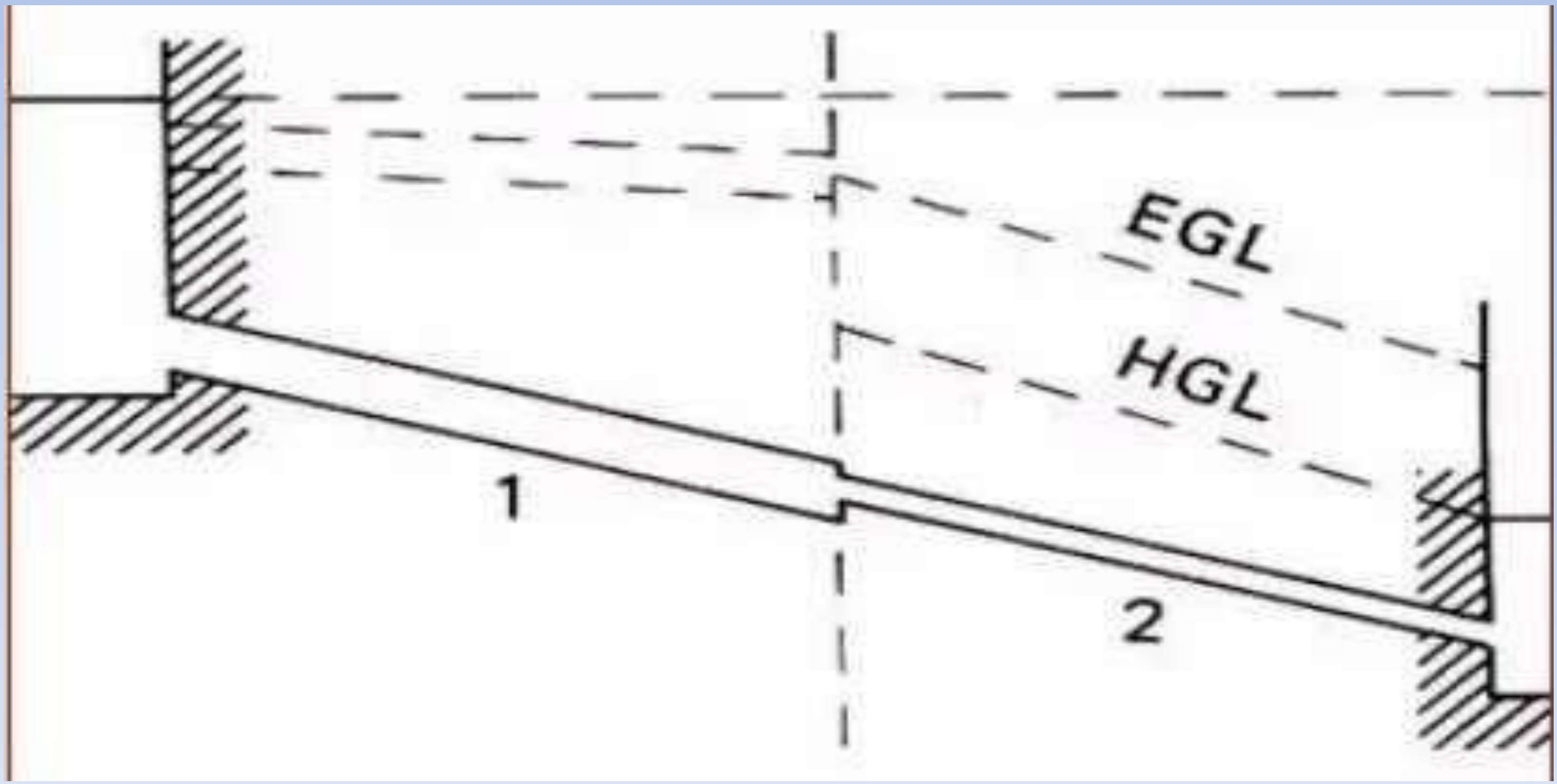
Flow through pipe in series and parallel

Flow through pipe in series

- When pipes of different diameters are connected end to end to form a pipe line, they are said to be in series. The total loss of energy (or head) will be the sum of the losses in each pipe plus local losses at connections.

- An arrangement of such pipe line between two reservoir is shown in fig.



- Total head loss for the system H = height difference of reservoirs
- h^{f1} = head loss for 200mm diameter section of pipe
- h^{f2} = head loss for 250mm diameter section of pipe
- h^{entry} = head loss at entry point
- h^{enp} = head loss at join of the two pipes
- h^{exit} = head loss at exit point

$H = h^{f1} + h^{f2} + h^{entry} + h^{enp} + h^{exit}$

$$= \frac{4 f l_1 1}{d_1} \cdot \frac{v_1^2}{2g} + \frac{4 f l_2 2}{d_2} \cdot \frac{v_2^2}{2g} + 0.5 \frac{v_1^2}{2g} + \frac{(v_1 - v_2)^2}{2g} + \frac{v_2^2}{2g}$$

- Applying Bernoulli's equation for point (1) and (2)

$$\frac{p_1}{w} + z_1 + \frac{v_a^2}{2g} = \frac{p_2}{w} + z_2 + \frac{v_b^2}{2g} + 0.5\frac{v_1^2}{2g} + —$$

As p1=p2

And

$$v_1 = v_2 = 0$$

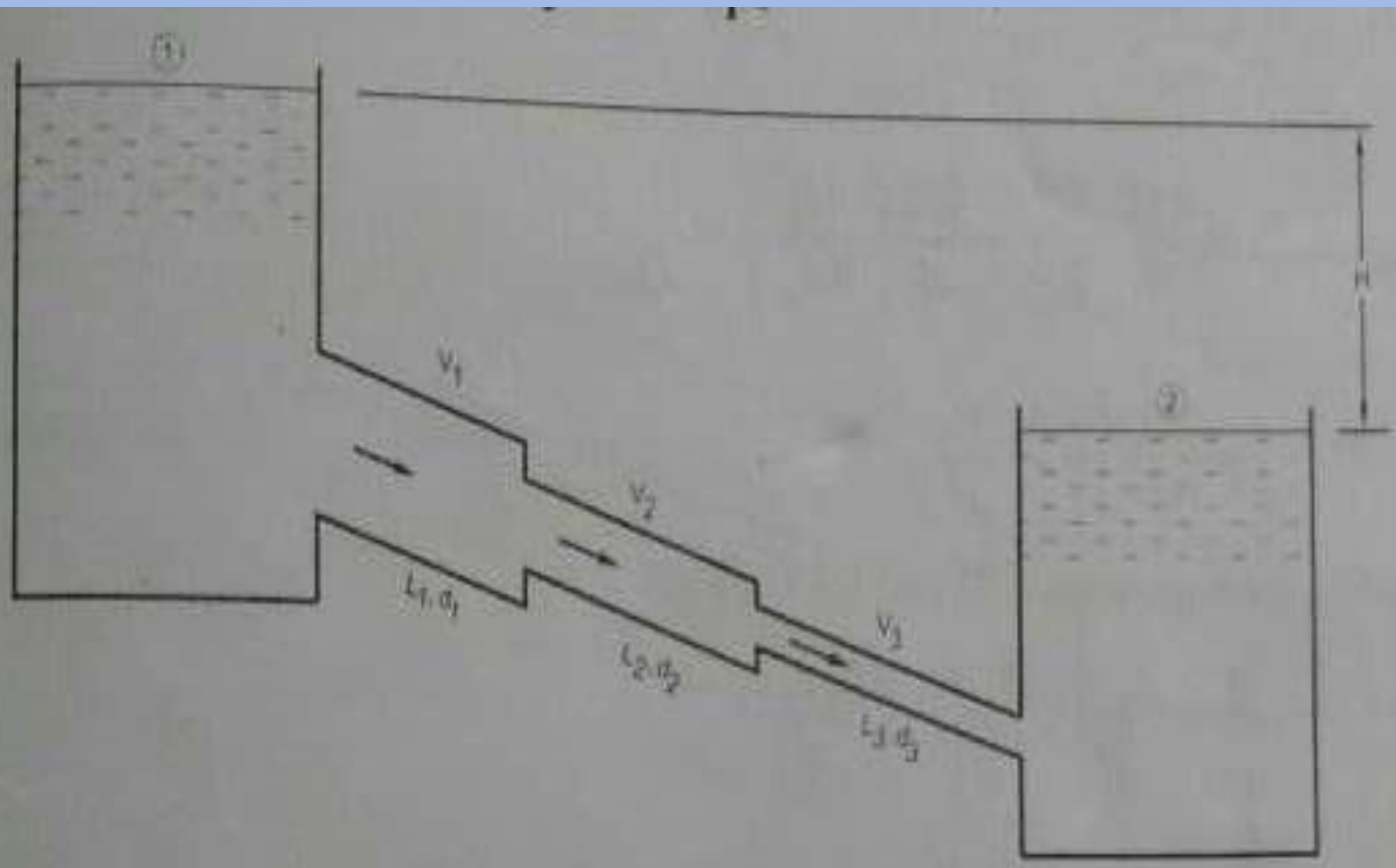
$$h_{totalloss} = z_1 - z_2 = H$$

Flow through pipe in parallel

- Many times the flow from one reservoir to another reservoir is increased by connecting number of pipes in parallel as shown in fig.

- Assume Q1 and Q2 are the discharges through the pipes 1
and 2

$$Q = Q_1 + Q_2$$



1
2
H
V1
V2
V3
L1, d1
L2, d2
L3, d3

▶Neglecting all minor losses.

Head loss in pipe 1 = Head loss in pipe 2

$$\frac{4 f_1 l_1}{d_1} * \frac{v_1^2}{2g} = \frac{4 f_2 l_2}{d_2} * \frac{v_2^2}{2g}$$

If f1=f2 then

$$\frac{f_1 l_1 v_1^2}{d_1} = \frac{f_2 l_2 v_2^2}{d_2}$$

$$\frac{l_1 v_1^2}{d_1} = \frac{l_2 v_2^2}{d_2}$$

PROBLEM

Flow through pipe in series

Example : Consider the two reservoirs shown in figure 16, connected by a single pipe that changes diameter over its length. The surfaces of the two reservoirs have a difference in level of 9m. The pipe has a diameter of 200mm for the first 15m (from A to C) then a diameter of 250mm for the remaining 45m (from C to B). Find Q?

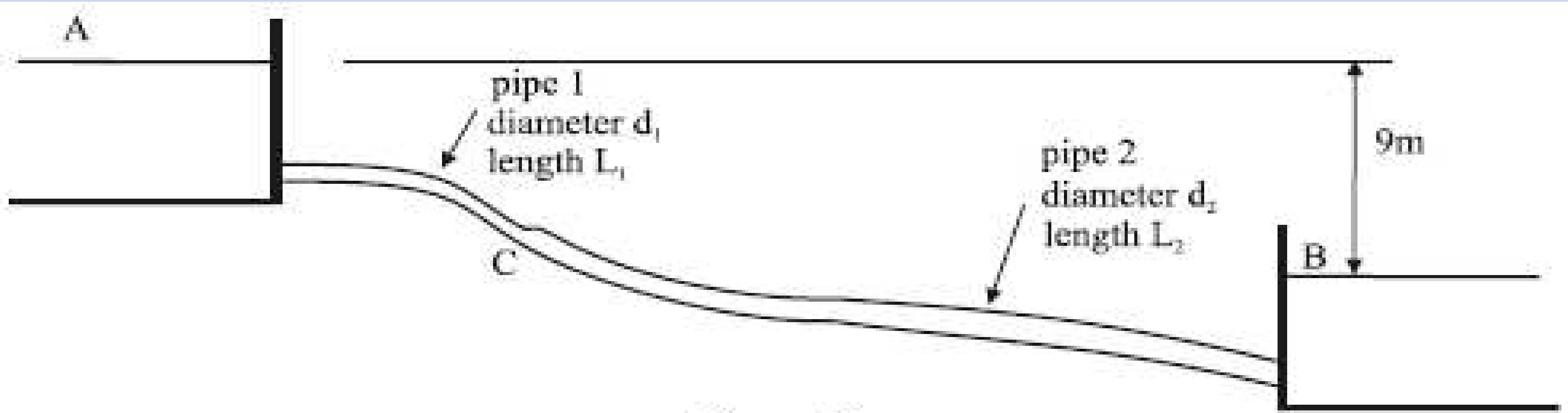


Figure 16:

For both pipes use f = 0.01.

- Total head loss for the system H = height difference of reservoirs
- h^{f1} = head loss for 200mm diameter section of pipe
- h^{f2} = head loss for 250mm diameter section of pipe
- h^{entry} = head loss at entry point
- h^{anp} = head loss at join of the two pipes
- h^{exit} = head loss at exit point

$H = h^{f1} + h^{f2} + h^{entry} + h^{enp} + h^{exit} = 9m$

$$h_{f1} = \frac{f l_1 Q^2}{3 d_1^{\ 2}} \qquad h_{f2} = \frac{f l_2 Q^2}{3 d_2^{\ 2}}$$

$$h_{entry} = 0.5\frac{u_1^2}{2g} = 0.5\frac{1}{2g}\left(\frac{4Q}{\Pi d_1^{\,2}}\right) = 0.5 * 0.0826\frac{Q^2}{d_1^4} = 0.0413\frac{Q^2}{d_1^4}$$

$$h_{exit} = 1.0\frac{u_1^2}{2g} = 1.0 * 0.0826\frac{Q^2}{d_2^{\,4}} = 0.0413\frac{Q^2}{d_2^{\,4}}$$

$$h_{anp} = \frac{(u_1 - u_2)^2}{2g} = \left(\frac{4Q}{\Pi}\right)\frac{\left(\frac{1}{d_1^2} - \frac{1}{d_2^{\,2}}\right)}{2g} = 0.0826Q^2\left(\frac{1}{d_1^2} - \frac{1}{d_2^{\,2}}\right)$$

Substitute value in equation

$H = h^{f1} + h^{f2} + h^{entry} + h^{enp} + h^{exit} = 9m$

and solve for Q, to give $Q = 0.158\ m^3/s$

Flow through pipe in parallel

- Example :
- Two pipes connect two reservoirs (A and B) which have a height difference of 10m. Pipe 1 has diameter 50mm and length 100m. Pipe 2 has diameter 100mm and length 100m. Both have entry loss $k^L = 0.5$ and exit loss $k^L=1.0$ and Darcy f of 0.008.

Calculate:

a) rate of flow for each pipe

b) the diameter D of a pipe 100m long that could replace the two pipes and provide the same flow

- Apply Bernoulli to each pipe separately. For pipe 1:

$$\frac{p_a}{\rho g} + \frac{u_a^2}{2g} + z_a = \frac{p_b}{\rho g} + \frac{u_b^2}{2g} + z_b + 0.5\frac{u_1^2}{2g} + \frac{4flu_1^2}{2gd_1} + 1.0\frac{u_1^2}{2g}$$

$$z_1 - z_2 = \left(0.5 + \frac{4fl}{d_1} + 1.0\right)\frac{v_1^2}{2g}$$

$$10 = \left(1.0 + \frac{4*0.008*100}{0.05}\right)\frac{u_1^2}{2*9.81}$$

$$u_1 = 1.731 m/s$$

$$10 = \left(1.0 + \frac{4*0.008*100}{0.05}\right)\frac{u_1^2}{2*9.81}$$

$$u_1 = 1.731 m/s$$

$$Q_2 = u_2 \frac{\Pi d_2^2}{4} = 0.0190 m^3/s$$

For pipe 2:

$$\frac{p_a}{\rho g} + \frac{u_a^2}{2g} + z_a = \frac{p_b}{\rho g} + \frac{u_b^2}{2g} + z_b + 0.5\frac{u_2^2}{2g} + \frac{4flu_2^2}{2gd_2} + 1.0\frac{u_2^2}{2g}$$

Again p^A and p^B are atmospheric, and as the reservoir surface move s slowly u_A and u_B are negligible, so

$$Q = Au = \frac{\Pi D^2}{4} u$$

$$u = \frac{4Q}{\Pi D^2} = \frac{0.02852}{D^2}$$

$$Q_2 = u_2 \frac{\Pi d_2^{\ 2}}{4} = 0.0190 m^3 / s$$

- b) Replacing the pipe,

we need $Q = Q^1 + Q^2 = 0.0034 + 0.0190 = 0.0224$ m³/s

For this pipe, diameter D, velocity u , and making the same assumptions about entry/exit losses, we have

$$\frac{p_a}{\rho g} + \frac{u_a^2}{2g} + z_a = \frac{p_b}{\rho g} + \frac{u_b^2}{2g} + z_b + 0.5\frac{u_1^2}{2g} + \frac{4flu_1^2}{2gd_1} + 1.0\frac{u_1^2}{2g}$$

$$z_a - z_b = \left(0.5 + \frac{4fl}{D} + 1.0\right)\frac{u_2^2}{2g}$$

$$10 = \left(1.0 + \frac{4 * 0.008 * 100}{D}\right)\frac{u^2}{2 * 9.81}$$

$$196.2 = \left(1.0 + \frac{3.2}{D}\right)u^2$$

- The velocity can be obtained from Q

$$Q = Au = \frac{\Pi D^2}{4}u$$

$$u = \frac{4Q}{\Pi D^2} = \frac{0.02852}{D^2}$$

$$196.2 = \left(1.0 + \frac{3.2}{D}\right)\left(\frac{0.02852}{D^2}\right)^2$$

$$0 = 241212D^5 - 1.5D - 3.2$$

which must be solved iteratively

D=0.1058m