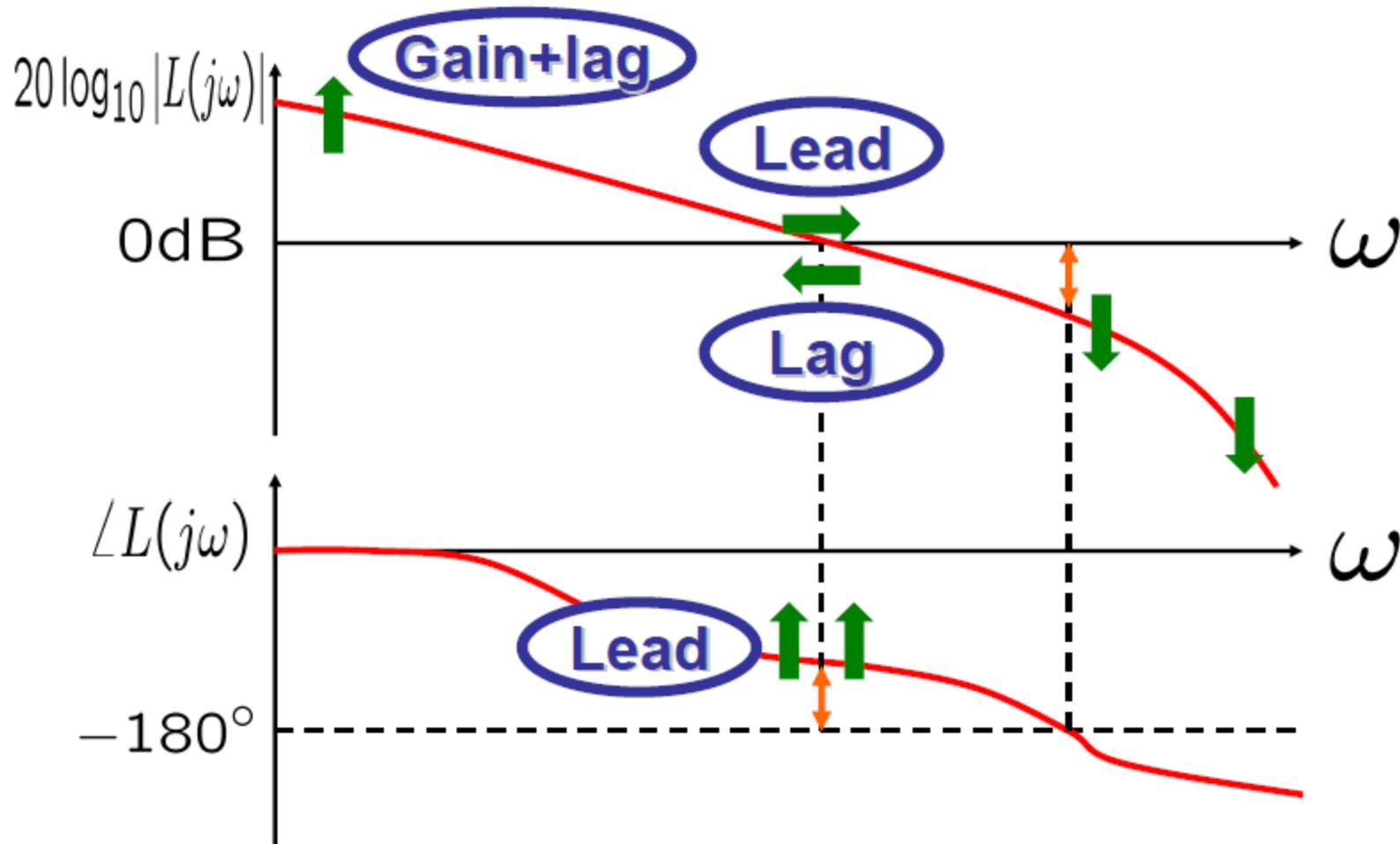
MEMORIZE THESE SHAPES!!!



Straight-line approximations



Guideline of lead/lag design (review)



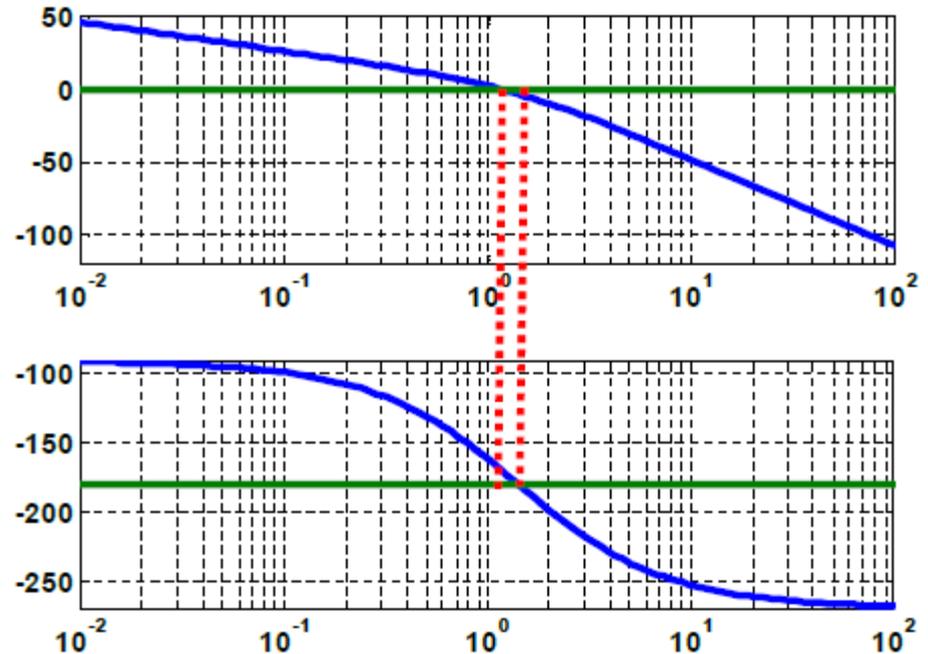
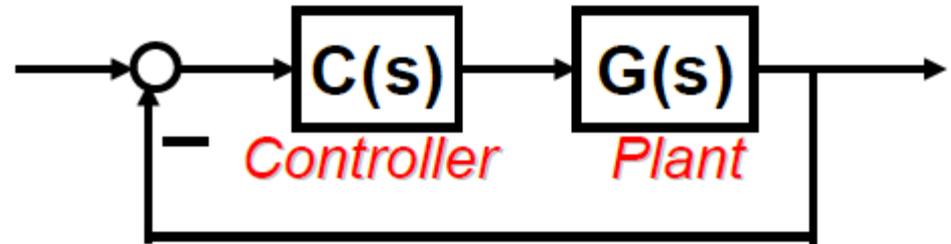
An example

- Consider a system

$$G(s) = \frac{4}{s(s+1)(s+2)}$$

- Analysis for $C(s)=1$
 - Stable
 - PM at least 12 deg
 - GM at least 3.5 dB

These values are too small for good transient response!



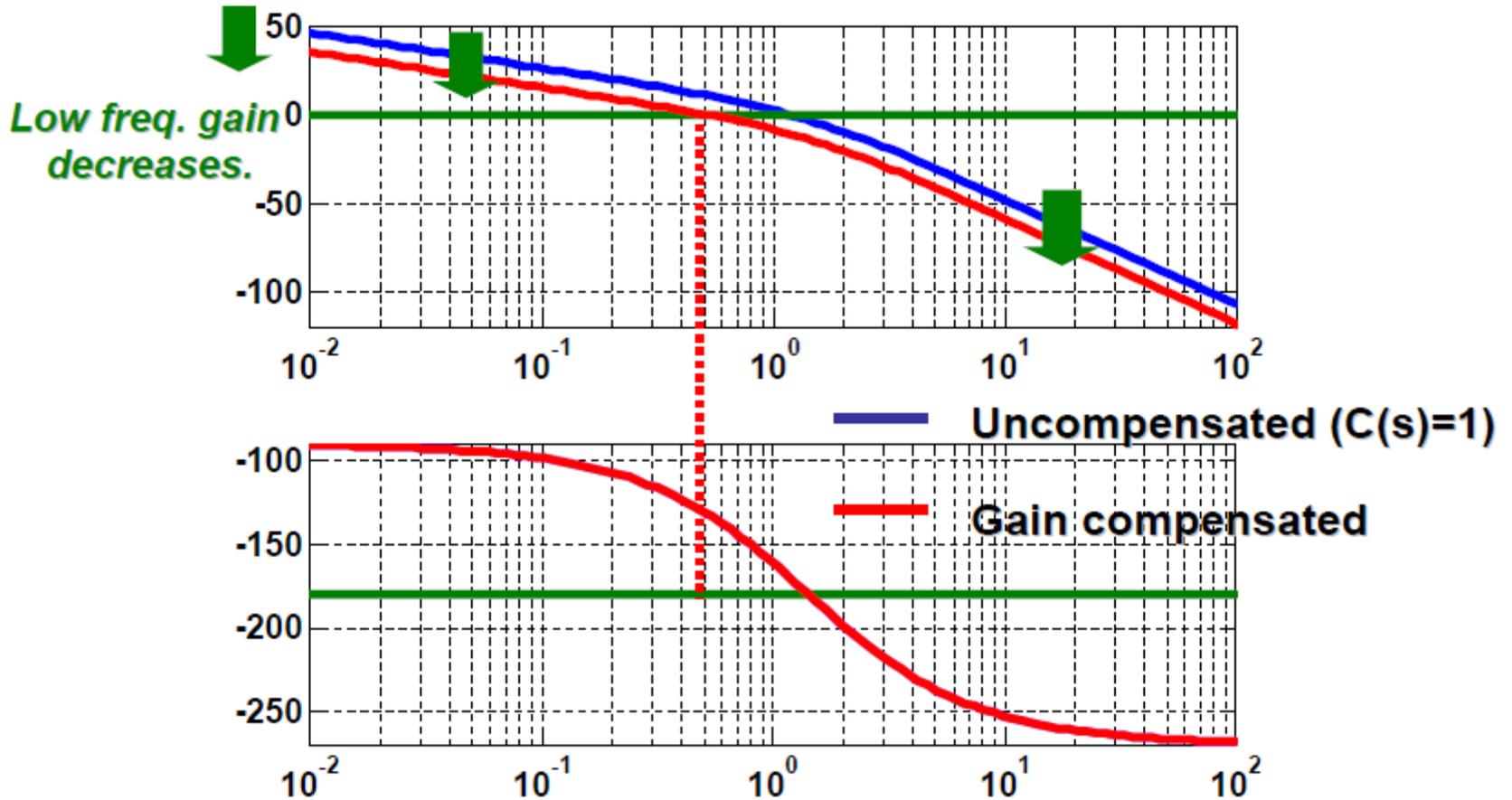
Gain compensation

- PM is specified to be 50 deg.
- In this example, to **increase PM** by gain compensation, we need to lower the gain curve.

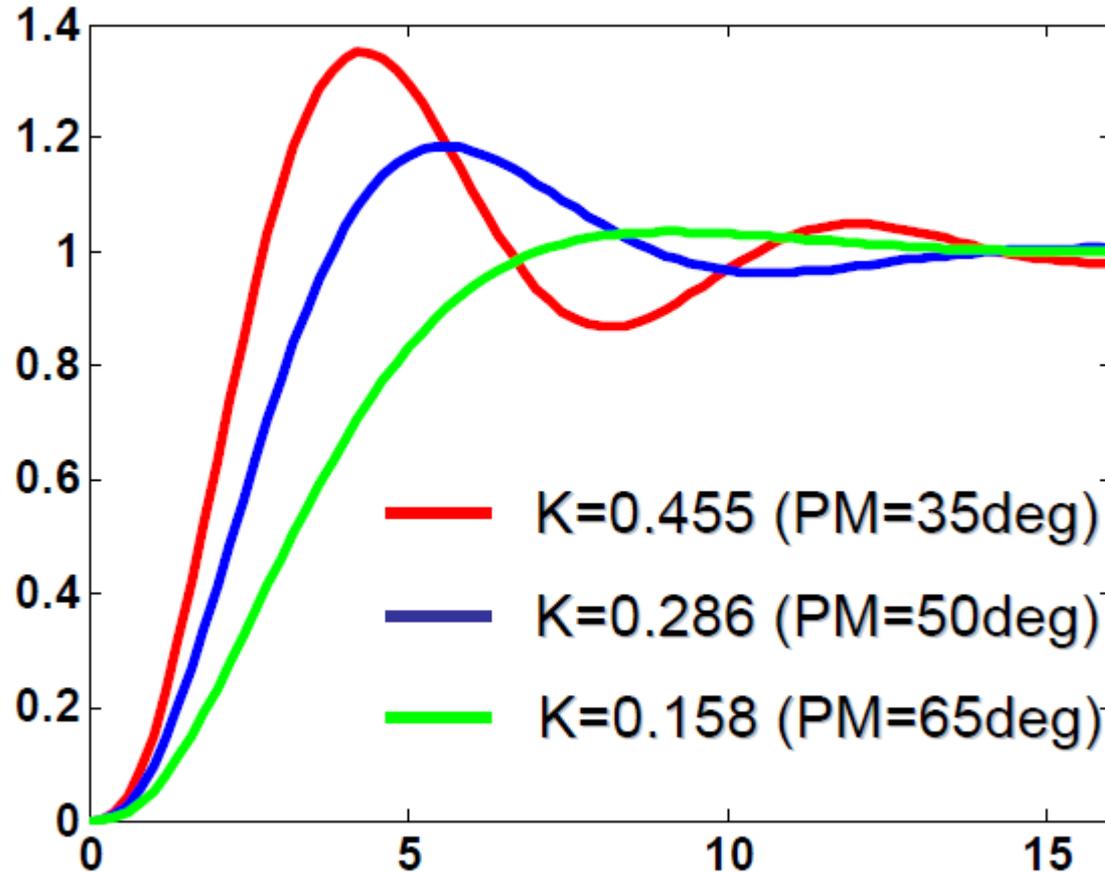
K	PM deg	Overshoot %	ω_g rad/sec	Rise time sec
0.455	35	35	0.7	1.7
0.286	50	18	0.5	2.4
0.158	65	3.7	0.3	4.5



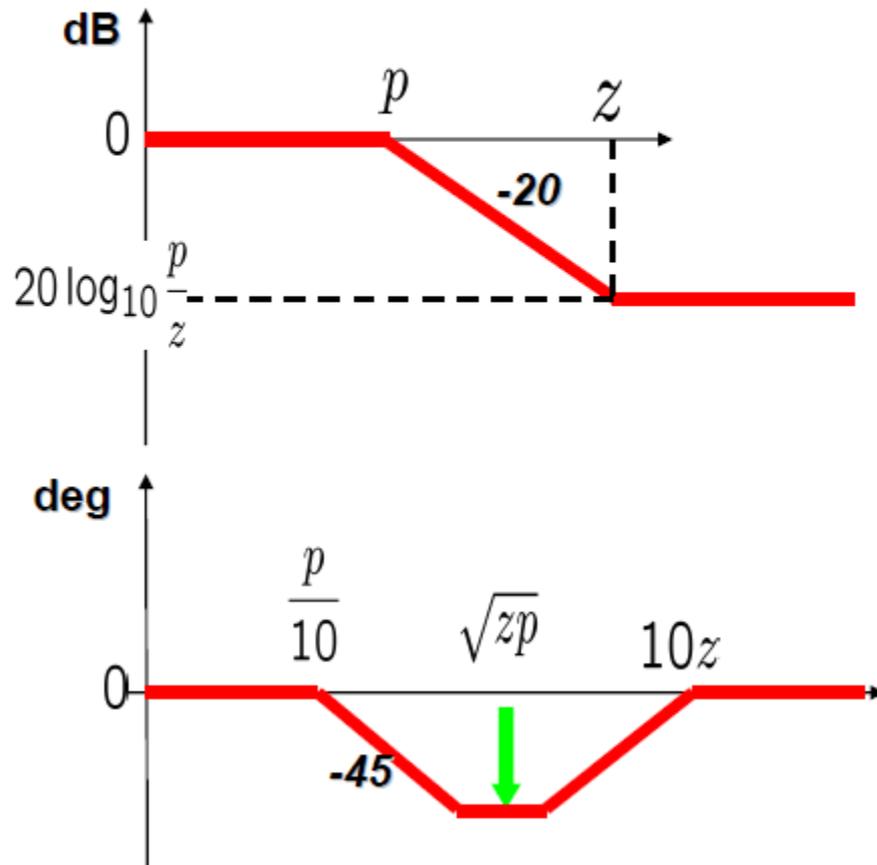
Bode plot for $C(s)=0.286$



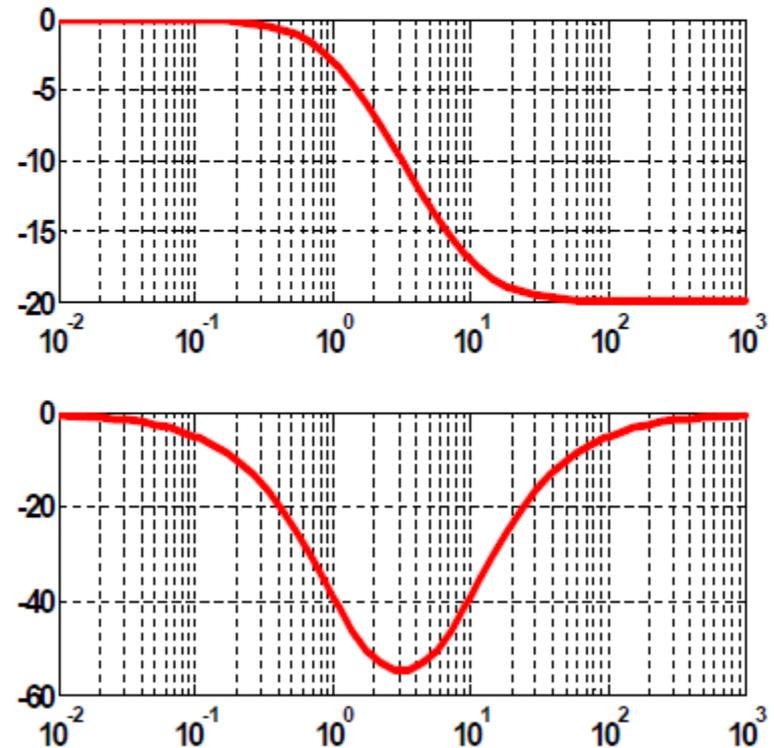
Step responses



Phase-lag compensator (review)



$$G(s) = \frac{\frac{s}{z} + 1}{\frac{s}{p} + 1}, \quad 0 < p < z$$



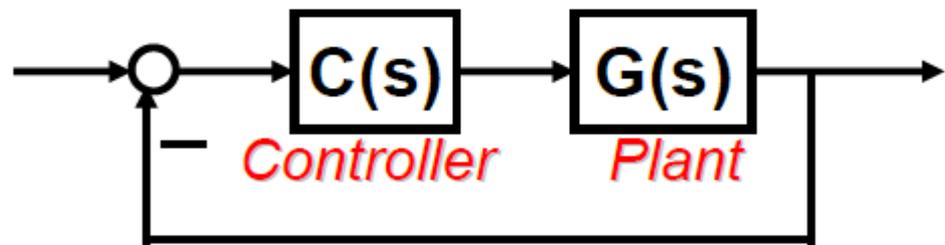
Phase-lag C(s) design

We try to design phase-lag C(s) which gives

- PM 50deg
- Low frequency gain same as the original plant.

1. To satisfy low frequency requirement, adjust DC gain of OL system by a constant gain K.

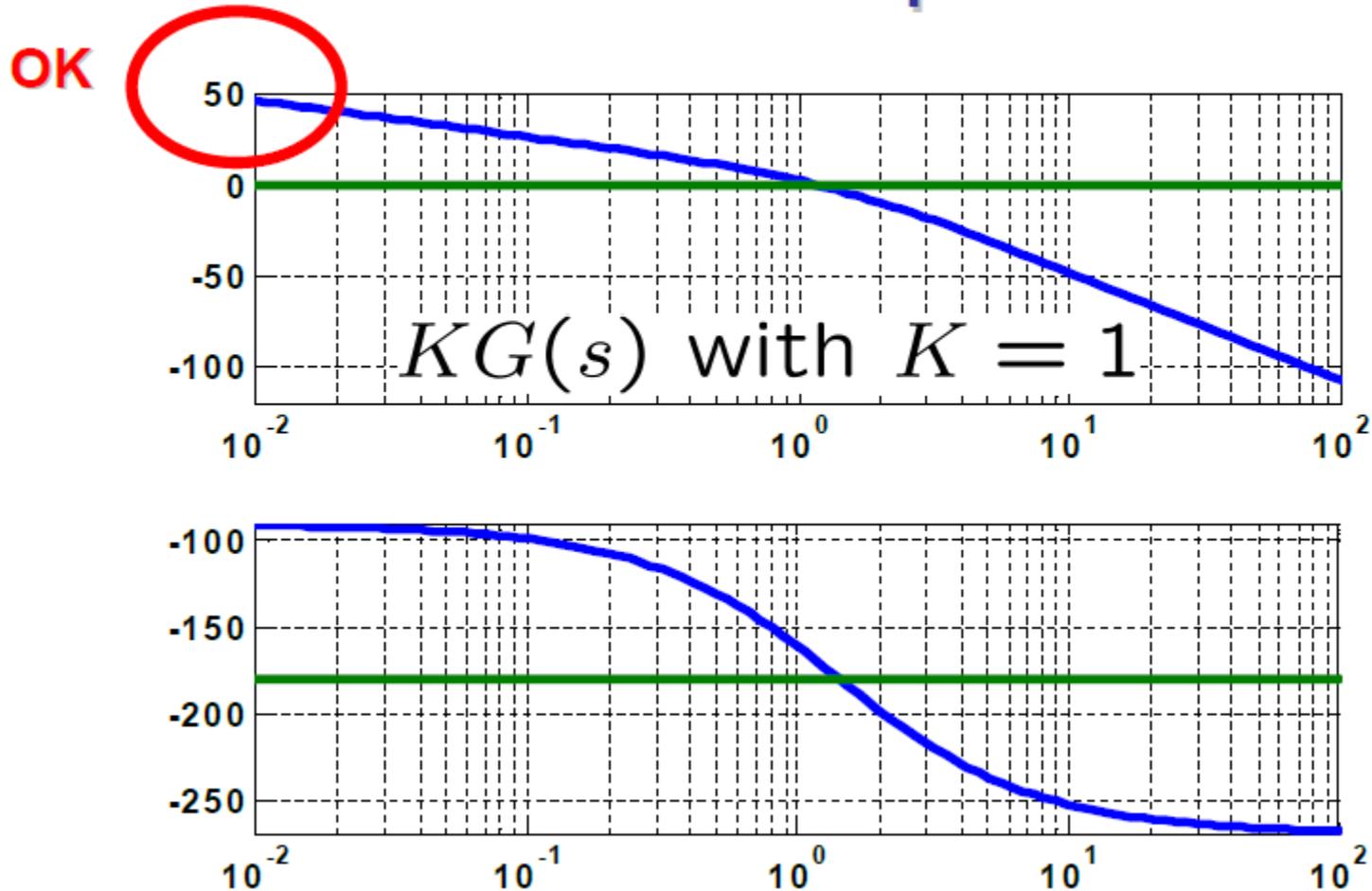
- Analysis for C(s)=1
 - Stable
 - PM at least 12 deg
 - GM at least 3.5 dB



$$G(s) = \frac{4}{s(s+1)(s+2)}$$



After Step 1



Phase-lag C(s) design

2. Find the frequency ω_g (which will become gain crossover frequency after compensation) where

$$\angle G(j\omega_g) = -180^\circ + \phi_m + 5^\circ, \quad \phi_m : \text{required PM}$$

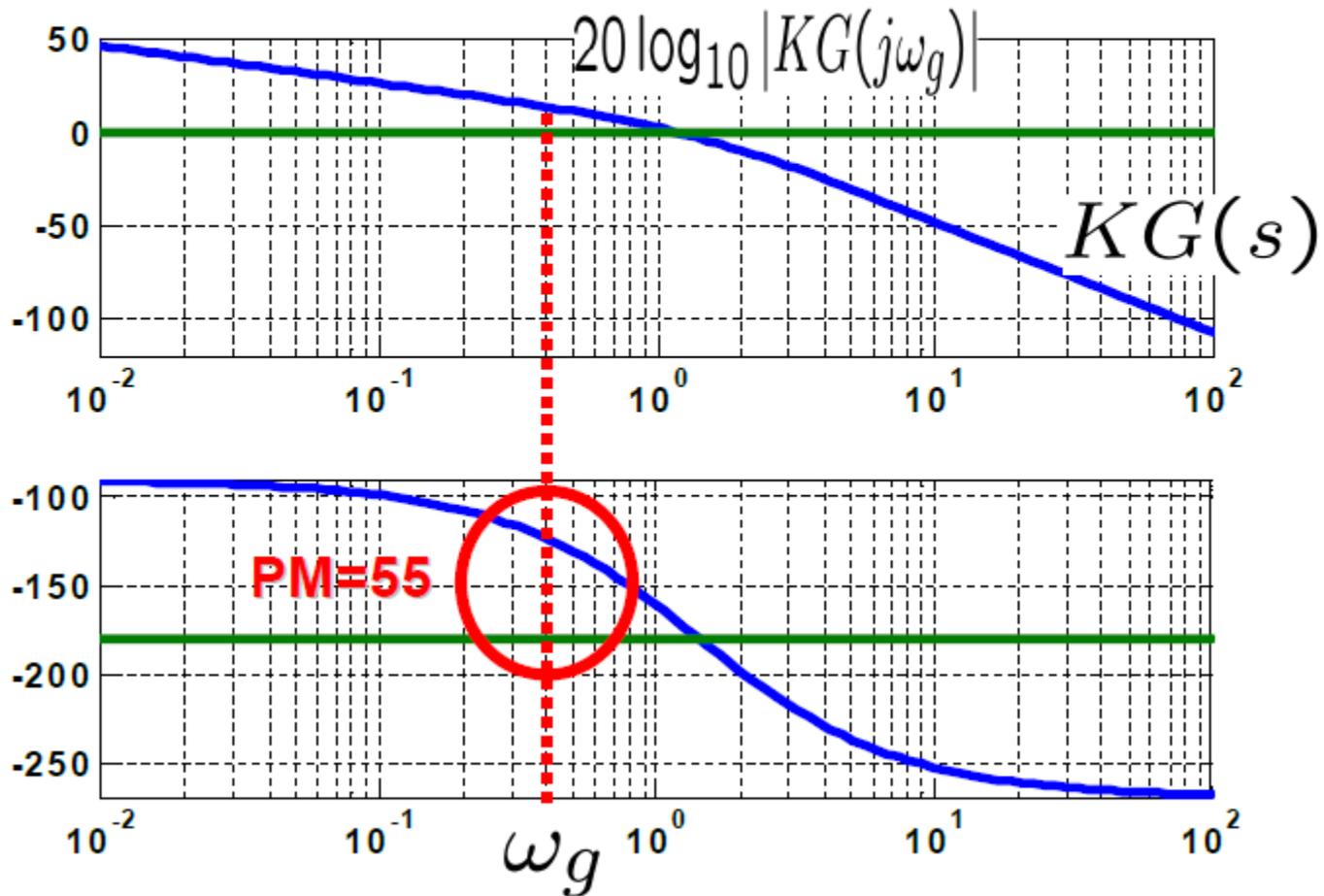
In this example,

$$\angle G(j\omega_g) = -180^\circ + \underbrace{50^\circ}_{\phi_m} + 5^\circ = -125^\circ \quad \longrightarrow \quad \omega_g = 0.4$$

Note: The reason of +5 deg is explained later.



After Step 2



Phase-lag C(s) design

3. Set z and p as

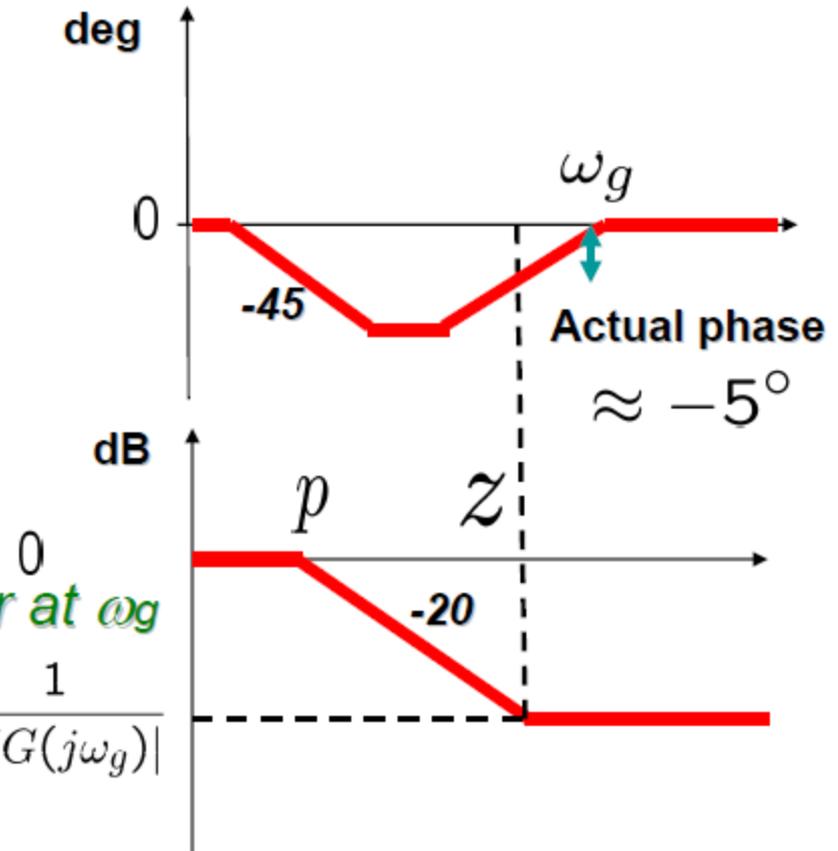
$$z = 0.1\omega_g (= 0.04)$$

For small phase lag at ω_g

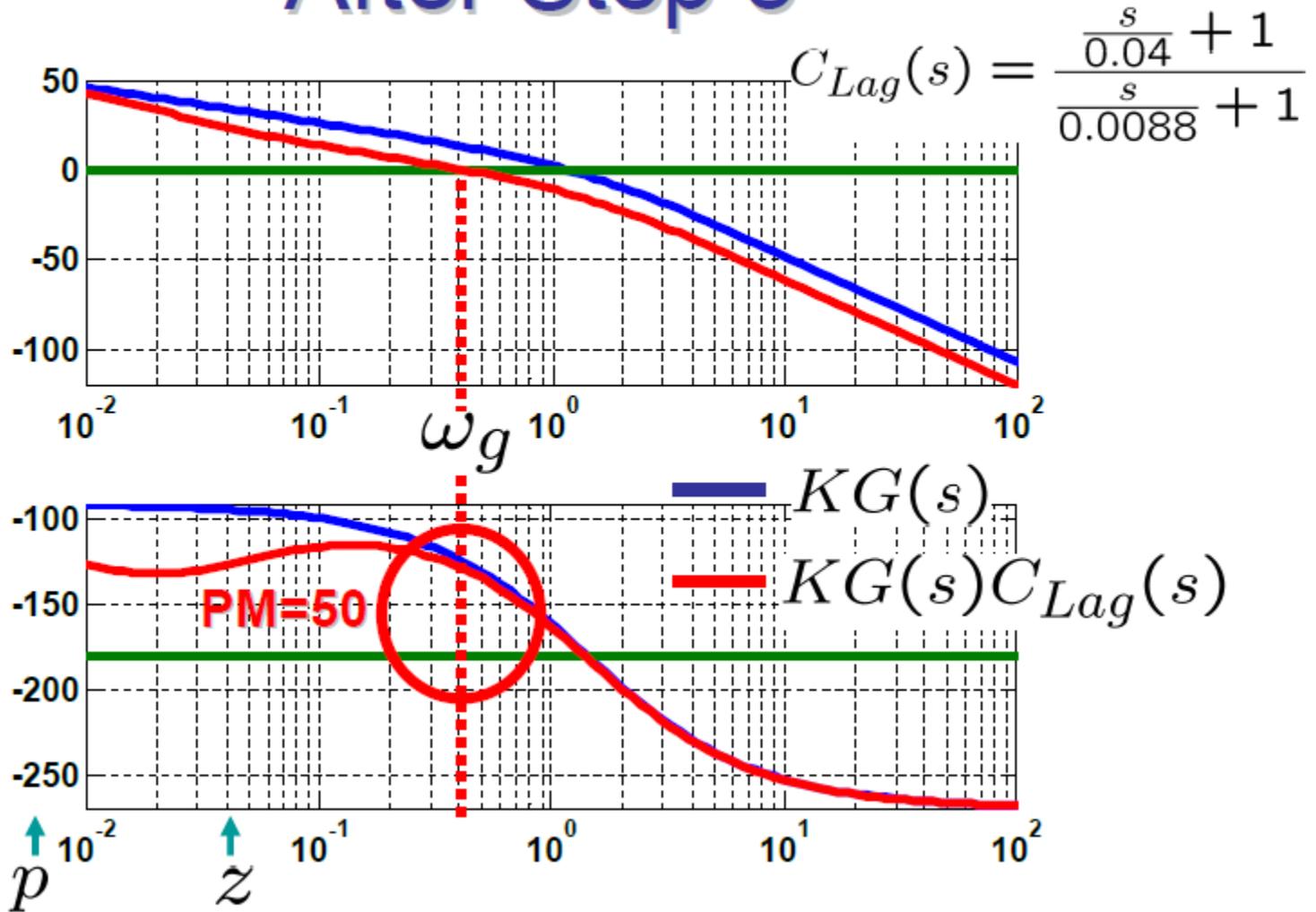
$$p = \frac{0.1\omega_g}{|KG(j\omega_g)|} \left(= \frac{0.04}{4.55} \right)$$

For setting new gain crossover at ω_g

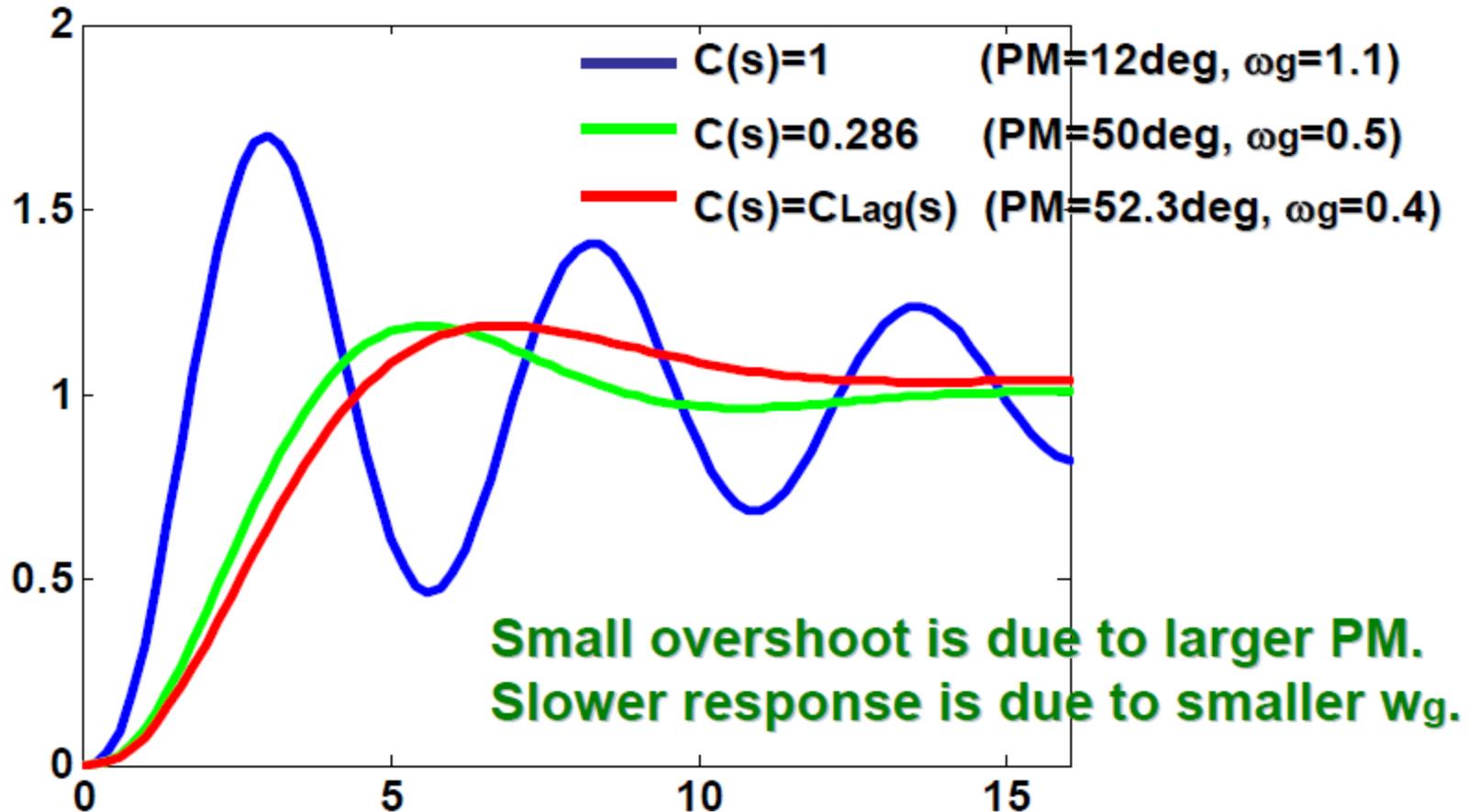
$$20 \log_{10} \frac{p}{z} = 20 \log_{10} \frac{1}{|KG(j\omega_g)|}$$



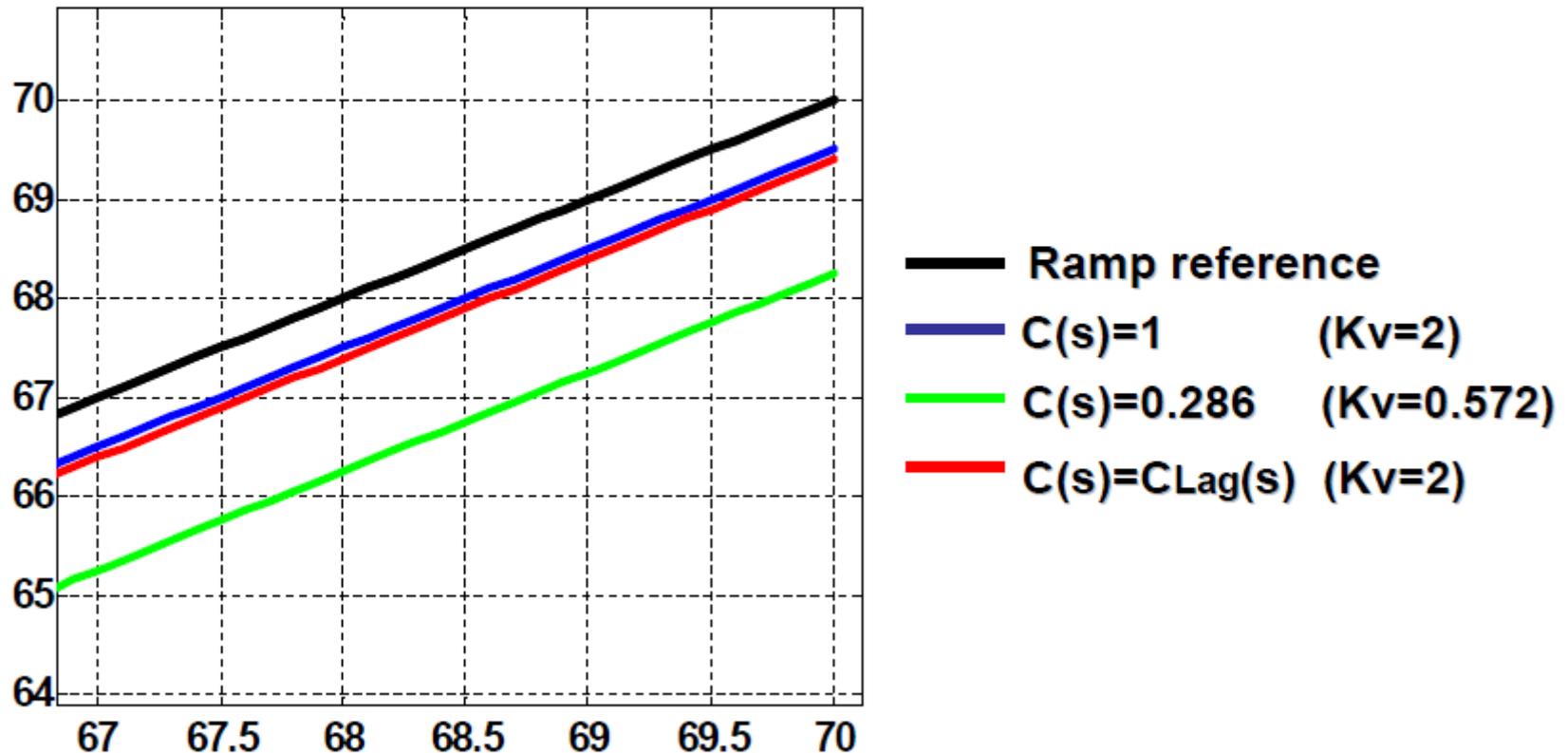
After Step 3



Step responses



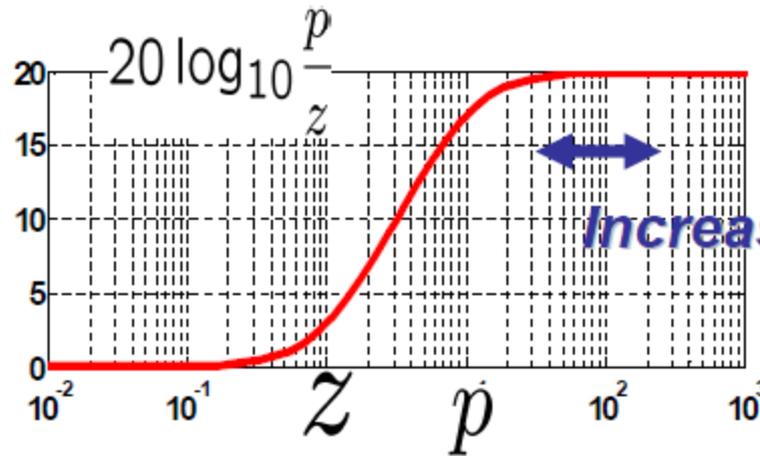
Ramp responses



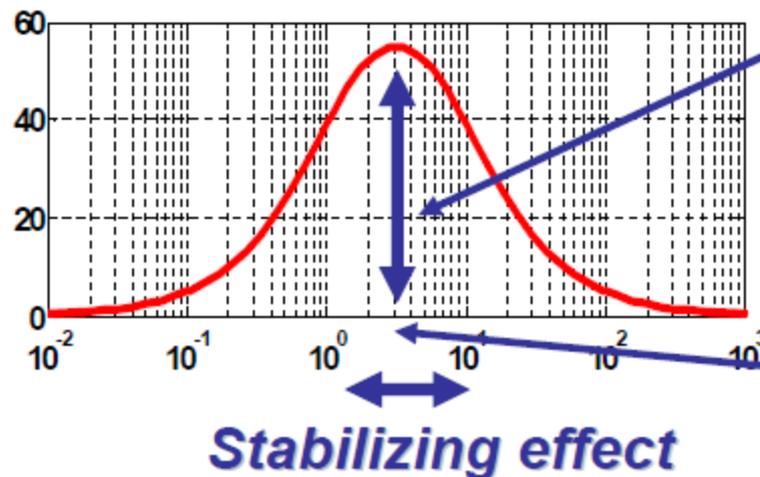
Smaller steady-state error is due to larger K_v .



Phase-lead compensator (review)



$$G(s) = \frac{\frac{s}{z} + 1}{\frac{s}{p} + 1}, \quad 0 < z < p$$



$$\theta_m := \sin^{-1} \frac{\frac{p}{z} - 1}{\frac{p}{z} + 1}$$



$$\frac{p}{z} = \frac{1 + \sin \theta_m}{1 - \sin \theta_m}$$

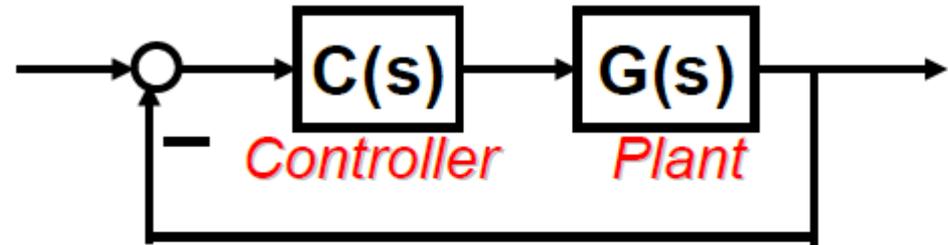
Max phase lead at \sqrt{zp}



An example: Revisited

- Consider a system

$$G(s) = \frac{4}{s(s+1)(s+2)}$$



- Specs
 - Stable
 - PM at least 50 deg
 - Settling time < 4s
- Note large settling time by previous (gain and lag compensator) designs

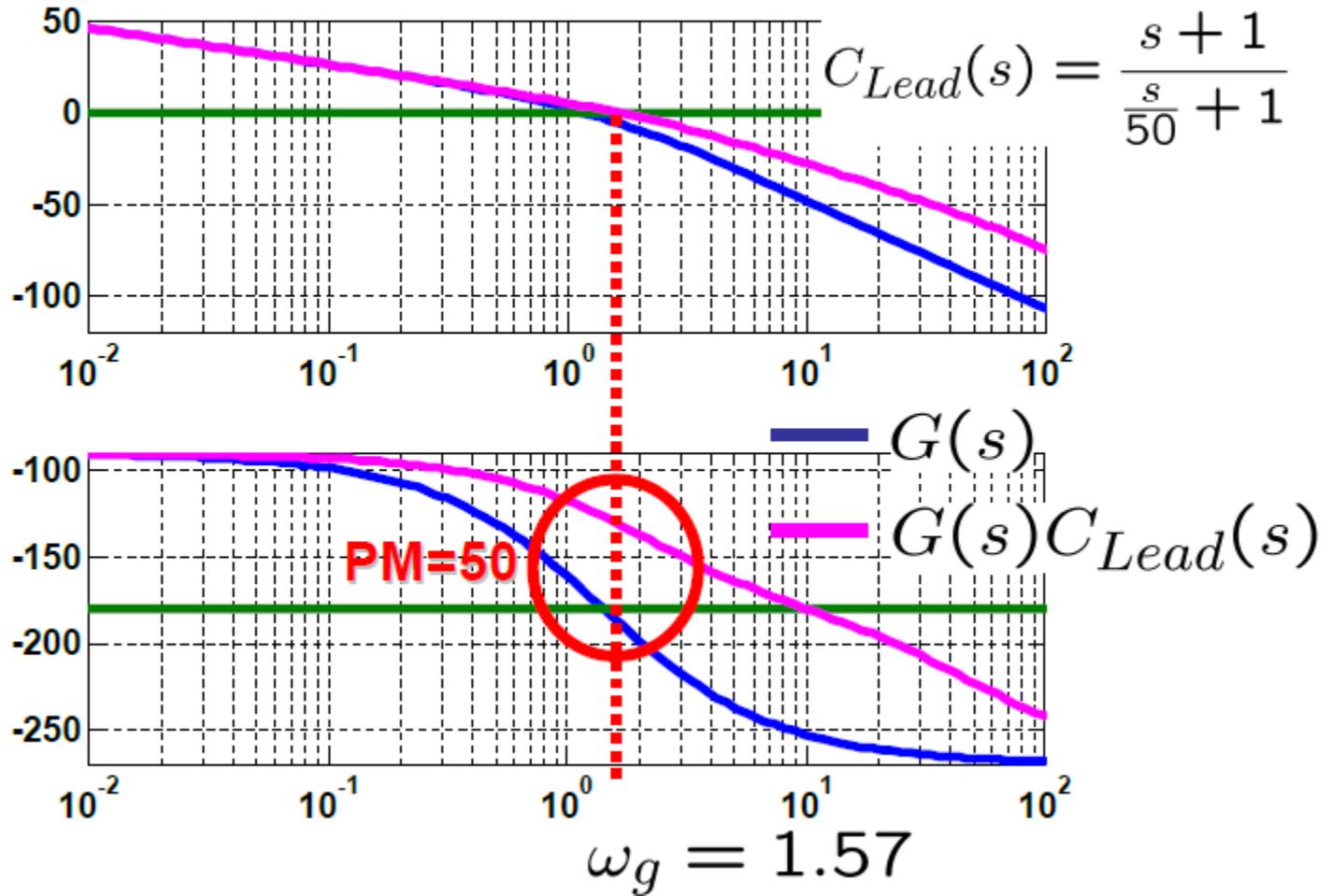


Phase-lead design procedure

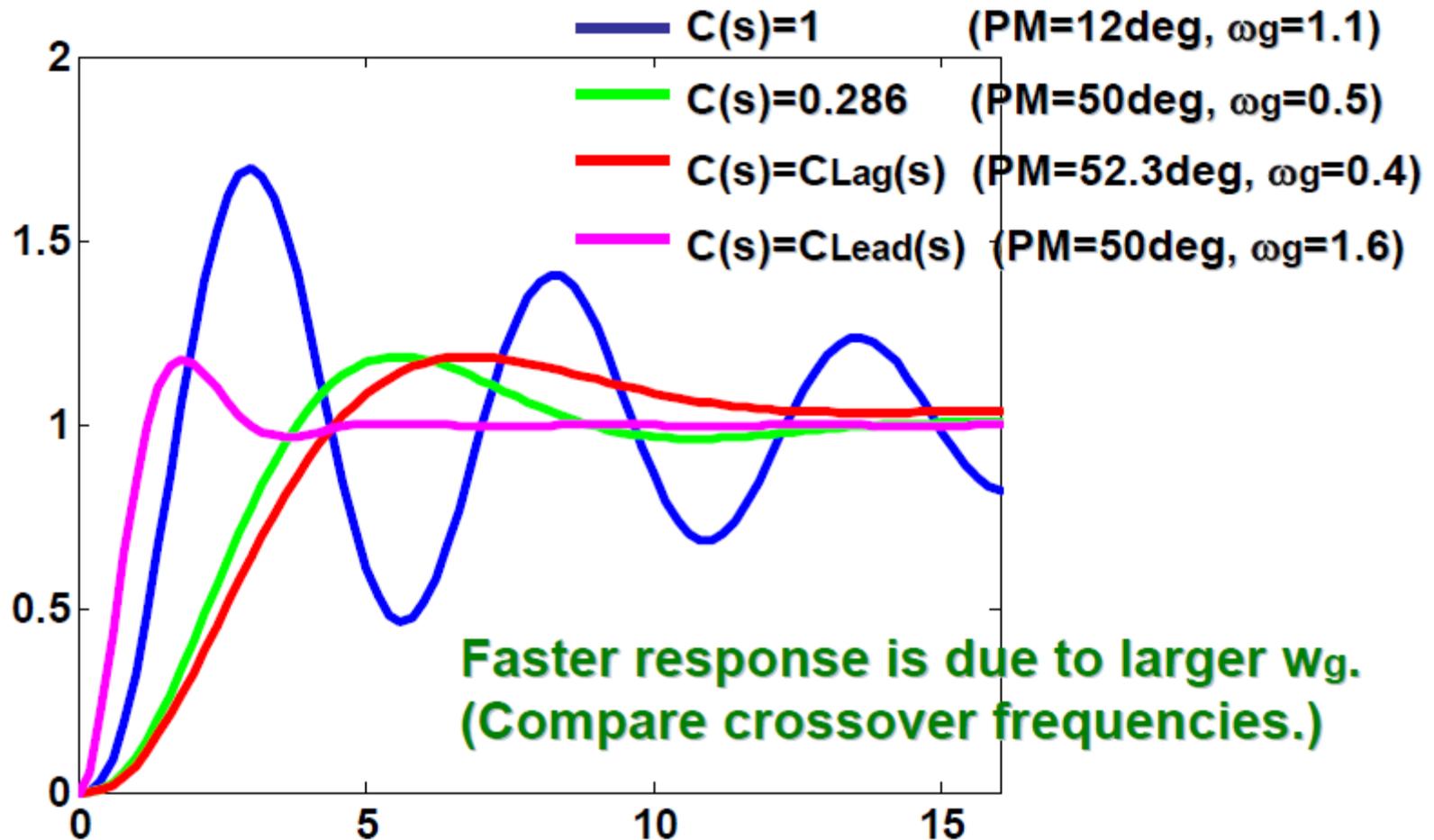
1. Select z near uncompensated ω_g .
In the example, $\omega_g=1.14$. So, select, for example, $z=1$.
2. Select $p>z$ by **trial-and-error**.
3. Check PM and settling time. If not satisfactory, move the pole p . If moving pole does not give the desired results, try to move the zero z .



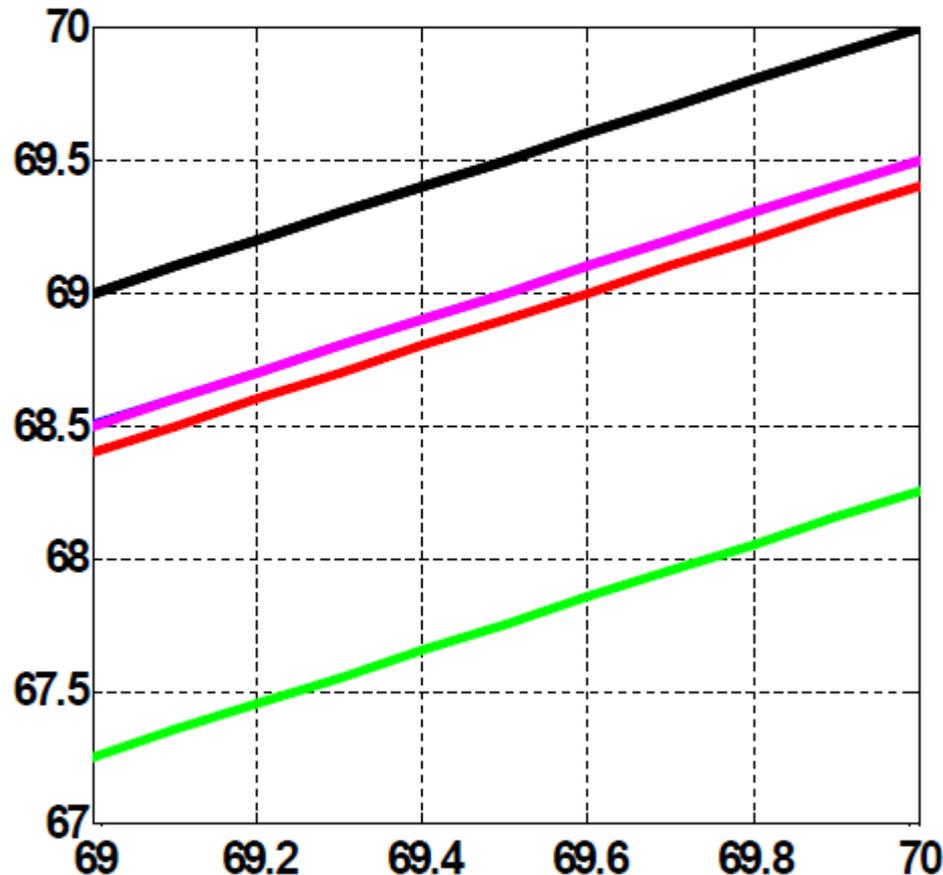
After lead C(s) design



Step responses



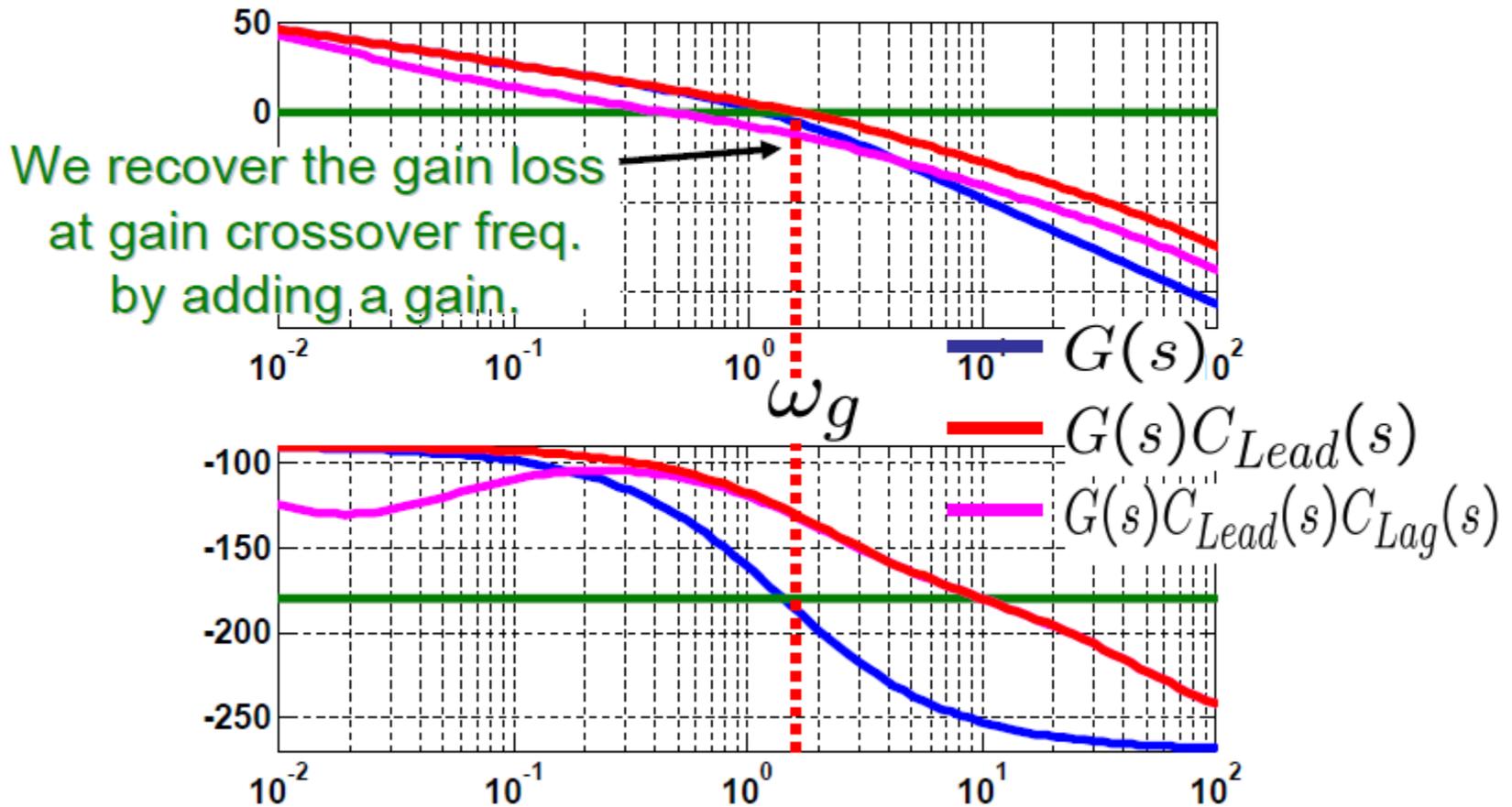
Ramp responses



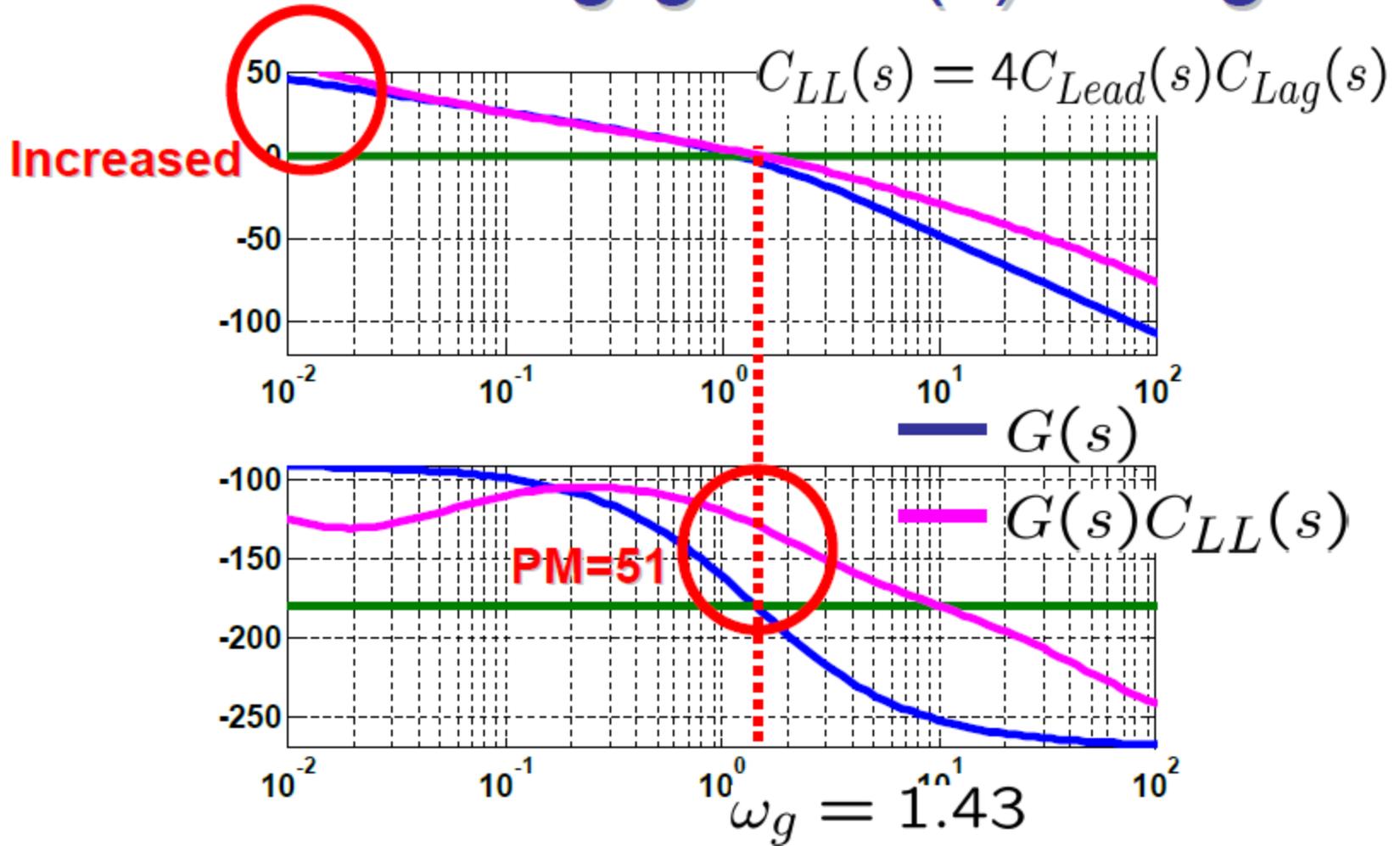
- Ramp reference**
- $C(s)=1$ ($K_v=2$)**
- $C(s)=0.286$ ($K_v=0.572$)**
- $C(s)=C_{Lag}(s)$ ($K_v=2$)**
- $C(s)=C_{Lead}(s)$ ($K_v=2$)**



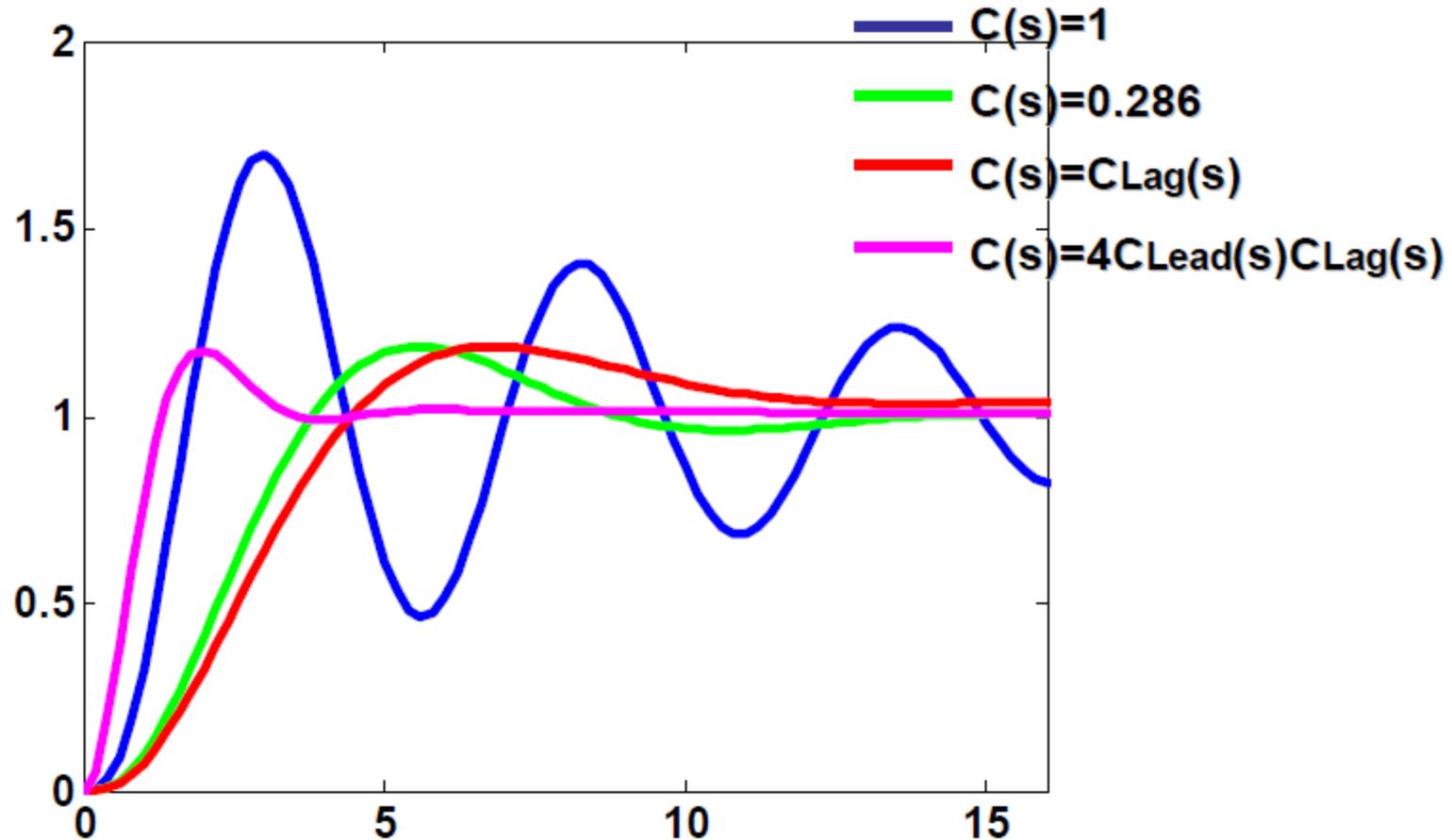
After lead-lag C(s) design



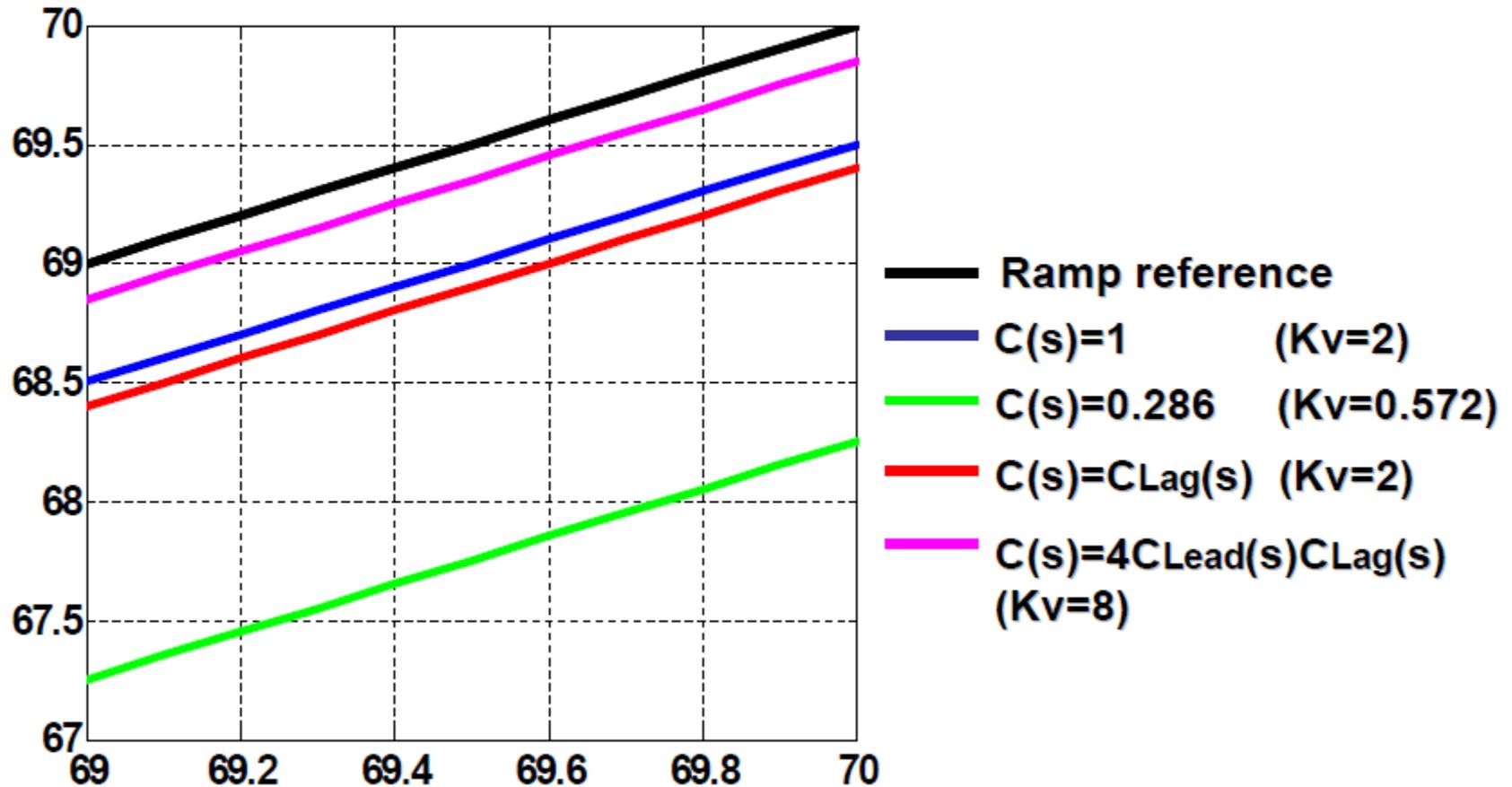
After lead-lag-gain $C(s)$ design



Step responses (Lead-lag)



Ramp responses (Lead-lag)



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Modeling

- ✓ Laplace transform
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- ✓ Models for systems
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Analysis

- ✓ Time response
 - ✓ Transient
 - ✓ Steady state
- ⇨ ✓ Frequency response
 - ✓ Bode plot
- ✓ Stability
 - ✓ Routh-Hurwitz
 - ✓ Nyquist

Design

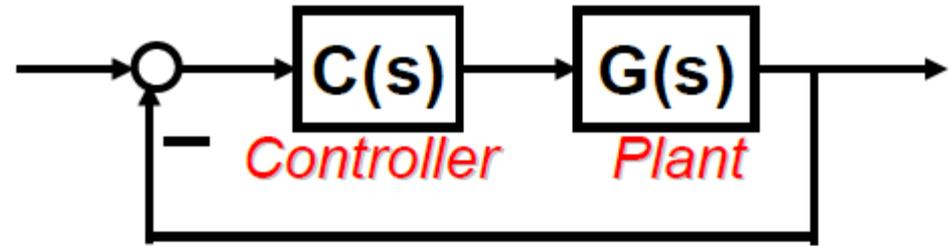
- ✓ Design specs
- ✓ Root locus
- ⇨ ✓ Frequency domain
- ✓ PID & Lead-lag
- Design examples



An example: Revisited

- Consider a system

$$G(s) = \frac{4}{s(s+1)(s+2)}$$



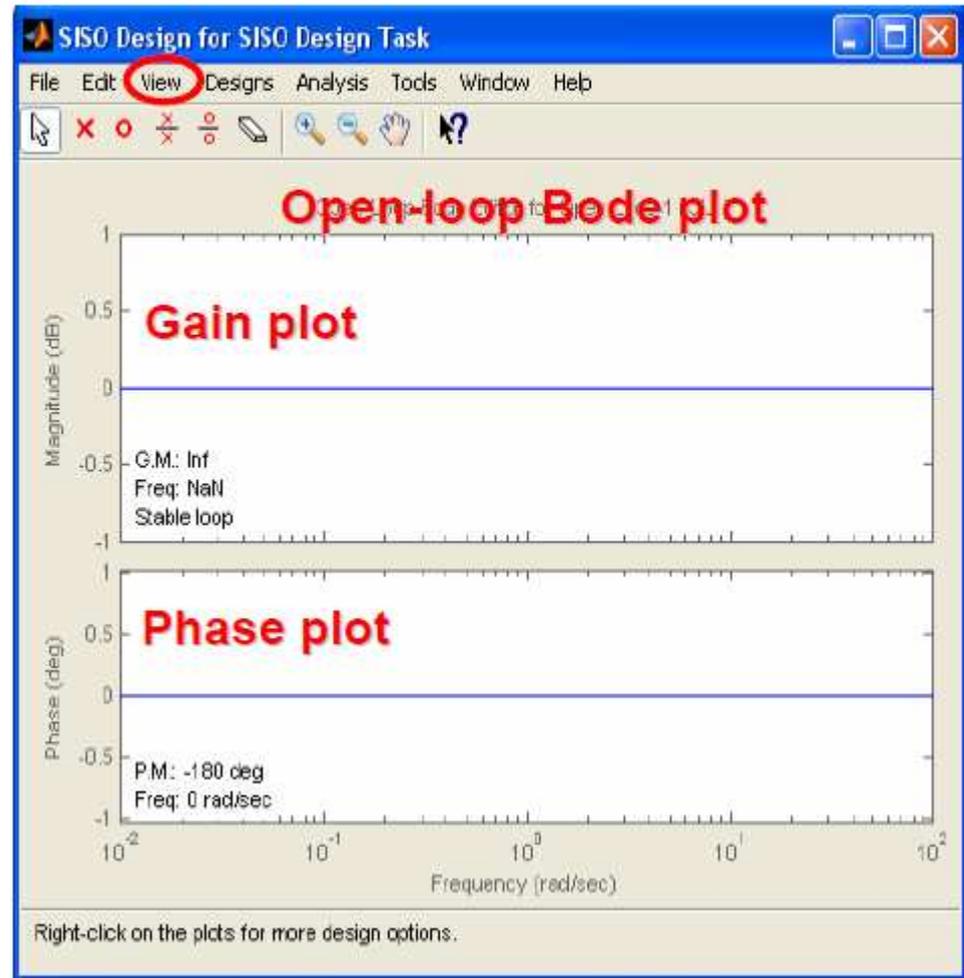
- Specs

- Stable
- PM at least 50 deg
- Settling time < 4s
- Steady-state error
 - For unit step input = 0
 - For unit ramp input < 0.3



SISO Design Tool in Matlab

- Graphical-user interface (GUI) that allows you to design compensators.
- Type “sisotool” in Matlab prompt.
`>> sisotool`
- Select “Open-Loop Bode” from *View* → *Design Plots* configuration.

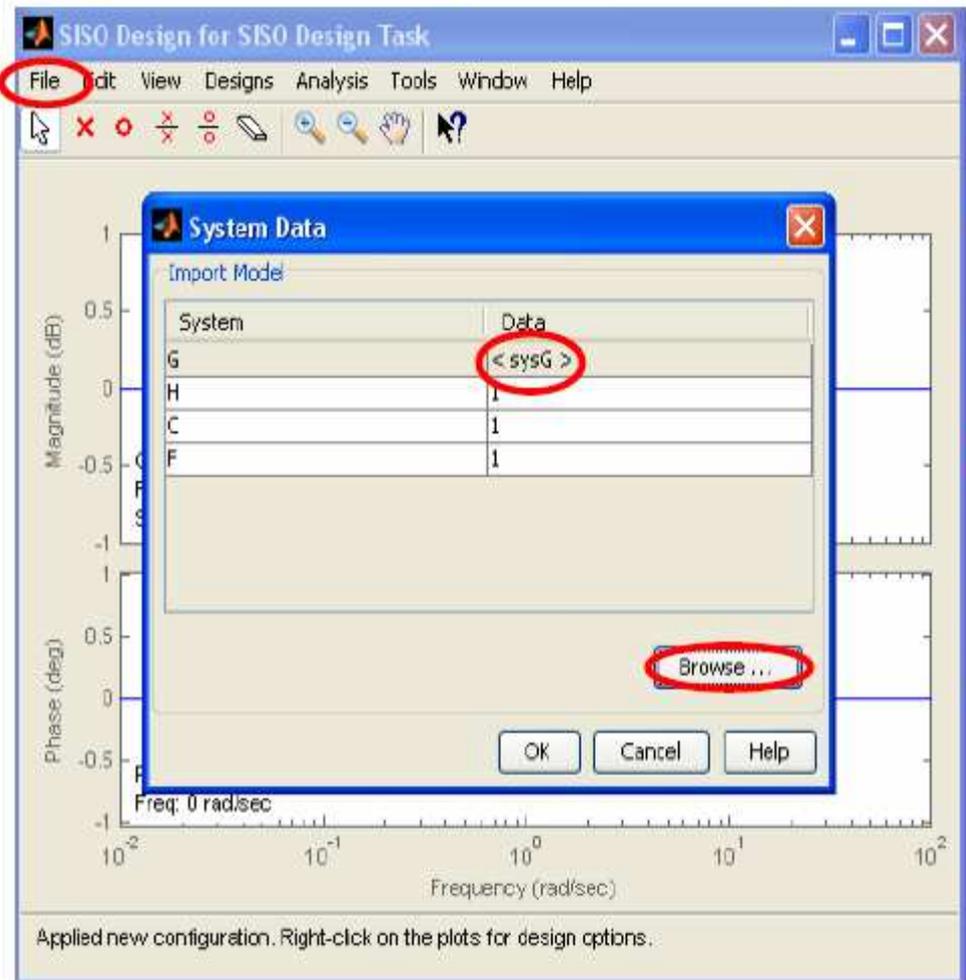


SISO Design Tool (cont'd)

- Input the plant

```
>> num=4;  
>> den=[1 3 2 0];  
>> sysG=tf(num,den);
```

- Import the plant *sysG* from *File* → *Import*

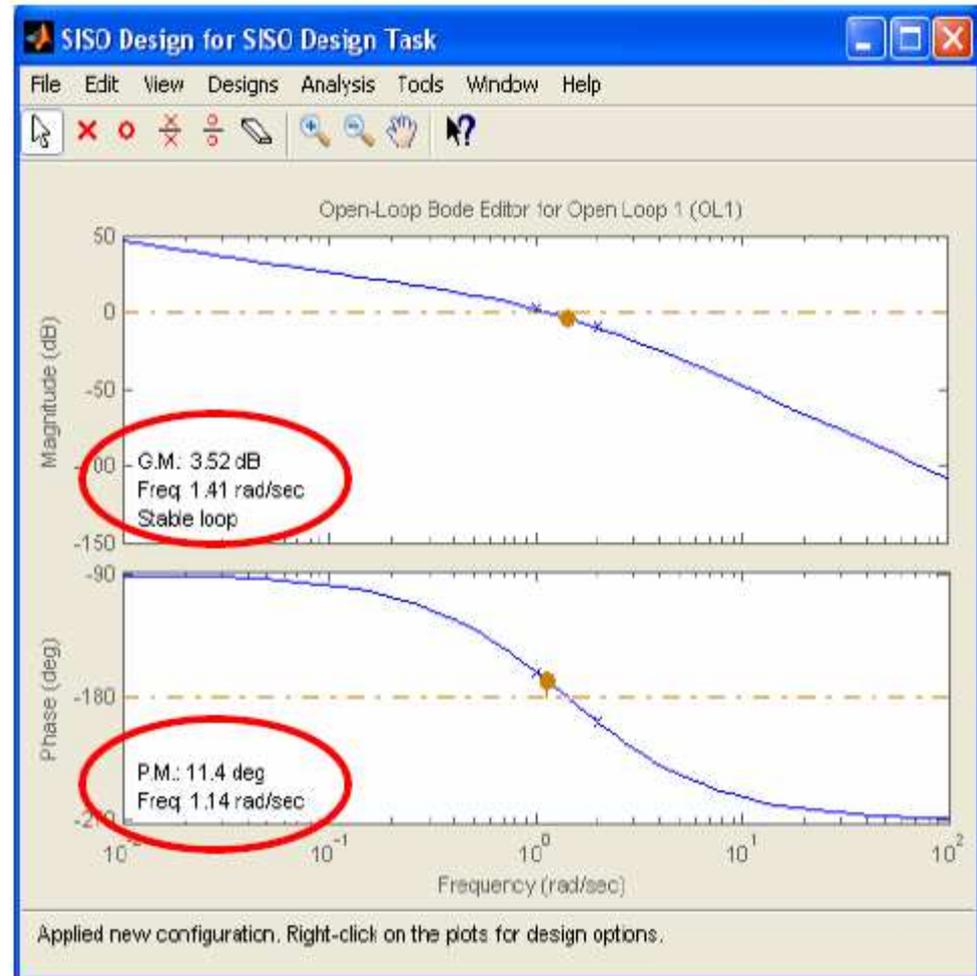


SISO Design Tool (cont'd)

- You will see the Bode plots for

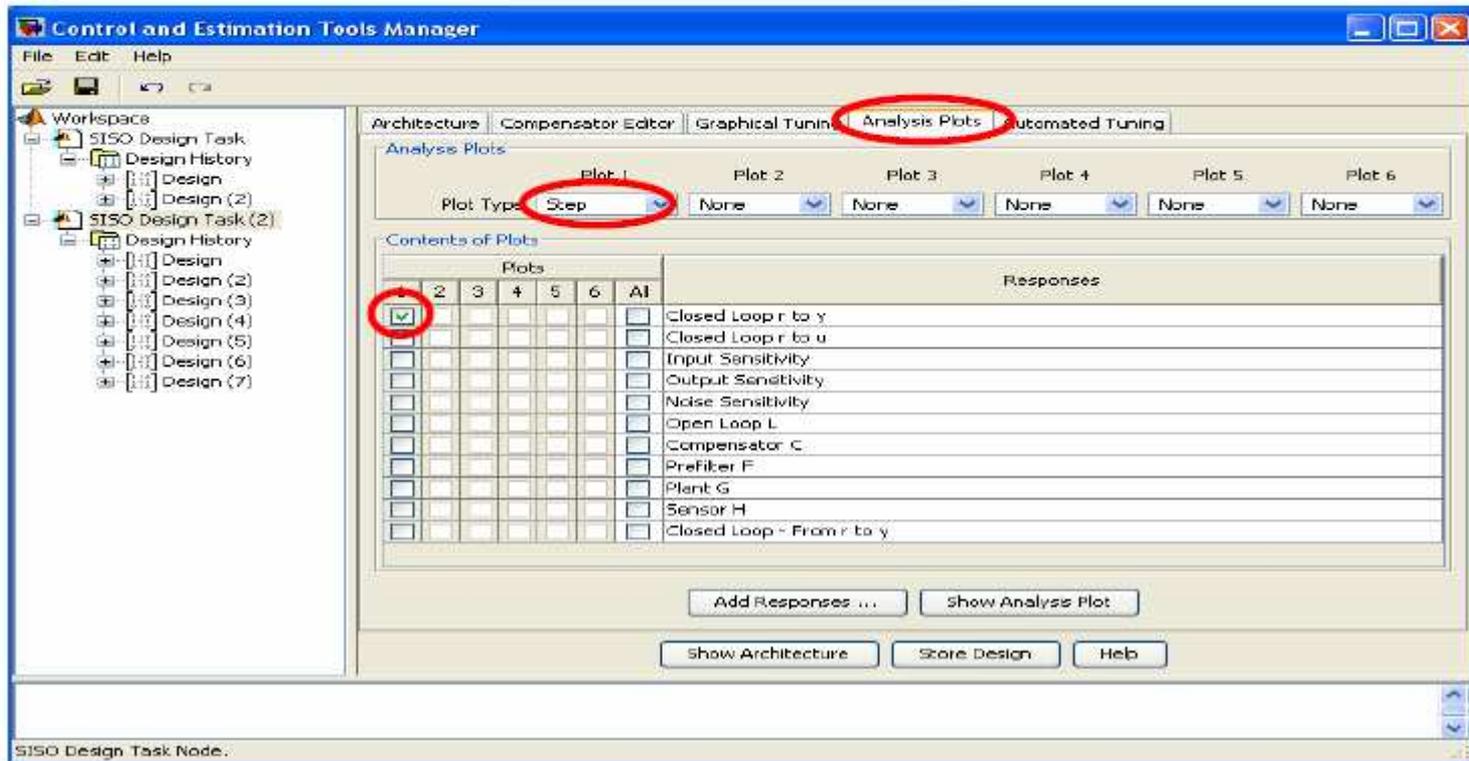
$$G(s) = \frac{4}{s(s+1)(s+2)}$$

- The default setting $C(s)=1$.
- In the red circles:
 - GM
 - PM
 - Gain & phase crossover freq.



SISO Design Tool (cont'd)

- See the step response
 - *Designs* → *Edit Compensator...* → *Analysis Plots*



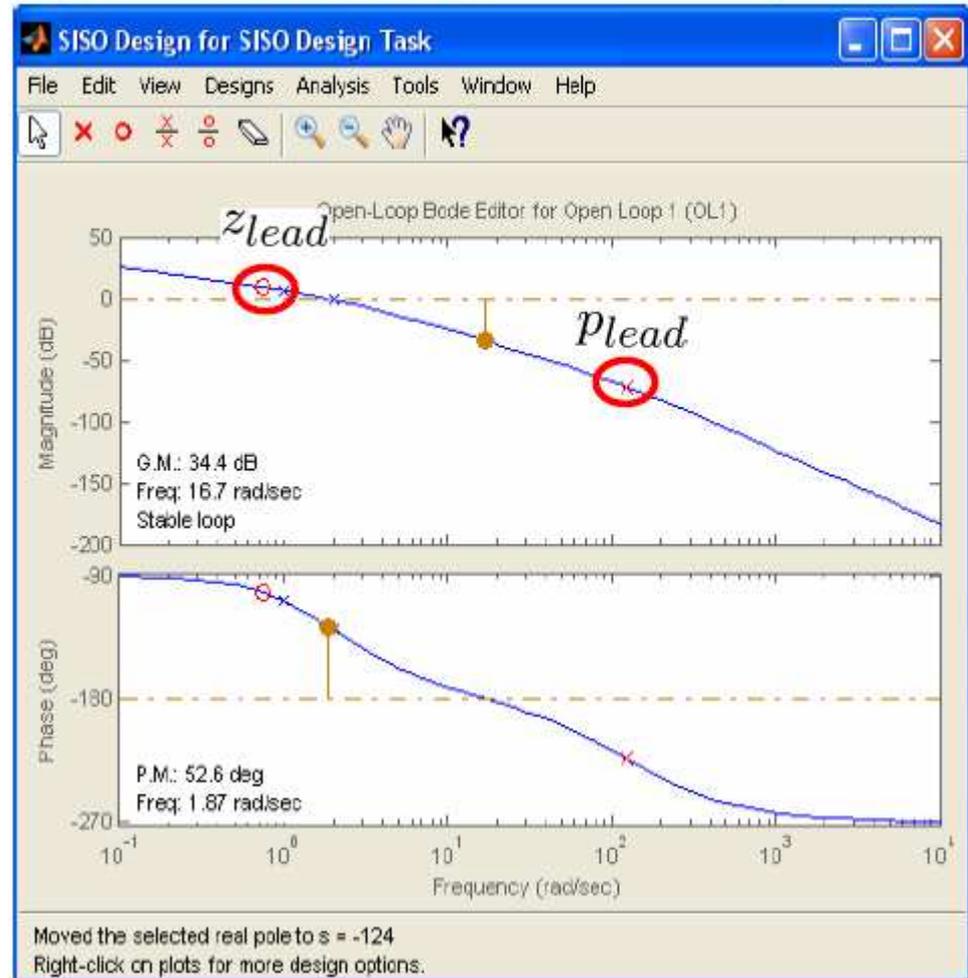
SISO Design Tool (cont'd)

- Add a pole & a zero of a compensator:

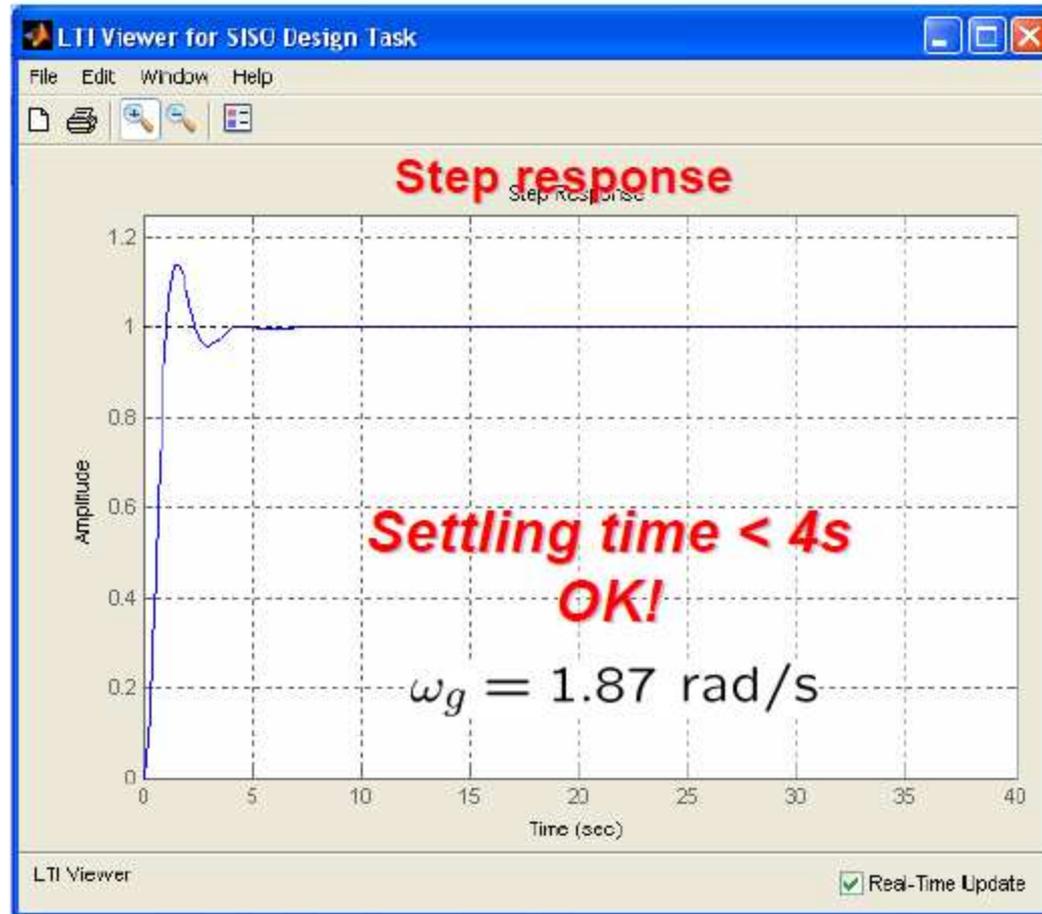
$$C_{Lead}(s) = \frac{\frac{s}{z_{lead}} + 1}{\frac{s}{p_{lead}} + 1}$$

- If necessary, move the pole and zero
 - by click-and-drag, or
 - *Design* → *Edit Compensator...*

PM > 50 degree OK!

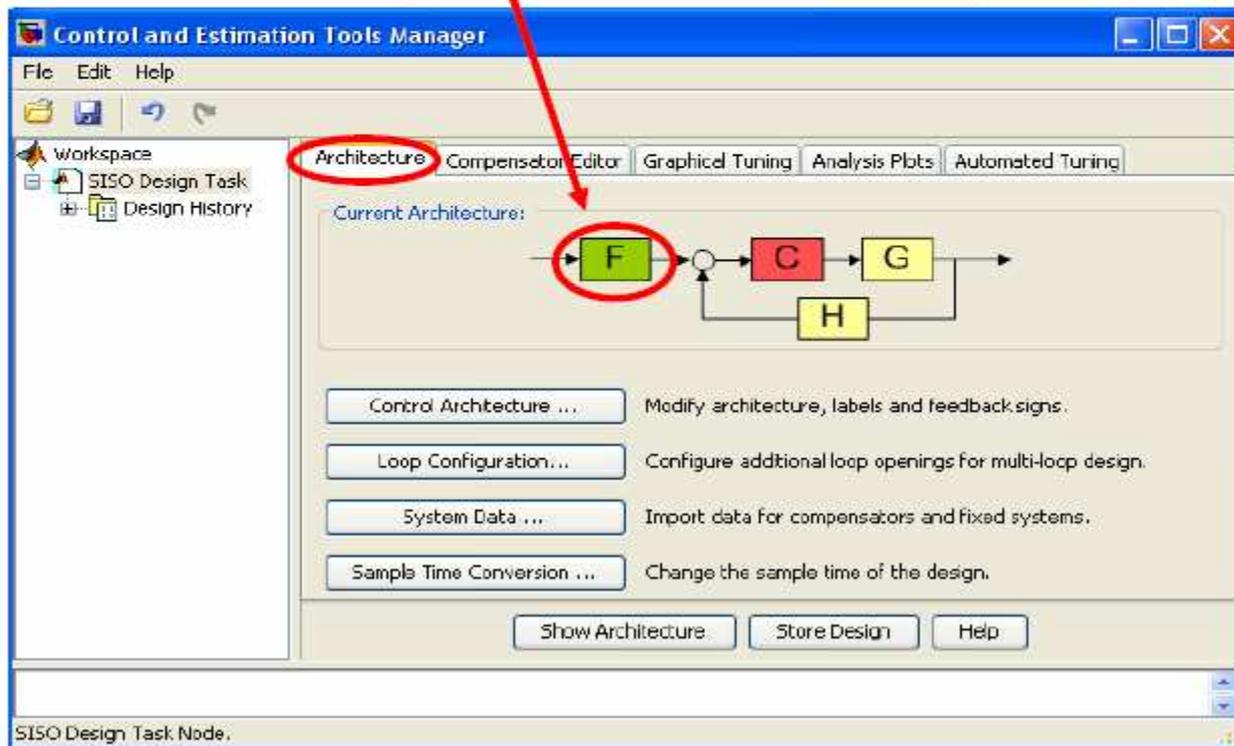


SISO Design Tool (cont'd)



SISO Design Tool (cont'd)

- Ramp response is not supported by sisotool.m.
- We use **an integrator** to show ramp response.

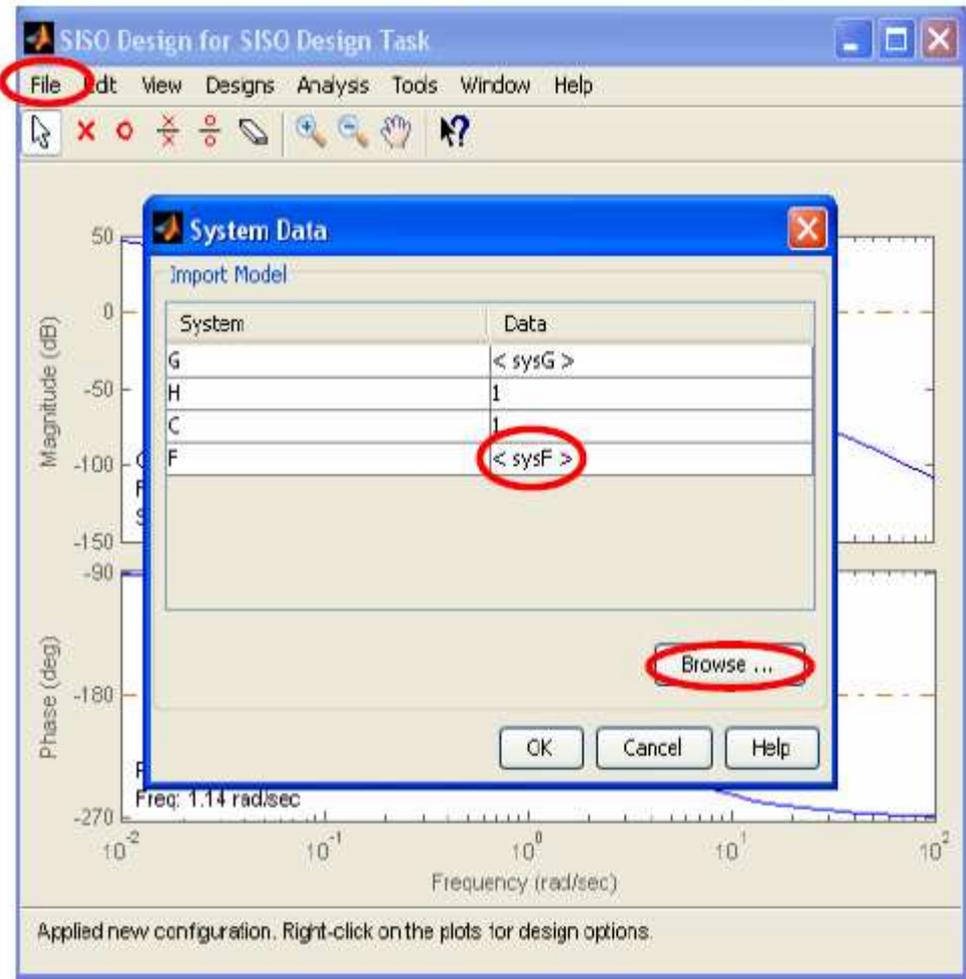


SISO Design Tool (cont'd)

- Input an integrator

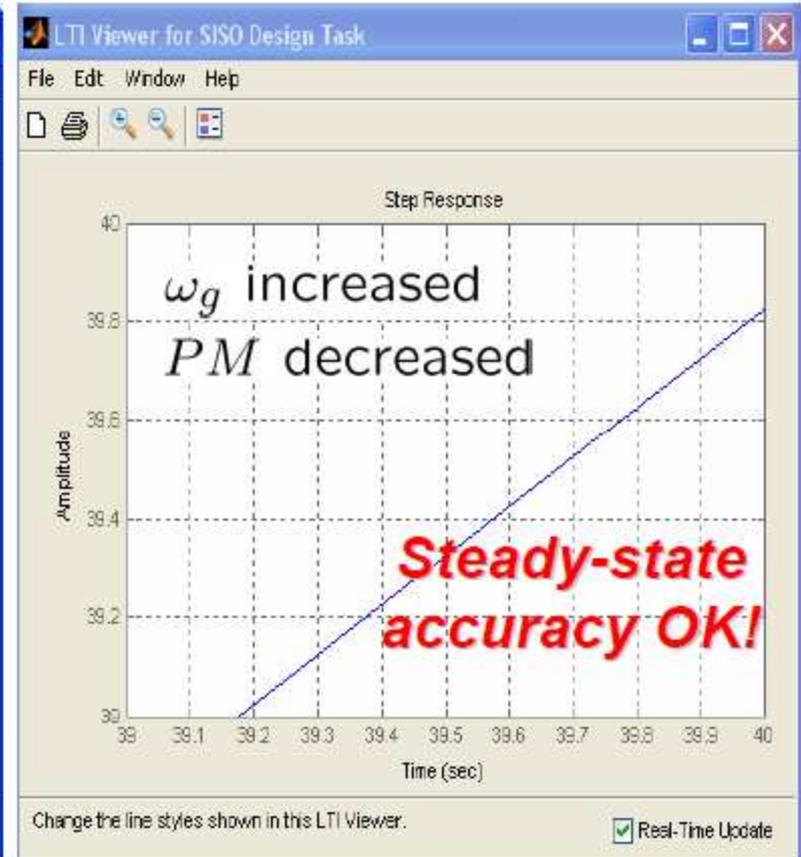
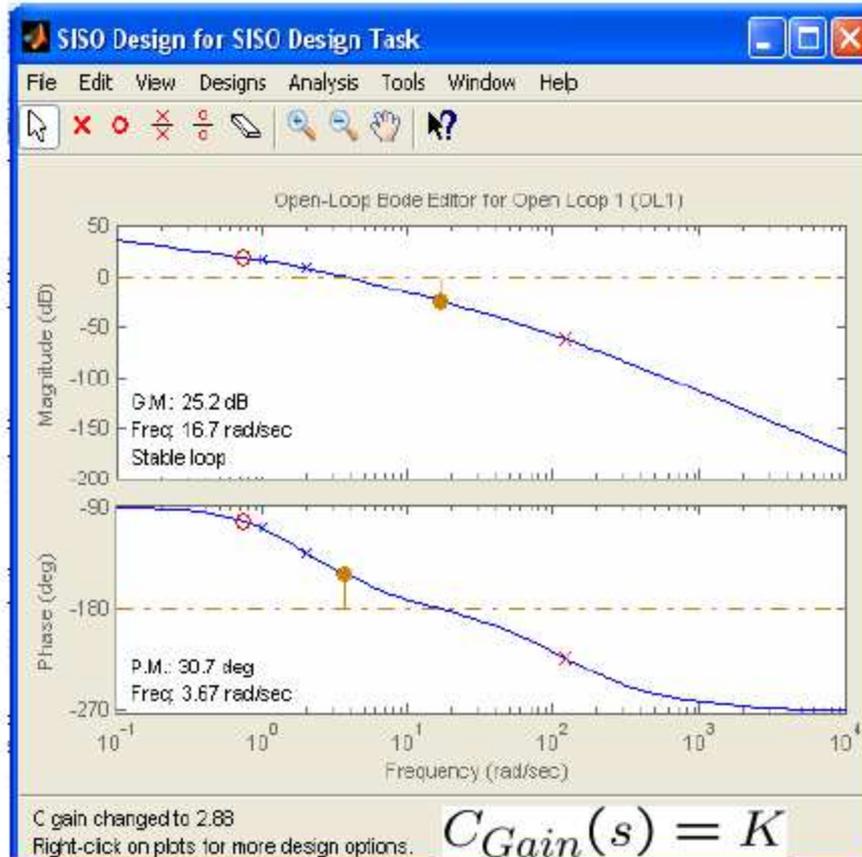
```
>> num=1;  
>> den=[1 0];  
>> sysF=tf(num,den);
```

- Import the plant `sysF` from *File* → *Import*



SISO Design Tool (cont'd)

- Increase gain until good SS error is obtained.

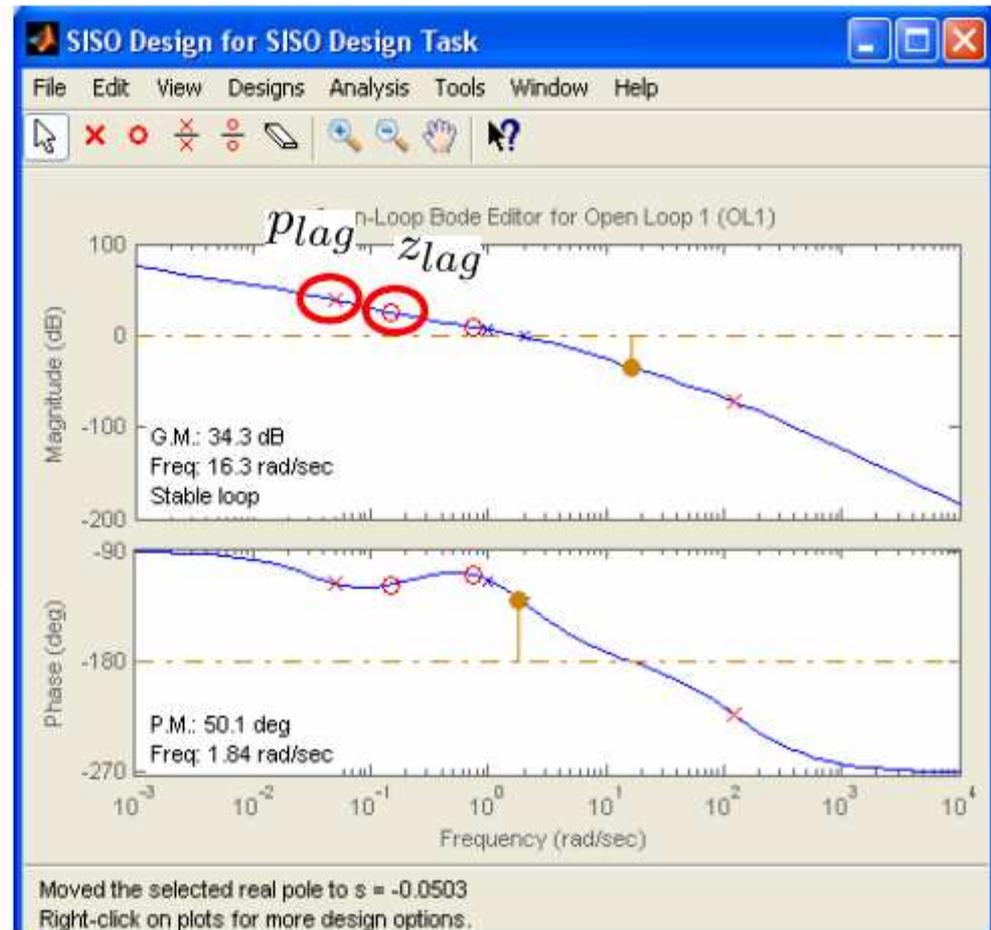


SISO Design Tool (cont'd)

- Add a pole & a zero of a compensator:

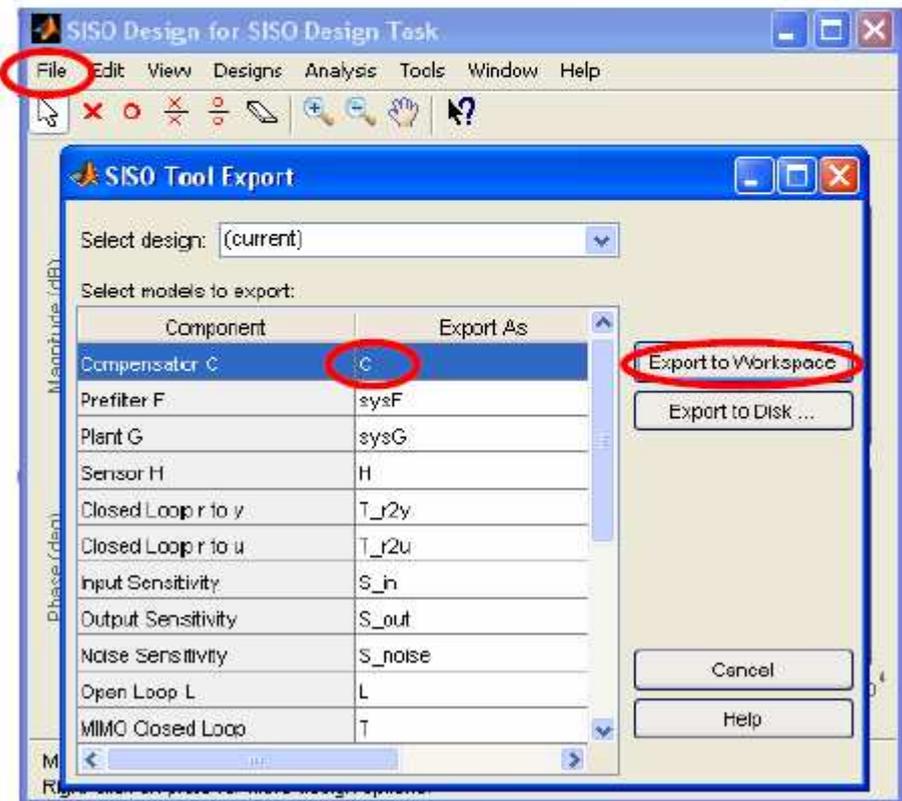
$$C_{Lag}(s) = \frac{\frac{s}{z_{lag}} + 1}{\frac{s}{p_{lag}} + 1}$$

- If necessary, move the pole and zero
 - by click-and-drag, or
 - *Design* → *Edit Compensator...*



SISO Design Tool (cont'd)

- Final controller is $KC_{Lead}(s)C_{Lag}(s) = K \cdot \frac{\frac{s}{z_{lead}} + 1}{\frac{s}{P_{lead}} + 1} \cdot \frac{\frac{s}{z_{lag}} + 1}{\frac{s}{P_{lag}} + 1}$
- One can export the controller to workspace from *File* → *Export*



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Outline

- System description and control objective
- Modeling
- Analysis
 - Stability analysis via Routh-Hurwitz criterion
 - Steady-state error analysis
- Design of lead-lag compensators
 - Root locus
 - Frequency response



Radio telescope antenna

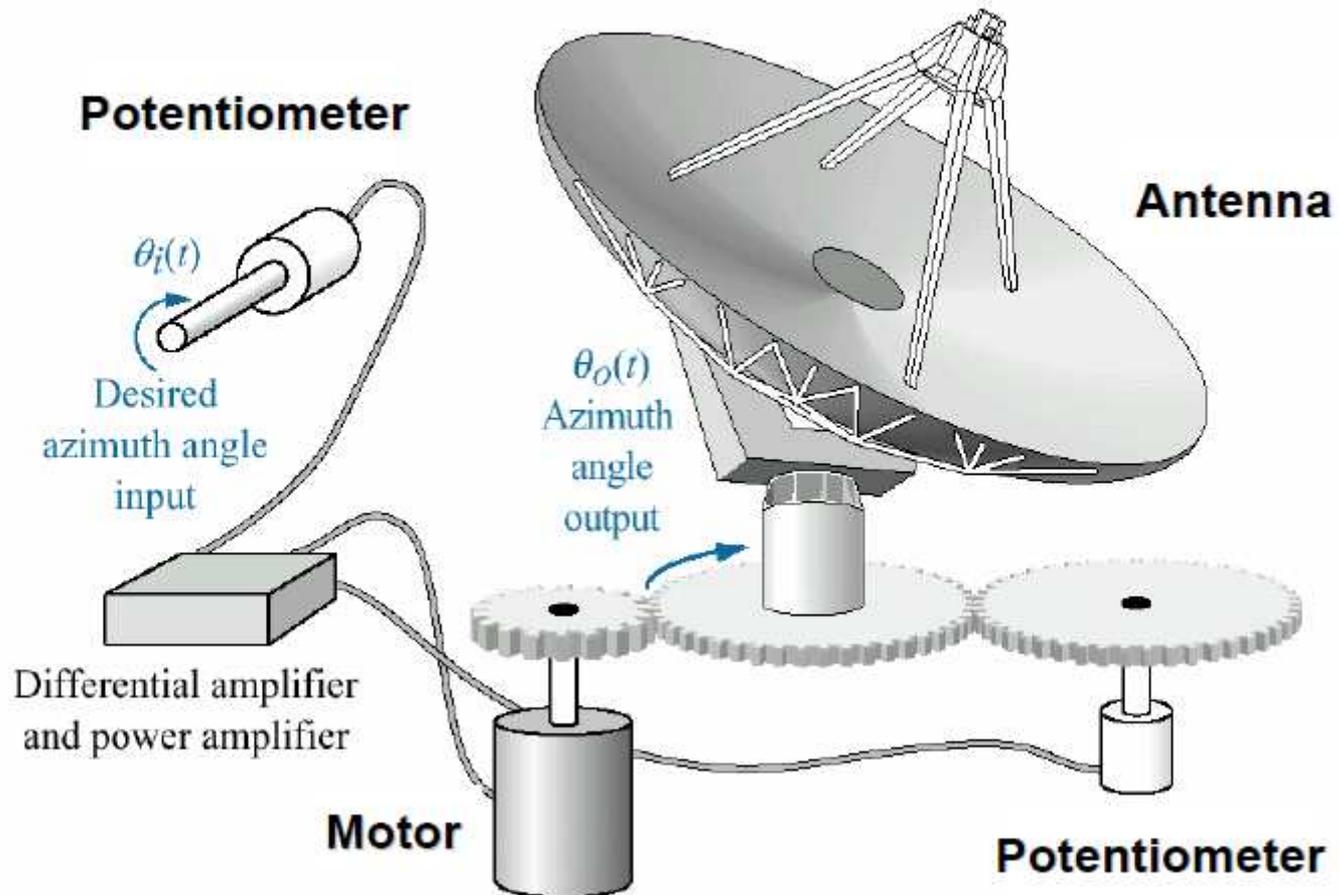
- Large & parabolic
- Receive radio wave
- Radio astronomy
- Tracking
- Collecting data
 - Satellite
 - Space probe



Objective: Control the antenna azimuth to follow a position input command.



Antenna azimuth position control



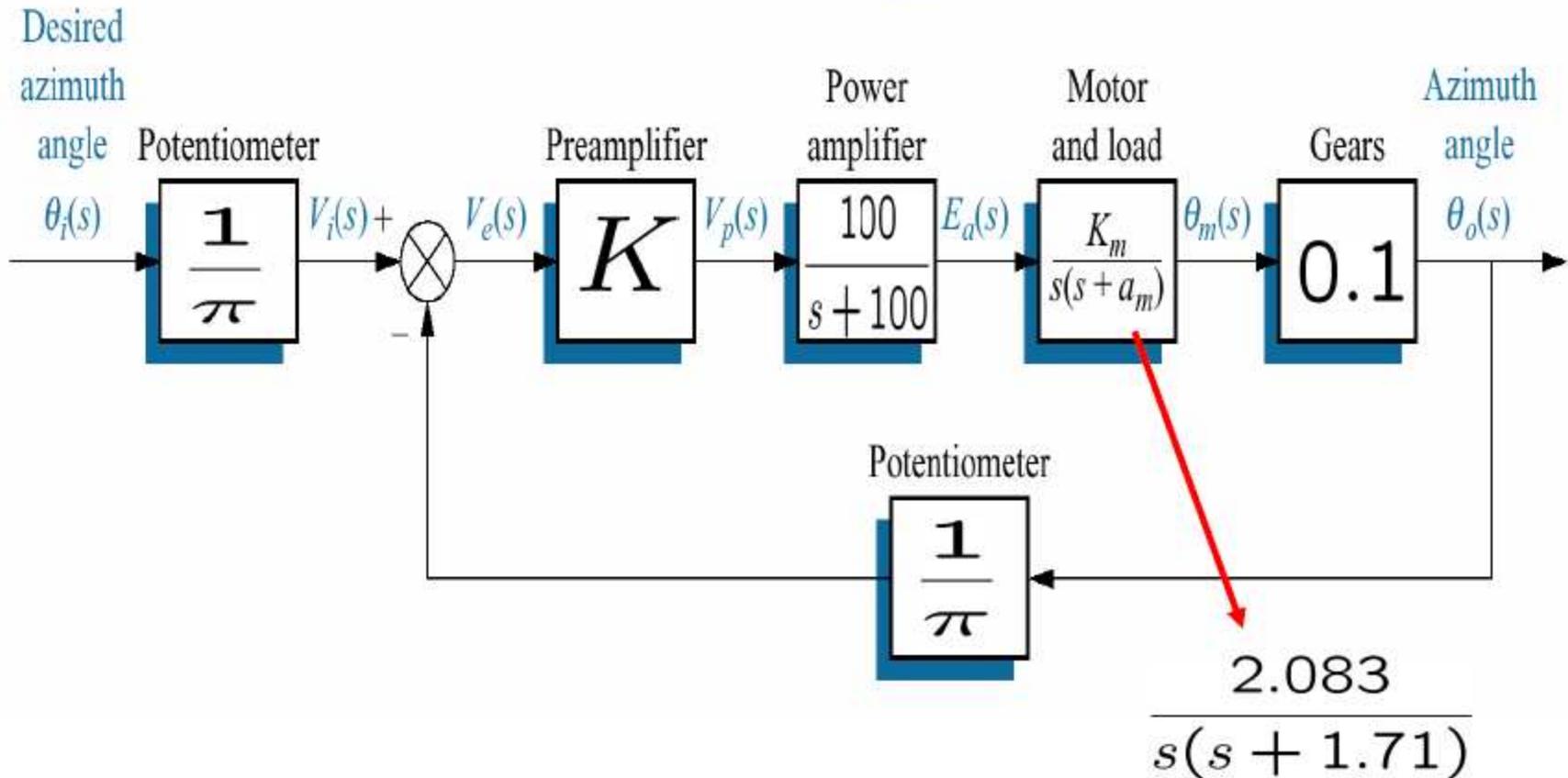
(Taken from Nise's book)

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Antenna azimuth position control Block diagram



Closed-loop transfer function

- From $\theta_i(s)$ to $\theta_o(s)$

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{K \frac{10}{s+100} \cdot \frac{2.083}{s(s+1.71)}}{1 + K \frac{10}{s+100} \cdot \frac{2.083}{s(s+1.71)} \cdot \frac{1}{\pi}} \cdot \frac{1}{\pi}$$

→
$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{6.63K}{s^3 + 101.71s^2 + 171s + 6.63K}$$



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Stability of feedback system

- Routh array

s^3	1	171
s^2	101.71	$6.63K$
s^1	$17392.41 - 6.63K$	0
s^0	$6.63K$	

CL system is stable



$$0 < K < 2623$$



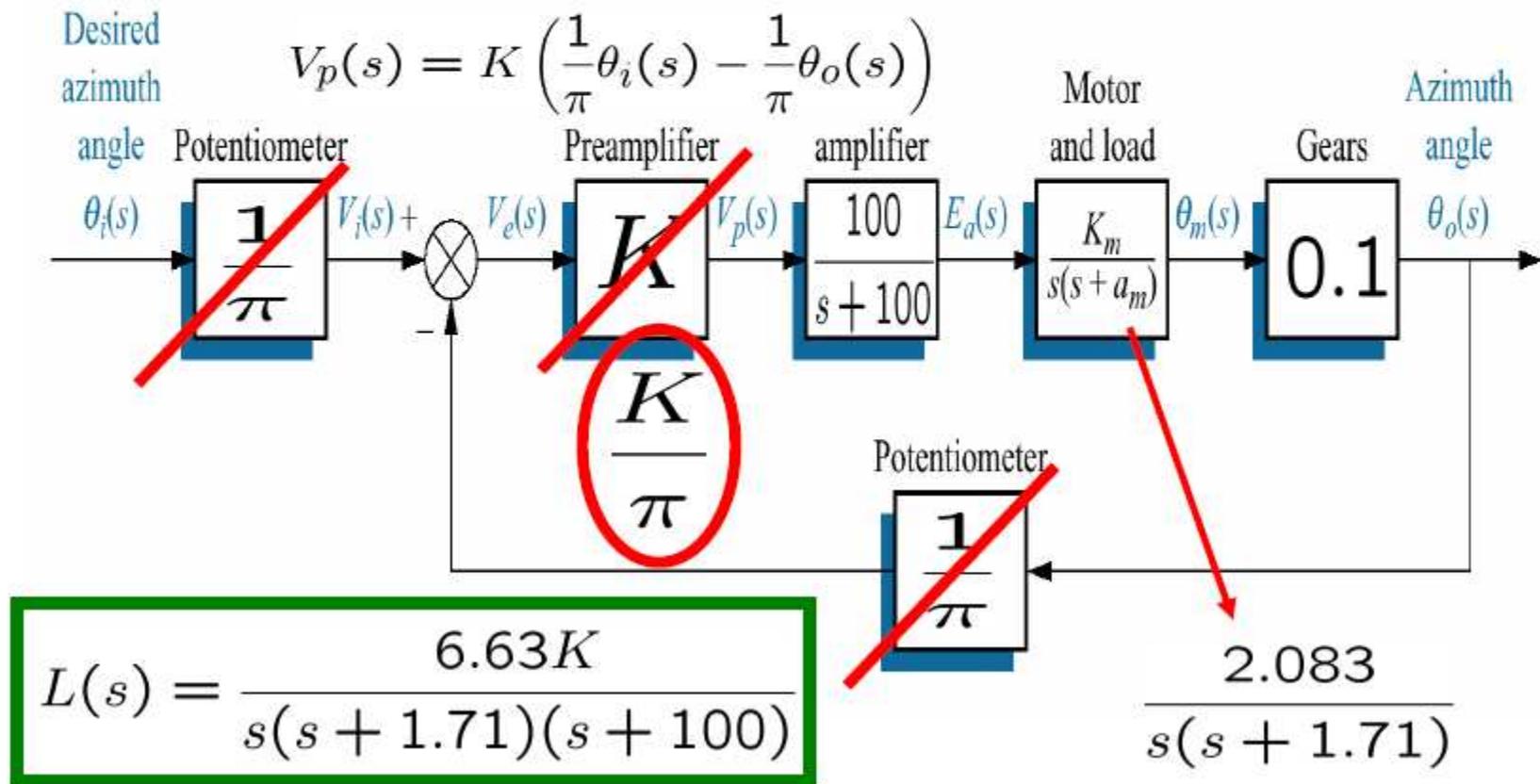
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Steady-state error

- Transform block diagram into a **unity feedback**:



Steady-state error (cont'd)

- For unit step:

$$K_p = L(0) = \infty$$

$$e_{ss} = \frac{1}{1 + K_p} = 0$$

- For unit ramp:

$$K_v = \lim_{s \rightarrow 0} sL(s) = \frac{K}{25.79}$$

$$e_{ss} = \frac{1}{K_v} = \frac{25.79}{K}$$

- For unit parabolic:

$$K_a = \lim_{s \rightarrow 0} s^2 L(s) = 0$$

$$e_{ss} = \frac{1}{K_a} = \infty$$



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Gain design via root locus

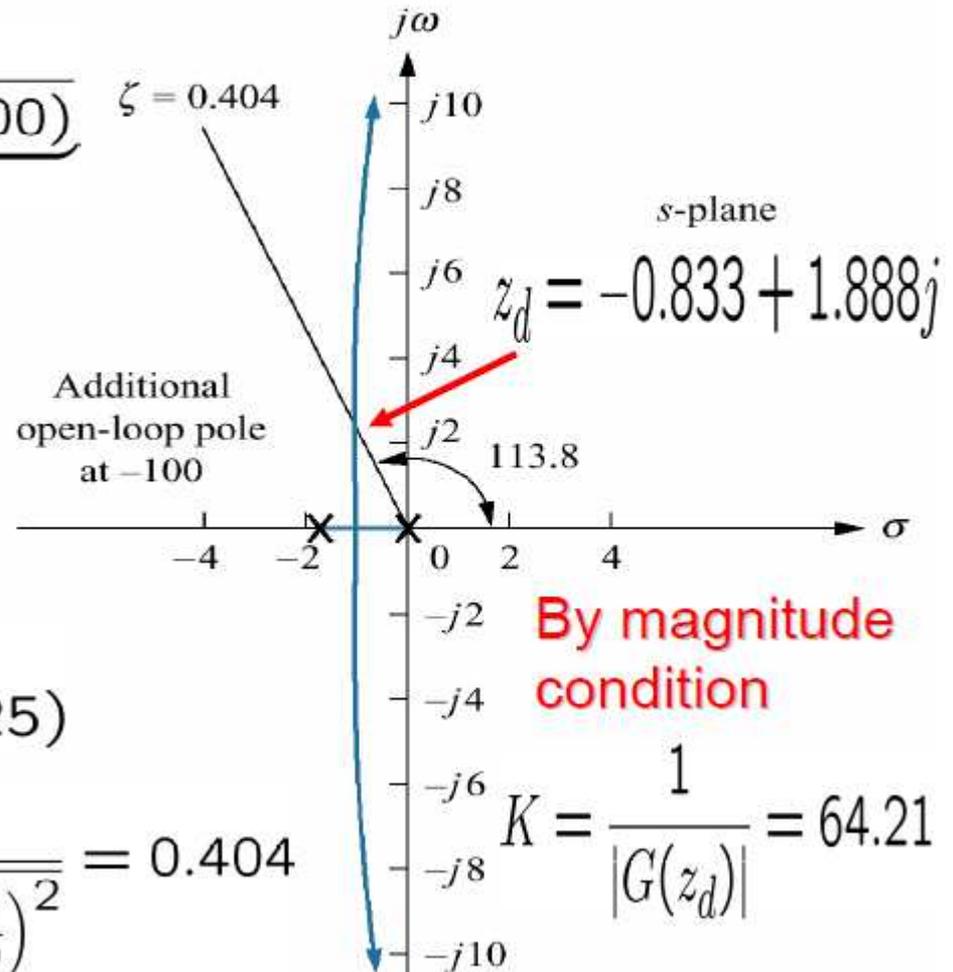
$$L(s) = K \frac{6.63}{\underbrace{s(s + 1.71)(s + 100)}_{G(s)}} \quad \zeta = 0.404$$

- Find K required for 25% overshoot.

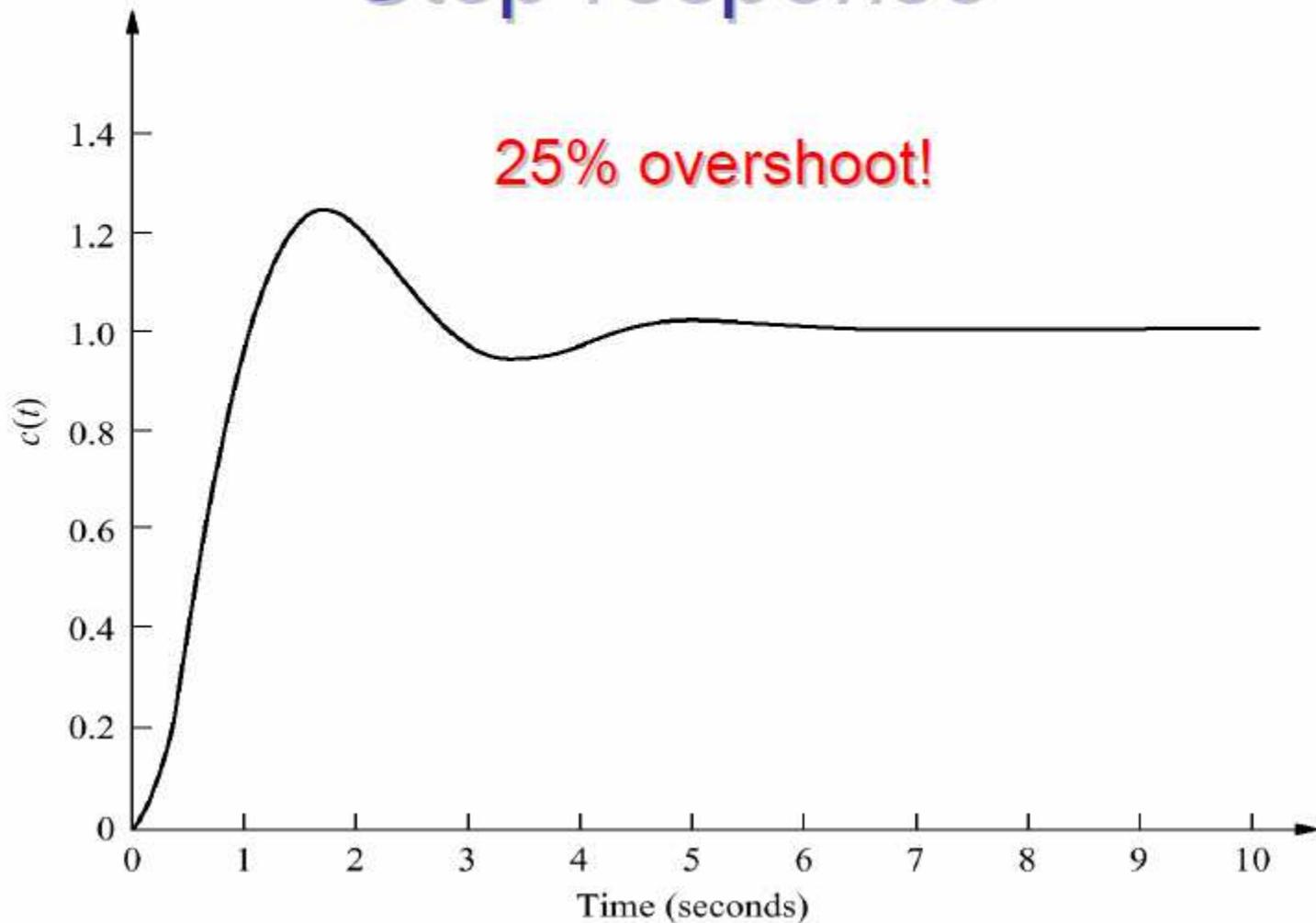
For 2nd order systems

$$PO = 100e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} (= 25)$$

$$\zeta = \frac{-\log \frac{PO}{100}}{\sqrt{\pi^2 + \left(\log \frac{PO}{100}\right)^2}} = 0.404$$



Step response



Lead-lag design via root locus

- Design specifications
 - 25% overshoot
 - 2% settling time 2 seconds
 - $K_v=20$
- In the previous gain design,
 - 25% overshoot
 - 2% settling time 4.8 seconds
 - $K_v=2.49$ (Verify this by yourself!)

Not satisfactory!



Lead compensator design

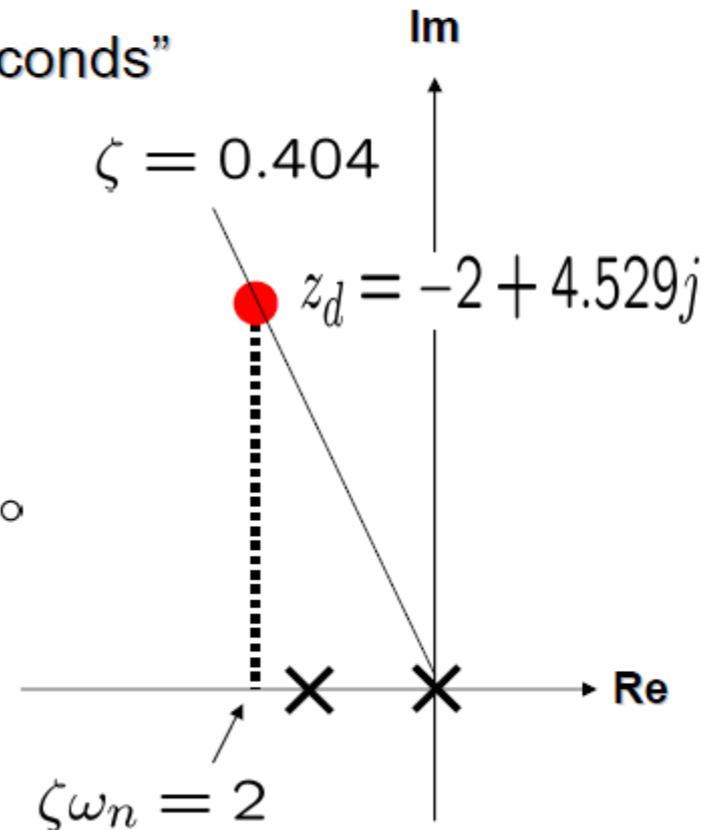
- Locate desired poles
 - Due to “2% settling time 2 seconds”

$$\frac{4}{\zeta\omega_n} = 2 \iff \zeta\omega_n = 2$$

- Angle condition

$$\begin{cases} \angle L(z_d) = \angle G(z_d)C(z_d) = 180^\circ \\ \angle G(z_d) = 149.86^\circ \end{cases}$$

$$\implies \angle C(z_d) = 30.14^\circ$$



Lead compensator design

$$C_{Lead}(s) = K \frac{s + z}{s + p}$$

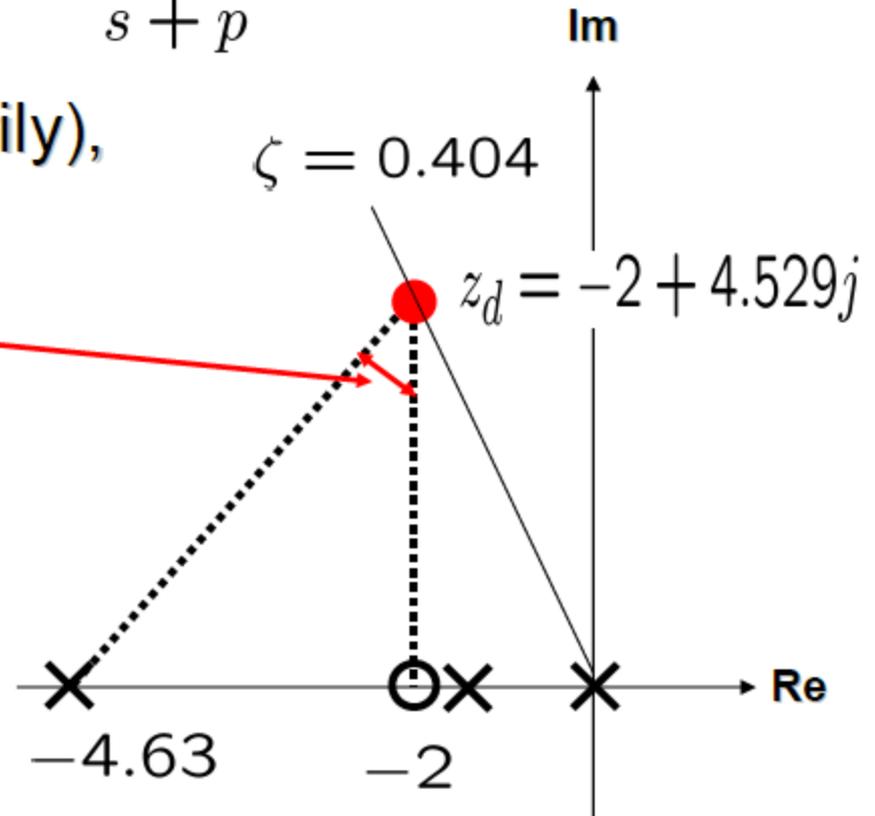
- Fix $z = -2$ (rather arbitrarily), and obtain p s.t.

$$\angle C_{Lead}(z_d) = 30.14^\circ$$

➔ $p = -4.63$

- Find K

$$K = \frac{1}{\left| G(z_d) \frac{z_d + z}{z_d + p} \right|} = 384.46$$



Lag compensator design

$$G(s)C_{Lead}(s) = \frac{2549(s + 2)}{s(s + 1.71)(s + 100)(s + 4.63)}$$

$$C_{Lag}(s) = \frac{s + z}{s + p}$$

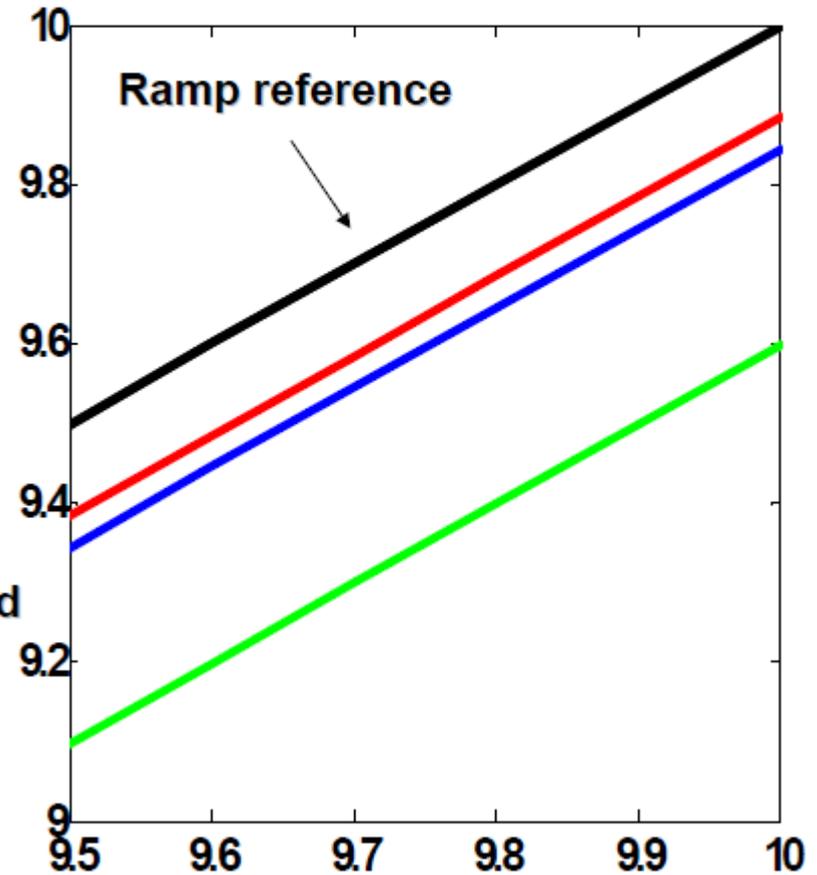
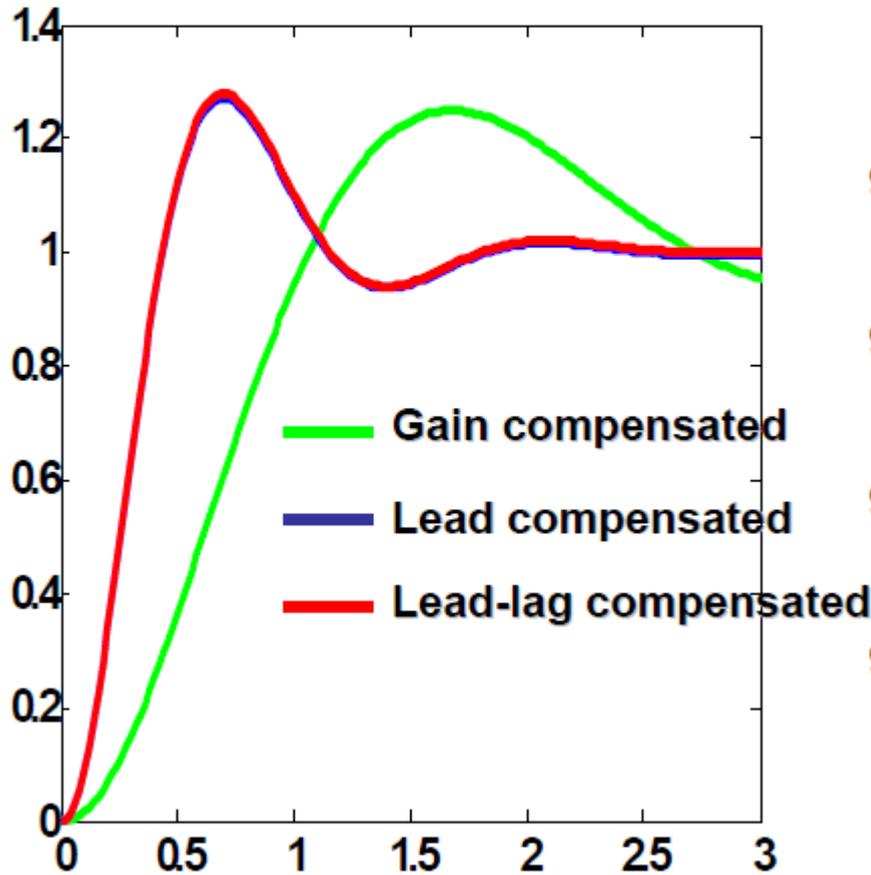
- Error constant K_v

$$K_v = \lim_{s \rightarrow 0} sG(s)C_{Lead}(s)C_{Lag}(s) = 6.44 \frac{z}{p} = 20$$

$$\rightarrow \frac{z}{p} = \frac{20}{6.44} = 3.1 \quad \rightarrow \quad p = -0.01, \quad z = -0.031$$



Step and ramp responses



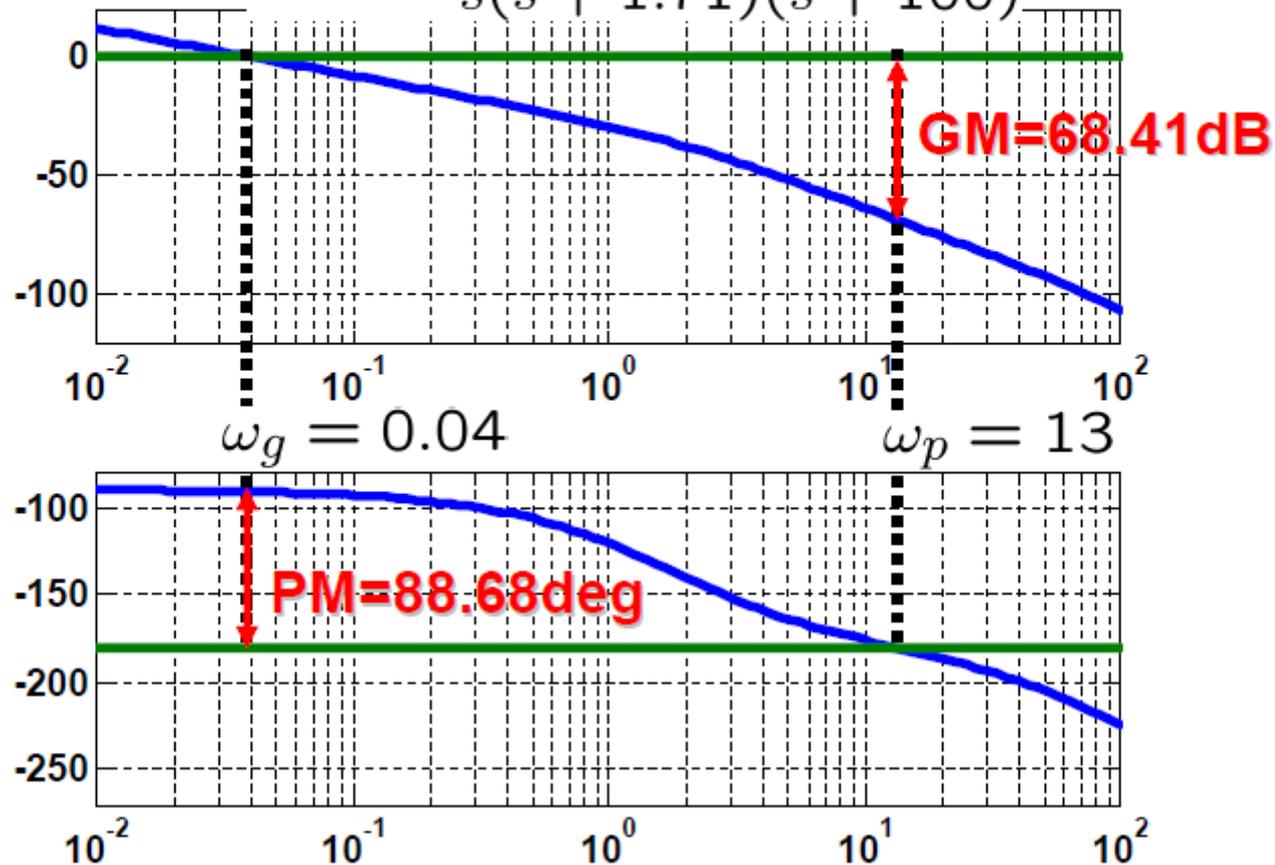
Outline

- System description and control objective
- Modeling
- Analysis
 - Stability analysis via Routh-Hurwitz criterion
 - Steady-state error analysis
- Design of lead-lag compensators
 - Root locus
 - Frequency response



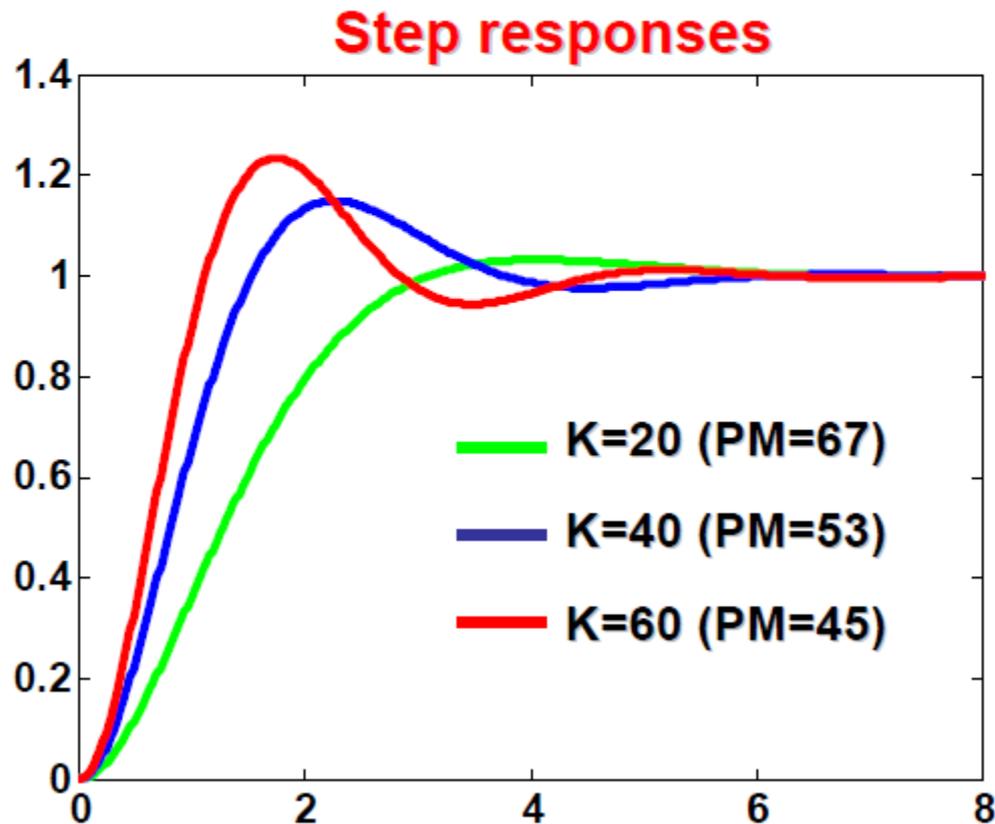
Open-loop frequency response

$$G(s) = \frac{6.63}{s(s + 1.71)(s + 100)}$$



Gain, PM, and step response

- Change preamplifier gain K to 20, 40, 60.



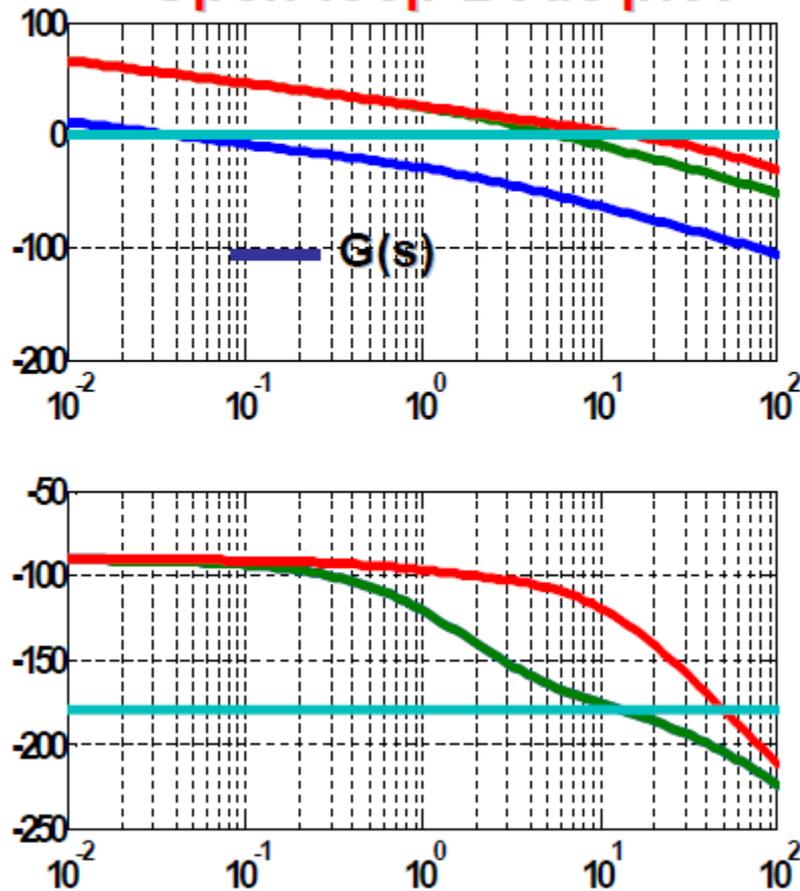
Lead-lag design via freq. response

- Design specifications (same as before)
 - 25% overshoot
 - 2% settling time 2 seconds
 - $K_v=20$
- One possible procedure
 1. Use the gain to satisfy K_v .
 2. Use lead compensator to increase PM (and gain crossover frequency).
 3. Use lag compensator to decrease gain crossover frequency (to increase PM).

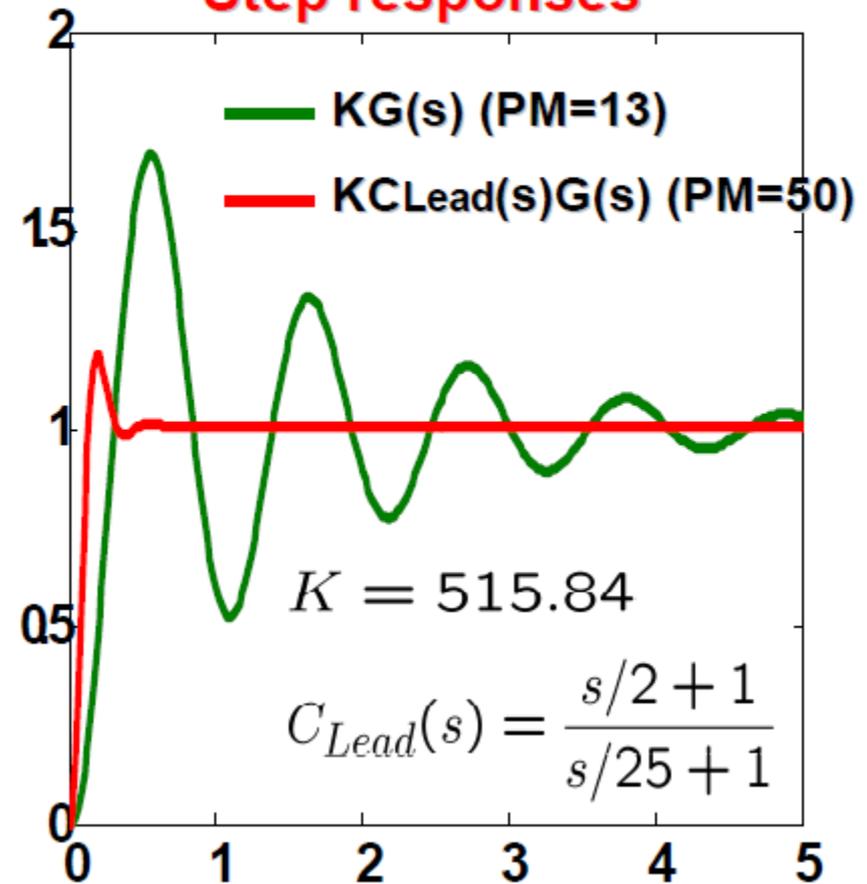


Gain+lead compensation

Open-loop Bode plot

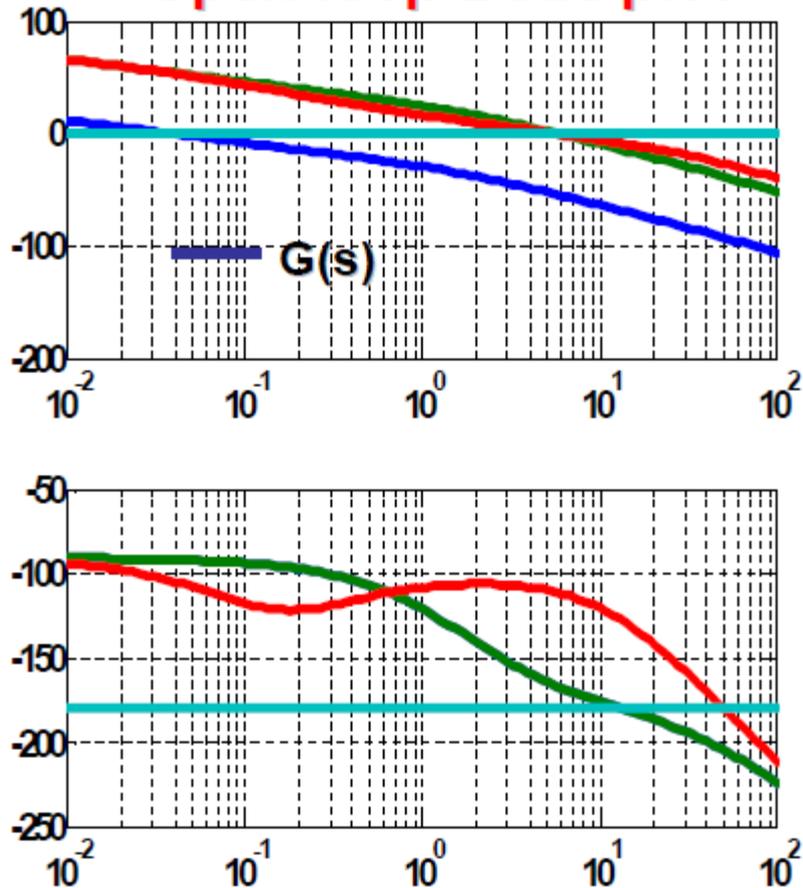


Step responses

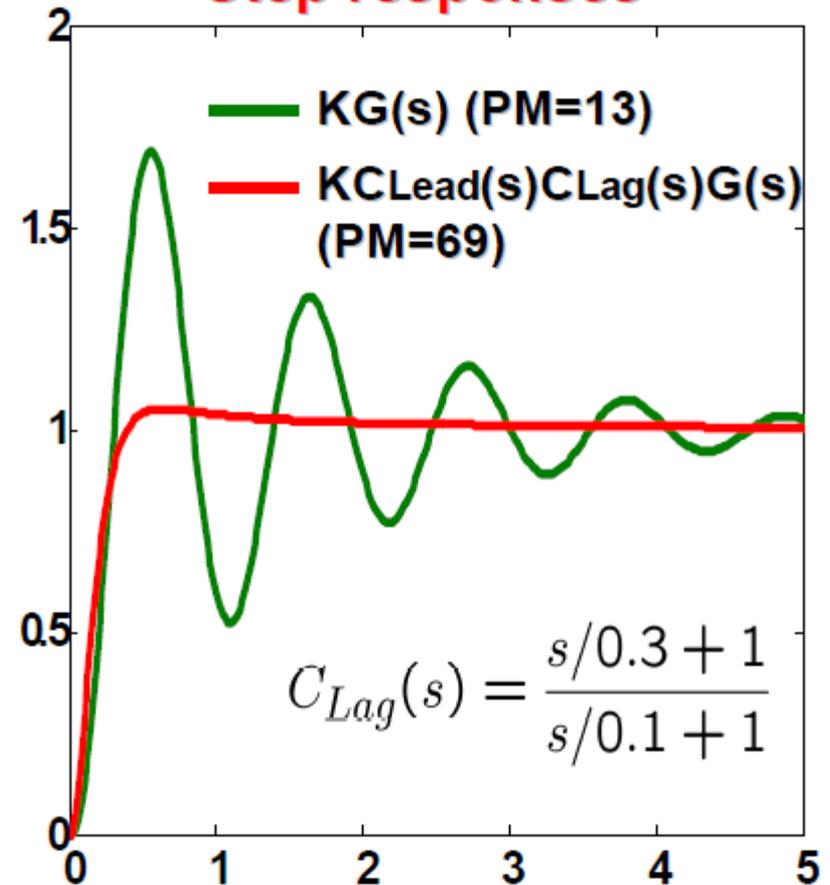


Gain+lead/lag compensation

Open-loop Bode plot



Step responses



Summary

- **Lag compensator** can be used for improving
 - PM by maintaining low freq. gain or
 - Low freq. gain by maintaining PM
- **Lead compensator** can be used for improving
 - Gain crossover frequency ω_g and
 - PM, by maintaining low frequency gain.
- **Lead-lag compensator** can improve
 - Transient (ω_g for speed, PM for overshoot)
 - Steady state (low frequency gain for error constant)
- **Frequency response shaping in Matlab**
- **SISO Design Tool**
- **Case Study**
 - Antenna azimuth position control

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- **Next**
 - Course summary
 - Problem solving



End of Lecture 11

