The Public Authority for Applied Education and Training

College of Technological Studies

Department of Mechanical Power and Refrigeration

Fluid Machinery

Lecture Notes

(MEE 221/66)

by

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Chapter (2)

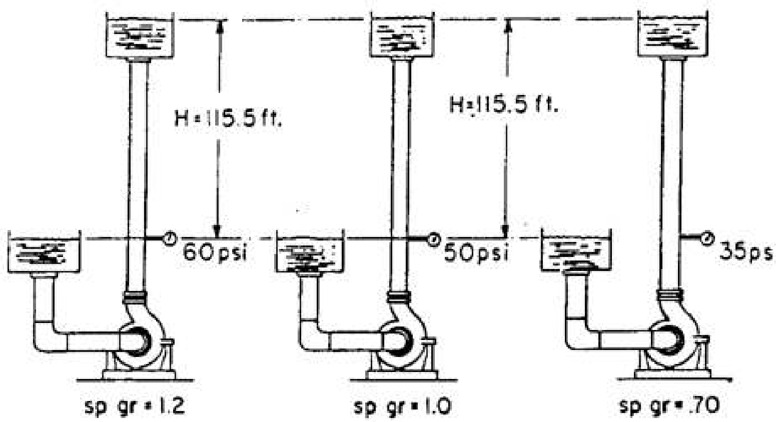
Dynamic Pumps

Pump Performance

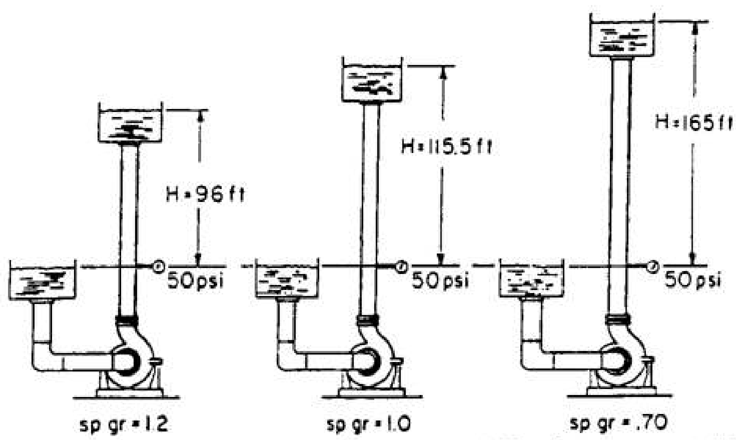
Pump: is a device that converts mechanical energy into hydraulic energy.

Pressure versus head:

$$P = \gamma h = \rho g h \qquad (pa)$$

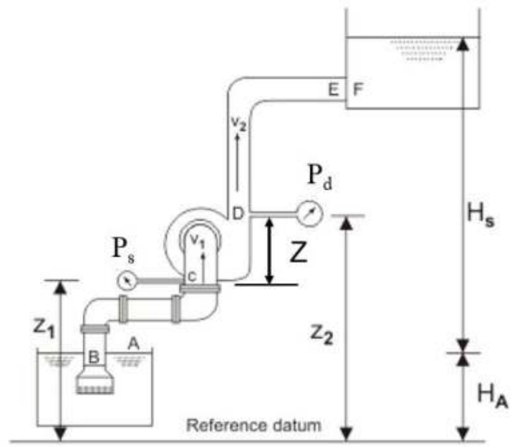


Pressure-head relationship of identical pumps handling liquids of different specific gravities.



Pressure-head relationship of pumps delivering same pressure handling liquids of different specific gravities.

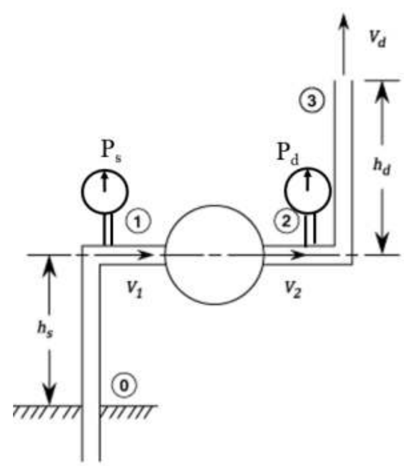
Head delivered by a pump (pump head):



$$h = H_d - H_s$$

$$h = \frac{P_d}{\gamma} + \frac{V_d^2}{2g} + Z_d - \left(\frac{P_s}{\gamma} + \frac{V_s^2}{2g} + Z_s\right)$$

If the pump inlet and outlet are located at the same level:



Then:

$$Z_d = Z_s$$

$$h = \frac{P_d}{\gamma} + \frac{V_d^2}{2g} - \left(\frac{P_s}{\gamma} + \frac{V_s^2}{2g}\right)$$

Pump efficiency (η):

$$\eta = \frac{Hydraulic\ Power}{Mechanical\ Power} = \frac{\gamma Q h}{Mechanical\ Power}$$

Mechanical Power:

$$Mechanical\ Power = \frac{\gamma\left(\frac{N}{m^3}\right) Q \left(\frac{m^3}{s}\right) h(m)}{\eta} \qquad \left(\frac{N.m}{s} = \frac{J}{s} = W\right)$$

To convert from (W) to horse power (hp):

$$1000\ W = 1.341\ hp \Rightarrow hp = \frac{1000}{1.341} = 745.7\ W$$

$$Mechanical\ Power = \frac{\gamma\left(\frac{N}{m^3}\right) Q \left(\frac{m^3}{s}\right) h(m)}{\eta * 745.7} \qquad (hp)$$

Hydraulic efficiency (η_h):

Varies from (0.6-0.9)

$$\eta_h = \frac{Hydraulic\ Power}{Power\ input\ to\ the\ pump} = \frac{\gamma Q h}{P_{in}}$$

Volumetric efficiency (η_V):

Volumetric losses are the leakage between the casing and the impeller and the leakage from the seals. Varies from (0.97-0.98).

$$\eta_V = \frac{Q}{Q + \Delta Q}$$

where, ΔQ: amount of leakage

Q: actual discharge

Mechanical efficiency (η_m):

Varies from (0.95-0.98).

$$\eta_m = \frac{Actual\ power\ input\ to\ impeller}{Power\ input\ to\ the\ shaft}$$

$$Power\ input\ to\ the\ shaft\ (brake\ power) = T\omega$$

where, T: Torque exerted on the shaft of the pump by the motor that drives the shaft.

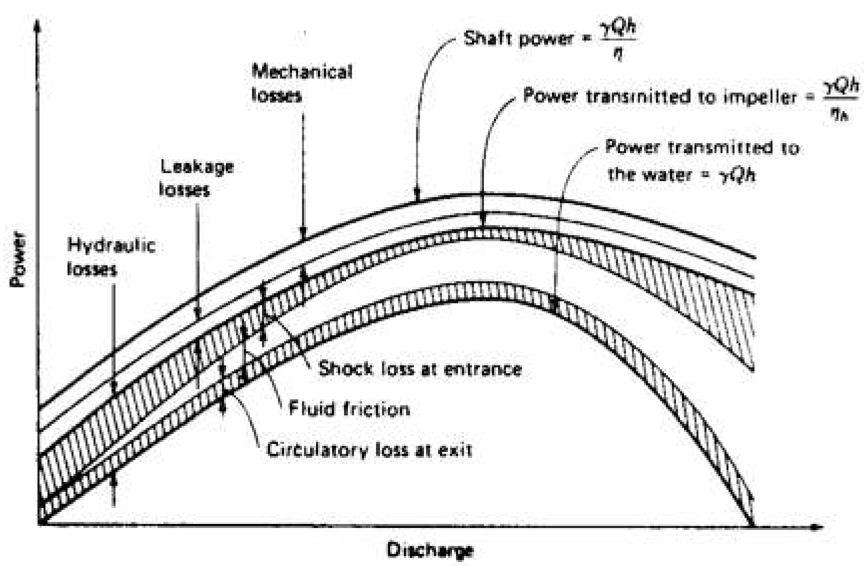
ω: The rate of rotation of the shaft in radians per second.

Overall efficiency (η_O):

Varies from (0.71-0.86)

$$\eta_O = \frac{Hydraulic\ Power}{Power\ input\ to\ the\ shaft} = \frac{\gamma Qh}{P_{in}}$$

$$\eta_O = \eta_h * \eta_V * \eta_m$$



Example-1:

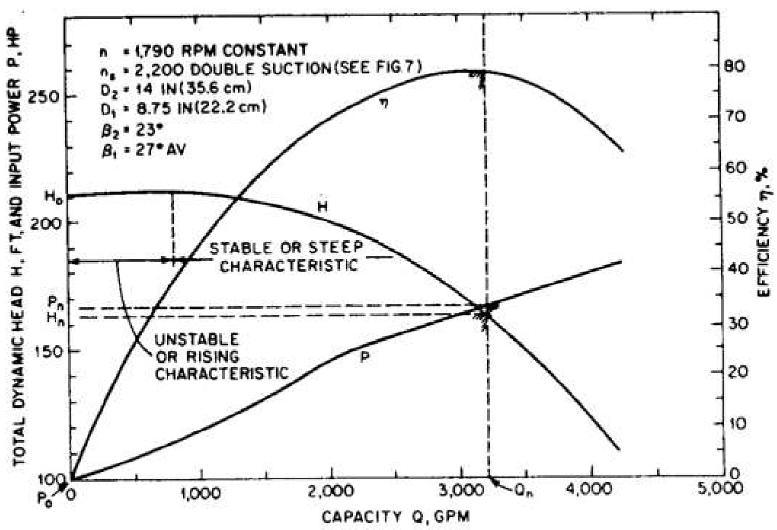
A pump has an intake pipe diameter of 0.2m and a discharge pipe diameter of 0.15m. The pressure gage at discharge reads 207 kPa and the vacuum gage at the intake reads 33.9 kPa. If Q=85 L/s of water and brake power is 35 kW, find the pump efficiency. The intake and discharge at the same elevation.

Solution:

Performance characteristics of pumps at constant speed:

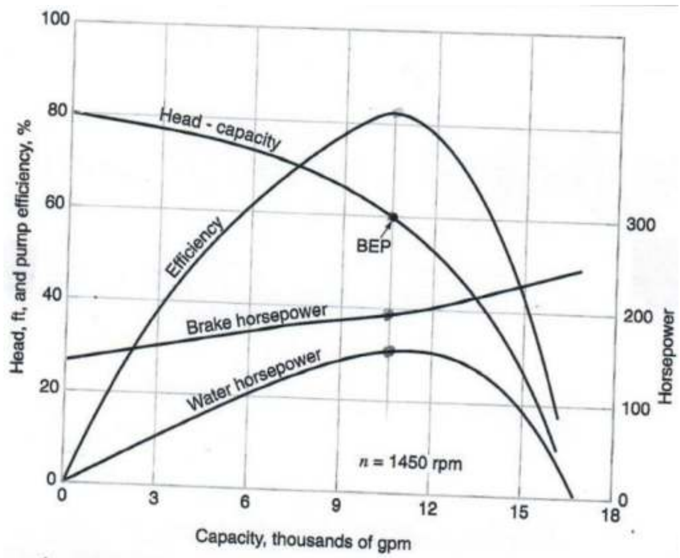
- The efficiency of a pump varies considerably, depending upon the conditions under which it must operate.
- The performance characteristics of any pump can be obtained by laboratory tests where "h", "η", "power (brake power)" as function of "Q" are obtained.

- Pump performance curves: are the curves that shows the relationship between pump flow rate "Q" and other pump parameters (i.e., head (h), efficiency (η) and power (P)).



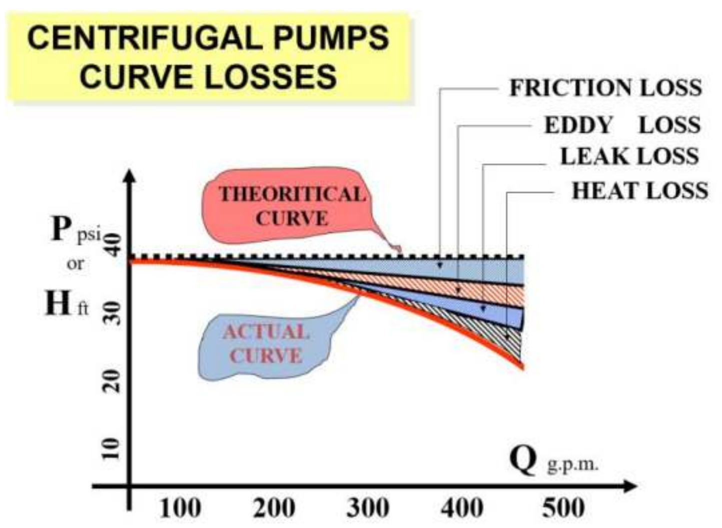
Typical pump performance curves.

- The "head (h) – capacity (Q)" curve is called "pump characteristic curve".
- Best Efficiency Point (BEP): is the point where the pump efficiency reaches its maximum value.
- "Normal capacity" is the flow rate (discharge) that corresponds to the Best Efficiency Point (BEP).
- The pump characteristic curve and other performance curves for a typical mixed-flow pump is as follows:



- This pump has a normal capacity (Q) of 10,500 gpm when developing a normal head (h) of 60 ft at an operating speed (n) of 1450 rpm.

Centrifugal pumps curve losses:



Performance curves for different pump speeds:

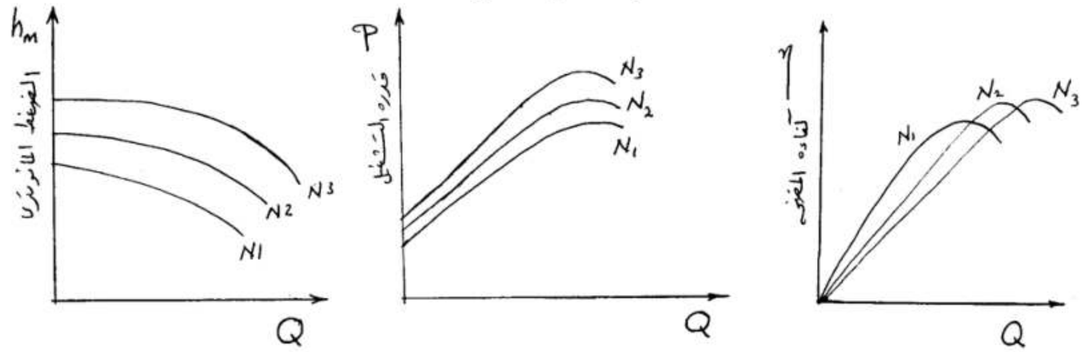
Pump performance curves at any speed (N_2) can be determined from performance curves at certain speed (N_1) using the affinity laws which are as follows:

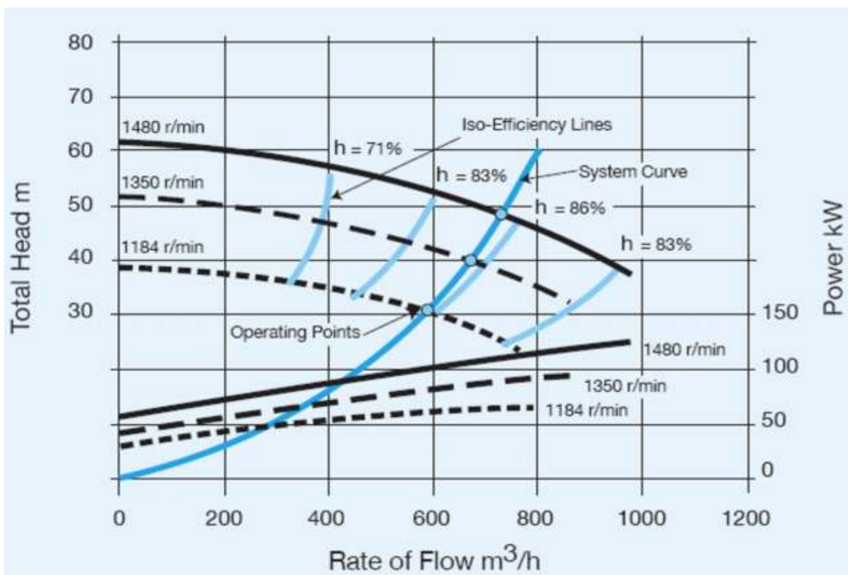
$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2}$$

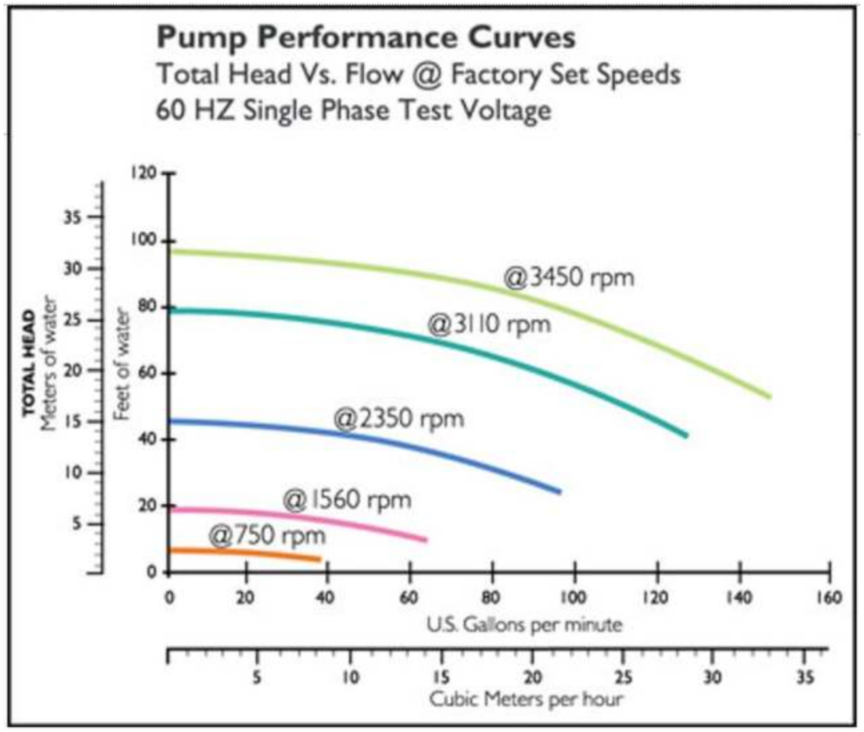
$$\frac{h_1}{h_2} = \left(\frac{N_1}{N_2}\right)^2$$

$$\frac{P_{in,1}}{P_{in,2}} = \left(\frac{N_1}{N_2}\right)^3$$

$$N_3 > N_2 > N_1$$







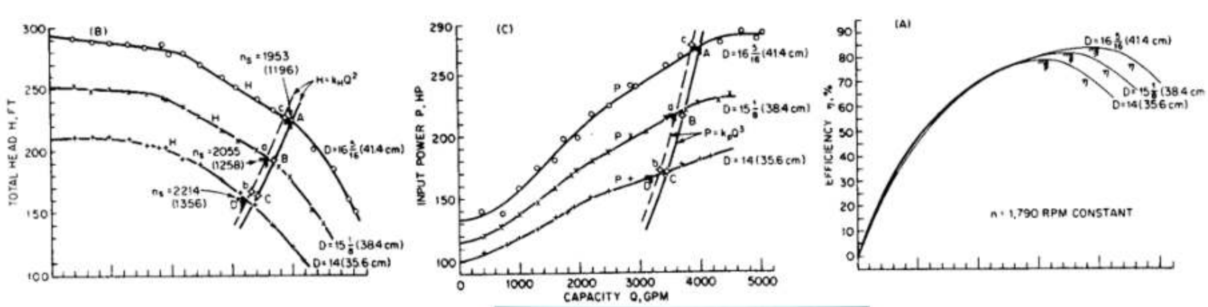
Performance curves for different impeller diameters:

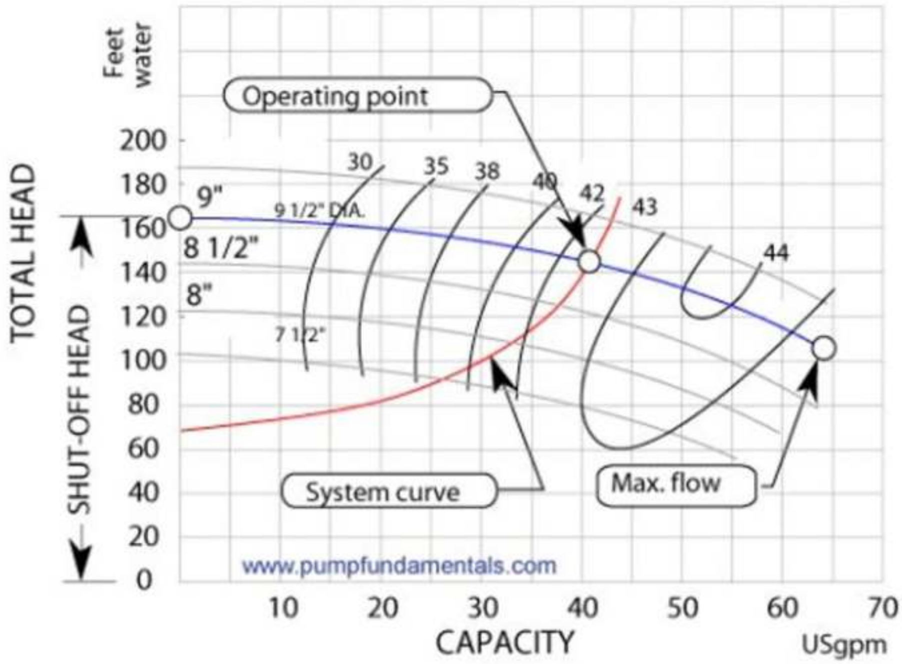
The effect of impeller diameter changes on pump performance curves is equivalent to the effect of speed changes, i.e., if the impeller diameter changes from (D_1) to (D_2), the following relations can be derived:

$$\frac{Q_1}{Q_2} = \frac{D_1}{D_2}$$

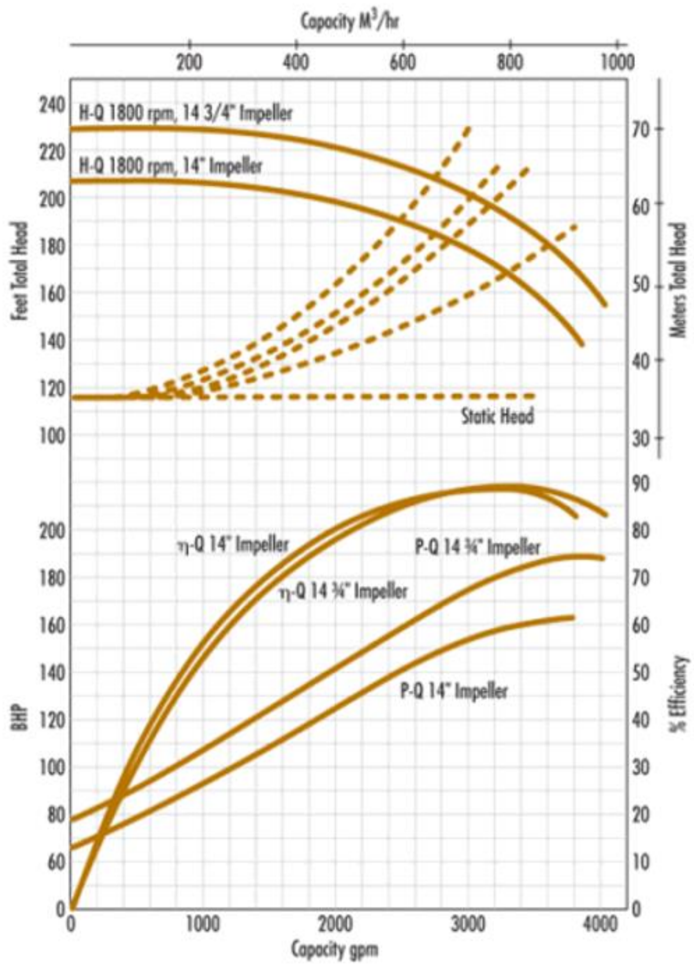
$$\frac{h_1}{h_2} = \left(\frac{D_1}{D_2}\right)^2$$

$$\frac{P_{in,1}}{P_{in,2}} = \left(\frac{D_1}{D_2}\right)^3$$





Feet water
Operating point
TOTAL HEAD
SHUT-OFF HEAD
9"
9 1/2" DIA.
8 1/2"
8"
7 1/2"
30
35
38
40
42
43
44
System curve
Max. flow
www.pumpfundamentals.com
CAPACITY
USgpm



Capacity M3/hr
H-Q 1800 rpm, 14 3/4" Impeller
H-Q 1800 rpm, 14" Impeller
Feet Total Head
Meters Total Head
Static Head
η-Q 14" Impeller
η-Q 14 ¾" Impeller
P-Q 14 ¾" Impeller
P-Q 14" Impeller
BHP
% Efficiency
Capacity gpm

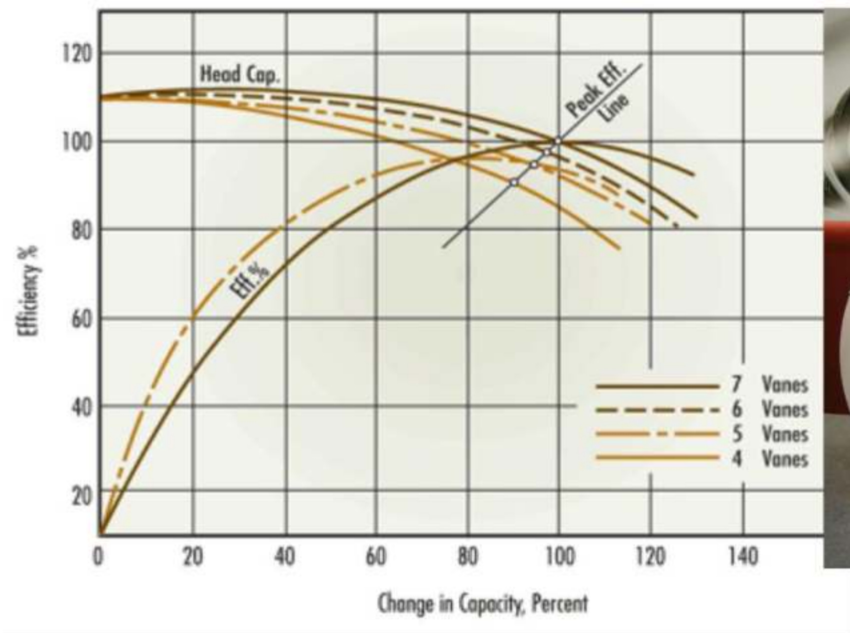
Example-2:

A 33cm diameter centrifugal pump impeller discharges 4.5 m^3/min of water at a head of 35m when running at 1200 rpm.

a) If its efficiency is 88%, what is the horse power input to the shaft of the pump?
b) If the pump diameter is modified to 40cm, what would be "h", "Q" and the "brake horse power" delivered by the shaft for homologous conditions?

Solution:

Influence of number of impeller vanes on pump performance:



Probable effect of reversed mounting of impeller:

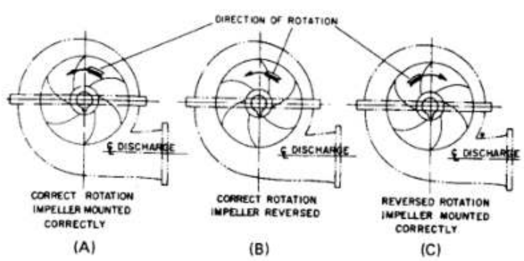
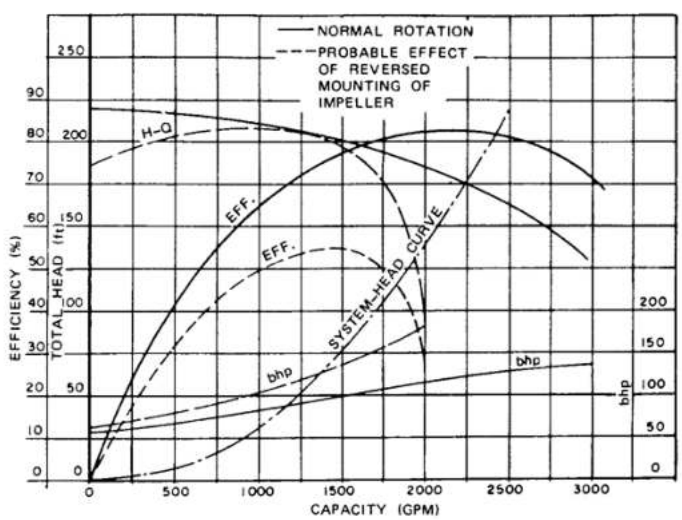
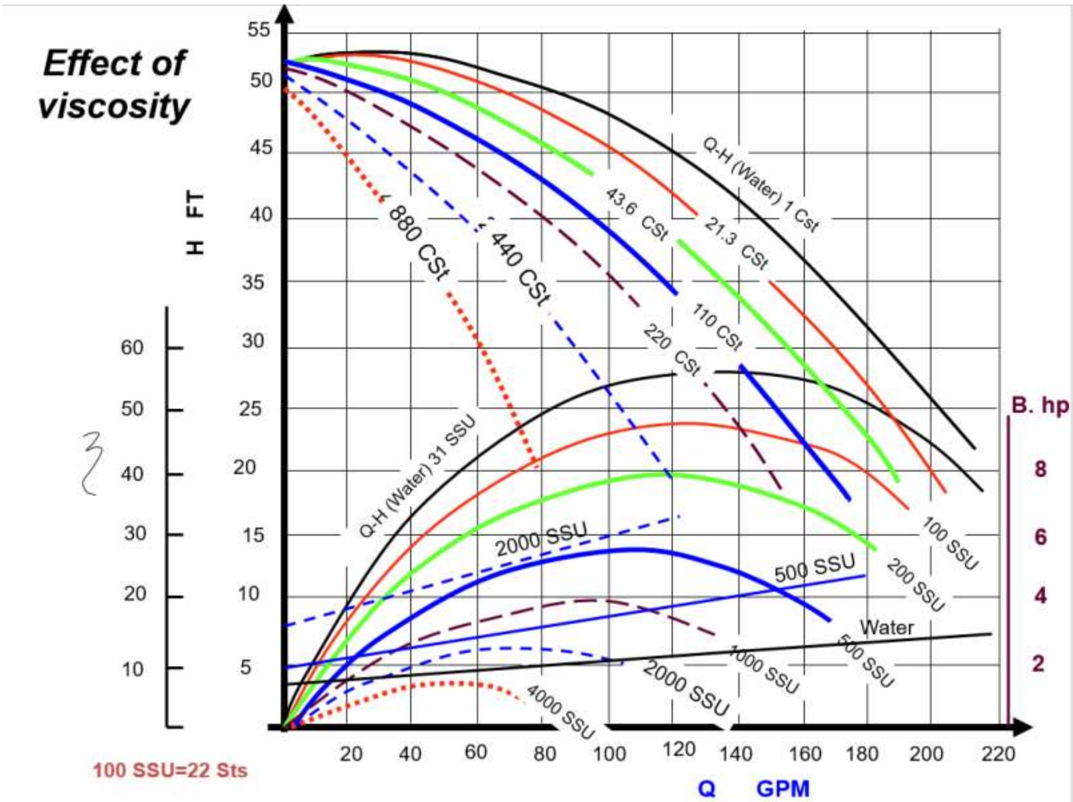


Figure 6.11 Mounting of impeller in a volute casing.

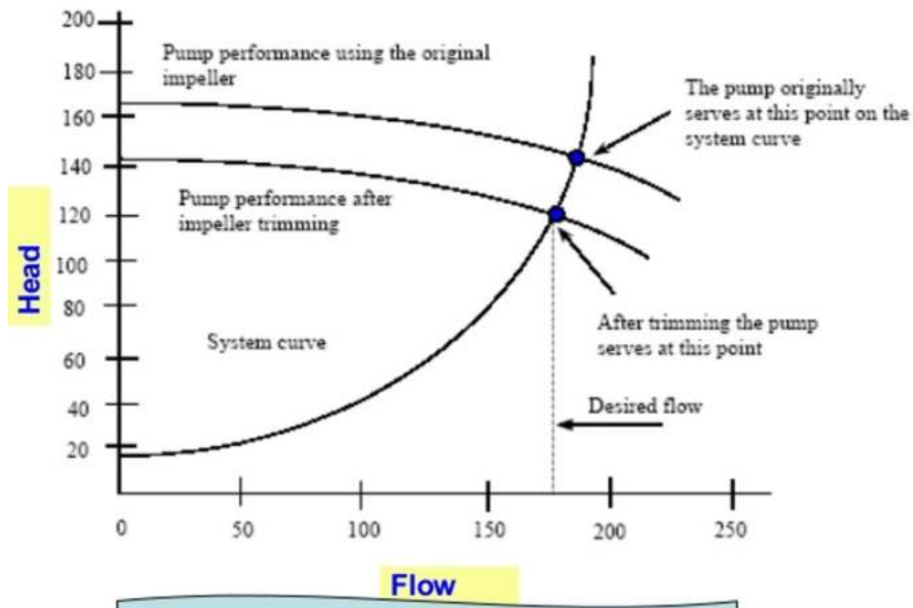


Effect of viscosity:



Pump impeller trimming:

Flow control for permanent flow reduction



Situation After Impeller Trimming

Pipe line system curve:

$$h_{sys} = h_s + h_L$$

where, h_{sys}: system head, h_s: static head and h_L: head losses.

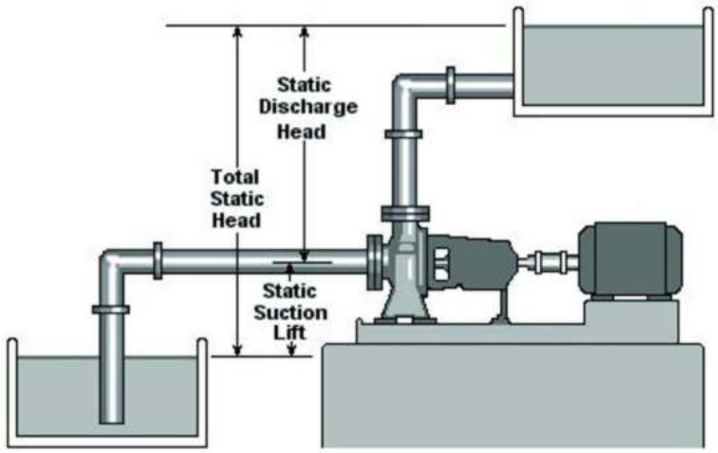
Static lift:

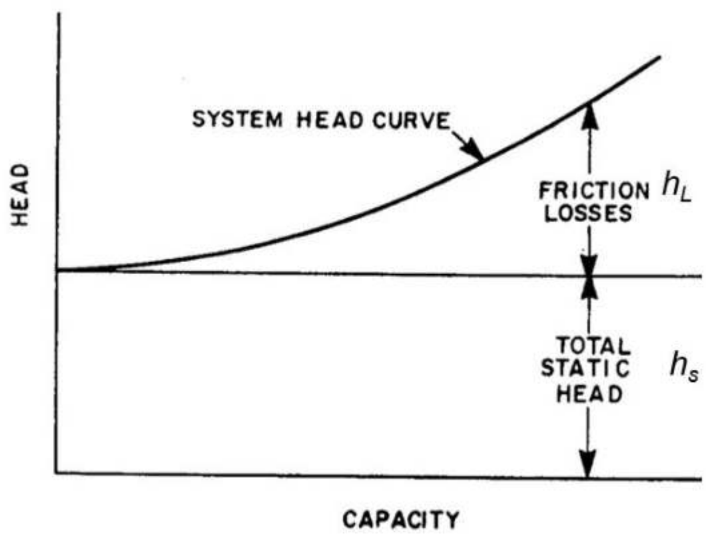
Elevation difference between pump center line and the suction water surface. (If the pump is higher than the suction water surface, then the static lift is positive. But if the pump is lower than the suction water surface, then the static lift is negative).

Static discharge:

Elevation difference between pump center line and the discharge point. (If the pump is higher than the discharge point, then the static discharge is negative. But if the pump is lower than the discharge point, then the static discharge is positive).

$$Static\ head = static\ lift + static\ discharge$$





<u>The head losses in pipe lines are two types:</u>

1) **<u>Friction losses (h_f):</u>**

This is due to friction between fluid layers and between fluid and the conduit (pipe) wall.

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

where,

h_f: head loss due to friction (m)

f: friction factor, which depends on flow type (laminar or turbulent).

L: pipe length (m)

D: pipe diameter (m)

V: fluid velocity (m/s)

Flow type:

Flow type can be determined using a dimensionless parameter called "Reynolds number (Re)".

$$Re = \frac{\rho V D}{\mu}$$

where,

Re: Reynolds number

ρ: fluid density $\frac{kg}{m^3}$

V: fluid velocity (m/s)

D: pipe diameter (m)

μ: fluid viscosity (N.s/m^2)

If:

Re<2000 (laminar flow)

2000<Re<4000 (the flow is in transition period)

Re> 4000 (turbulent flow)

For laminar flow:

$$f = \frac{64}{Re}$$

For turbulent flow:

- "f" depends on Reynolds number (Re) and pipe relative roughness $\left(\frac{\varepsilon}{D}\right)$.
- "f" can be determined using "Moody diagram".

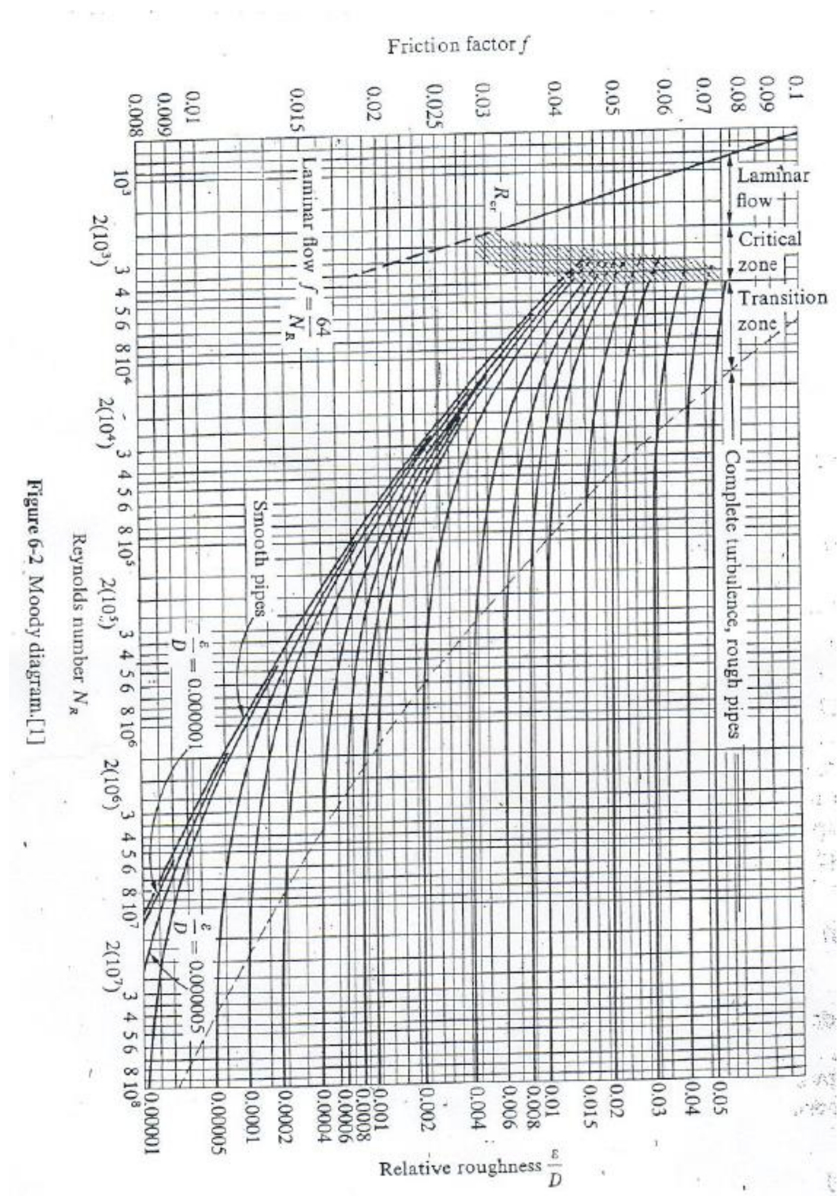


Figure 6-2 Moody diagram.[1]

2) **<u>Miror losses (h_m):</u>**

There is also some energy loss whenever there is a sudden change in flow pattern, such as an obstruction in the path of the flow or a change in fluid velocity or direction. These losses are often smaller than friction losses, so they are commonly referred to as “minor losses”.

- There are two methods of calculating minor losses:

a) <u>Losses coefficient (K):</u>

$$h_m = \sum K_m \frac{V^2}{2g}$$

where,

h_m : minor losses (m)

K_m: minor loss coefficient, which depends on the type of fitting.

V: fluid velocity (m/s)

<u>Pipe line system curve:</u>

$$h_{sys} = h_s + h_L$$

$$h_{sys} = h_s + (h_f + h_m)$$

$$h_{sys} = h_s + \left(f\frac{L}{D} + \sum K_m\right)\frac{V^2}{2g}$$

$$Q = VA \Longrightarrow V = \frac{Q}{A}$$

$$\Longrightarrow h_{sys} = h_s + \frac{\left(f\frac{L}{D}+\sum K_m\right)Q^2}{2gA^2}$$

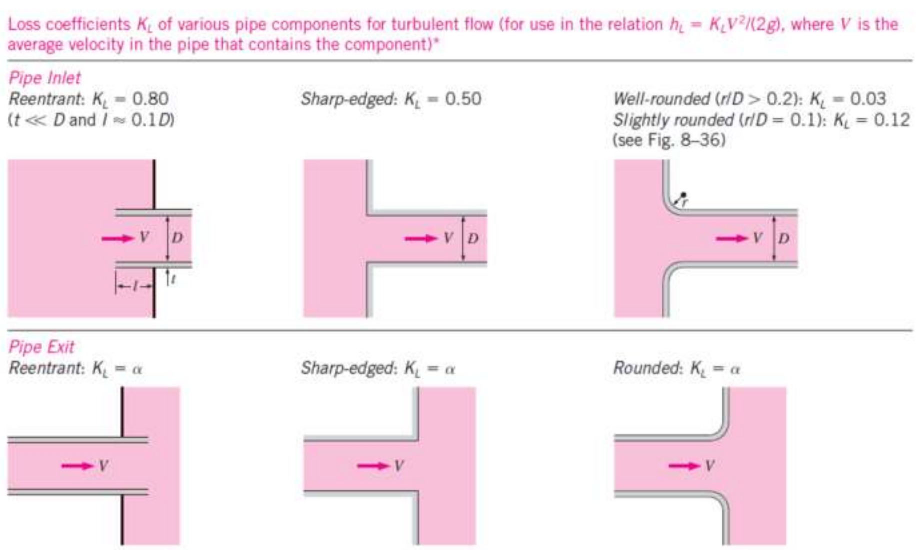
$$h_{sys} = h_s + KQ^2$$

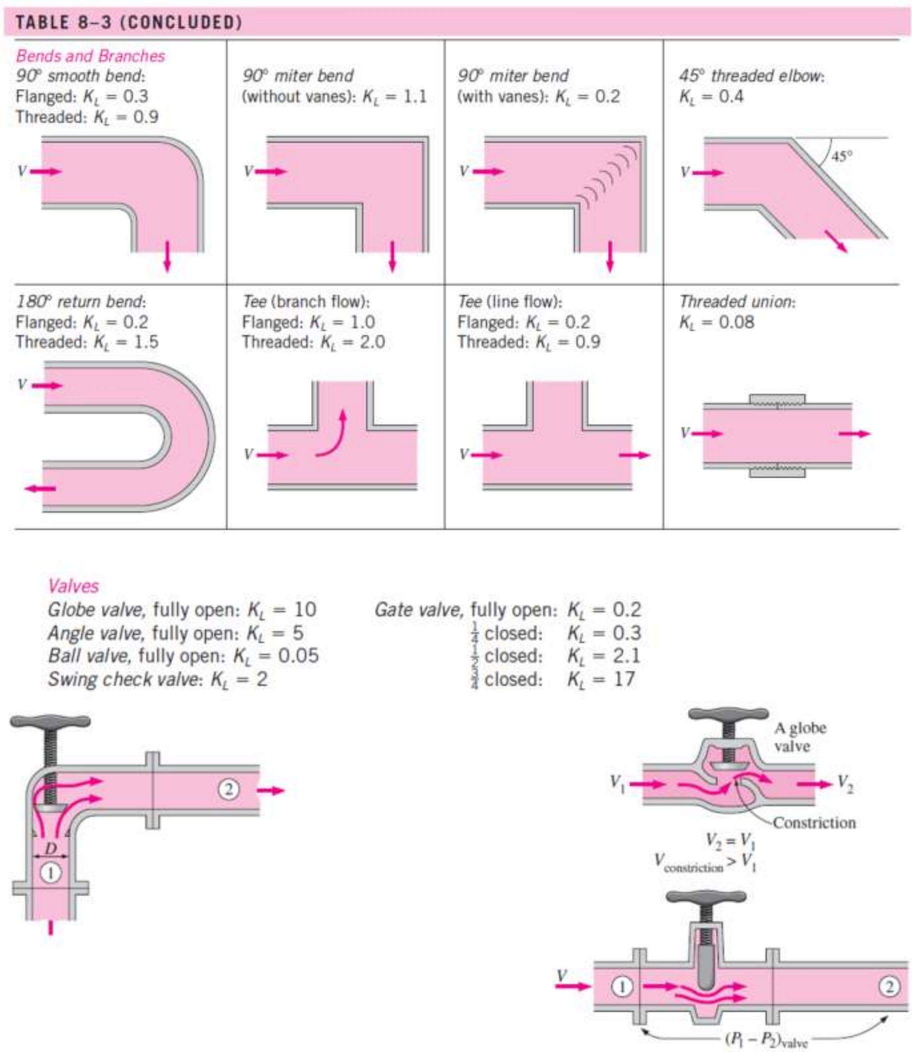
$$h_L = KQ^2$$

$$h_s = \Delta Z$$

$$K = \frac{\left(f\frac{L}{D}+\sum K_m\right)}{2gA^2}$$

Fitting Losses (K_m or K_L)





b) Equivalent length (L_{eq}):

In this method, the minor losses are replaced by an equivalent length of a straight pipe which gives the same losses.

$$h_m = f\frac{L_{eq}}{D}\frac{V^2}{2g}$$

$$h_{sys} = h_s + h_L$$

$$h_{sys} = h_s + (h_f + h_m)$$

$$h_{sys} = h_s + \left(f\frac{(L+L_{eq})}{D}\right)\frac{V^2}{2g}$$

$$h_{sys} = h_s + \left(f\frac{(L+L_{eq})}{D}\right)\frac{Q^2}{2gA^2}$$

$$h_{sys} = h_s + KQ^2$$

$$h_s = \Delta Z$$

$$K = f\frac{(L + L_{eq})}{2gDA^2}$$

Equivalent Length of Straight Pipe for Valves and Fittings (feet)												
Screwed Fittings		Pipe Size										
		1/4	3/8	1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	4
Elbows	Regular 90 deg	2.3	3.1	3.6	4.4	5.2	6.6	7.4	8.5	9.3	11.0	13.0
	Long radius 90 deg	1.5	2.0	2.2	2.3	2.7	3.2	3.4	3.6	3.6	4.0	4.6
	Regular 45 deg	0.3	0.5	0.7	0.9	1.3	1.7	2.1	2.7	3.2	4.0	5.5
Tees	Line flow	0.8	1.2	1.7	2.4	3.2	4.6	5.6	7.7	9.3	12.0	17.0
	Branch flow	2.4	3.5	4.2	5.3	6.6	8.7	9.9	12.0	13.0	17.0	21.0
Return Bends	Regular 180 deg	2.3	3.1	3.6	4.4	5.2	6.6	7.4	8.5	9.3	11.0	13.0
Valves	Globe	21.0	22.0	22.0	24.0	29.0	37.0	42.0	54.0	62.0	79.0	110.0
	Gate	0.3	0.5	0.6	0.7	0.8	1.1	1.2	1.5	1.7	1.9	2.5
	Angle	12.8	15.0	15.0	15.0	17.0	18.0	18.0	18.0	18.0	18.0	18.0
	Swing Check	7.2	7.3	8.0	8.8	11.0	13.0	15.0	19.0	22.0	27.0	38.0
Strainer			4.6	5.0	6.6	7.7	18.0	20.0	27.0	29.0	34.0	42.0

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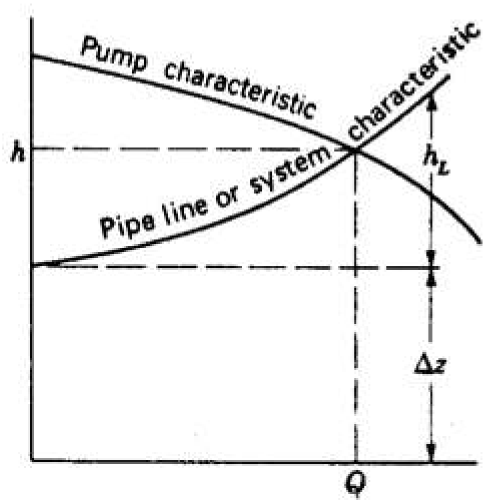
Operating point of a pump:

- The manner in which a pump operates depends not only on the pump performance characteristics but also on the characteristics of the system in which it is to operate (system demand).
- For a particular pump, draw both the pump operating characteristics curve [h versus Q] for a selected speed of operation and the system characteristics curve [i.e., the required pumping head versus Q].



- **Pump operating point:** the point where the pump curve and the system curve intersect.
- The value of (h) and (Q) determined by the intersection, may or may not be those for the maximum efficiency. If they are not, this means that the pump is not exactly suited for the specific conditions.

Example-3:

Water is pumped between two reservoirs ($\Delta Z = 15m$) in a pipe line with the following characteristics ($D = 300mm, L = 70m, f = 0.025\ and\ \sum K = 2.5$). The radial flow pump characteristic curve is approximated by the following formula:

$$h_P = 22.9 + 10.7Q - 111Q^2$$

where, h_P is in meters and Q is in m^3/s. Determine the discharge (Q) and the pump head (h) corresponding to the operating point.

Solution:

Example-4:

A centrifugal pump has the following performance at 1750 rpm:

Q (L/s)	0	2	4	6	8	10	12
H (m)	22	24	23.5	21.5	18.5	14	9.5
η (%)	0	31	54	72	84	84	74

If this pump is interposed in a pipe line of 50 m long and 5 cm diameter, draw the pipe line system curve. The pipe line has two bends, one in the suction side ($K_1 = 0.8$) and one in the delivery side ($K_2 = 0.85$). the pipe entrance loss coefficient is ($K_3 = 1.2$). The suction level is 2 m above the pump level and the delivery level is 5 m above the pump. Pipe friction losses coefficient (f) is 0.04.

Solution:

Q (L/s)	0	2	4	6	8	10	12
H (m)	22	24	23.5	21.5	18.5	14	9.5
η (%)	0	31	54	72	84	84	74
h_{sys} (m)							

Specific speed (N_s):

- Specific speed is used to compare the performance of different types of pumps.
- Specific speed is an index relating flow rate, total head and rotative speed of the impeller for pumps of similar geometries.

$$N_s = \frac{N\sqrt{Q}}{(gH)^{3/4}}$$

where,

N_s: specific speed (dimensionless)

N: rotational speed of the impeller (rps)

Q: flow rate (m^3/s)

H: total head (m)

Note: Both the flow rate (Q) and the total head (H) are measured at the Best Efficiency Point (BEP).

- Specific speed is an index used to predict desired pump or turbine performance [i.e., it predicts the general shape of a pump's impeller].

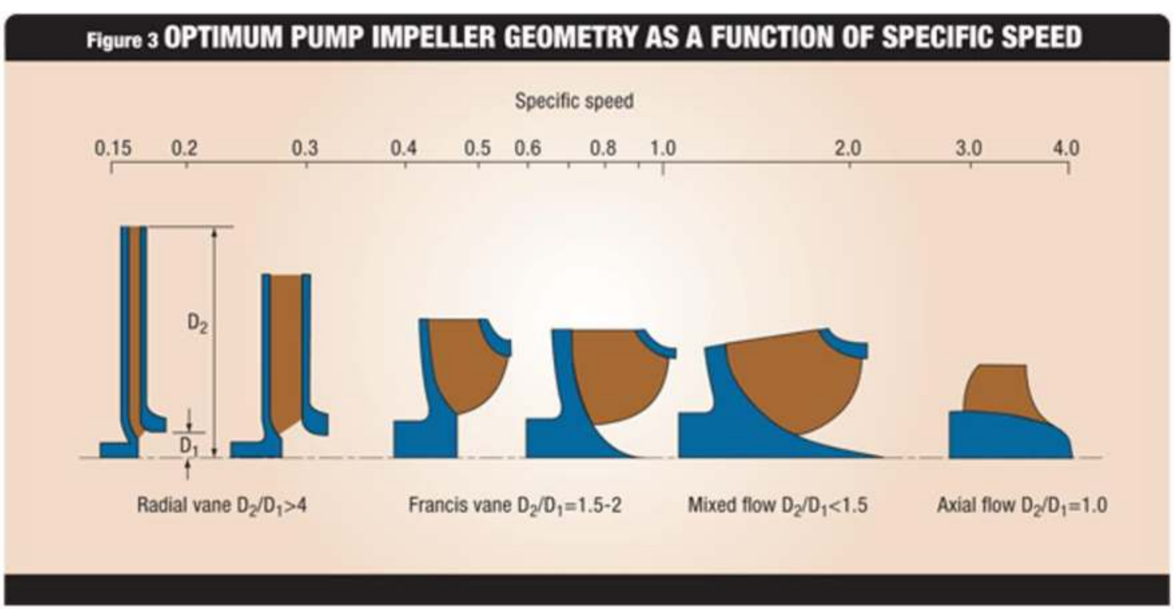
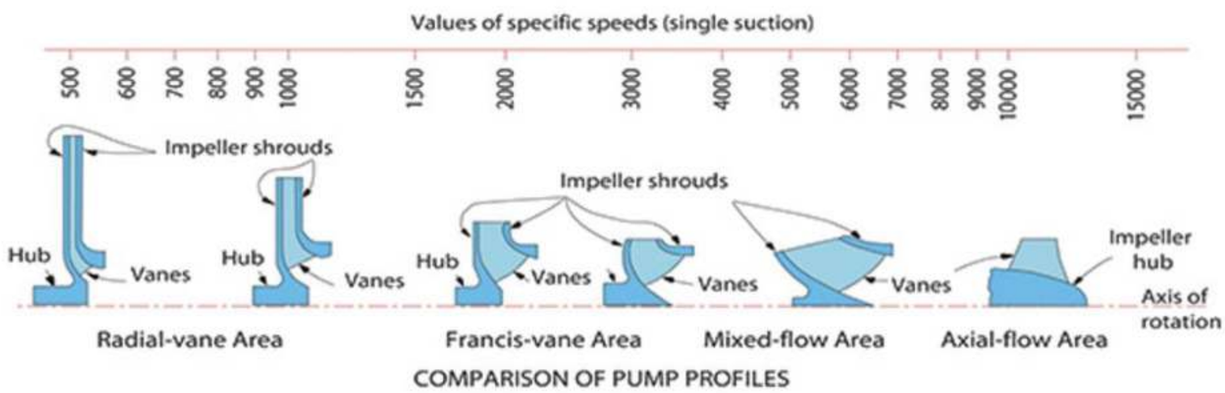


Figure 3 OPTIMUM PUMP IMPELLER GEOMETRY AS A FUNCTION OF SPECIFIC SPEED



Example-5:

The characteristic of a 0.6 m diameter centrifugal pump impeller running at 1500 rpm are as follows:

Q (m^3/min)	0	9	17.5	26.5	35	44	53	62	70.5
H (m)	120	119.4	118.8	117.3	111.7	98.8	75.3	42.6	0
η (%)	0	41	60	74	83	83	74	51	0

If the pump is used to transfer water between two reservoirs having a difference in level of 60 m, determine:

a) The pump operating point if the head loss to frictional and other resistances is 50.4 m at 30 m^3/min.
b) The power consumed by the pump.
c) Pump specific speed.

Solution:

Q (m^3/min)	0	9	17.5	26.5	35	44	53	62	70.5
h_{sys} (m)									

Cavitation:

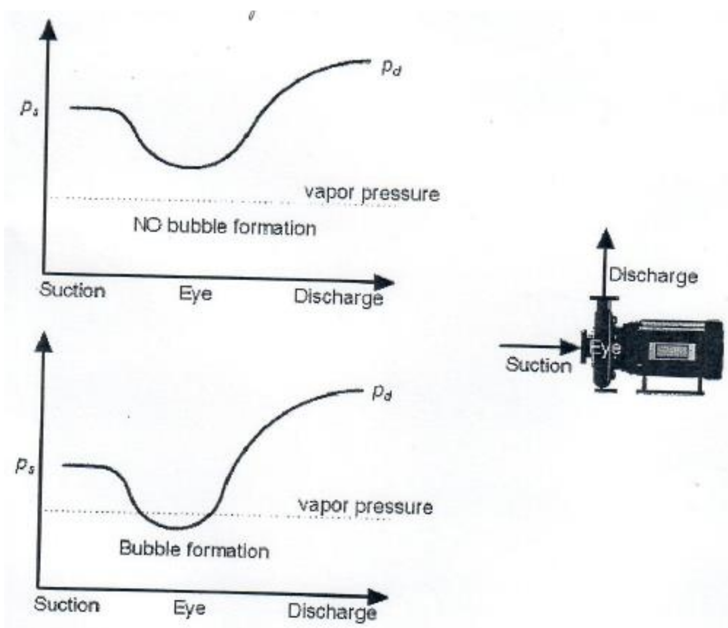
- Cavitation can be occur in turbomachines when the local pressure [absolute pressure] drops to the vapor pressure of the liquid, and as a result, vapor filled cavities (bubbles) are formed.
- As the vapor filled bubbles are transported through the turbomachine into regions of greater pressure (pressure above vapor pressure of the liquid being pumped), they will collapse rapidly generating extremely high localized pressure.
- Those vapor bubbles that collapse close to the impeller cause pitting and cavitation erosion to the impeller surface material.
- Cavitation not only cause damage to the impeller material, but also lower the head-capacity and mechanical efficiency curves.
- Signs of cavitation in turbopumps include noise, internal rattle and increased vibration.

Net positive suction head (NPSH):

NPSH: is the minimum head required at the pump suction side to keep the pump away from cavitation.

- Low pressure at the suction side of a pump can cause the fluid to start boiling.
- Boiling (bubble formation) starts when the pressure of the liquid is reduced to the vapor pressure of that liquid at the actual temperature.



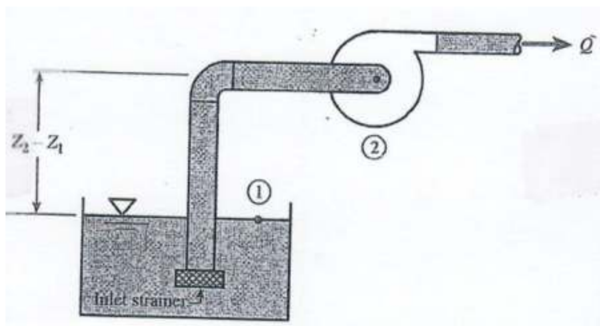
NPSH: is the difference between total head (velocity head and pressure head) at the pump suction side and the liquid vapor pressure.

$$NPSH = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} - \frac{P_v}{\gamma}$$

where, P_v is the vapor pressure of the fluid.

Available NPSH:

The available $(NPSH)_A$ can be obtained by applying Bernoulli equation between points 1 and 2.



$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$P_1 = P_{atm}$$

$V_1 \cong 0$ $(velocity\ of\ a\ large\ water\ surface\ is\ very\ small\ which\ can\ be\ neglected)$

From the definition of NPSH:

$$(NPSH)_A = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} - \frac{P_v}{\gamma}$$

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} = (NPSH)_A + \frac{P_v}{\gamma}$$

By substituting in Bernoulli equation:

$$\frac{P_{atm}}{\gamma} + Z_1 = (NPSH)_A + \frac{P_v}{\gamma} + Z_2 + h_L$$

$$(NPSH)_A = \frac{P_{atm}}{\gamma} + Z_1 - \frac{P_v}{\gamma} - Z_2 - h_L$$

$$(NPSH)_A = \frac{P_{atm} - P_v}{\gamma} - \Delta Z - h_L$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} + \sum K_m \frac{V^2}{2g}$$

- Pump manufacturers specify a required NPSH ($(NPSH)_R$).
- To a void cavitation $\Longrightarrow$ $(NPSH)_A \geq (NPSH)_R$

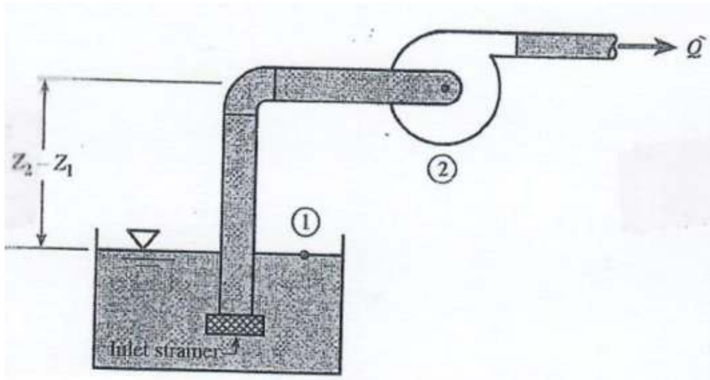
Example-6:

A pump required to deliver water against a head of 150 m. the height of the pump relative to water surface at the intake is 3 m. The barometric pressure is 98.6 kPa and the vapor pressure of water is 3.5 kPa. Assume the friction losses in the intake piping are 1.5 m. Find the available NPSH.

Solution:

Example-7:

A centrifugal pump is installed above water tank as shown in the figure. The pump is used to provide a flow rate of 0.015 m^3/s of water at 20 ℃. Under this flow conditions, the pump manufacturer specifies $(NPSH)_R$ of 5 m. If the atmospheric pressure is 101 kPa, determine the maximum elevation height $(Z_2 - Z_1)$ that the pump can be installed above the water free surface without causing pump cavitation. The head loss (h_L) in the 100 mm diameter suction pipe line is due to a pump inlet strainer having a loss coefficient of 15. Other head losses are negligibly small. ($P_{v@20℃} = 2.34\ kPa$).

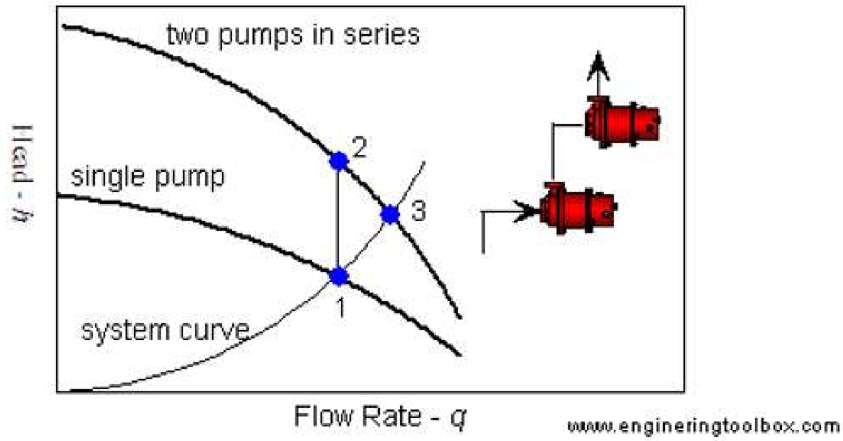


Solution:

Pumps series and parallel operation:

Pumps is series:

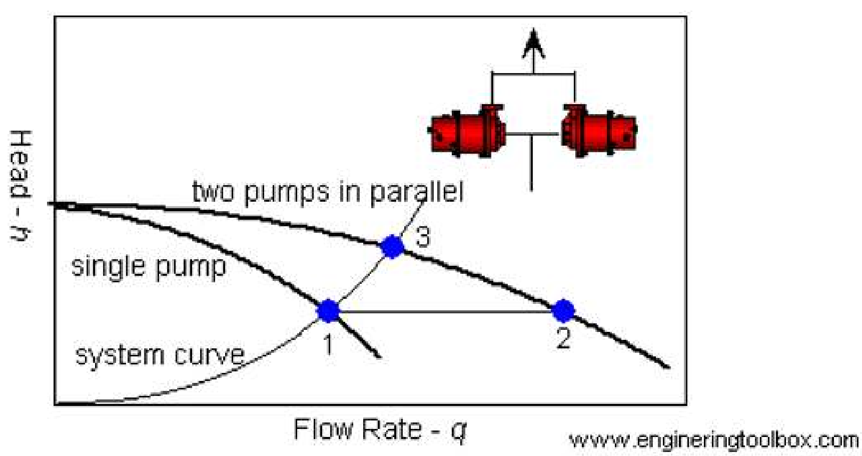
- Pumps can be arranged in series or parallel to provide an additional head or flow rate capacity.
- Centrifugal pumps arranged in series are used to overcome larger system head loss than one pump can handle a lone.



- For two identical pumps in series, the head will be twice the head of a single pump at the same flow rate as indicated by point (2).
- In practice, the combined head and flow rate moves along the system curve to point (3).
- Point (3) is where the system operates with both pumps running.
- Point (1) is where the system operates with one pump running.
- The overall efficiency of pumps arranged in series is given by:

$$\eta = \frac{\gamma Q \sum h}{\sum P_{in}}$$

<u>Pumps is parallel:</u>



- For two identical pumps arranged in parallel and the head is kept constant, the flow rate doubles as indicated by point (2) compared to a single pump.
- Point (3) is where the system operates with both pumps running.
- Point (1) is where the system with one pump running.
- The overall efficiency of pumps arranged in parallel is given by:

$$\eta = \frac{\gamma h \sum Q}{\sum P_{in}}$$

Example-8:

Two reservoirs A and B are connected with a long pipe which has characteristic such that the head loss is expressed as $h_L = 350Q^2$ where h_L in meters and Q in m^3/s and the static head is 25 m.

The pump characteristic is:

Q (m^3/s)	0	0.075	0.15	0.2	0.25	0.3
H (m)	51	50	48	44	38	29
η (%)	0	58	80	72	58	35

Find:

a) Point of operation.
b) Shaft power.
c) If two identical pumps are operated in series, find the point of operation's head, discharge, efficiency and power required by two pumps.
d) If two identical pumps are operated in parallel, find the point of operation's head, discharge, efficiency and power required by two pumps.

Solution:

Q (m^3/s)	0	0.075	0.15	0.2	0.25	0.3
h_{sys} (m)						
$2Q$ (m^3/s)						
2h (m)						

Pump similarity laws:

- Similarity laws can be used to predict the performance of a prototype pump from the test of a scaled model.
- The three major reasons for running tests on a model rather than the prototype are:
  1) Similarity laws enable us to make experiments with a convenient fluid such as water and then apply the results to a fluid that is less convenient to work.
  2) The model can be made smaller than the prototype in size, so the tests are easier to perform.
  3) The model can be made of a different material than the prototype (for example wood instead of metal) and thus can be less expensive to be manufactured.

Required similarity conditions:

a) **Geometric similarity:**

The model and its prototype should be identical in shape but differ only in size.

b) **Dynamic similarity:**

The ratio of the corresponding forces in the model and the prototype should be the same. For example, Reynolds number of the model during the tests must equal the Reynolds number of the prototype.

- The following **four** similarity laws can be applied to centrifugal pumps based solely on geometric similarity. Subscripts "m" and "P" of each dimensionless term stand for "model" and "prototype", respectively.
  1) Pump flow rate similarity law:

$$\left(\frac{Q}{ND^3}\right)_m = \left(\frac{Q}{ND^3}\right)_P$$

  2) Pump head similarity law:

$$\left(\frac{gh}{N^2D^2}\right)_m = \left(\frac{gh}{N^2D^2}\right)_P$$

  3) Pump input shaft power similarity law:

$$\left(\frac{P}{\rho N^3 D^5}\right)_m = \left(\frac{P}{\rho N^3 D^5}\right)_P$$

4) Pump overall efficiency similarity law:

$$\eta_m = \eta_P$$

where,

Q: volume flow rate (m^3/s)

N: pump speed (rpm)

D: impeller diameter (m)

h: pump head (m)

P: shaft input power (W)

ρ: fluid density (kg/m^3)

η: overall efficiency

Example-9:

A model centrifugal pump has a scale of 1:15. The model when tested at 3600 rpm, delivered 0.1 m^3/s of water at a head of 40 m. Assuming the prototype has an efficiency of 88 %, what will be its speed, capacity, and power requirement at a head of 50 m?

Solution:

Pump similarity laws:

If pumps (1) and (2) are from the same geometric design family and are operating at similar kinematic and dynamic operating conditions, the flow rate, pump head, and pump power for the pumps will be related according to the following expressions:

$$\frac{Q_2}{Q_1} = \left(\frac{N_2}{N_1}\right)\left(\frac{D_2}{D_1}\right)^3$$

$$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2\left(\frac{D_2}{D_1}\right)^2$$

The above equations are used to predict the flow rate and pump head for a design change in pump speed (N) and impeller diameter (D).

$$\frac{P_{in,2}}{P_{in,1}} = \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{N_2}{N_1}\right)^3\left(\frac{D_2}{D_1}\right)^5$$

The above equation is used to predict the pump power for a design change in fluid density (ρ), pump speed (N) and impeller diameter (D).

- If the impeller diameter changes from D_1 to D_2 while the pump speed is kept constant:

$$\frac{Q_2}{Q_1} = \left(\frac{D_2}{D_1}\right)^3$$

$$\frac{H_2}{H_1} = \left(\frac{D_2}{D_1}\right)^2$$

$$\frac{P_{in,2}}{P_{in,1}} = \left(\frac{D_2}{D_1}\right)^5$$

- If the pump speed changes from N_1 to N_2 while the impeller diameter is kept constant:

$$\frac{Q_2}{Q_1} = \left(\frac{N_2}{N_1}\right)$$

$$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2$$

$$\frac{P_{in,2}}{P_{in,1}} = \left(\frac{N_2}{N_1}\right)^3$$

<u>Example-10:</u>

At its optimum point of operation, a given centrifugal pump with an impeller diameter of 50 cm delivers 3.2 m^3/s of water against a head of 25 m when running at 1500 rpm.

1) If its efficiency is 82%, what is the brake power of the driving shaft?
2) What is the specific speed of the pump?
3) If a kinematically similar pump with an impeller diameter of 80 cm is rotating at 1200 rpm, what would be the pump discharge, head, shaft power, and specific speed? Assume both pumps operate at the same efficiency.

Solution:

Chapter (3)

Compressors

Compressor: is a device used to raise the pressure of a gas. (The function of a compressor is to take a definite quantity of a fluid (usually a gas, and most often air) and deliver it at a required pressure.

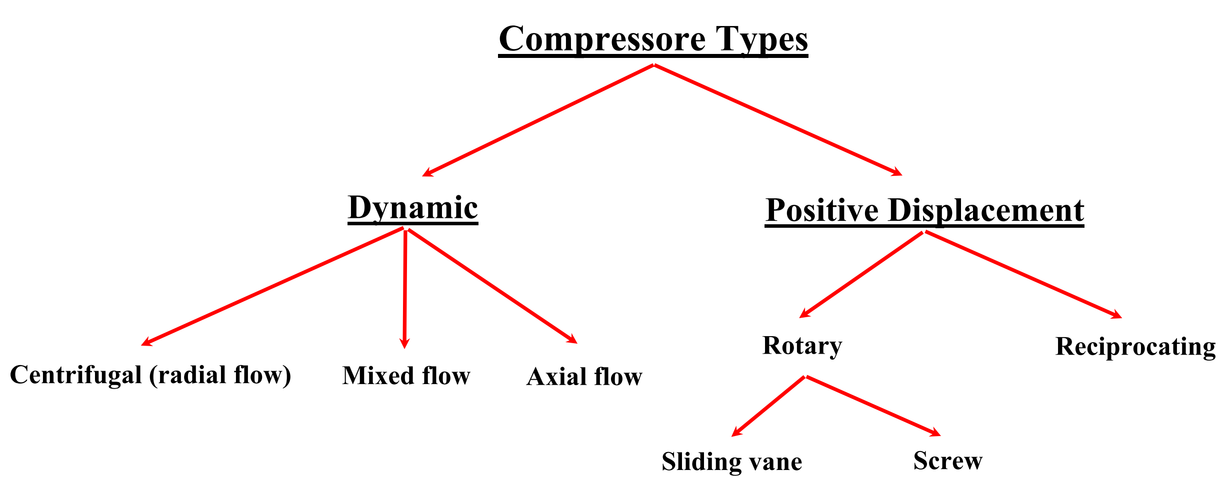
Compressor types:

Compressors can be classified based on the type of operation:

1) **Dynamic Compressors:** do their work by using inertial forces applied to the gas by means of rotating bladed impellers.
2) **Positive displacement compressors:** trap the gas by the action of mechanical components and restrict its escape as compression takes place through direct volume reduction.

Compressors can also be classified based on their compression ratio:

1) **Fans:** provide a very small compression ratio in the range of (1-1.14).
2) **Blowers:** provide a compression ratio in the range of (1.14-4).
3) **Compressors:** provide a high compression ratio starting from 4 up to 7000.



- Positive displacement compressors divided into two types, reciprocating and rotary.
- The differences between the two types are:

a) **Reciprocating compressor:**

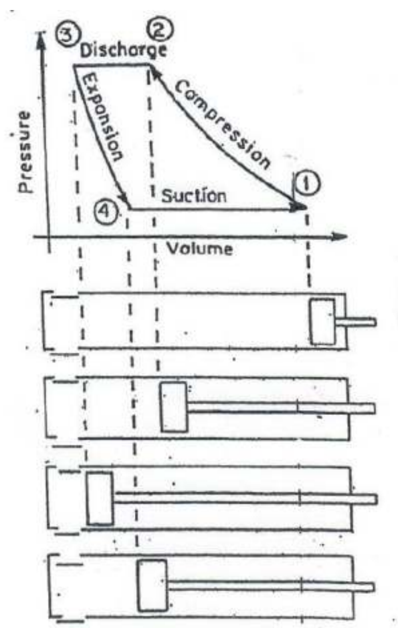
Have the characteristics of low mass flow rate and high pressure ratios.

b) Rotary compressor:

- Have the characteristics of high mass flow rate and low pressure ratios.
- Rotary compressors are smaller in size, lighter in weight and mechanically simpler than the reciprocating compressors.

Reciprocating compressor:

- Reciprocating compressor consists of piston, connecting rod, crank and the cylinder arrangement.
- The cycle takes one revolution of the crank shaft for completion and the indicator diagram will be as follows:



- The line (4-1): represents the suction stroke.
- The line (1-2): represents the compression stroke.
- The line (2-3): represents the discharge stroke.
- The line (3-4): represents the expansion stroke.

Compression ratio (r):

$$r = \frac{P_{discharge}}{P_{suction}} = \frac{P_d}{P_s}$$

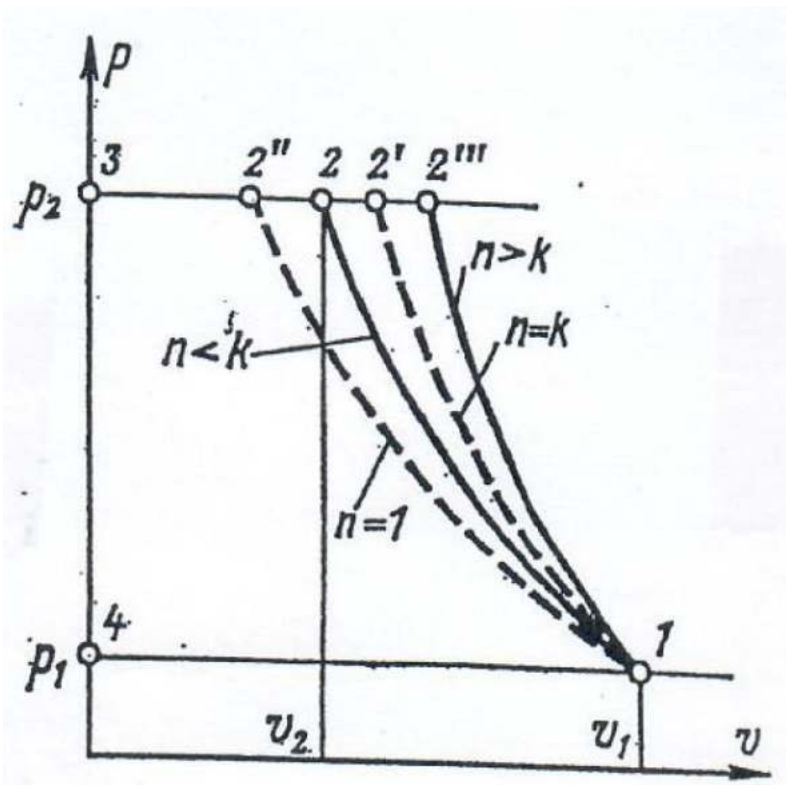
Compression processes:

- Polytropic process ($PV^n = C$)
- Isentropic process ($PV^k = C$)
- Isothermal process ($PV = C$)

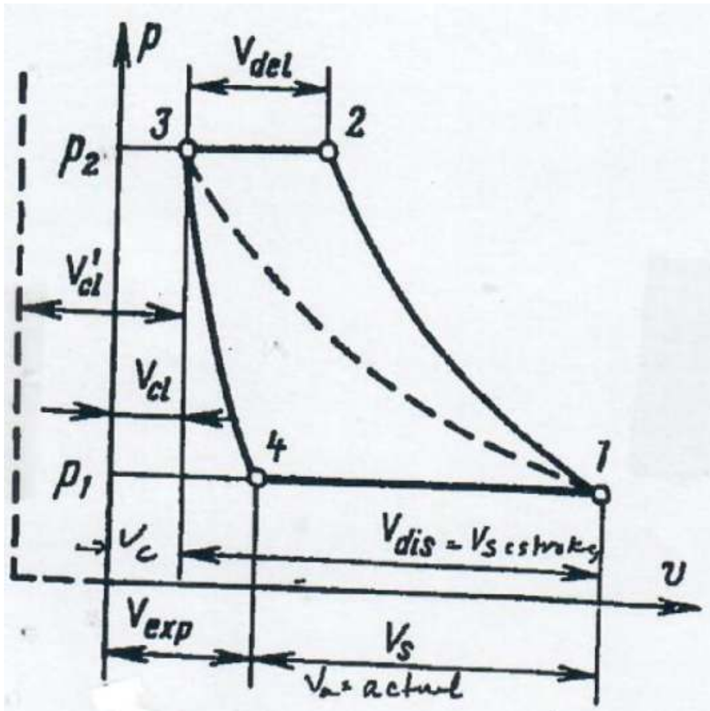
where, "n" and "k" are constants.

For reciprocating compressors:

$$1.2 \leq n \leq 1.35$$



Cylinder clearance and capacity:



Clearance volume (V_c):

Is the volume trapped gas at the end of discharge stroke ($V_c = V_3$).

Effective swept volume (V_s):

$$V_s = V_1 - V_4$$

Displacement volume or swept volume (V_d):

$$V_d = V_1 - V_3$$

$$V_d = \frac{\pi}{4} D^2 S$$

where, D: piston diameter, S: piston stroke.

Relative clearance (RC):

$$RC = \frac{V_c}{V_d}$$

Total volume: $V_1 = V_c + V_d$

To find $V_2 \Longrightarrow use\ P_1 V_1^n = P_2 V_2^n$

Compressor capacity (Q):

$$Q = V_s N \qquad (m^3/min)$$

where, N: speed of the compressor (rpm), V_s: effective swept volume (m^3).

Volumetric efficiency (η_{vol}):

$$\eta_{vol} = \frac{V_s}{V_d}$$

or: $$\eta_{vol} = 1 - RC[(r)^{\frac{1}{n}} - 1]$$

if:

RC $\Uparrow, \eta_{vol} \Downarrow$

r $\Uparrow, \eta_{vol} \Downarrow$

$\eta_{vol} \Downarrow, V_s \Downarrow$

Indicated power:

- Compressor cycle is accurately calculated with thermodynamic equation for real gases.
- Perfect gas equations are very close to the actual value for discharge pressure up to 10 MPa.

Indicated power ($\dot{W}$):

$$\dot{W} = \left(\frac{n}{n-1}\right) P_s \dot{V}_s [(r)^{\frac{n-1}{n}} - 1]$$

Actual power ($\dot{W}_a$):

$$\dot{W}_a = \frac{\dot{W}}{\eta_m}$$

where, η_m: compressor mechanical efficiency ($\eta_m = 0.8 - 0.93$).

Isothermal efficiency (η_T):

Isothermal power ($\dot{W}_T$):

$$\dot{W}_T = P_s \dot{V}_s \ln(r)$$

Isothermal efficiency (η_T):

$$\eta_T = \frac{\dot{W}_T}{\dot{W}}$$

η_T: depends on the rate of compressor cooling ($\eta_T = 0.65 - 0.85$).

Example-1:

A single stage single acting reciprocating compressor has a bore of 0.2 m, a stroke of 0.3 m, and a speed of 500 rpm. The clearance volume is 5% of the displacement volume and the polytropic index (n=1.3). Intake conditions are 97 kPa and 20 °C and the compression pressure is 550 kPa. Determine:

a) The free air delivered in m^3/min (free air condition are 101.325 kPa, and 15°C).
b) The volumetric efficiency.
c) The air delivery temperature.
d) The indicated power.
e) Mass flow rate.
f) The isothermal efficiency.

g) The power of the motor required to drive the compressor if the compressor mechanical efficiency is 85%.

Solution:

Example-2:

A single stage double acting reciprocating air compressor has a free air delivery of 14 m^3/min measured at 1.013 bar and 15 °C. The pressure and temperature in the cylinder during induction are 0.95 bar and 32 °C, respectively. The delivery pressure is 7 bar and the index of compression and expansion is (n=1.3). The compressor speed is 300 rpm. The stroke/bore ratio is 1.1. The clearance volume is 5% of the displacement volume. Determine:

a) The volumetric efficiency.
b) The bore and the stroke.
c) The indicated power.

Solution:

Multi-stage compression:

The following considerations set a limit on the compression ratio in one cylinder compressor:

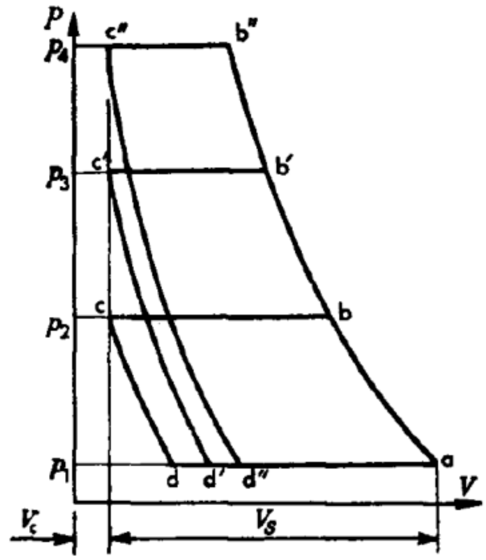
- Volumetric efficiency decreases with the increase of the compression ratio:

$$\eta_{vol} = 1 - RC\left[(r)^{\frac{1}{n}} - 1\right], as\ r\ \Uparrow, \eta_{vol}\ \Downarrow$$

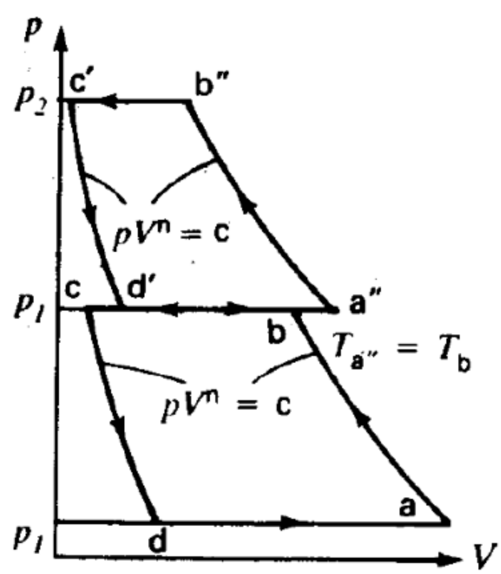
- Mechanical stresses increase with higher pressure ratios.
- Temperature increases with increasing pressure ratios and could reach compressor oil flash point temperature (493-533 K).

$\frac{P_2}{P_1}$	Final air temperature, K		
	Adiabatic compression	Polytropic compression with cooling of cylinder	Polytropic compression with cooling of cylinder and head
2	358	337	325
4	438	402	372
6	493	454	409
8	536	493	443

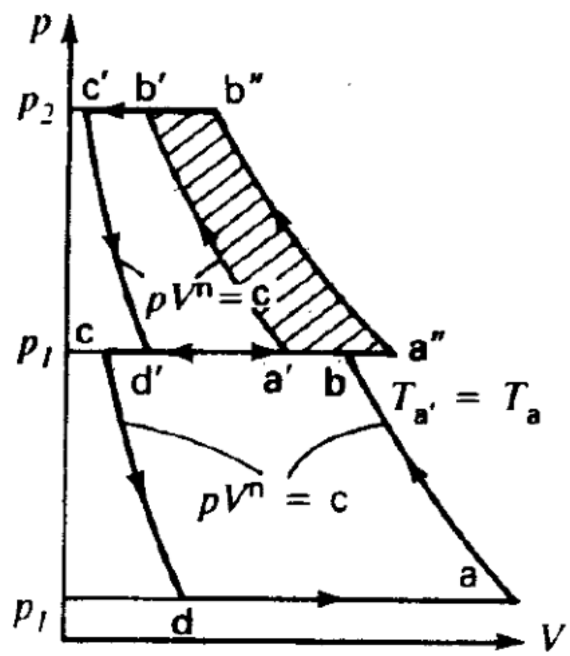
- For the compression from P_1 to P_2, the cycle is (a-b-c-d) and the free air delivery (F.A.D.) per cycle is $(V_a - V_d)$ as shown in the figure below. For the compression from P_1 to P_3, the cycle is (a-b'-c'-d') and the free air delivery (F.A.D.) per cycle is $(V_a - V_d')$. For the compression from P_1 to P_4, the cycle is (a-b''-c''-d'') and the free air delivery (F.A.D.) per cycle is $(V_a - V_d'')$. Therefore, for a required (F.A.D.), the cylinder size would have to increase as the pressure ratio increases.



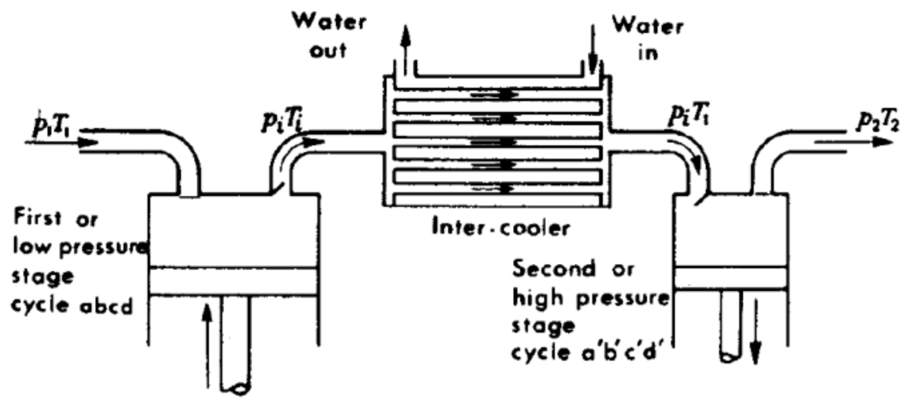
- The volumetric efficiency can be improved by carrying out the compression in two stages. After the first stage of compression, the fluid is passed into a smaller cylinder in which the gas is compressed to the required final pressure.
- If the machine has two stages, the gas will be delivered at the end of the second stage, but it could be delivered to a third cylinder for a higher pressure ratio.
- The indicator diagram for a two-stage compressor is shown in the figure below. In this diagram, it is assumed that the delivery process from the first (or low pressure) stage and the induction process of the second (or high pressure) stage are done at the same pressure.



- Multi-stage compression with intercooling brings the compressor cycle near to isothermal process. Therefore, a considerable saving in the driving power can be achieved for a given compression ratio (i.e., to have a minimum work input, the compression process should be isothermal).
- The ideal isothermal compression can only be obtained if the cooling is continuous which is difficult to obtain during normal compression.
- With multi-stage compression, the gas can be cooled down as it is being transferred from one stage (or cylinder) to the next, by passing it through an intercooler.
- If intercooling is complete, the gas will enter the second stage at the same temperature at which it entered the first stage.
- The saving in work input obtained by intercooling is shown by the shaded area in the figure below.



- The two indicator diagrams (a-b-c-d) and (a'-b'-c'-d') are shown with a common pressure (P_i). This does not occur in a real machine as there is a small pressure drop between the cylinders.
- The figure below shows the intercooler:



- The number of stages required to achieve a given compression ratio are shown in the following table:

Pressure ratio	Number stages
Up to 6	1
6 to 30	2
30 to 100	4
100 to 150	5
Over 150	6 and more

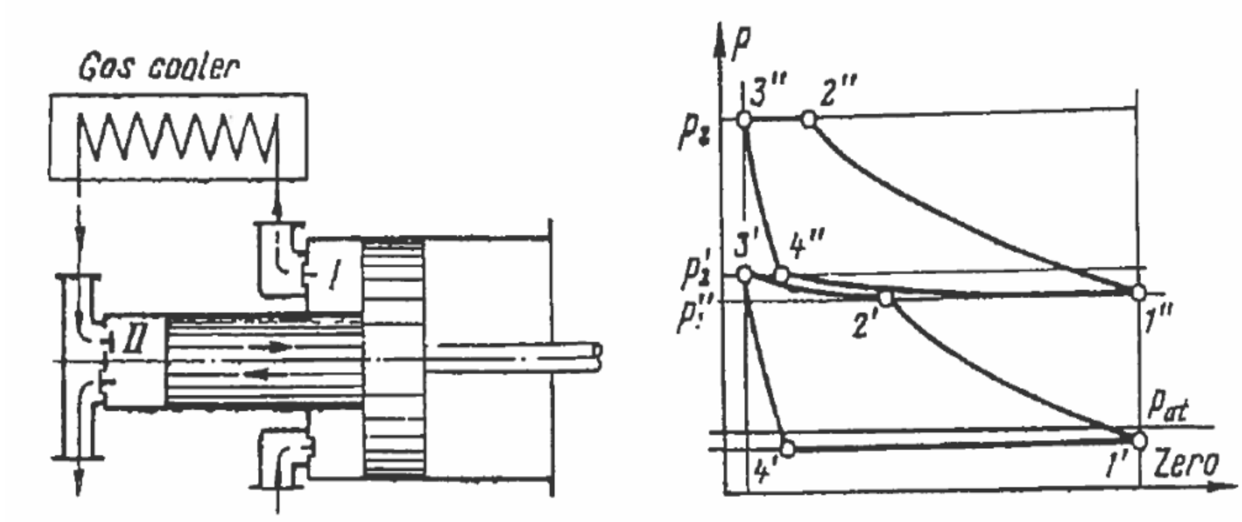
- Anyway, increasing the number of stages complicates the design and raises the cost of the compressor which restricts the maximum number of stages in modern compressors.
- Modern water cooled compressor has a pressure ratio less than 7.
- Large capacity design has a pressure ratio ≤ 4.

Multi-Stage Type Design:

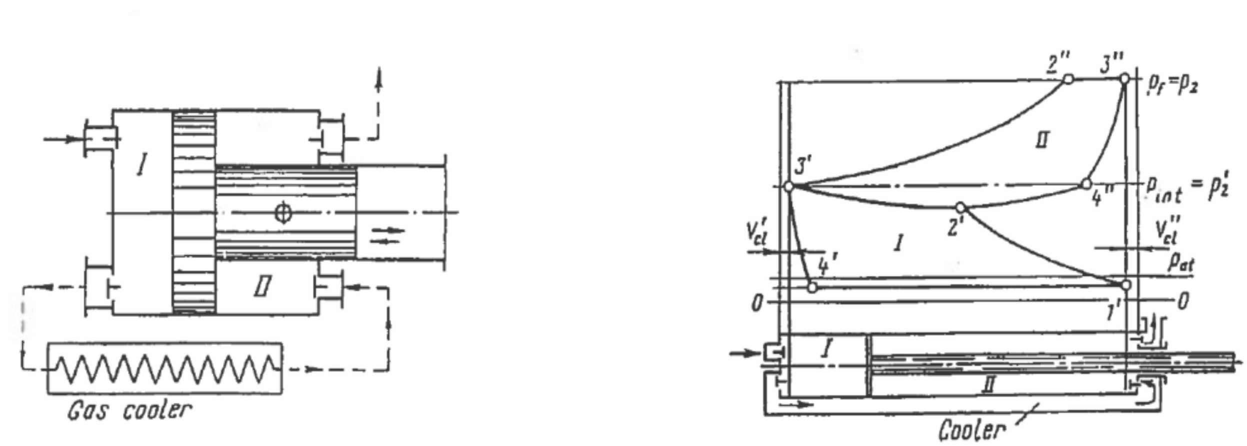
Multi-stage compressors are divided into two main categories:

1) **Differential pistons and several compression stages in one cylinder:**

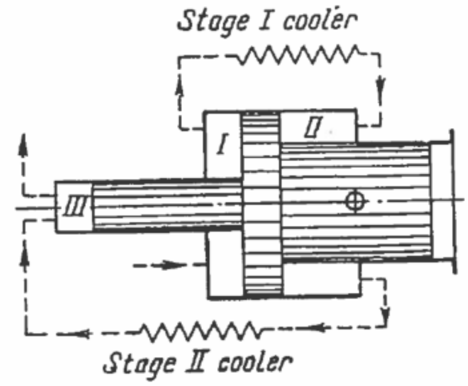
   a) **Two-stage single acting differential piston compressor:**

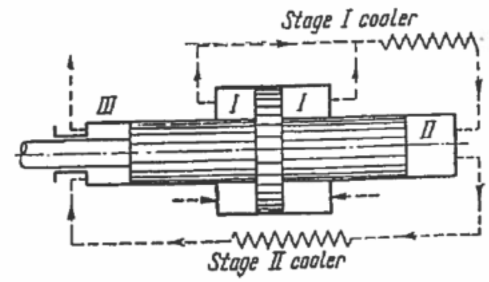


   b) **Two-stage double acting differential piston compressor:**

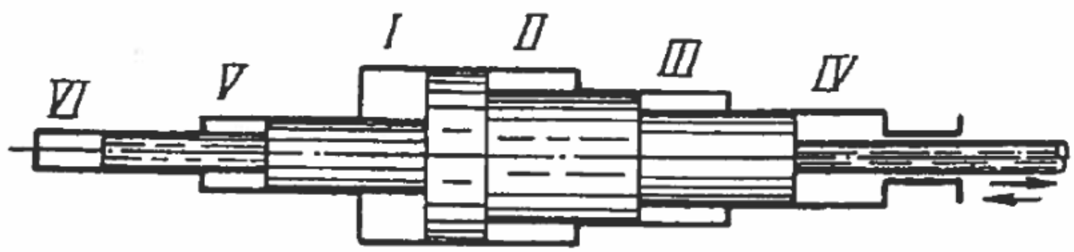


c) Three-stage differential piston compressor:



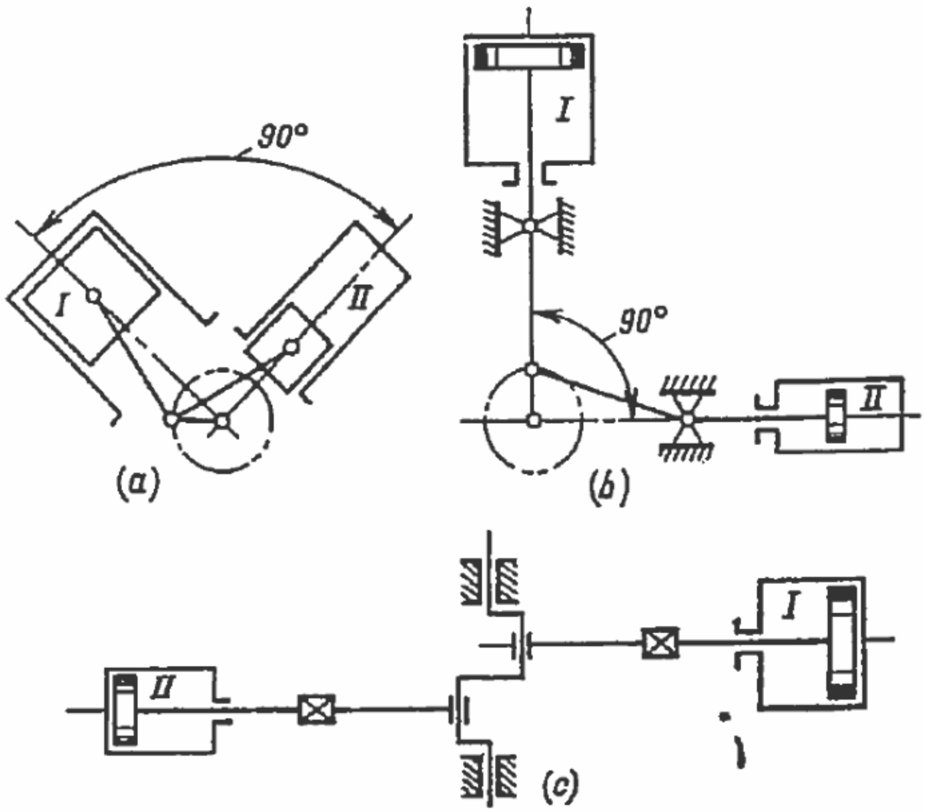


d) Multi-stage differential piston compressor:



Six-stage differential-piston compressor. Schematic

2) Compression stages realized in separate cylinders:



Two-stage compressor designs with pressure stages in individual cylinders

Example-3:

Two-stage air compressor has a free air delivery of 15 m^3/hr measured at 1.013 bar and 20°C is used to compress air from 1 bar and 30°C to 7 bar and 80°C. If the clearance ratio is 0.05, compressor speed is 1300 rpm, and the air exits the first stage at 3 bar and 90 °C. Calculate the following:

a) The index of compression for each stage.
b) The volumetric efficiency for each stage.
c) The displacement volume for each stage.
d) The work required per kg mass of air.

e) Draw the indicator (P-V) diagram.

Assume ideal inter-staging and inter-cooling.

<u>Solution:</u>