UBT
جامعة الأعمال و التكنولوجيا
UNIVERSITY OF BUSINESS AND TECHNOLOGY

CE | College Of Engineering
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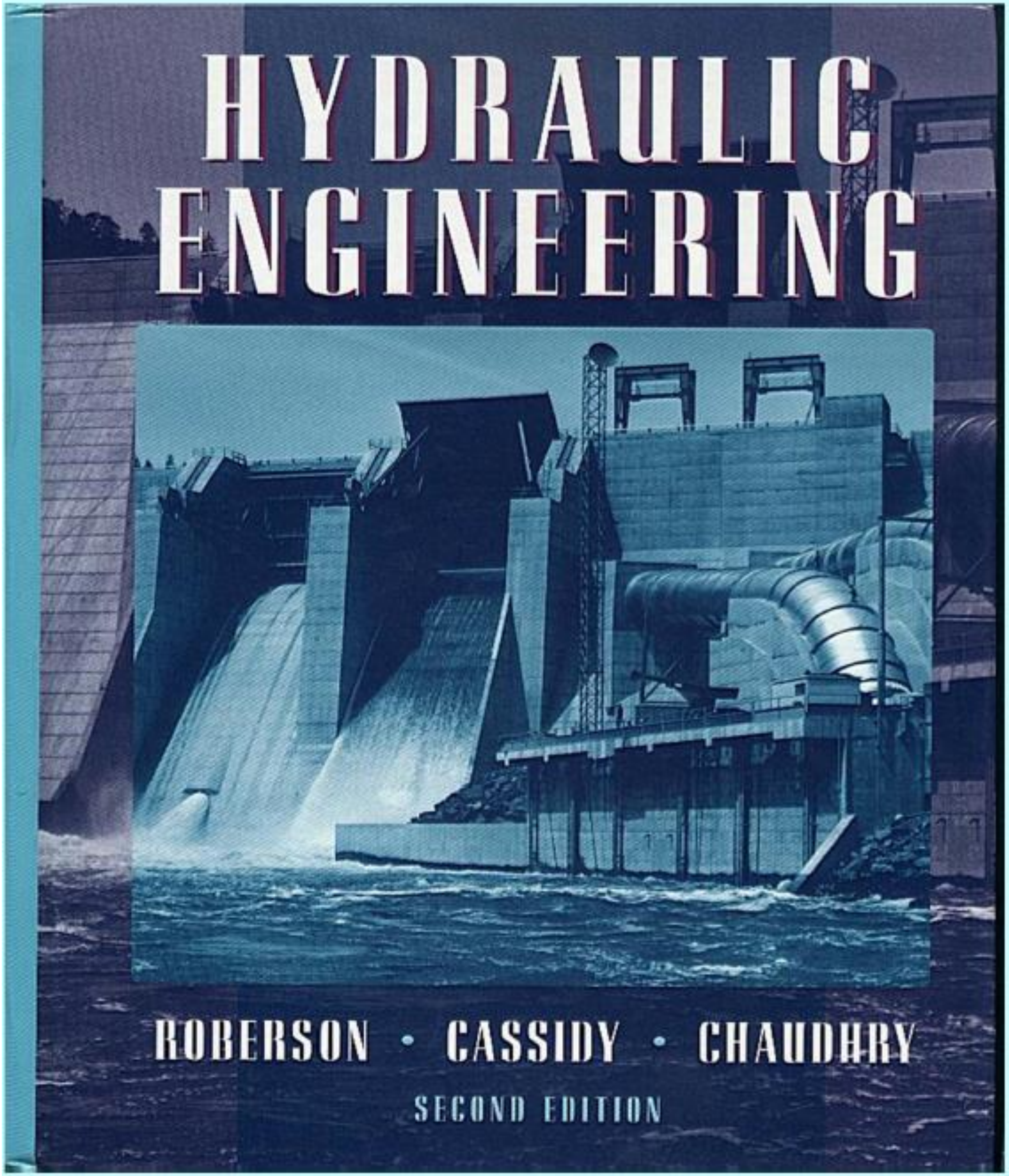
HYDRAULICS CE-322

SUMMER 2021

Text book


HYDRAULIC
ENGINEERING
ROBERSON · CASSIDY · CHAUDHRY
SECOND EDITION

Contents

Course Objectives

- Reviewing the basic properties of water
- Reviewing the basic principles of hydrostatics
- Reviewing the basic principles of fluid in motion (conservation of mass, momentum and energy)
- Analyzing, understanding and designing of water flow in pipes and pipes networks.
- Analyzing, understanding and designing water flow in open channels under different conditions.
- Learning the basic theory of hydromachines (pumps in particular).
- Learning different techniques for measuring velocity, pressure and flow rate in both pipe and open channel flows.
- Understanding similitude and its applications in hydraulic modeling

Week	Topics	Chapter Objectives	CLOs ref.	Students Work /due dates
1	History of Hydraulics			
1-2	Closed Conduit Flow: Single, branching and looped pipes, pipe network analysis and design.	Analyzing and designing of water flow in pipes and pipes networks. Apply hydraulic software to solve water pipe line networks	2.3, 2.34.2	HW 1&2
3	Hydraulic Machinery: pumps and turbines, performance curves, sizing and operating conditions	Apply the basic theory of hydro-machines (pumps in particular).	2.1,2.2, 2.3	HW 3
4	Open Channel Flow: steady flow, non-uniform water surface profiles, hydraulic jump, structures	Analyzing and designing water flow in open channels under different conditions. Identify the different techniques for measuring velocity, pressure and flow rate in both pipe and open channel flows.	2.1,2.2, 2.3	HW 4&5
5-6	Unsteady Closed Conduit Flow: water hammer problems and mitigation, pipeline start-up, draining	Reviewing the basic principles of fluid in motion (conservation of mass, momentum and energy)	2.1,2.2,	HW 6
7	Groundwater Hydraulics: Darcy's law, one-dimensional and radial flow, drawdown near wells	analyze hydrological data in order to evaluate water resource in an area	2.3	HW 7

Hydraulic

- *Hydraulic*

  Comes from the Greek word *hydraulikos,* meaning water.

- **Hydraulics** is the science of studying the mechanical behaviour of water at rest or in motion

- Hydraulic Engineering is the application of fundamental principles of fluid mechanics on water.

- Hydraulic systems

  Systems which are designed to accommodate water at rest and in motion.

- *hydraulic engineering systems,*

Involve the application of engineering principles and methods to :

  - planning,
  - control,
  - transportation,
  - conservation, and
  - utilization of water.

Examples of Hydraulic Projects

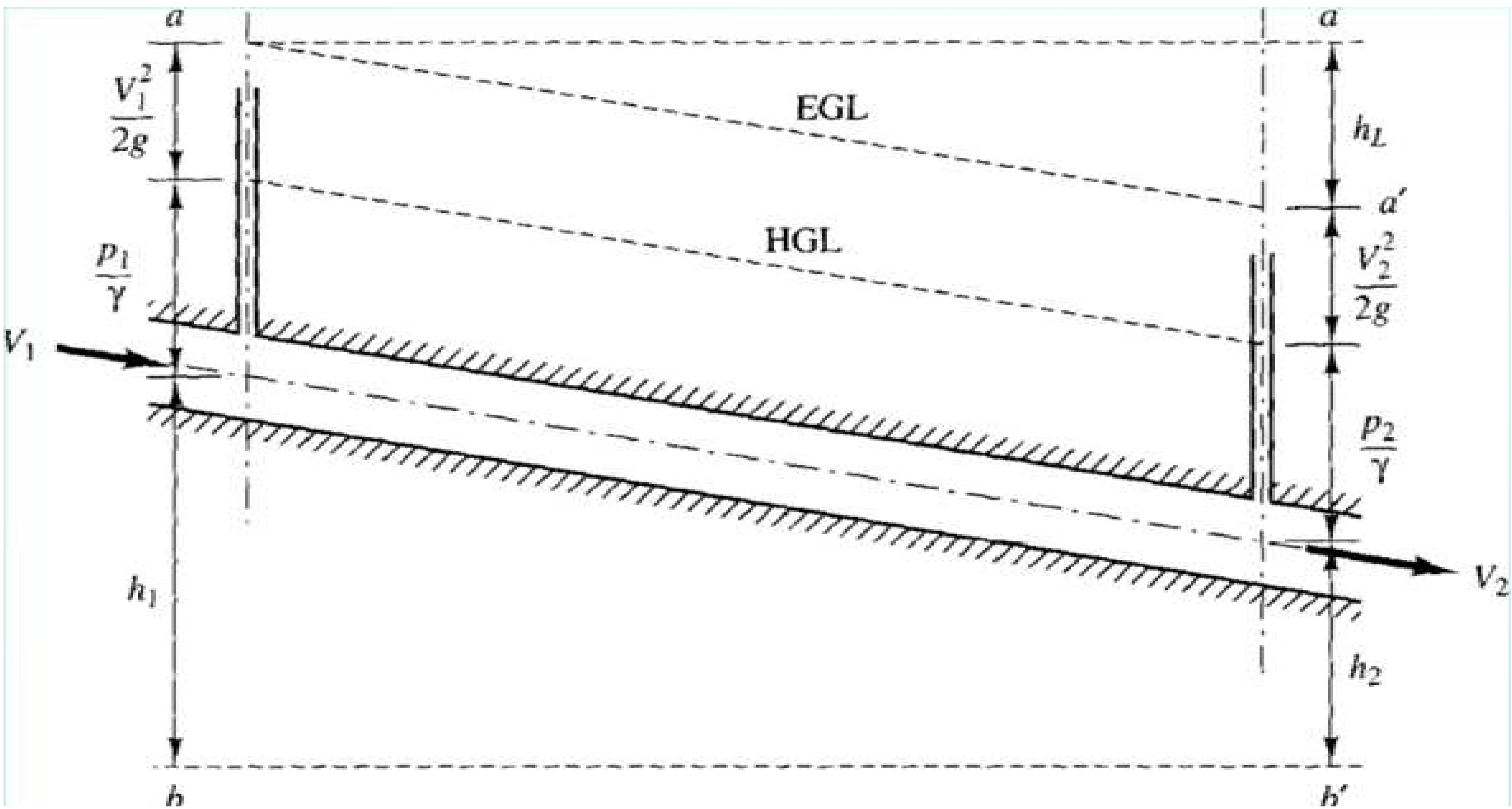
- Water pipelines
- Water distribution systems
- Sewer systems
- Dams and water control structures
- Storm sewer systems
- Rivers and manmade canals
- Coastal and Harbour structures
- Irrigation and Drainage Projects
- Cooling systems
- Etc.

Parameter	SI Unit	US Unit
Length	m	ft
Time	s	s
Area	m^2	ft^2
Volume	m^3	ft^3
Mass	kg	$lb\text{-}s^2/ft$ or slug
Force	kg . m/s^2 or N	lb
Weight	kg . m/s^2 or N	lb
Pressure	N/m^2	lb/ft^2
Density	kg/m^3	$slugs/ft^3$
Specific weight	N/m^3	lb/ft^3
Surface tension	mN/m	mlb/ft
Dynamic viscosity	$N.s/m^2$ or Pa.s or kg/(m.s)	$lb.s/ft^2$ or slug/(ft.s)
Kinematic viscosity	m^2/s	ft^2/s
Energy	N.m or Joule (J)	lb.ft
Power	N.m/s or J/s	lb.ft/s

The Bernoulli Equation

$$\frac{P_1}{\rho_1 g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho_2 g}+\frac{V_2^2}{2g}+z_2$$

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h$$

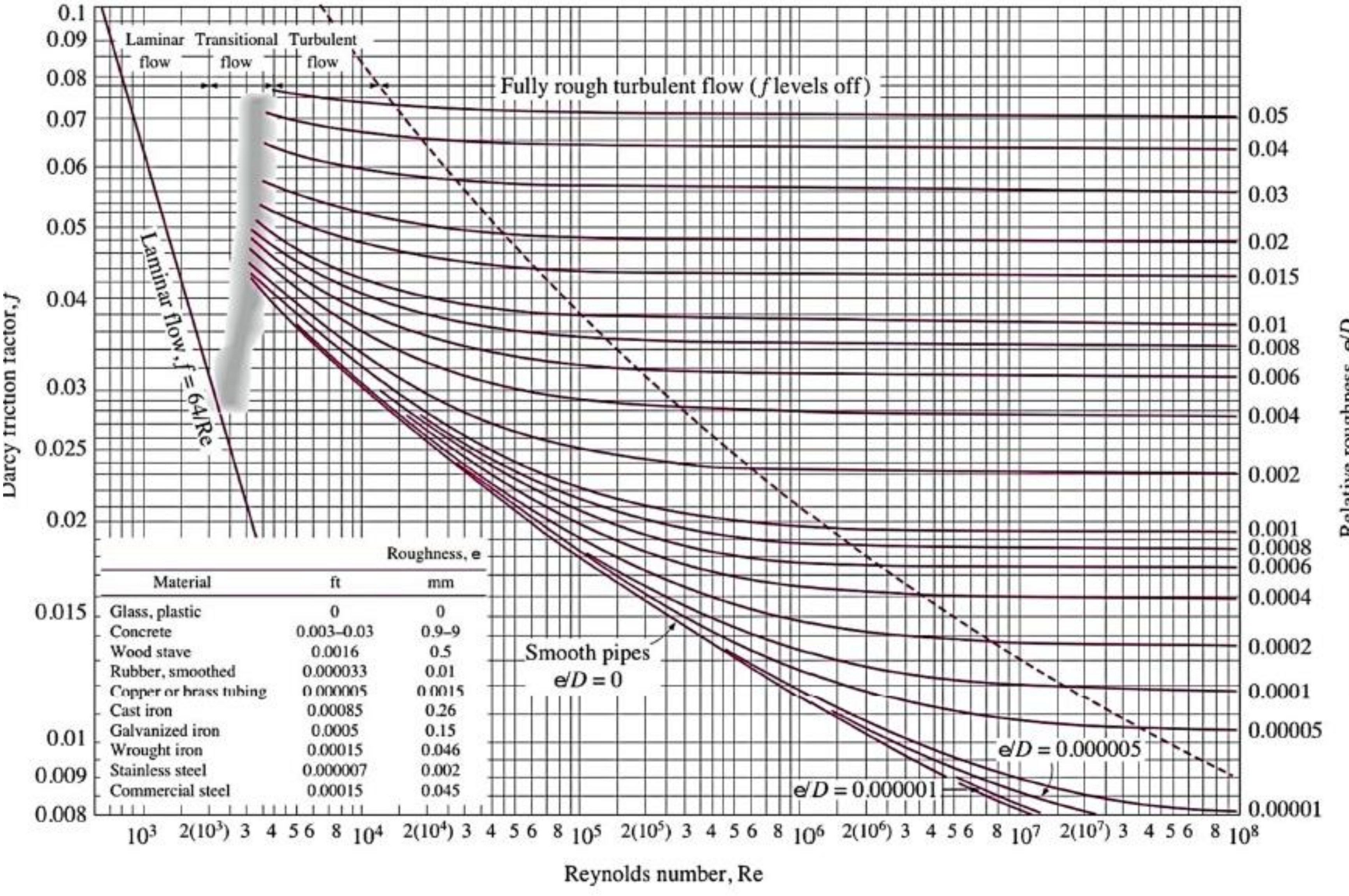


Reynolds number

$$N_R = \frac{VD\rho}{\mu} = \frac{VD}{\nu}$$

where	V:	mean velocity in the pipe	[L/T]
	D:	pipe diameter	[L]
	ρ:	density of flowing fluid	$[M/L^3]$
	μ:	dynamic viscosity	[M/LT]
	ν:	kinematic viscosity	$[L^2/T]$

The Moody Chart



Material	Roughness, e ft	Roughness, e mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

HYDRAULICS OF FLOW AND DESIGN OF PRESSURE PIPES

- The pressure pipes can be laid at any depth below the hydraulic gradient line.
- If the pipe goes much below the hydraulic gradient line, the pressure developed in the pipe may be too high for ordinary pipe material.
- It will demand the construction of thicker and stronger pipes, which may create enormous constructional difficulties and increased cost.
- The size and shape of supply pipes must be determined by hydraulic, structural and economical considerations

- The hydraulic gradient should be such as to generate velocities which are neither so small as to require large dia pipe, nor so large as to excessive loss of pressure head.
- The velocities should also be non-silting and non-scouring.
- The flow velocities are normally kept between 0.9 m/s to 1.5 m/s though velocities up to 3 m/s to 6 m/s can be resisted by the commonly available pipes or pipe materials.
- Design should be in such a way that the available pressure head between the source and the city is just lost in overcoming the frictional resistance offered to the flow by the pipe interior.

- In addition to the head lost due to pipe friction(H_L), there are minor losses caused by the abrupt changes in the flow geometry due to
  - Changes in pipe size
  - Bends
  - Valves and fittings
- In long pipes, the minor losses can be neglected.
- The head loss due to pipe friction can be found by using
  - Darcy-Weisbach formula
  - Manning’s formula
  - Hazen-William’s formula and
  - Modified Hazen-Willaim’s formula

Darcy-Weisbach formula

$$H_L = \frac{f'.L.V^2}{2gd}$$

H_L = Head loss in metres

L = Length of pipes in metres

d= Diameter of the pipe in metres

V = Mean velocity of flow through pipe in m/s

g = Acceleration due to gravity

f' = Dimensionless friction factor generally varying between 0.02 (for smooth new pipes) and 0.075 (for old rough pipes).

- The dimensionless friction factor depends upon Reynold number and relative roughness of the pipe.
- Relative roughness of the pipe depends upon the absolute roughness (e) of the inside surface and the diameter of the pipe(d).
- Reynold number, $R_e = \frac{Vd}{v}$
- Relative roughness, $\delta = \frac{2e}{d}$

a) For laminar flows, $R_e = 2000$

$$f' = \frac{64}{R_e}$$

b) Above $R_e = 4000$, turbulent flow is fully established.

- Relation suggested by Nikuradse and others

$$\left[\frac{1}{\sqrt{f'}} = 2log_{10} Re\sqrt{f'} - 0.8\right] \text{ for smooth pipes}$$

$$\left[\frac{1}{\sqrt{f'}} = 2log_{10} \frac{d}{2e} + 1.74\right] \text{ for rough pipes}$$

- For smooth pipes and R_e = 20000 to 2000000, slitcher has given the formula

$$f' = 0.005 + \frac{0.396}{{R_e}^{0.3}}$$

S.No	Material of pipe	Value of 2e in mm x 10^{-3}
1	Glass, copper, lead, or asbestos	1.524
2	Steel or wrought iron	45.75
3	Asphalted cast iron	122
4	Galvanized iron	152
5	Cast iron	300
6	Concrete	300 – 3000
7	Riveted steel	900 - 9000

3. Manning's formula

$$V = \frac{k}{n} R_h^{2/3} \cdot S^{1/2}$$

V = velocity of flow (m/s)

k =conversion factor of 1.486 $(ft/m)^{1/3}$

n =Manning coefficient

R_h = hydraulic radius (m)

S = slope of the water surface

* The value of "n" is calculated by kutter's formula

4. Kutter's formula

$$C = \frac{k_1 + \frac{k_2}{S} + \frac{k_3}{n}}{1 + \frac{n}{\sqrt{R}} \cdot \left(k_1 + \frac{k_2}{S}\right)}$$

Where

- C = Chézy's roughness coefficient
- S = Friction slope
- R = Hydraulic radius (m,)
- n = Kutter's roughness (unit less)
- k_1 =Constant (23.0 SI,)
- k_2=Constant (0.00155 SI,)
- k_3= Constant (1.0 SI,)

Manning's formula

- Used for gravity conduits
- Also applicable to turbulent flow in pressure conduits
- Yields good result provided the roughness coefficient *n* is accurately estimated.
- Head loss, is given by

$$H_L = \frac{n^2 . V^2 . L}{R^{4/3}}$$

where, n = Manning rugosity coefficient

L = Length of pipe in meters

V = Flow velocity through pipe in m/s

R = Hydraulic mean depth of pipe

$$= \left[\frac{A}{P} = \frac{\frac{\pi d^2}{4}}{\pi d} = \frac{d}{4}\right] in\ metres.$$

Circular section running partially full (Refer fig. 8.6),

Let a = area of cross - section

b = wetted perimeter

r = H.M.D. (Hydraulic Mean Depth)

v = velocity of flow

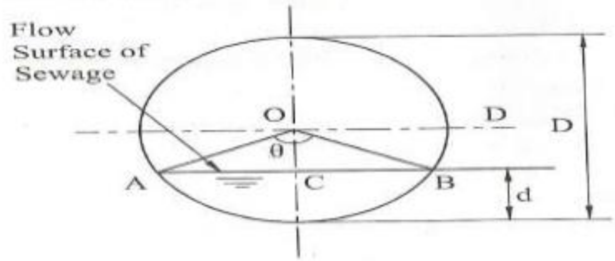


Fig. 8.6

Central angle is given by $\cos \frac{1}{2}\theta = 1 - \frac{2d}{D}$

(1) Depth $d = \frac{D}{2} - \frac{D}{2}\cos\frac{\theta}{2} = \frac{D}{2}(1 - \cos\theta/2)$

Proportional depth $\frac{d}{D} = \frac{1}{2}(1 - \cos\theta/2)$

(2) Area $a = \frac{\pi}{4} D^2 \times \frac{\theta}{360^\circ} - \frac{D}{2}\cos\frac{\theta}{2} \cdot \frac{D}{2}\sin\frac{\theta}{2}$

$$a = \frac{\pi}{4} D^2 \left[\frac{\theta}{360^\circ} - \frac{\sin\theta}{2\pi}\right]$$

$\therefore$ Proportional Area $= \frac{a}{A}\left[\frac{\theta}{360^\circ} - \frac{\sin\theta}{2\pi}\right]$

(3) Wetted perimeter :

$$P = \pi D \cdot \frac{\theta}{360^\circ}$$

$\therefore$ Proportional wetted perimeter :

$$\frac{P}{P} = \frac{\theta}{360^\circ}$$

(4) H.M.D. :

$$r = \frac{a}{p} = \frac{\frac{\pi D^2}{4}\left[\frac{\theta}{360^\circ} - \frac{\sin\theta}{2\pi}\right]}{\pi D \frac{\theta}{360^\circ}}$$

$$\therefore \quad r = \frac{D}{4}\left[1 - \frac{360^\circ \sin\theta}{2\pi\theta}\right]$$

$$\text{Proportionate HMD} = \frac{r}{R} = \frac{\frac{D}{4}\left[1 - \frac{360^\circ \sin\theta}{2\pi\theta}\right]}{\frac{D}{4}}$$

$$= \left[1 - \frac{360^\circ \sin\theta}{2\pi\theta}\right]$$

Table 8.1

Hydraulic Elements of circular sewer Running Partially full

d/D	a/A	p/p	r/R	For N/n = 1.0		N/n	For Variation N/n	
				v/V	q/Q		v/V	q/Q
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	1	1	1	1	1	1	1	1
0.9	0.949	0.795	1.192	1.124	1.61	0.94	1.057	1.002
0.8	0.858	0.705	1.217	1.14	0.988	0.88	1.003	0.869
0.7	0.748	0.631	1.185	1.12	0.838	0.85	0.952	0.712
0.6	0.626	0.564	1.11	1.072	0.671	0.83	0.89	0.557
0.5	0.5	0.5	1	1	0.5	0.81	0.81	0.405
0.4	0.373	0.436	0.857	0.902	0.337	0.79	0.713	0.266
0.3	0.252	0.369	0.684	0.776	0.196	0.78	0.605	0.153
0.2	0.143	0.295	0.482	0.625	0.088	0.79	0.486	0.07
0.1	0.052	0.205	0.254	0.401	0.021	0.82	0.329	0.017
0	0				0			

(6) Discharge :

$q = a \times v$ Taking $\frac{N}{n} = 1.0$, we get

Proportional discharge $= \frac{q}{Q} = \frac{a.v}{AV} = \frac{a}{A} \times \left(\frac{r}{R}\right)^{2/3}$

$$\therefore \frac{q}{Q} = \left[\frac{\theta}{360^\circ} - \frac{\sin\theta}{2\pi}\right]\left[1 - \frac{360^\circ \sin\theta}{2\pi\theta}\right]$$

$$= \frac{\theta}{360^\circ}\left[1 - \frac{360^\circ \sin\theta}{2\pi\theta}\right]^{5/3}$$

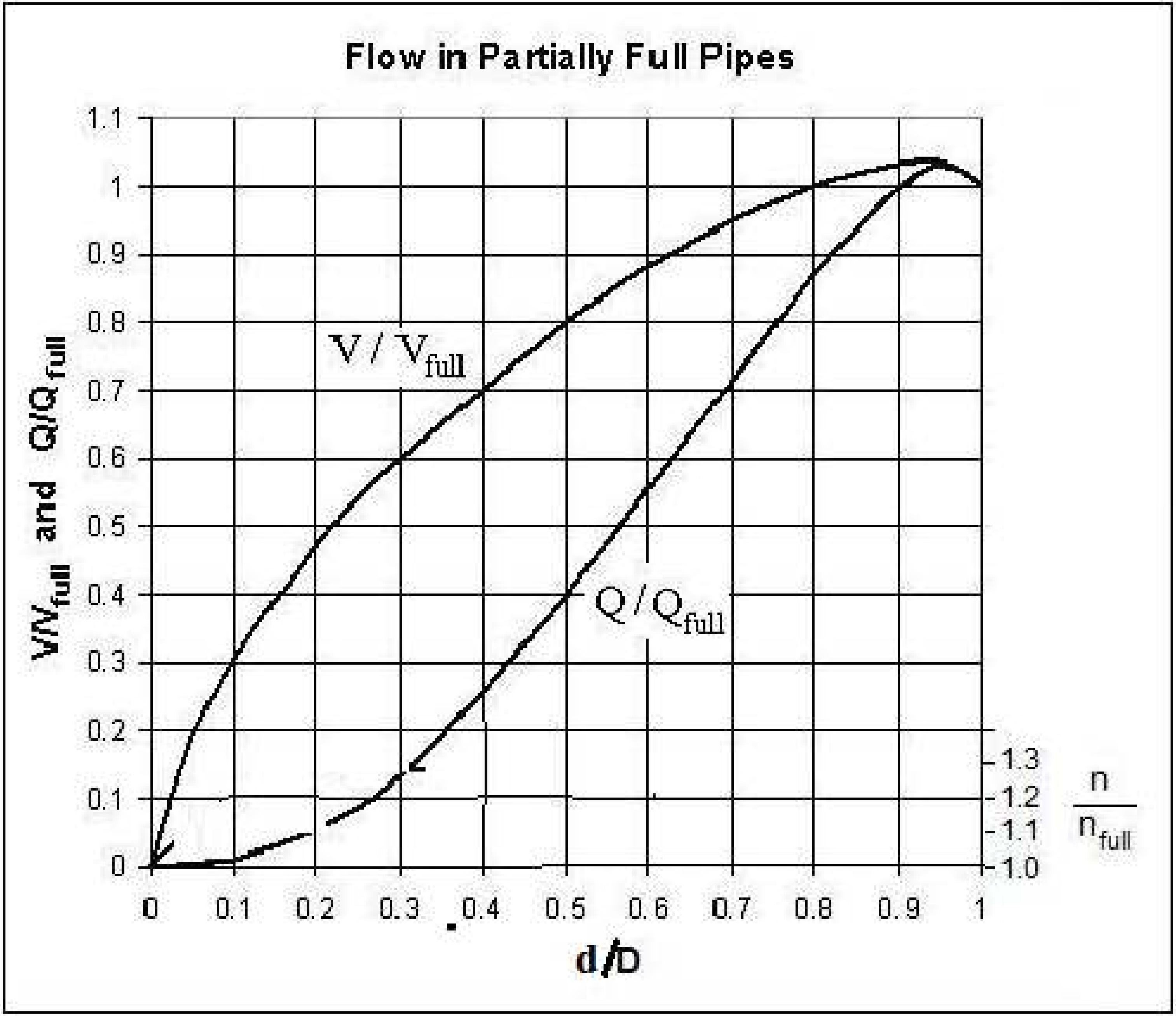
For variable value of $\frac{N}{n}$, we get

$$\frac{q}{Q} = \frac{N}{n}\left(\frac{a}{A}\right)\left(\frac{r}{R}\right)^{2/3}$$

Practice problems

Problem – 8.1 : Calulate the velocity of flow and corresponding discharge in a sewer of circular section having diameter = 1.2 m, laid at a gradient 1 in 400. The sewer runs at 0.7 depth. Use Manning's formula taking $N = 0.018$

Problem – 8.2 : Determine the size of a circular sewer for a discharge of 450 lit/s running half-full Assume $s = 1/4500$ and $N = 0.016$.


Flow in Partially Full Pipes
V/Vfull and Q/Qfull
V / Vfull
Q / Qfull
d/D
n/nfull
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.0
1.1
1.2
1.3

HAZEN WILLIAM'S FORMULA

$$V = 0.85\, CH . R^{0.63} . S^{0.54}$$

Where,

CH = Coefficient of hydraulic capacity

R = Hydraulic mean depth of pipe in metres

$= \left(\frac{d}{4}\right)$ for circular pipes flowing full

S = Slope of energy line

V = Flow velocity through the pipe in m/s.

Pipe material	Value of C_H Depending upon the smoothness of the pipe material
Concrete(regardless of age)	130
Cast Iron	
New	130
5 years old	120
20 years old	100
Welded steel (new)	120
Riveted steel (new)	110
Vitrified clay	110
Brick sewers	100
Asbestos-Cement	140

Limitations of Hazen-Williams formula

- C_H is not dimensionless but has the units of $L^{-0.37} T^{-1}$.Its value changes with change in employed units.
- The numerical constant of 0.85 (in MKS units) in Hazen-Williams formula has been calculated for an assumed hydraulic mean depth(R) of 0.3m(1ft) and friction slope of 1/1000. This formula is used for all ranges of pipe dia and friction slopes. This practice may lead to an error upto
  - ±30% in evaluation of velocity
  - ±55% in head loss(HL) due to pipe friction

- The Hazen-William’s coefficient is the representative of friction conditions of a pipe and hence depend on pipe roughness and reynold’s number.
- The Hazen william’s ceofficient is independent of pipe dia, velocity of flow and viscosity.

Hydraulic formulae

1. Chezy's formula

$$v = C\sqrt{Ri},$$

where
V= is the mean velocity [m/s],
C= is the Chézy coefficient [$m^{1/2}$/s],
R= is the hydraulic radius (~ water depth) [m],
i= is the bottom slope[m/m].

- Constant (C) is very complex. Depends on size, shape and smoother roughness of the channel, the mean depth etc.
- C can be calculated by using Bazin's formula.

2. Bazin's formula

$$C = \frac{157.6}{[1.81 + (K/R^{1/2})]}$$

Where,

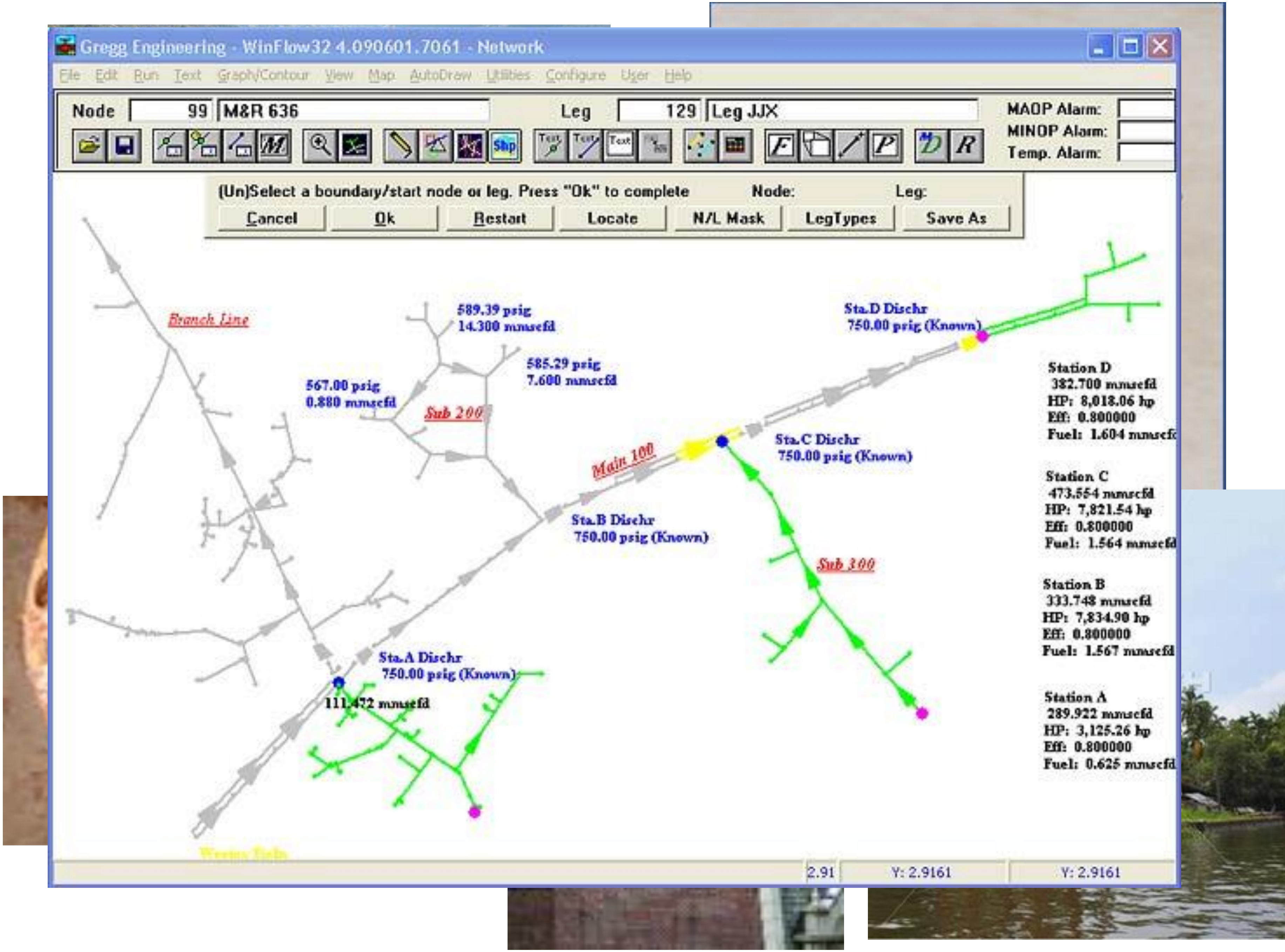
K= Bazin's constant

R= hydraulic radius

Sr. No.	Inside nature of the sewer	K values
1.	Very smooth	0.109
2.	Smooth: bricks & concrete	0.290
3.	Smooth: rubble masonry	0.833
4.	Good, earthen material	1.540
5.	Rough: bricks & concrete	0.500
6.	Rough earthen material	3.170

Overview of Pipe Networks

- ‘Pipe flow’ generally refers to fluid in pipes and appurtenances flowing full and under pressure
- Examples: Water distribution in homes, industry, cities; irrigation
- System components
  - Pipes
  - Valves
  - Bends
  - Pumps and turbines
  - Storage (often unpressurized, in reservoirs, tanks, etc.)



Gregg Engineering - WinFlow32 4.090601.7061 - Network
File Edit Run Text Graph/Contour View Map AutoDraw Utilities Configure User Help
Node 99 M&R 636
Leg 129 Leg JJX
MAOP Alarm:
MINOP Alarm:
Temp. Alarm:
(Un)Select a boundary/start node or leg. Press "Ok" to complete
Node:
Leg:
Cancel
Ok
Restart
Locate
N/L Mask
LegTypes
Save As
Branch Line
589.39 psig
14.300 mmscfd
585.29 psig
7.600 mmscfd
567.00 psig
0.880 mmscfd
Sub 200
Main 100
Sta.D Dischr
750.00 psig (Known)
Sta.C Dischr
750.00 psig (Known)
Sta.B Dischr
750.00 psig (Known)
Sub 300
Sta.A Dischr
750.00 psig (Known)
111.472 mmscfd
Station D
382.700 mmscfd
HP: 8,018.06 hp
Eff: 0.800000
Fuel: 1.604 mmscfd
Station C
473.554 mmscfd
HP: 7,821.54 hp
Eff: 0.800000
Fuel: 1.564 mmscfd
Station B
333.748 mmscfd
HP: 7,834.90 hp
Eff: 0.800000
Fuel: 1.567 mmscfd
Station A
289.922 mmscfd
HP: 3,125.26 hp
Eff: 0.800000
Fuel: 0.625 mmscfd
2.91
Y: 2.9161
Y: 2.9161

Energy Relationships in Pipe Systems

- Energy equation between any two points:

$$E_2 = E_1 + h_{pump} - h_{turb} - \sum h_{L,f}$$

$$z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_{pump} - h_{turb} - \sum h_{L,f}$$

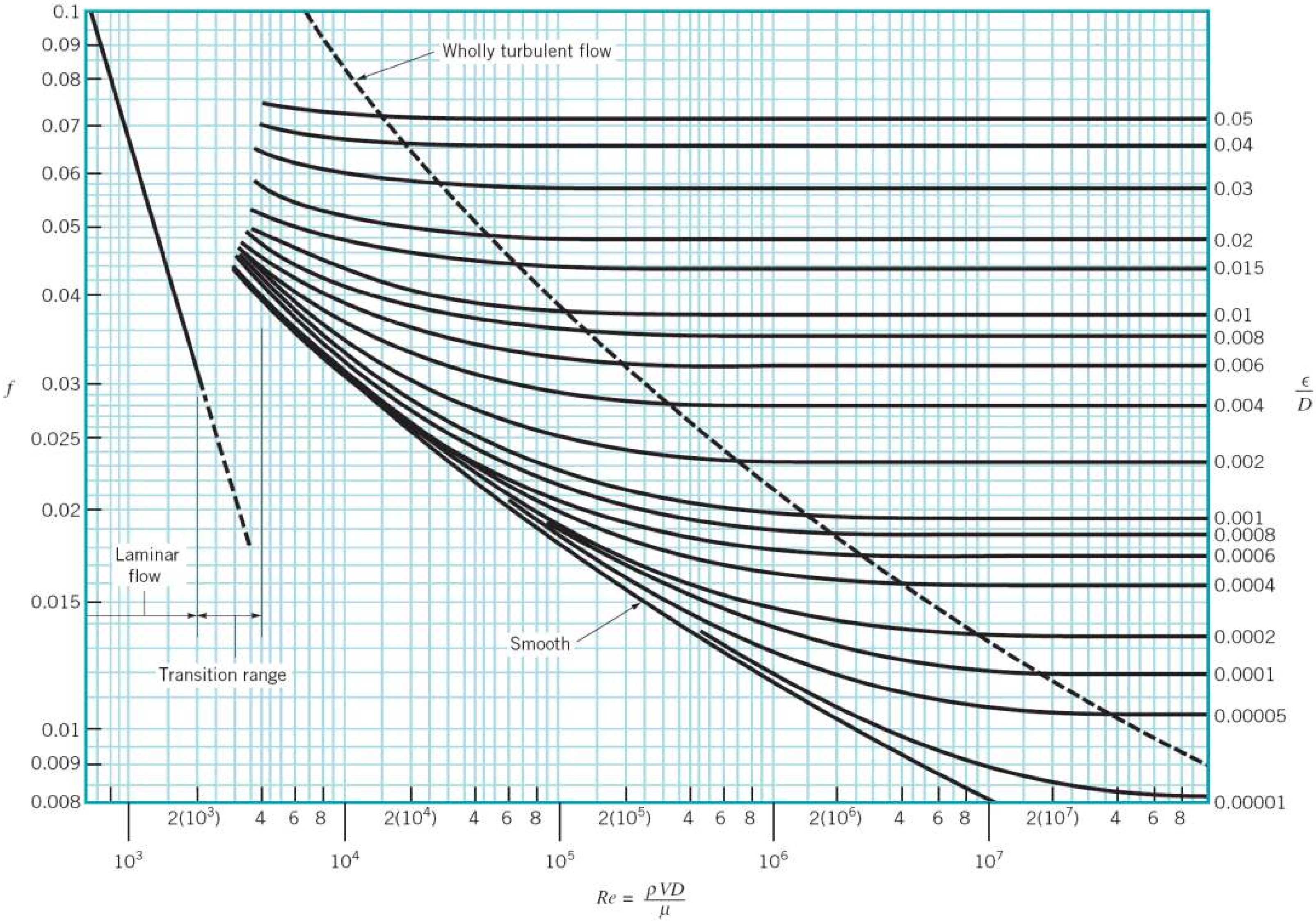
- Analysis involves writing expressions for h_L in each pipe and for each link between pipes (valves, expansions, contractions), relating velocities based on continuity equation, and solving subject to system constraints (Q, p, or V at specific points).

Energy Losses in Piping Systems

- Darcy-Weisbach equation for headlosses in pipes (major headlosses):

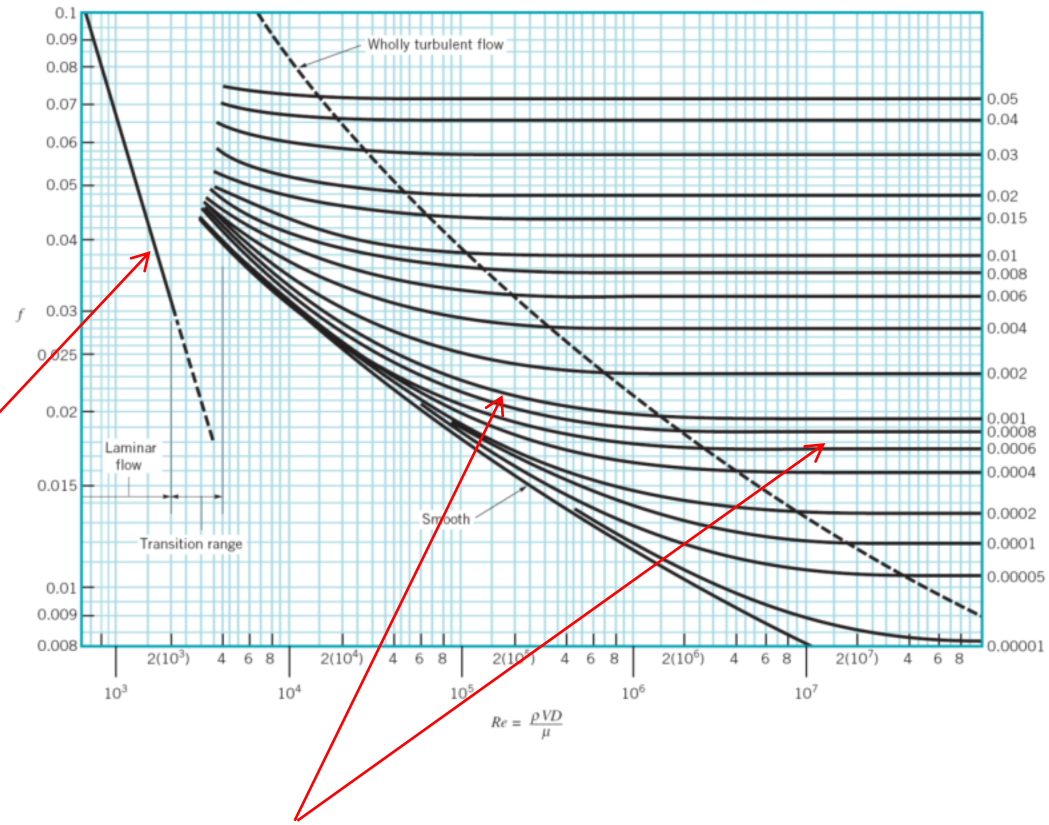
$$h_L = f\frac{l}{D}\frac{V^2}{2g}$$

Estimating *f* Graphically



Trends in *f*

- *f* declines with increasing Re, e.g., increasing *V* at fixed *D*.
- In laminar region, $f = 64/\text{Re}$



- In turbulent region, for given ε/D, *f* declines more slowly than in laminar region; eventually, the decline stops altogether.

Mathematical Expressions for *f*

- Colebrook and Haaland eqns yield good estimates of *f* in turbulent flow

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.71}{\mathrm{Re}\sqrt{f}}\right)$$

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right]$$

- Useful for calculations in spreadsheets or special software for pipe flow analysis

Example

- Compare the velocity and pressure heads for typical conditions in a street main:
  V = 1.5 m/s; D = 0.5 m; p = 500 kPa

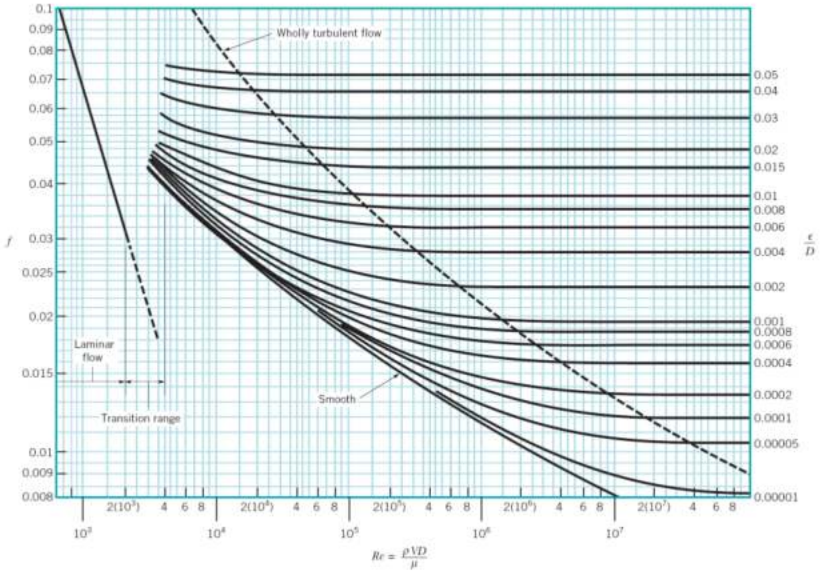
$$\frac{V^2}{2g} = \frac{\left(1.5\ \text{m/s}\right)^2}{2\left(9.8\ \text{m/s}^2\right)} = 0.115\ \text{m}$$

$$\frac{p}{\gamma} = \frac{\left(500\ \text{kPa}\right)\left[\left(1000\ \text{N/m}^2\right)/\text{kPa}\right]}{9800\ \text{N/m}^3} = 51.0\ \text{m}$$

- If f = 0.02, h_L for each 0.5 m of pipe is 2% of the velocity head, or 0.0023 m, corresponding to 0.0045% of the pressure head.

Typical Pipe Flow Problems

- Type I: Pipe properties (ε, D, l) and V known, find h_L.
- Determine f from Moody diagram or an equivalent equation, and h_L from the DW eqn



$$h_L = f\frac{l}{D}\frac{V^2}{2g}$$

Example

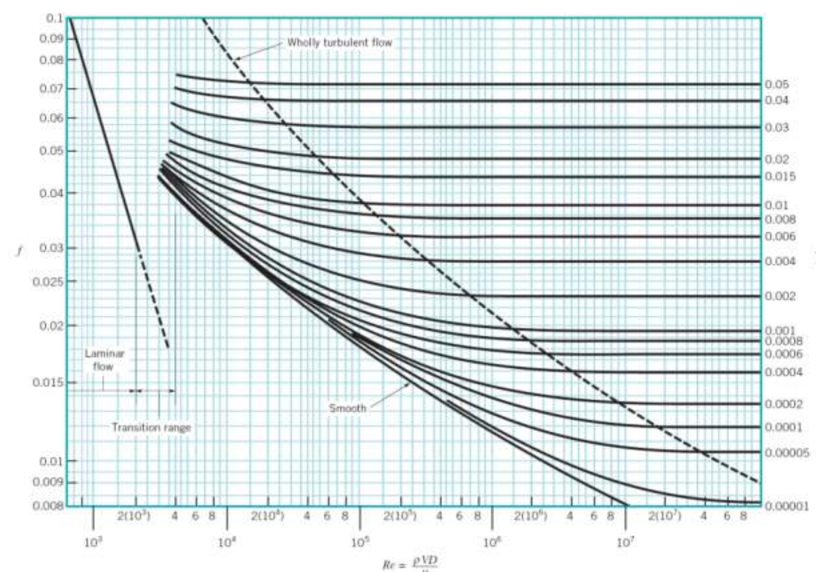
A 20-in-diameter galvanized pipe (ε = 0.0005 ft) 2 miles long carries 4 cfs at 60°F. Find h_L using (a) the Moody diagram and (b) the Colebrook eqn.

a) $$V = \frac{Q}{A} = \frac{4\ \text{ft}^3/\text{s}}{\pi(1.67\ \text{ft})^2/4} = 1.83\ \text{ft/s}$$

$$\text{Re} = \frac{DV}{\nu} = \frac{(1.67\ \text{ft})(1.83\ \text{ft/s})}{1.22\text{x}10^{-5}\ \text{ft}^2/\text{s}} = 2.51\text{x}10^5$$

$$\frac{\varepsilon}{D} = \frac{0.0005\ \text{ft}}{1.67\ \text{ft}} = 0.00030$$

$$f = 0.017$$



$$h_L = f\frac{l}{D}\frac{V^2}{2g} = (0.017)\frac{2(5280\text{ ft})}{1.67\text{ ft}}\frac{(1.83\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = 5.59\text{ ft}$$

b) Colebrook eqn: $$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.71}{\text{Re}\sqrt{f}}\right)$$

	F	G	H
9	e/D	0.0003	0.0003
10	Re	251000	2.51E+05
11	f	0.03	0.03
12	LHS	=1/SQRT(G11)	5.774
13	RHS	=-2*LOG(G9/3.7 + 2.71/G10*G12)	7.687
14	LHS - RHS	=G12-G13	-1.913

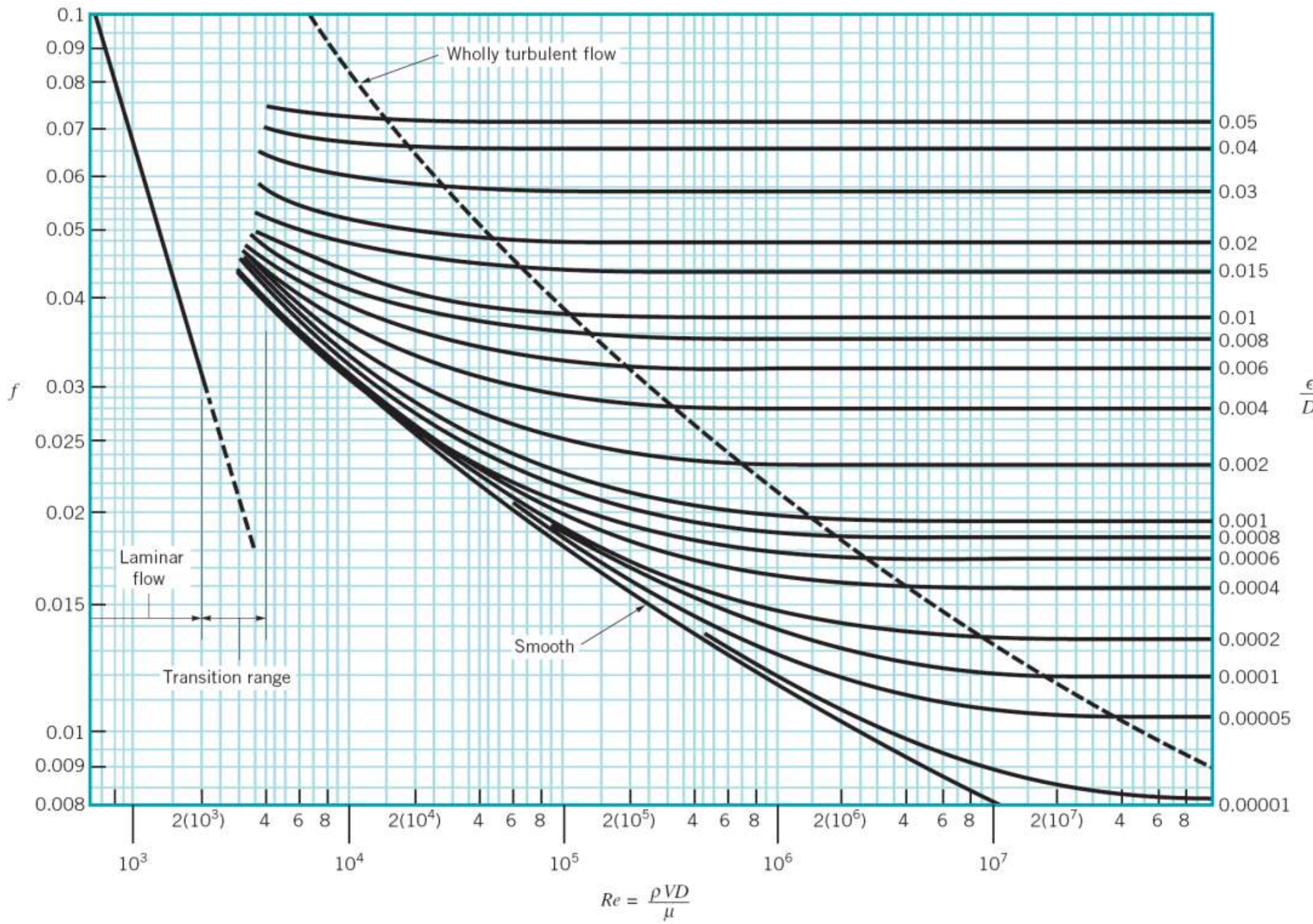
e/D	0.0003
Re	2.51E+05
f	**0.017422**
LHS	7.576
RHS	7.576
LHS - RHS	2.55E-07

$$h_L = f\frac{l}{D}\frac{V^2}{2g} = (0.0174)\frac{2(5280\text{ ft})}{1.67\text{ ft}}\frac{(1.83\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = 5.72\text{ ft}$$

Typical Pipe Flow Problems

- Type II: Pipe properties (ε, *D*, *l*) and h_L known, find *V*.
- Guess *V*, determine *f* and h_L as in Type I, iterate until h_L equals known value, ***or***
- Solve Colebrook and DW eqns simultaneously to eliminate *V*, yielding:

Solving Type II Pipe Problems: Iterative Approach



$$h_L = f \frac{l}{D} \frac{V^2}{2g}$$

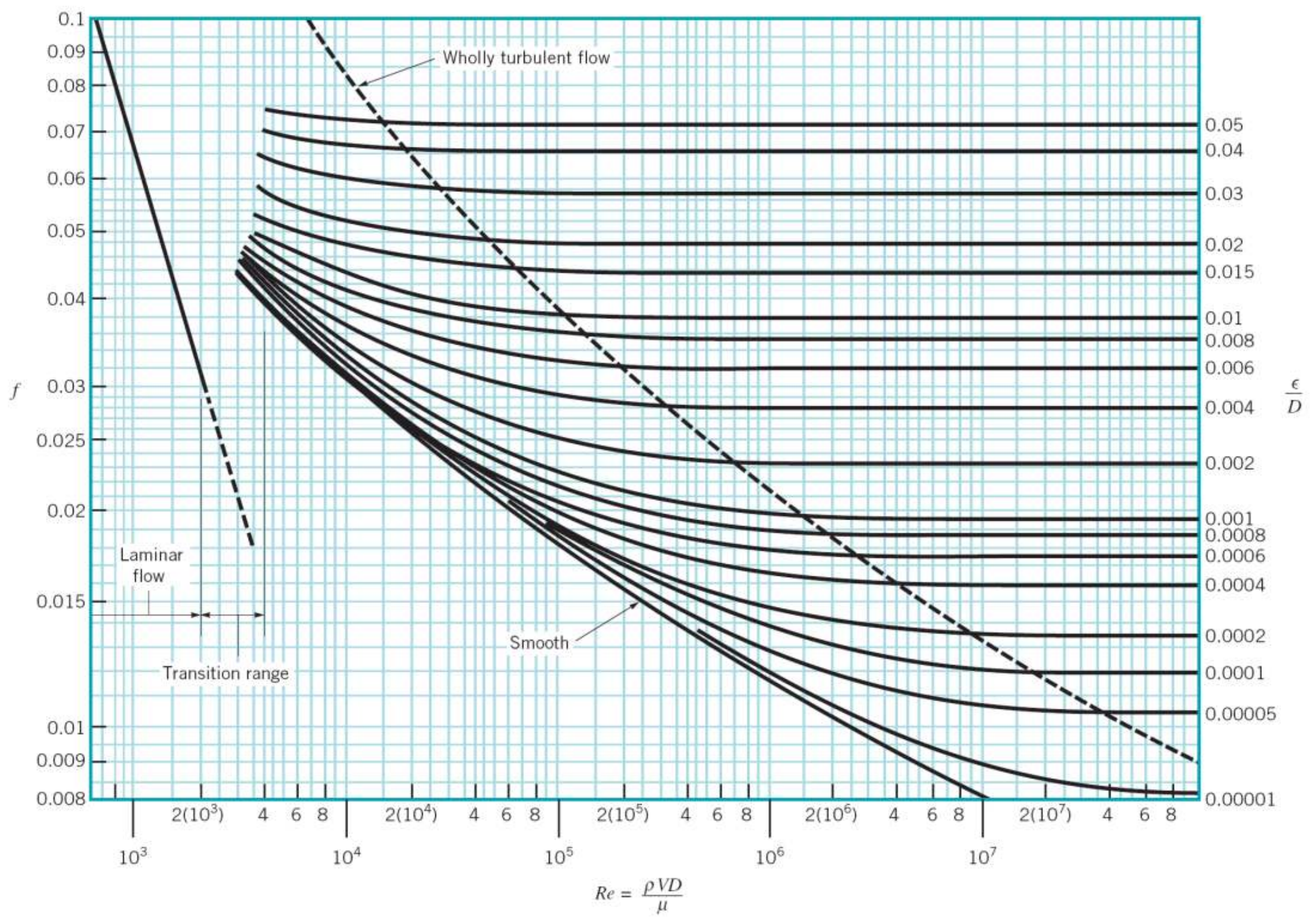
$$f = \frac{0.25}{\left\{ \log \left[\left(0.317 \frac{gD^3}{\nu^2 l} h_L \right)^{-1/2} + \frac{e/D}{3.7} \right] \right\}^2}$$

Rearranged D-W eqn: $V = \left(\frac{2h_L gD}{fl} \right)^{1/2}$

$$V = -2\sqrt{\frac{2gDh_L}{l}} \log \left(\frac{\varepsilon/D}{3.7} + \frac{2.51\nu}{D} \sqrt{\frac{l}{2gDh_L}} \right)$$

Example

For the pipe analyzed in the preceding example, what is the largest flow rate allowable if the total frictional headloss must remain <8 ft?



Example

For the pipe analyzed in the preceding example, what is the largest flow rate allowable if the total frictional headloss must remain <8 ft?

$$V = -2\sqrt{\frac{2gDh_L}{l}}\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51\nu}{D}\sqrt{\frac{l}{2gDh_L}}\right)$$

Substituting known values, $V = 2.19\ \text{ft/s}$

$$Q = VA = (2.19\ \text{ft/s})\frac{\pi(1.67\ \text{ft})^2}{4} = 4.80\frac{\text{ft}^3}{\text{s}}$$

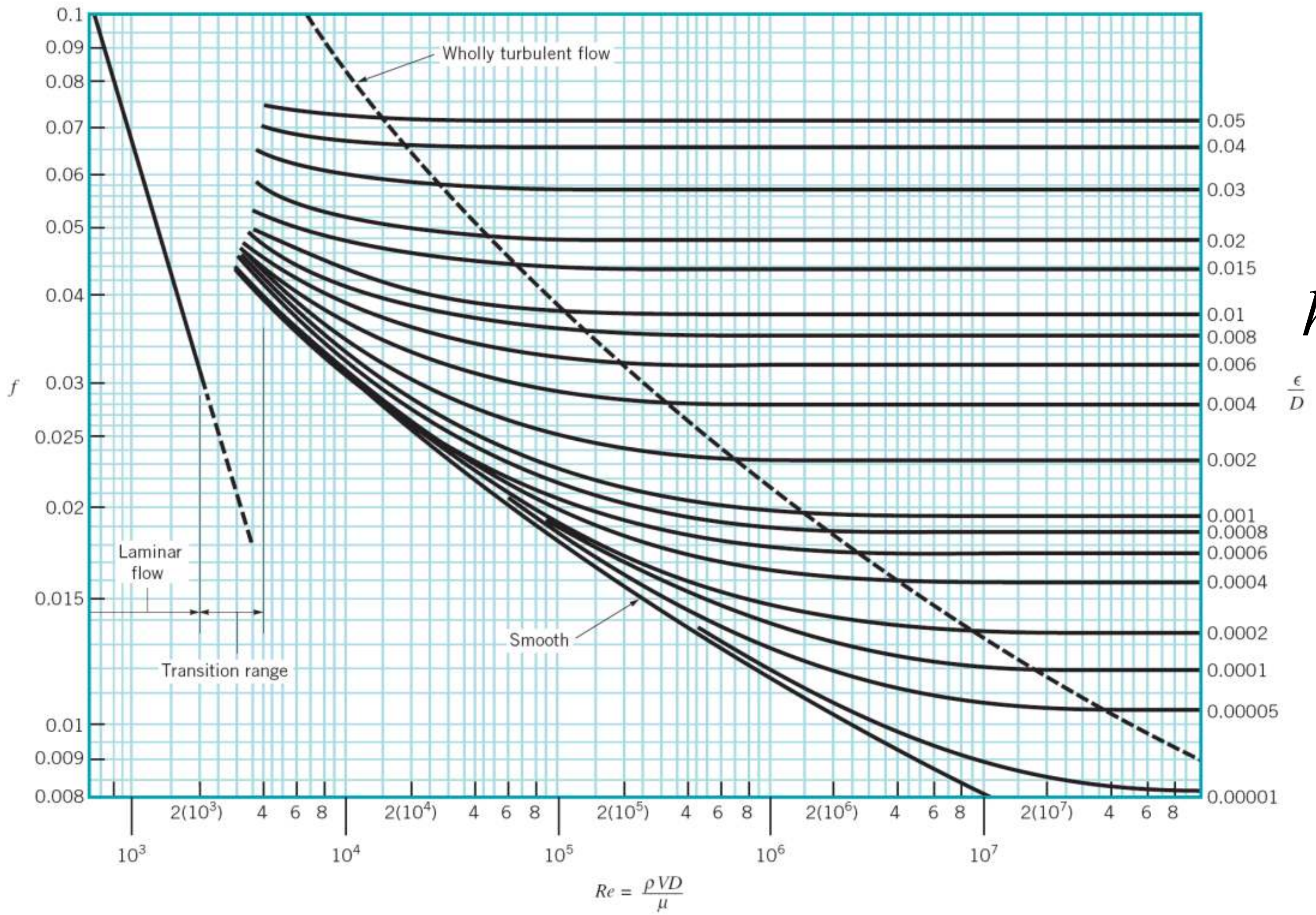
Typical Pipe Flow Problems

- Type III: ε, l, V, and h_L known, find D.
- Several approaches, all iterative; e.g., Guess D, determine V as in Type II, iterate until V equals known value

Example

What diameter galvanized pipe would be required in the preceding examples if a flow rate of 10 cfs was needed, while keeping the total frictional headloss at <8 ft?

Solving Type III Pipe Problems: Iterative Graphical Approach



$$h_L = f\frac{l}{D}\frac{V^2}{2g}$$

$$Q = -2\sqrt{\frac{2gDh_L}{l}}\left\{\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51\nu}{D}\sqrt{\frac{l}{2gDh_L}}\right)\right\}\left(\frac{\pi D^2}{4}\right)$$

g	32.2
hL	8
l	10560
eps	0.0005
nu	1.22E-05
D_guess	2
LHS = Q	10
RHS	7.72E+00
LHS – RHS	2.28E+00

g	32.2
hL	8
l	10560
eps	0.0005
nu	1.22E-05
D_guess	**2.206594**
LHS = Q	10
RHS	1.00E+01
LHS – RHS	-8.22E-07

Comparison of Equations for Transitional and Turbulent Curves on the Moody Diagram

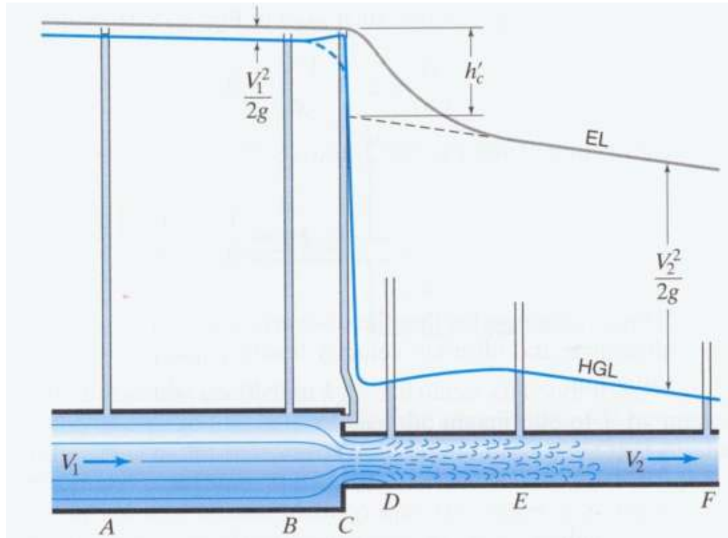
	D-W	H-W*	Manning*
V	$\sqrt{2gD\frac{h_f}{l}\frac{1}{f}}$ $=\sqrt{2g}\ D^{0.50}S^{0.50}f^{-0.50}$	$0.849C_{HW}R_h^{0.63}S^{0.54}$ $=0.354D^{0.63}S^{0.54}C_{HW}$	$\frac{1}{n}R_h^{0.67}S^{0.50}$ $=0.397D^{0.67}S^{0.50}\frac{1}{n}$
Q	$\frac{\pi\sqrt{2g}}{4}D^{2.50}S^{0.50}f^{-0.50}$	$0.278D^{2.63}S^{0.54}C_{HW}$	$0.312D^{2.67}S^{0.50}\frac{1}{n}$
***h_L* (=*S*l*)**	$\frac{8}{\pi^2 g}Q^2\frac{l}{D^5}f$	$10.7Q^{1.85}\frac{l}{D^{4.87}}\frac{1}{C_{HW}^{1.85}}$	$10.3Q^2\frac{l}{D^{5.33}}\frac{1}{n^2}$

*Coefficients shown are for SI units (*V* in m/s, and *D* and R_h in m); for BG units (ft/s and ft), replace 0.849 by 1.318; 0.354 by 0.550; 0.278 by 0.432; 10.7 by 4.73; 1/*n* by 1.49/*n*; 0.397 by 0.592; 0.312 by 0.465; and 10.3 by 4.66.

Energy Losses in Bends, Valves, and Other Transitions ('Minor Losses')

- Minor headlosses generally significant when pipe sections are short (e.g., household, not pipeline)
- Caused by turbulence associated with flow transition; therefore, mitigated by modifications that 'smooth' flow patterns
- Generally much greater for expansions than for contractions
- Often expressed as multiple of velocity head: $h_L = K_{minor} \frac{V^2}{2g}$
- *K* is the ratio of energy lost via friction in the device of interest to the kinetic energy of the water (upstream or downstream, depending on geometric details)

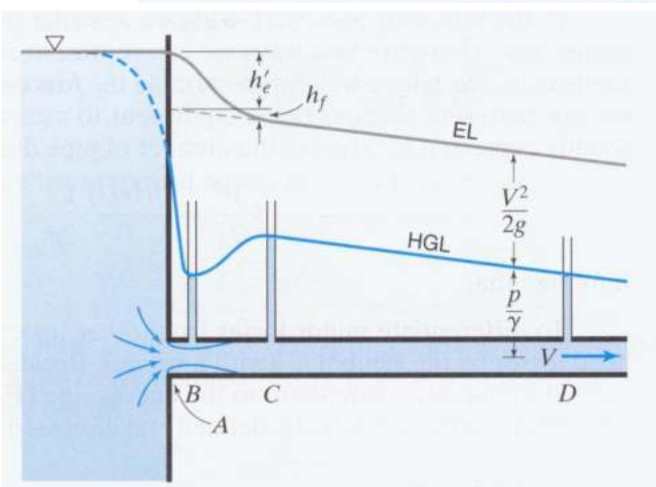
Energy Losses in Contractions

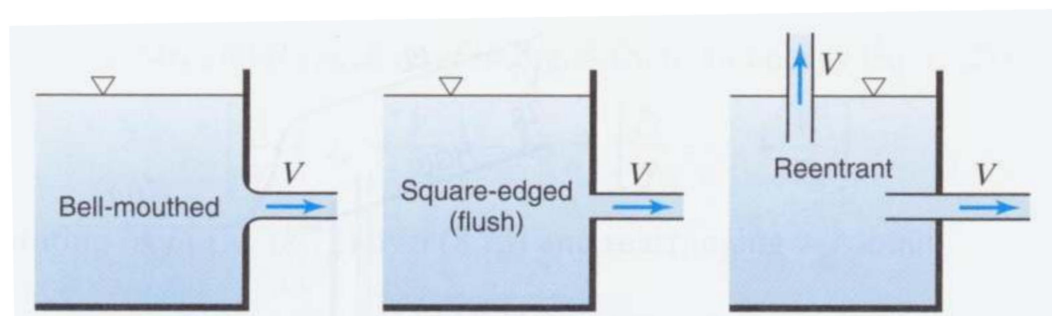


$$h_c = k_c \frac{V_2^2}{2g}$$

TABLE 8.2 Loss coefficients for sudden contraction

D_2/D_1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
k_c	0.50	0.45	0.42	0.39	0.36	0.33	0.28	0.22	0.15	0.06	0.00

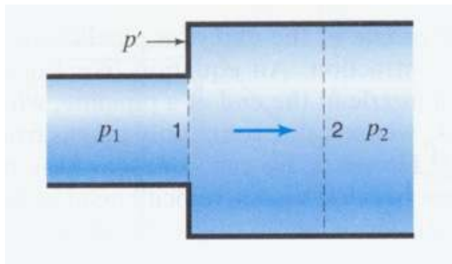




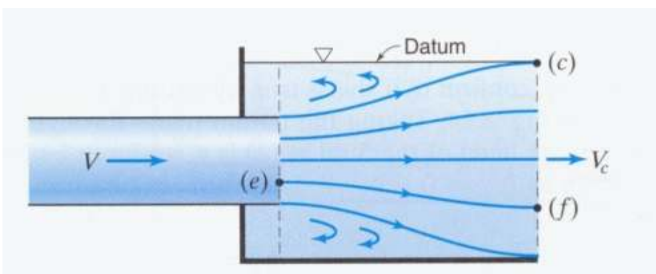
(*a*) $k_e = 0.04$ (*b*) $k_e = 0.5$ (*c*) $k_e \approx 0.8$

All images from Finemore & Franzini (10e, 2002)

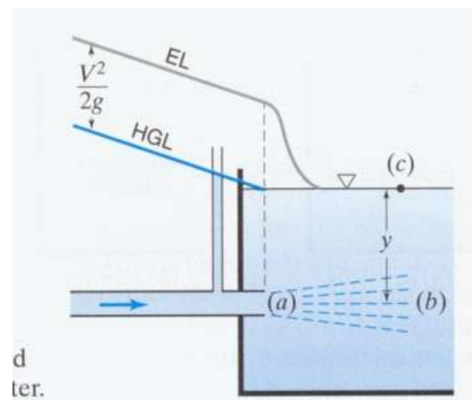
Energy Losses in Expansions



$$h_x = \frac{\left(V - V_c\right)^2}{2g}$$



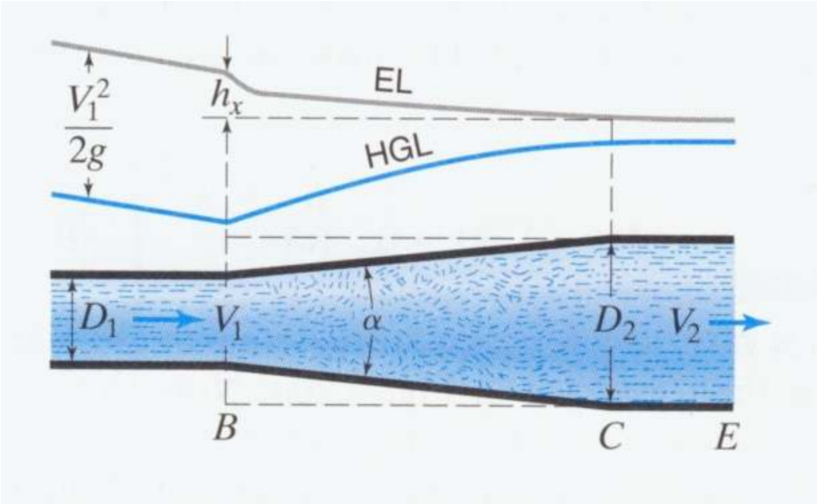
$$h_{x,discharge} = \frac{V^2}{2g} - \frac{V_c^2}{2g}$$



$$h_{x,discharge} = \frac{V^2}{2g} - \cancel{\frac{V_c^2}{2g}}$$

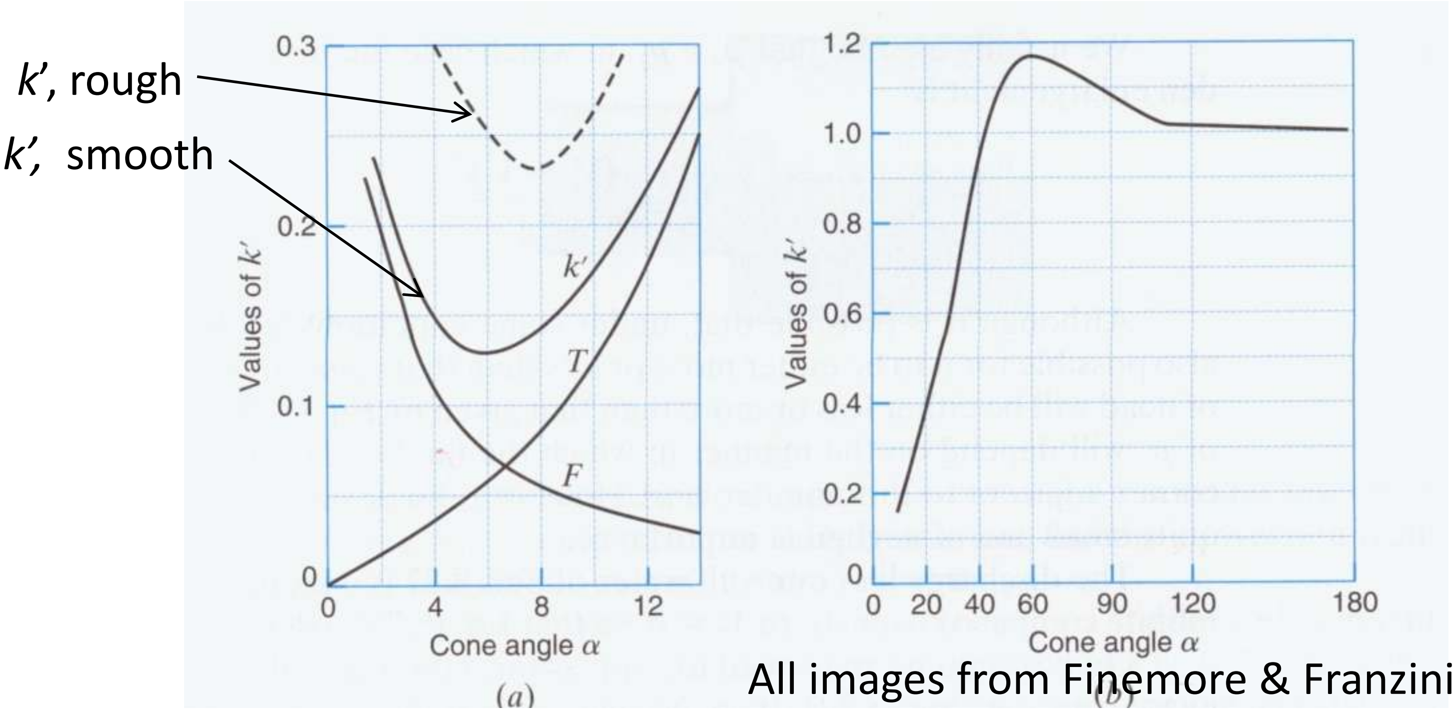
All images from Finemore & Franzini (10e, 2002)

Energy Losses in Expansions



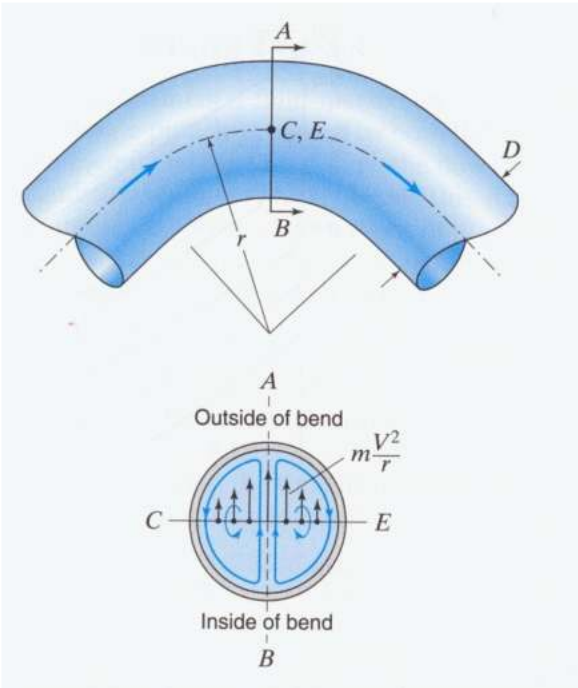
Conical diffuser

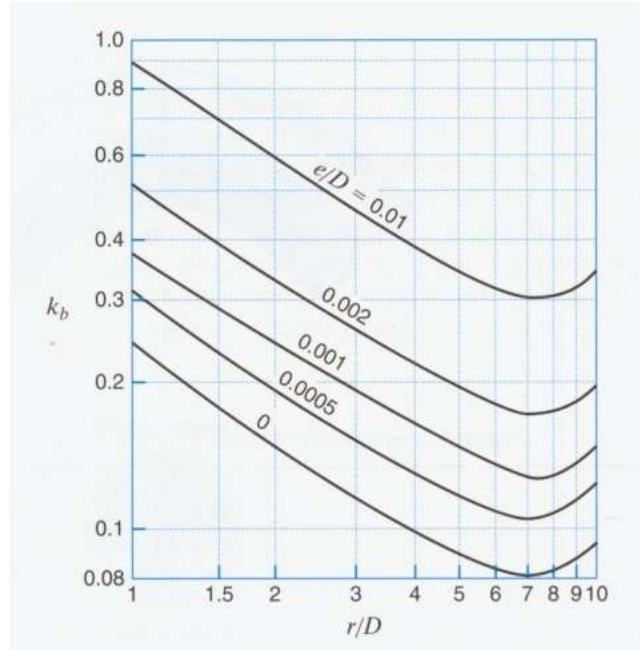
$$h_{cone} = k_{cone} \frac{(V_1 - V_2)^2}{2g}$$



All images from Finemore & Franzini (10e, 2002)

Energy Losses in Pipe Fittings and Bends





$$h_b = k_b \frac{V^2}{2g}$$

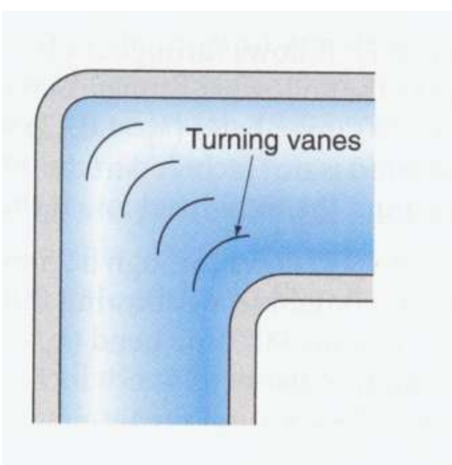


TABLE 8.3 **Values of loss factors for pipe fittings**

Fitting	k
Globe valve, wide open	10
Angle valve, wide open	5
Close-return bend	2.2
T, through side outlet	1.8
Short-radius elbow	0.9
Medium-radius elbow	0.75
Long-radius elbow	0.60
45° elbow	0.42
Gate valve, wide open	0.19
half open	2.06

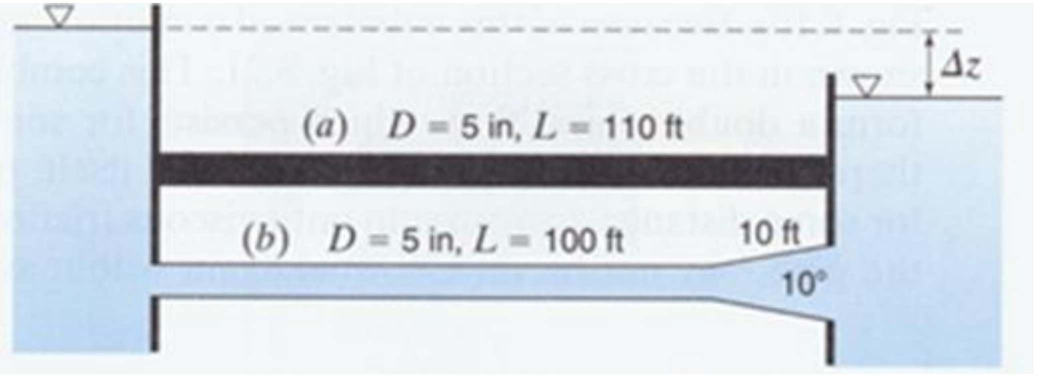
All images from Finemore & Franzini (10e, 2002)

Example

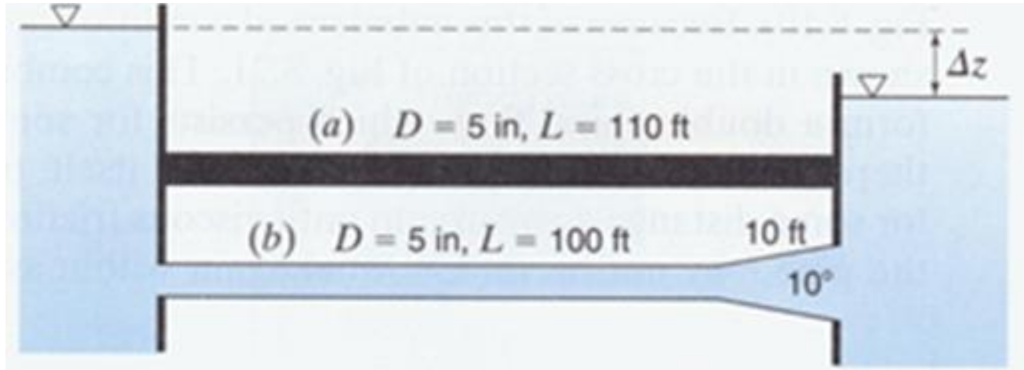
A 5-in-diameter pipe with an estimated *f* of 0.033 is 110 feet long and connects two reservoirs whose surface elevations differ by 12 feet. The pipe entrance is flushed, and the discharge is submerged.

(a) Compute the flow rate.

(b) How much would the flow rate change if the last 10 ft of the pipe were replaced with a smooth conical diffuser with a cone angle of 10°?



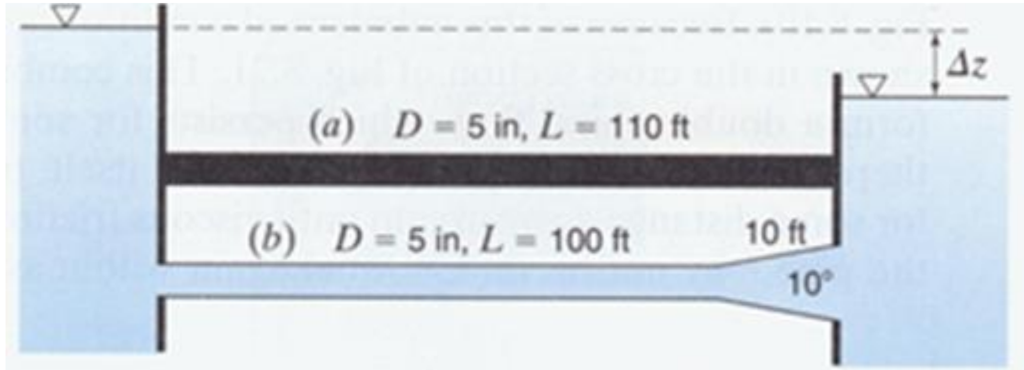
$$5'' = 0.417 \text{ ft}$$



$$(\mathrm{a})\quad h_{L,tot} = h_{L,pipe} + h_{L,minor} = f\frac{l}{D}\frac{V^2}{2g} + (0.5+1)\frac{V^2}{2g} = \left(f\frac{l}{D}+1.5\right)\frac{V^2}{2g}$$

$$V = \sqrt{\frac{2gh_{L,tot}}{\frac{fl}{D}+1.5}} = \sqrt{\frac{2\left(32.2\ \mathrm{ft/s^2}\right)\left(12\ \mathrm{ft}\right)}{\frac{(0.033)(110\ \mathrm{ft})}{0.417\ \mathrm{ft}}+1.5}} = 8.70\ \mathrm{ft/s}$$

$$Q = VA = \left(8.70\ \mathrm{ft/s}\right)\frac{\pi\left(0.417\ \mathrm{ft}\right)^2}{4} = 1.19\ \mathrm{ft^3/s}$$



$$\text{(b)}\quad h_{L,tot} = h_{L,pipe} + h_{L,entrance} + h_{L,cone} + h_{L,exit}$$

$$= f\frac{l_1}{D_1}\frac{V_1^2}{2g} + k_{entrance}\frac{V_1^2}{2g} + k_{cone}\frac{\left(V_1 - V_2\right)^2}{2g} + k_{exit}\frac{V_2^2}{2g}$$

$$D_2 = D_1 + 2L_{cone}\tan 5^{\text{o}} = 0.417\ \text{ft} + 2\left(10\ \text{ft}\right)\left(0.0875\right) = 2.17\ \text{ft}$$

$$\frac{V_2}{V_1} = \left(\frac{D_1}{D_2}\right)^2 = \left(\frac{0.417\ \text{ft}}{2.17\ \text{ft}}\right)^2 = 0.0370$$

From graph, for a smooth, 10$^{\text{o}}$ cone, k_{cone} = 0.175

$$h_{L,tot} = f\,\frac{l_1}{D_1}\frac{V_1^2}{2g} + k_{entrance}\,\frac{V_1^2}{2g} + k_{cone}\,\frac{\left(V_1 - V_2\right)^2}{2g} + k_{exit}\,\frac{V_2^2}{2g}$$

$$= \left(0.033\right)\frac{100 \text{ ft}}{0.417 \text{ ft}}\,\frac{V_1^2}{2g} + 0.5\,\frac{V_1^2}{2g}$$

$$+\left(0.175\right)\frac{\left(V_1 - 0.037V_1\right)^2}{2g} + 1.0\,\frac{\left(0.037V_1\right)^2}{2g}$$

$$V_1 = 9.49 \text{ ft/s}$$

$$Q = V_1 A = \left(9.49 \text{ ft/s}\right)\frac{\pi\left(0.417 \text{ ft}\right)^2}{4} = 1.29 \text{ ft}^3/\text{s}$$