

DATA STRUCTURE

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Recursion in Python

- **Recursion**
- Recursion is a method of solving problems that involves breaking the problem into smaller and smaller subproblems until we get to small enough problem that can be solved trivially.
- **In programming**
- Another approach of programming whereby a method directly or indirectly **calls itself**
- **How to stop recursion?**
- using a condition called "**base case**". It is the case that can return the result directly without a recursive call.

Summation of a list

$$\text{total} = (1 + (3 + (5 + (7 + 9))))$$

$$\text{total} = (1 + (3 + (5 + 16)))$$

$$\text{total} = (1 + (3 + 21))$$

$$\text{total} = (1 + 24)$$

$$\text{total} = 25$$

Example: Factorial

- The factorial is defined in this way:

$$n! = \begin{cases} 1 & n = 0 \\ 1 \times 2 \times \cdots \times n & n > 0 \end{cases}$$

This can be computed by a loop.

- We can also define the factorial as:

$$n! = \begin{cases} 1 & n = 0 \\ n \times (n - 1)! & n > 0 \end{cases}$$

So, if we know how to compute the factorial of $n - 1$, we know how to compute the factorial of n .

Factorial

- All what I Know that factorial(1) is 1, thus I can calculate any factorial based on this rule.
- To see how the calculation is done. Trace factorial(4):

$$\begin{aligned}\text{Factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * 1)) \\ &= 4 * (3 * 2) \\ &= 4 * 6 \\ &= 24\end{aligned}$$

Factorial

```
def factorial(n):  
    if n == 0 or n == 1:  
        return 1  
    else:  
        return n * factorial(n-1)  
  
#----Test ----  
  
f = factorial(5)  
print(f)
```

each recursive method MUST have a condition called "base case" to determine the ending point of recursive call

The Three Laws of Recursion

- All recursive algorithms must obey three important laws:
 1. A recursive algorithm must have a base case.
 2. A recursive algorithm must change its state or input and move toward the base case.
 3. A recursive algorithm must call itself, recursively.

Example

- How many recursive calls are made when computing Sum [2,4,6,8,10]
- Answer?

Stack Overheads in Recursion

- We need to pay attention to two important results:
 1. The depth of recursion
 2. The stack overhead in recursion
- Every recursive call keeps a copy of the recursive method in a special memory space → called (Recursion Stack).
- If the recursion stack got full then it will cause the freeze of the program
 - sometimes this error called → **"stack overflow"**.

Recursion Stack

Factorial (0)

Factorial (1)

Factorial (2)

Factorial (3)

Factorial (4)

Is it a Good idea to write a Recursive Function?

- Recursion enables us to write a program in a natural way → it helps programmers to generate realistic looking scenery for computer generated movies.
- The speed of the recursive program is slower because of “**Stack Overhead**”
- In a recursive program you have to specify recursive conditions, terminating conditions, and recursive expressions

Summation of a list

- Here is both implementations of getting a summation of a list. First is the iterative code and the second is using recursive code.

total = (1 + (3 + (5 + (7 + 9))))

total = (1 + (3 + (5 + 16)))

total = (1 + (3 + 21))

total = (1 + 24)

total = 25

```
def sumList(lst):
    total = 0
    for item in lst:
        total += item

    return total

def RecursiveSumList(lst):
    if len(lst) == 1:
        return lst[0]
    else:
        return lst[0] + RecursiveSumList( lst[1:] )

#----Test ----

L = [1,3,5,7,9]
print( sumList(L) )
print( RecursiveSumList(L) )
```

GCD in Recursion

- $\text{gcd}(a, 0) = a$
- $\text{gcd}(a, b) = \text{gcd}(b, a \bmod b)$
- **def** RecursiveGCD(a,b):
 - **if** (b == 0):
 - **return** a
 - **else:**
 - **return** RecursiveGCD(b, a % b)
- r = RecursiveGCD(24,12)
- **print** (r)
- r = RecursiveGCD(565,15)
- **print** (r)

Fibonacci in Recursion

- $\text{Feb}(0) = 0$
- $\text{Feb}(1) = 1$
- $\text{Feb}(n) = \text{Feb}(n-1) + \text{Feb}(n-2)$

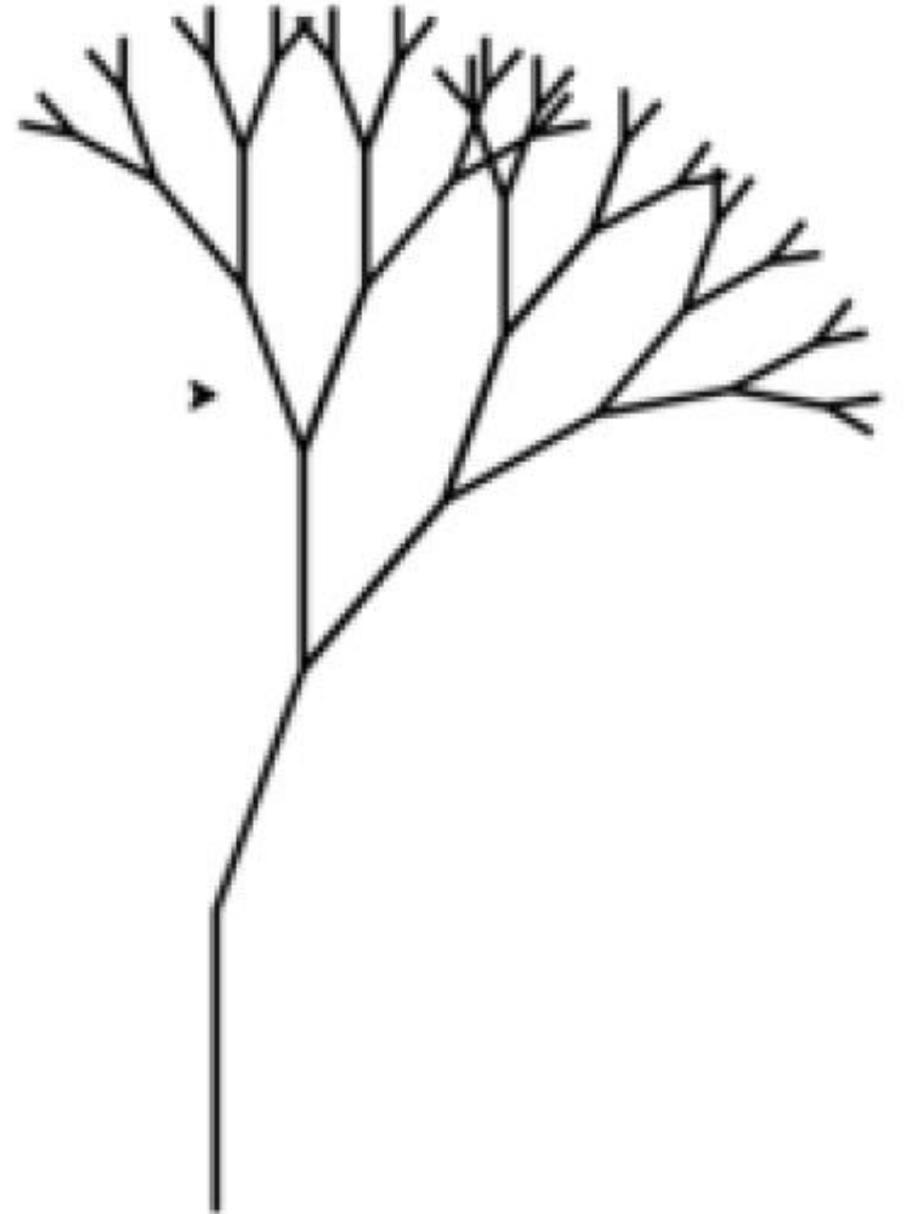
```
def feb(n):  
    if n == 0:  
        return 0  
  
    if n == 1:  
        return 1  
  
    return feb(n-1) + feb(n-2);  
  
#----Test ----  
  
f = feb(10)  
print(f)
```

Visualizing Recursion

- We can see the mental model to visualize what is happening in recursion functions
- We use Python's turtle graphics model → **Turtle**
- The idea is that the turtle will move forward, backward, left, right ..etc.
- You can change the color of the ink when the turtle moves its tail.

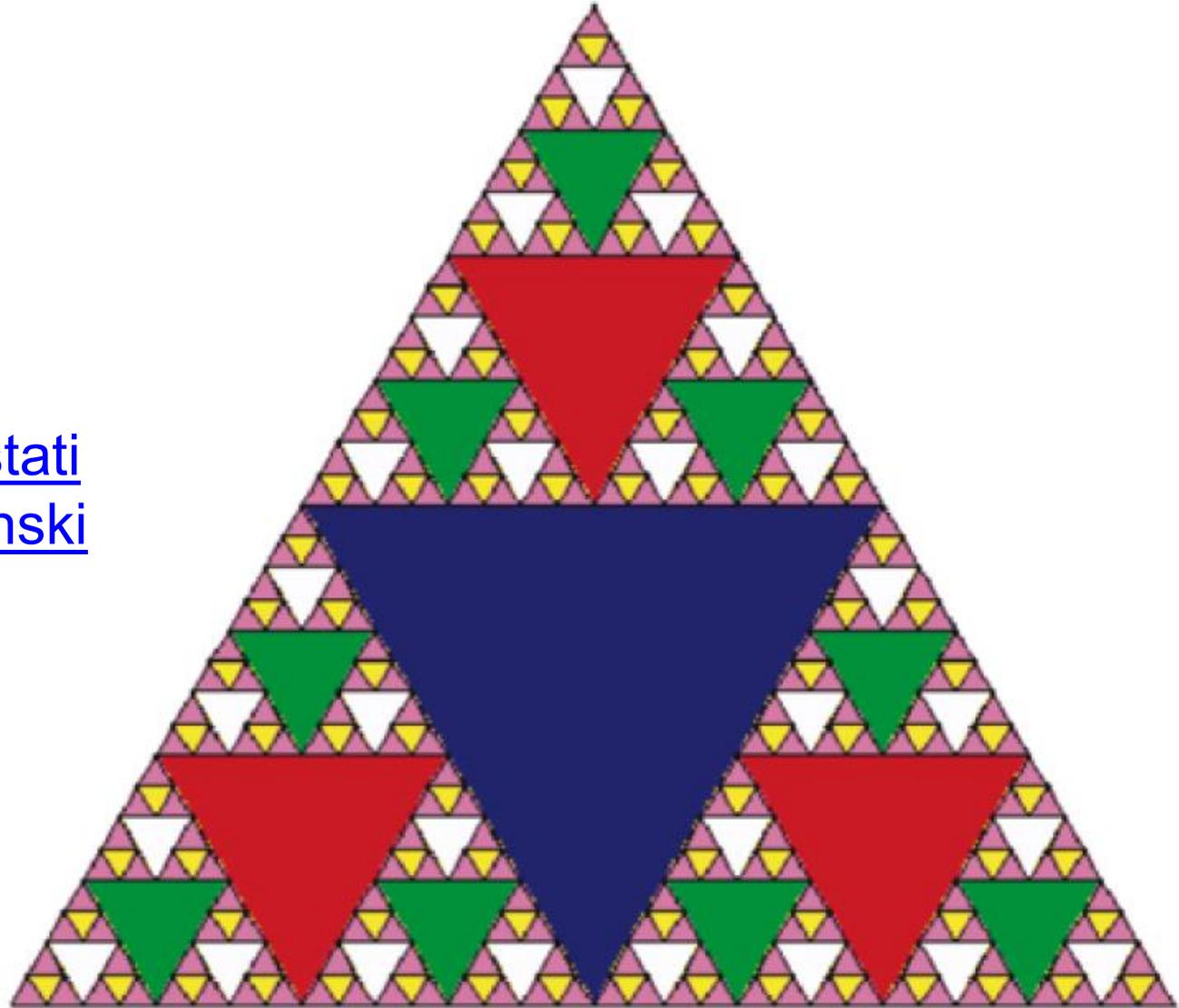
The First Half of the Tree

- <http://interactivepython.org/courselib/static/pythonds/Recursion/pythondsintro-VisualizingRecursion.html>



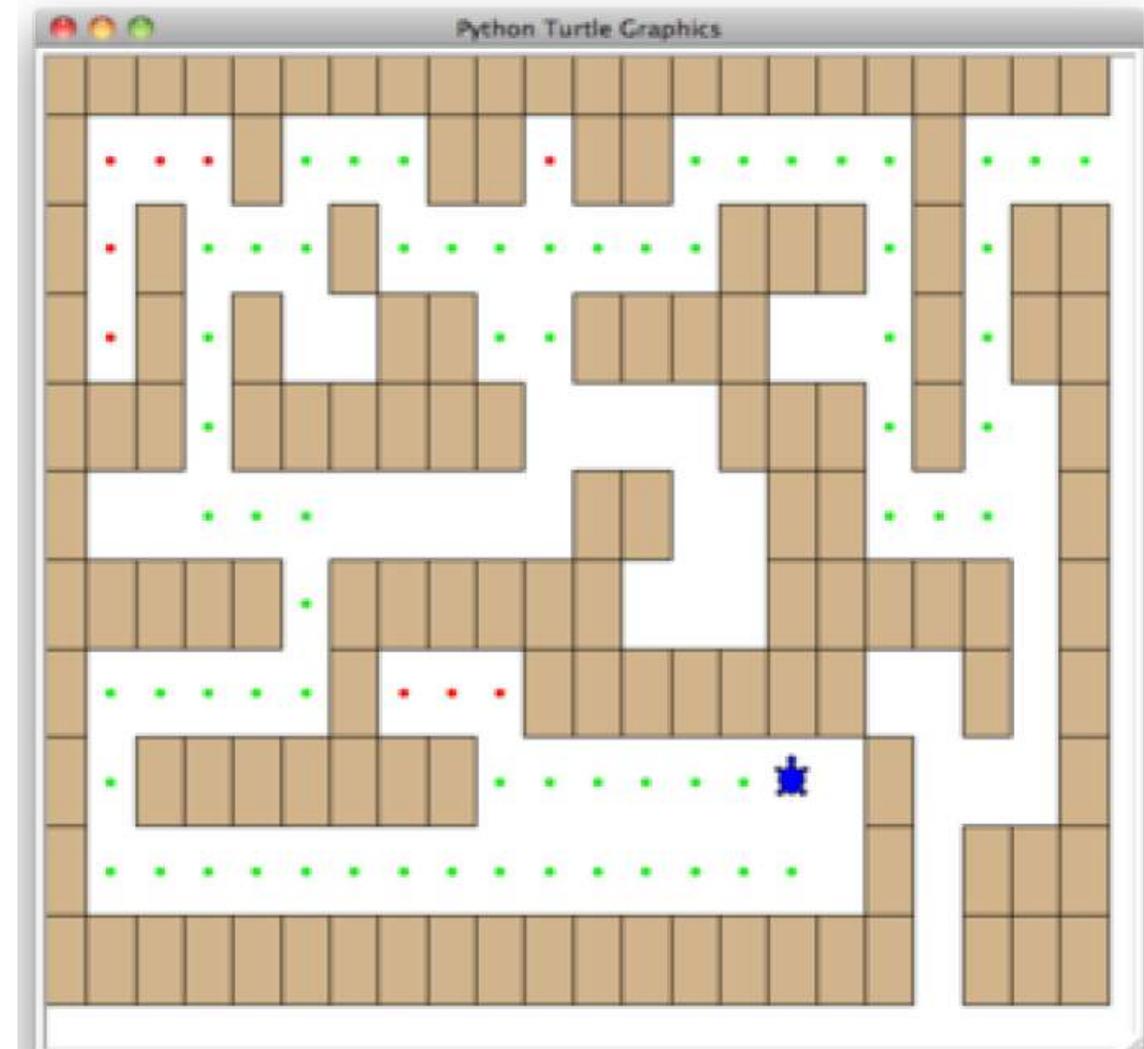
The Sierpinski Triangle

- <http://interactivepython.org/courselib/static/pythonds/Recursion/pythondsSierpinskiTriangle.html>



The Finished Maze Search Program

- <http://interactivepython.org/courselib/static/pythonds/Recursion/pythondsSierpinskiTriangle.html>



Recursion Examples

- Write a recursion function to find the sum of every number in an integer number.
 - Example: $n=9182 \Rightarrow \text{SumDigits} = 9+1+8+2 = 20$

```
def sumDigits(n):  
    if n < 10:  
        return n  
  
    lastDigit = n % 10  
    rest = n//10  
    return lastDigit + sumDigits(rest)  
  
#----Test ----  
print( sumDigits(9182) )
```

Types Of Recursion

- Linear Recursion
- Binary Recursion
- Tail Recursion
- Exponential Recursion
- Nested Recursion
- Mutual Recursion