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PE 437

NUMERICAL METHODS IN PETROLEUM ENGINEERING

PART 1

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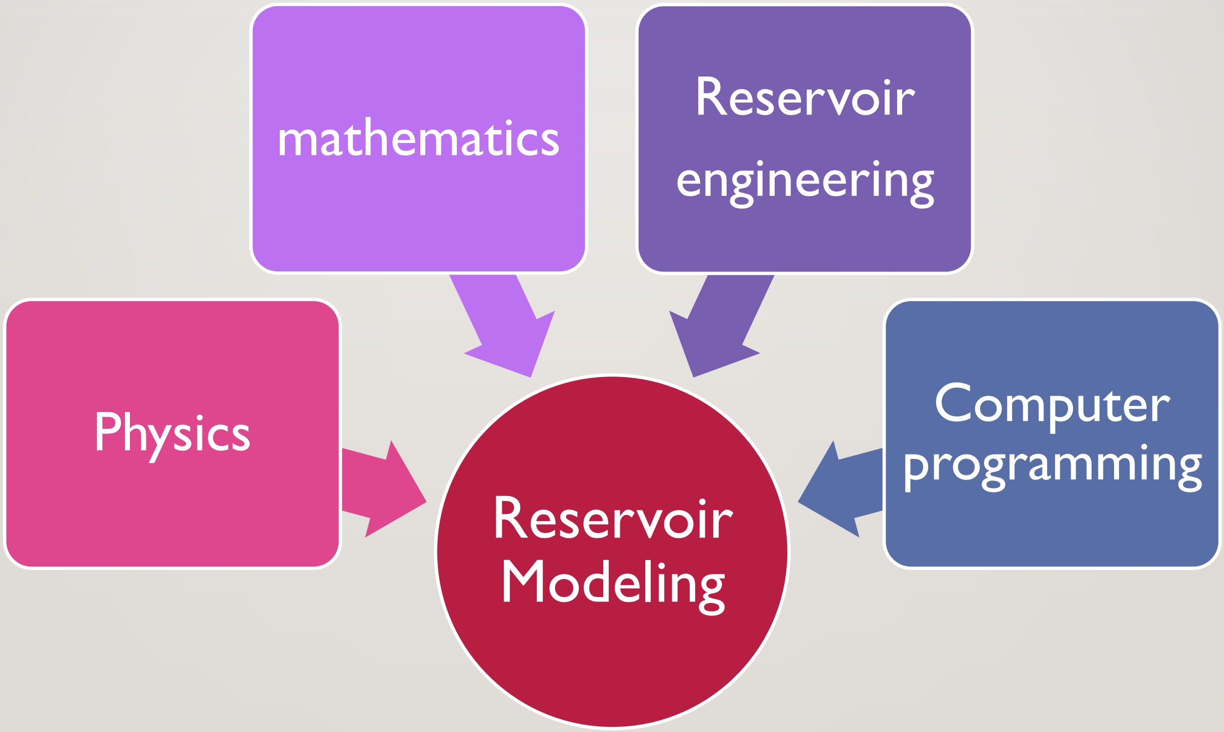
CHAPTER 1
INTRODUCTION TO RESERVOIR MODELLING



Need for Reservoir Modelling (Simulation)

The desire of the petroleum engineer to obtain accurate performance predictions of the entire reservoir under different operating conditions. So, the high risk element is minimized.

High risk comes from the following:

1- heterogeneous rock properties
2- variations of fluid properties
3- complexity of the recovery mechanism
4- availability of predictive methods

Classification of Reservoir Simulators

- black-oil reservoir simulator

- gas reservoir simulators
    - single-phase (gas)
    - two-phases (gas & water)

- compositional reservoir simulator

- thermal simulator

- IMPES, full implicit

- single-porosity or dual-porosity

- Some refer to the application (e.g., thermal)
- Others to the model formulation (e.g., implicit)
- Yet others to an attribute of the reservoir formation (e.g., dual-porosity)

Single Phase Flow in Porous Media

Single phase Darcy Flow

Field units : $$v = -0.001127\frac{k}{\mu}\frac{dP}{dl} \quad (1\text{-}1)$$

where: v = RB/(D-ft^2), *k* = md, μ = cp, P = psia, L = ft

Lab Units: $$v = -\frac{k}{\mu}\frac{dP}{dl} \quad (1\text{-}2)$$

where: v = cm/s, k = Darcy, μ = cp, P = atm, L = cm

$\frac{dP}{dl}$ pressure gradient in the direction of flow.

Darcy velocity $(v) = \frac{q}{A}$ & Interstitial Velocity $(u) = \frac{q}{A\phi}$

Equations 1-1 & 1-2 without the gravity component

General Form of Darcy Equation with gravity component

$$v = -0.001127 \frac{k}{\mu} \left(\frac{\partial P}{\partial x} - \underbrace{\frac{\rho}{144} \frac{\partial D}{\partial x}}_{\text{Gravity term}} \right) \qquad (1\text{-}3)$$

Where,
v = RB/(D-ft^2), k = md, μ = cp, ρ = lbm/ft^3, p = psi, D = ft, x = ft ,

$\gamma = \rho/144$= (lbm/ft^3)/(144in^2/ft^2)=1/144 psi/ft

Substitute $\gamma = \rho/144$ in Eq. (1-3) yields the following:

$$v = -0.001127 \frac{k}{\mu} \left(\frac{\partial P}{\partial x} - \gamma \frac{\partial D}{\partial x} \right) \qquad (1\text{-}4)$$

Writing Eq. (1-4) in field units for all direction:

$$v_x = -0.001127\frac{k_x}{\mu}\left(\frac{\partial P}{\partial x} - \gamma\frac{\partial D}{\partial x}\right)$$

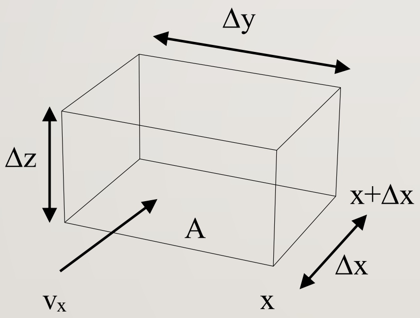
$$v_y = -0.001127\frac{k_y}{\mu}\left(\frac{\partial P}{\partial y} - \gamma\frac{\partial D}{\partial y}\right) \qquad (1\text{-}5)$$

$$v_z = -0.001127\frac{k_z}{\mu}\left(\frac{\partial P}{\partial z} - \gamma\frac{\partial D}{\partial z}\right)$$

CHAPTER2: EQUATION OF CONTINUITY

Material balance on a unit rock volume yield the following:

(Rate in) - (Rate out) + (Rate of Injected/produced fluid) = Rate of Accumulation



$$\rho v_x A\big|_x - \rho v_x A\big|_{x+\Delta x} + \overbrace{\bar{\rho}\hat{q}\underbrace{\Delta x \bar{A}}_{unit\,rock\,volume}}^{source\,term} B = \bar{A}\Delta x \frac{\left(\bar{\rho}\bar{\phi}\big|_{t+\Delta t} - \bar{\rho}\bar{\phi}\big|_{t}\right)}{\Delta t} \qquad (2\text{-}1)$$

$$\rho v_x A\big|_x - \rho v_x A\big|_{x+\Delta x} + \overbrace{\bar{\rho}\hat{q}\underbrace{\Delta x \bar{A}}_{unit\,rock\,volume}}^{source\,term} B = \bar{A}\Delta x \frac{\left(\bar{\rho}\bar{\phi}\big|_{t+\Delta t} - \bar{\rho}\bar{\phi}\big|_t\right)}{\Delta t} \qquad (2\text{-}1)$$

Check ($\rho v_x A\big|_x$; $\frac{m}{L^3}\frac{L}{T}L^2 = \frac{m}{T}$)

$\hat{q} \neq q$ q = STB/day and qB = rbbl/day

$\hat{q}$ = (STB/day)/unit rock volume ft^3 of rock

Rock Volume = $\bar{A}\Delta x$

Pore volume = $\bar{A}\Delta x\bar{\phi}$

Since $\frac{df}{dx} = \lim_{\Delta x \to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$, and $\frac{df}{dt} = \lim_{\Delta t \to 0}\frac{f(t+\Delta t) - f(t)}{\Delta t}$

Applying the above to eq. 2-1 yields the following:

$$\lim_{\Delta x \to 0}\frac{\rho\, v_x A\big|_x - \rho\, v_x A\big|_{x+\Delta x}}{\Delta x} + \bar{\rho}\bar{A}\hat{q}B = \bar{A}\lim_{\Delta t \to 0}\frac{\left(\bar{\rho}\bar{\phi}\big|_{t+\Delta t} - \bar{\rho}\bar{\phi}\big|_t\right)}{\Delta t}$$

Also, the average properties $\bar{A}, \bar{\rho}, \bar{\phi}$ will be replaced by A, ρ, ϕ

$$-\frac{\partial}{\partial x}\left(\rho A v_x\right) + \rho A\hat{q}B = A\frac{\partial}{\partial t}\left(\rho\phi\right)$$

Equation of Continuity (EOC) in 1D, 2D, and 3D:

$$-\frac{\partial}{\partial x}\left(\rho A v_x\right) + \rho A \hat{q} B = A\frac{\partial}{\partial t}\left(\rho\phi\right)$$ (1-D) (2-2a)

$$-\frac{\partial}{\partial x}\left(\rho h v_x\right) - \frac{\partial}{\partial y}\left(\rho h v_y\right) + \rho h \hat{q} B = h\frac{\partial}{\partial t}\left(\rho\phi\right)$$ (2-D) (2-2b)

$$-\frac{\partial}{\partial x}\left(\rho v_x\right) - \frac{\partial}{\partial y}\left(\rho v_y\right) - \frac{\partial}{\partial z}\left(\rho v_z\right) + \rho \hat{q} B = \frac{\partial}{\partial t}\left(\rho\phi\right)$$ (3-D) (2-2c)

1-D EOC for Single Phase, Slight Compressible fluid, Linear Flow

Start with $-\frac{\partial}{\partial x}(\rho A v_x)+\rho A\hat{q}B = A\frac{\partial}{\partial t}(\rho\phi)$ (2-2a)

since,

$v_x = -0.006328\frac{k_x}{\mu}\left(\frac{\partial P}{\partial x}-\gamma\frac{\partial D}{\partial x}\right)$,ft/day

$v_x = -0.001127\frac{k_x}{\mu}\left(\frac{\partial P}{\partial x}-\gamma\frac{\partial D}{\partial x}\right)$, bbl/(day-ft^2)

substitute for area A = $\Delta y\Delta z$ and for v_x (without gravity term and coefficient)

$$\Rightarrow -\frac{\partial}{\partial x}\left(\Delta y\Delta z\rho(-\frac{k_x}{\mu}\frac{\partial P}{\partial x})\right)+(\Delta y\Delta z)\rho\hat{q}B = (\Delta y\Delta z)\frac{\partial}{\partial t}(\rho\phi) \quad (2\text{-}3)$$

assume μ is constant, then multiply eq. 8 by μ and divide by $\Delta y\Delta z$,

$$\Rightarrow \frac{\partial}{\partial x}\left(\rho\, k_x \frac{\partial P}{\partial x}\right) + \rho\mu\hat{q}B = \mu \frac{\partial}{\partial t}(\rho\phi) \qquad (2\text{-}4)$$

Expansion of eq. 2-4:

Taking right hand side $\frac{\partial}{\partial t}(\rho\phi)$

$$\frac{\partial}{\partial t}(\rho\phi) = \phi\frac{\partial\rho}{\partial t} + \rho\frac{\partial\phi}{\partial t} = \phi\frac{\partial\rho}{\partial p}\frac{\partial p}{\partial t} + \rho\frac{\partial\phi}{\partial p}\frac{\partial p}{\partial t} = (\phi\frac{\partial\rho}{\partial p} + \rho\frac{\partial\phi}{\partial p})\frac{\partial p}{\partial t}$$

$$C_\phi = \text{compressibility of the rock} = \frac{1}{\phi}\left(\frac{\partial\phi}{\partial p}\right)\Bigg|_T$$

Since,

$$C = \text{fluid compressibility} = \frac{1}{\rho}\left(\frac{\partial\rho}{\partial p}\right)\Bigg|_T$$

$$C_t = \text{total compressibility} = C + C_\phi$$

then,

$$\frac{\partial}{\partial t}(\rho\phi) = (\phi\rho C + \rho\phi C_\phi)\frac{\partial p}{\partial t} = \rho\phi(C + C_\phi)\frac{\partial p}{\partial t} = \rho\,\phi\,C_t\frac{\partial p}{\partial t}$$, substitute in eq.2-4

$$\Rightarrow \frac{\partial}{\partial x}\left(\rho\, k_x \frac{\partial P}{\partial x}\right) + \rho\mu\hat{q}B = \mu\rho\phi\, C_t \frac{\partial P}{\partial t} \qquad (2\text{-}5)$$

now, we will apply product derivatives to the first part on L.H.S. of eq. 2-5:

$$\Rightarrow [\rho \frac{\partial}{\partial x}\left(k_x \frac{\partial P}{\partial x}\right) + k_x \frac{\partial P}{\partial x}\frac{\partial \rho}{\partial x}] + \rho\mu\hat{q}B = \mu\rho\phi\, C_t \frac{\partial P}{\partial t}$$

Since $\frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial p}\frac{\partial p}{\partial x} = \rho C \frac{\partial p}{\partial x}$

Then, $\Rightarrow [\rho \frac{\partial}{\partial x}\left(k_x \frac{\partial P}{\partial x}\right) + k_x \rho\, C\left(\frac{\partial P}{\partial x}\right)^2] + \rho\mu\hat{q}B = \mu\rho\phi\, C_t \frac{\partial P}{\partial t}$ (2-6)

The above equation is called **non-linear partial differential equation** (**N.L.P.D.E**) for 1-D, single phase, slight compressible fluid linear flow.

<u>Linearization</u>: The non-linear term $k_x \rho C \left(\frac{\partial P}{\partial x} \right)^2$ is very small compare to the linear term $k_x \frac{\partial P}{\partial x}$ for slight compressible fluid; therefore, we will set it to zero. Also we will divide eq. 2-6 by ρ ,

$$\Rightarrow \frac{\partial}{\partial x}\left(k_x \frac{\partial P}{\partial x} \right) + \mu \hat{q} B = \mu \phi C_t \frac{\partial P}{\partial t} \qquad (2\text{-}7)$$

The above equation is called **<u>linear partial differential equation</u>** (**L.P.D.E**) for 1-D, single phase, slight compressible fluid linear flow.

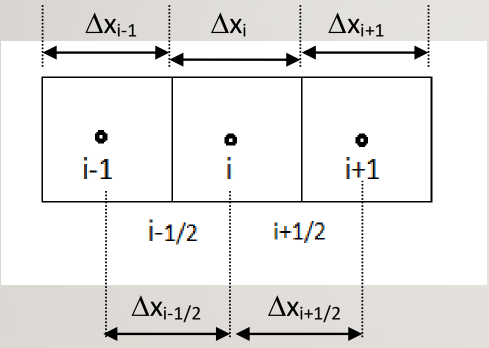
Now, multiply eq.2-7 by unit rock volume ($\Delta x \Delta y \Delta z$),

$$\Rightarrow (\Delta x \Delta y \Delta z) \frac{\partial}{\partial x}\left(k_x \frac{\partial P}{\partial x} \right) + \mu q B = (\Delta x \Delta y \Delta z) \mu \phi C_t \frac{\partial P}{\partial t} \qquad (2\text{-}8)$$

The above equation is called **<u>diffusivity equation</u>** for linear flow.

FINITE-DIFFERENCE SOLUTION OF DIFFUSIVITY EQ. FOR LINEAR FLOW

For one dimension, block centered, finite difference grid, we have:

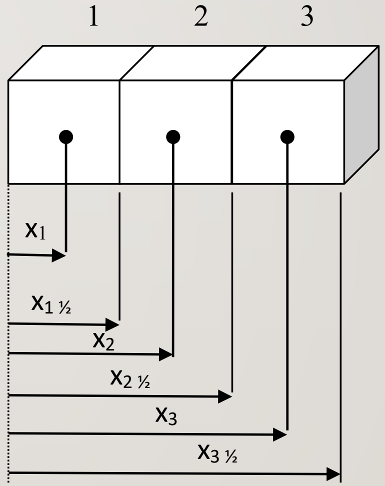


We have:

$$\Delta x_{1\frac{1}{2}} = x_2 - x_1$$

$$\Delta x_{2\frac{1}{2}} = x_3 - x_2 \qquad \Rightarrow \Delta x_{i+\frac{1}{2}} = x_{i+1} - x_i$$

For 3 node system, we have:



we have:

$$\Delta x_1 = x_{1\frac{1}{2}} - x_{\frac{1}{2}}$$

$$\Delta x_2 = x_{2\frac{1}{2}} - x_{1\frac{1}{2}} \qquad \Rightarrow \Delta x_i = x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}$$

$$\Delta x_3 = x_{3\frac{1}{2}} - x_{2\frac{1}{2}}$$

Also, by definition

$$x_{\frac{1}{2}} = x_w \ and \ x_{3\frac{1}{2}} = \ x_{IMAX+\frac{1}{2}}$$

i = distance spacing; node numbering
n = time spacing; n = current level & n +1 = next level (future)
Δt = time step
Δt_1 = first time step

We have,

$$\Rightarrow (\Delta x \Delta y \Delta z)\frac{\partial}{\partial x}\left(k_x \frac{\partial P}{\partial x}\right) + \mu q B = (\Delta x \Delta y \Delta z)\mu\phi C_t \frac{\partial P}{\partial t} \qquad (2\text{-}9)$$

Explicit Formulation

Applying finite difference using explicit method yields the following:

$$(\Delta x\ \Delta y\ \Delta z)_i \left\{\frac{1}{\Delta X_i}\left[K_{xi+\frac{1}{2}}\left(\frac{P_{i+1}^n - P_i^n}{\Delta X_{i+\frac{1}{2}}}\right) - K_{xi-\frac{1}{2}}\frac{P_i^n - P_{i-1}^n}{\Delta X_{i-\frac{1}{2}}}\right]\right\} + (\mu q B)_i =$$

$$(\Delta x\ \Delta y\ \Delta z)_i (\emptyset \mu C_t)_i \frac{P_i^{n+1} - P_i^n}{\Delta t}$$

$$P_i^{n+1} = \frac{(\Delta x\ \Delta y\ \Delta z)_i \left\{\frac{1}{\Delta X_i}\left[K_{xi+\frac{1}{2}}\frac{(P_{i+1}^n - P_i^n)}{\Delta X_{i+\frac{1}{2}}} - K_{xi-\frac{1}{2}}\frac{P_i^n - P_{i-1}^n}{\Delta X_{i-\frac{1}{2}}}\right]\right\} + (\mu q B)_i + \frac{(\Delta x\ \Delta y\ \Delta z)_i (\emptyset \mu C_t)_i P_i^n}{\Delta t}}{\frac{(\Delta x\ \Delta y\ \Delta z)_i (\emptyset \mu C_t)_i}{\Delta t}} \quad (2\text{-}10)$$

Note:

Equation 2-10 will solve for P in node i (P_i^{n+1}) only at each time step.

Disadvantage: - When time step is small $\rightarrow$ takes a lot of time to run

- Not stable

Implicit Formulation

Applying finite difference using implicit method to eq. 2-9 yields the following:

$$(\Delta x\ \Delta y\ \Delta z)_i \left\{\frac{1}{\Delta x_i}\left[K_{xi+\frac{1}{2}}\left(\frac{P_{i+1}^{n+1} - P_i^{n+1}}{\Delta X_{i+\frac{1}{2}}}\right) - K_{yi-\frac{1}{2}}\frac{(P_i^{n+1} - P_{i-1}^{n+1})}{\Delta X_{i-\frac{1}{2}}}\right]\right\} + \mu_i (qB)_i = (\Delta x \Delta y\ \Delta z)_i (\emptyset \mu C_t)_i \frac{1}{\Delta t}(P_i^{n+1} - P_i^n)$$

$$(\Delta y\ \Delta z)_i \left(\frac{K_x}{\Delta x}\right)_{i+\frac{1}{2}} (P_{i+1}^{n+1} - P_i^{n+1}) - (\Delta y\ \Delta z)_i \left(\frac{K_x}{\Delta x}\right)_{i-\frac{1}{2}} (P_i^{n+1} - P_{i-1}^{n+1}) + \mu_i (qB)_i = (\Delta x \Delta y \Delta z)_i \emptyset_i (\mu\ C_t)_i \frac{1}{\Delta t}(P_i^{n+1} - P_i^n) \quad \text{(2-11)}$$

The above equations have 3 unknowns with the general form as follows:

$$A_i\ P_{i-1}^{n+1} + B_i\ P_i^{n+1} + C_i\ P_{i+1}^{n+1} = D_i \quad \text{(2-12)}$$

$$A_i\, P_{i-1}^{n+1} + B_i\, P_i^{n+1} + C_i\, P_{i+1}^{n+1} = D_i \qquad (2\text{-}12)$$

Rewrite eq. 2-11 to look like eq. 2-12

$$(\Delta y\, \Delta z)_i \left(\frac{K_x}{\Delta x}\right)_{i-\frac{1}{2}} P_{i-1}^{n+1} - \left[(\Delta y\, \Delta z)_i \left(\frac{K_x}{\Delta x}\right)_{i-\frac{1}{2}} + (\Delta y\, \Delta z)_i \left(\frac{K_x}{\Delta x}\right)_{i+\frac{1}{2}} + (\Delta x \Delta y \Delta z)_i \emptyset_i (\mu C_t)_i \frac{1}{\Delta t}\right] P_i^{n+1} + (\Delta y \Delta z)_i \left(\frac{K_x}{\Delta x}\right)_{i+\frac{1}{2}} P_{i+1}^{n+1} = -\mu_i (qB)_i - (\Delta x \Delta y \Delta z)_i \emptyset_i (\mu C_t)_i \frac{1}{\Delta t} P_i^n \qquad (2\text{-}13)$$

Comparing eq. 2-12 with eq. 2-13

$$A_i = (\Delta y\, \Delta z)_i \left(\frac{K_x}{\Delta x}\right)_{i-\frac{1}{2}} \qquad (2\text{-}14)$$

$$C_i = (\Delta y\, \Delta z)_i \left(\frac{K_x}{\Delta x}\right)_{i+\frac{1}{2}} \qquad (2\text{-}15)$$

$$B_i = -\left[A_i + C_i + (\Delta x\, \Delta y\, \Delta z)_i \emptyset_i (\mu\, C_t)_i \frac{1}{\Delta t}\right] \qquad (2\text{-}16)$$

$$D_i = -\mu_i (qB)_i - (\Delta x\, \Delta y\, \Delta z)_i \emptyset_i (\mu\, C_t)_i \frac{1}{\Delta t} P_i^n \qquad (2\text{-}17)$$

Examples:
Closed boundary reservoir with 4 nodes

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G.E

$$A_i\, P_{i-1}^{n+1} + B_i P_i^{n+1} + \ C_i P_{i+1}^{n+1} = \ D_i$$

For node #1, i =1

$$A_1 P_0^{n+1} + B_1 P_1^{n+1} + \ C_1 P_2^{n+1} = \ D_1$$

For node #2, i =2

$$A_2 P_1^{n+1} + B_2 P_2^{n+1} + \ C_2 P_3^{n+1} = \ D_2$$

For node #3, i =3

$$A_3 P_2^{n+1} + B_3 P_3^{n+1} + \ C_3 P_4^{n+1} = \ D_3$$

For node #4, i =4

$$A_4 P_3^{n+1} + B_4 P_4^{n+1} + \ C_4 P_5^{n+1} = \ D_{4,} \quad C_4 = C_{IMAX} = 0$$

Presenting the above equation in matrix form,

$$\begin{bmatrix} B_1 & C_1 & 0 & 0 \\ A_2 & B_2 & C_2 & 0 \\ 0 & A_3 & B_3 & C_3 \\ 0 & 0 & A_4 & B_4 \end{bmatrix} \begin{bmatrix} P_1^{n+1} \\ P_2^{n+1} \\ P_3^{n+1} \\ P_4^{n+1} \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

Logarithm distribution of distance:

$X_{\frac{1}{2}} = X_w, \; X_{IMAX+\frac{1}{2}} = X_e$, IMAX = maximum number of nodes

Let: $\Delta U_d = \frac{1}{IMAX} \ln\left(\frac{X_e}{X_w}\right)$

Then

$X_{i+\frac{1}{2}} = X_w \, e^{i \, \Delta ud}$, I = 1, 2 …., IMAX

$X_i = X_1 \, e^{(i-1) \, \Delta ud}$, i = 1,2….., IMAX

$X_1 = X_w \, e^{(\Delta ud/2)}$

Logarithm distribution of time:

n_{max} = maximum number of time steps, t_{max} = maximum run time
Δt_1 = first time step

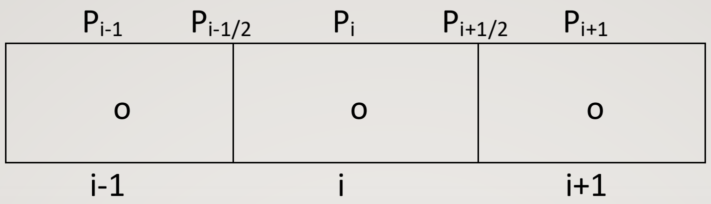
Let: $\Delta U_t \overset{def}{=} \frac{1}{n_{max}} \ln\left(\frac{t_{max}}{\Delta t_1}\right)$

Then, $\Delta t_i = \Delta t_1\left[e^{(i\Delta u)} - e^{(i-1)\Delta u}\right]$, i = 1,2, 3,....., n_{max}

(Homework # 1)

Harmonic Average Permeability (HAP):

The following figure is an example of grid centered block



Flow from i + 1 $\rightarrow$ i+ ½

$$\frac{q}{A} = -\frac{k_i+1}{\mu}\frac{\left(P_{i+1}-P_{i+\frac{1}{2}}\right)}{\left(x_{i+1}-x_{i+\frac{1}{2}}\right)} \rightarrow\ P_{i+1} - P_{i+\frac{1}{2}} = \frac{-q\mu}{A}\frac{\left(x_{i+1}-x_{i+\frac{1}{2}}\right)}{k_{i+1}} \qquad \text{(HAP-1)}$$

Flow from i + ½ $\rightarrow$ i

$$\frac{q}{A} = \frac{-k_i}{\mu}\frac{\left(P_{i+\frac{1}{2}}-P_i\right)}{\left(x_{i+\frac{1}{2}}-x_i\right)} \rightarrow \quad P_{i+\frac{1}{2}} - P_i = \frac{-q\mu}{A}\frac{\left(x_{i+\frac{1}{2}}-x_i\right)}{k_i} \qquad \text{(HAP-2)}$$

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Flow from i + 1 $\rightarrow$ i

$$\frac{q}{A} = \frac{-k_{i+\frac{1}{2}}}{\mu}\frac{(P_{i+1}-P_i)}{(x_{i+1}-x_i)} \rightarrow \quad P_{i+1} - P_i = \frac{-q\mu}{A}\frac{(x_{i+1}-x_i)}{k_{i+\frac{1}{2}}} \qquad \text{(HAP-3)}$$

Adding eq. HAP (1 & 2) $\rightarrow P_{i+1} - P_i = \frac{-q\mu}{A}\left[\frac{\left(x_{i+1} - x_{i+\frac{1}{2}}\right)}{k_{i+1}} + \frac{\left(x_{i+\frac{1}{2}} - x_i\right)}{k_i}\right]$ (HAP-4)

Comparing eq. HAP (2 with 4)

$$-\frac{q\mu}{A}\frac{(x_{i+1} - x_i)}{k_{i+\frac{1}{2}}} = \frac{-q\mu}{A}\left[\frac{\left(x_{i+1} - x_{i+\frac{1}{2}}\right)}{k_{i+1}} + \frac{\left(x_{i+\frac{1}{2}} - x_i\right)}{k_i}\right]$$

$$\frac{(x_{i+1} - x_i)}{k_{i+\frac{1}{2}}} = \frac{k_i\left(x_{i+1} - x_{i+\frac{1}{2}}\right) + k_{i+1}\left(x_{i+\frac{1}{2}} - x_i\right)}{k_{i+1}k_i}$$

$$k_{i+\frac{1}{2}} = \frac{k_i k_{i+1}(x_{i+1} - x_i)}{k_i\left(x_{i+1} - x_{i+\frac{1}{2}}\right) + k_{i+1}\left(x_{i+\frac{1}{2}} - x_i\right)} \qquad \text{(HAP-5)}$$

(Homework # 2)

Solving Tridiagonal Matrix:

$$A_i P_{i-1}^{n+1} + B_i\ P_i^{n+1} + C_i\ P_{i+1}^{n+1} = D_i$$

Replace the unknowns $P_{i-1}^{n+1} \rightarrow x_{i-1}$

$P_i^{n+1} \rightarrow x_i$

$P_{i+1}^{n+1} \rightarrow x_{i+1}$

$A_i\ x_{i-1} + B_i\ x_i + C_i\ x_{i-1} = D_i$ (S-1)

Let $x_{i-1} = E_i\ x_i + F_i$ (S-2), starting point where E_i & F_i are local variables

Substitute (S-2) $\rightarrow$ (S-1)

$A_i\ (E_i\ x_i + F_i) + B_i\ x_i + C_i\ x_{i+1} = D_i$ (S-3)

$(A_i\ E_i + B_i)\ x_i + C_i\ x_{i+1} + A_i\ F_i - D_i = 0$ (S-4)

Multiply eq. S-4 by $(A_i\ E_i + B_i)^{-1}$

$$(A_i E_i + B_i)^{-1}(A_i E_i + B_i)x = -(A_i E_i + B_i)^{-1} C_i x_{i+1} - (A_i E_i + B_i)^{-1}(A_i F_i - D_i)$$

$$x_i = -(A_i E_i + B_i)^{-1} C_i x_{i+1} - (A_i E_i + B_i)^{-1}(A_i F_i - D_i) \qquad \text{(S-5)}$$

Similar to eq. S-2, let $x_i = E_{i+1}\ x_{i+1} + F_{i+1}$ (S-2B)

Compare eq. (S-5) with (S-2B)

$E_{i+1} = -\ (A_i\ E_i + B_i)^{-1}\ \ C_i$ (S-6)

$F_{i+1} = -(A_i\ E_i + B_i)^{-1}\ (A_i\ F_i - D_i)$ (S-7)

Since $C_{IMAX} = 0$

Then from eq. S-6, we get

$E_{IMAX+1} = 0$

Substitute $E_{IMAX+1} = 0$ in eq. S-2

$x_{IMAX} = F_{IMAX+1}$ (S-8)

And $x_i = E_{i+1}\ x_{i+1} + F_{i+1}$, i = Imax -2, …., 3, 2, 1

<u>Algorithm:</u> (Tridiagonal Matrix)

1. Set: $A1 = 0$
   $C_{Imax} = 0$
   $E1 = 0$
   $F1 = 0$
2. Calculate E_{i+1}, F_{i+1} for $i = 1,2, 3...., Imax$
   $E_{i+1} = -(B_i + A_i E_i)^{-1}$
   $F_{i+1} = (B_i + A_i E_i)^{-1} (D_i - A_i F_i)$

3. Compute x_i for $i = 1, 2, 3...., Imax$
   Set $x_{Imax} = F_{Imax+1}$

$x_i = E_{i+1} x_{i+1} + F_{i+1}$, $i = Imax-1, Imax-2, ...3, 2, 1$

Summary of one dimension, single phase, slight compressible fluid for linear flow with coefficient in field units:

$$A_i = (\Delta y\ \Delta z)_i \left(\frac{k_x}{\Delta_y}\right)_{i-\frac{1}{2}}$$

$$C_i = (\Delta y\ \Delta z)_i \left(\frac{k_x}{\Delta_x}\right)_{i-\frac{1}{2}}$$

$$B_i = \left[A_i + C_i + (\Delta x \Delta y \Delta z)_i \emptyset_i (\mu C_t)_i \frac{1}{0.0002637\ \Delta t}\right]$$

$$D_i = \frac{\mu_i (qB)_i}{0.001127} - (\Delta x \Delta y \Delta z)_i \emptyset_i (\mu C_t)_i P_i^n \frac{0.006328}{24}$$

Where, 0.006328 = .001127*5.615
0.0002637 = (0.001127 * 5.615)/ 24

(Homework # 3)