15FELE121
Electromagnetism Fundamentals

Introduction

Chapter 1 – Vector Analysis

Dr. Michel Nahas

15FELE121 - Electromagnetism Fundamentals

- You MUST bring to every class:
  - Notebook / papers and pens
  - Calculator – recommended Casio FX350MS

- Be on time to the Class
  - 10 minutes late will be considered as absence

- Cell phones are not allowed in the Class
  - Turn your mobile phones off during the Class

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- In sections held 3 times a week:
  - Students are allowed to miss 9 sessions only
  - On the 10th absence, an "FA" grade will immediately be recorded for that unit

- In sections held 4 times a week:
  - Students are allowed to miss 12 sessions only
  - On the 13th absence, an "FA" grade will immediately be recorded for that unit

Student Attendance Policy - Video

15FELE121 - Electromagnetism Fundamentals

- Course Material (**MyLMS** page)

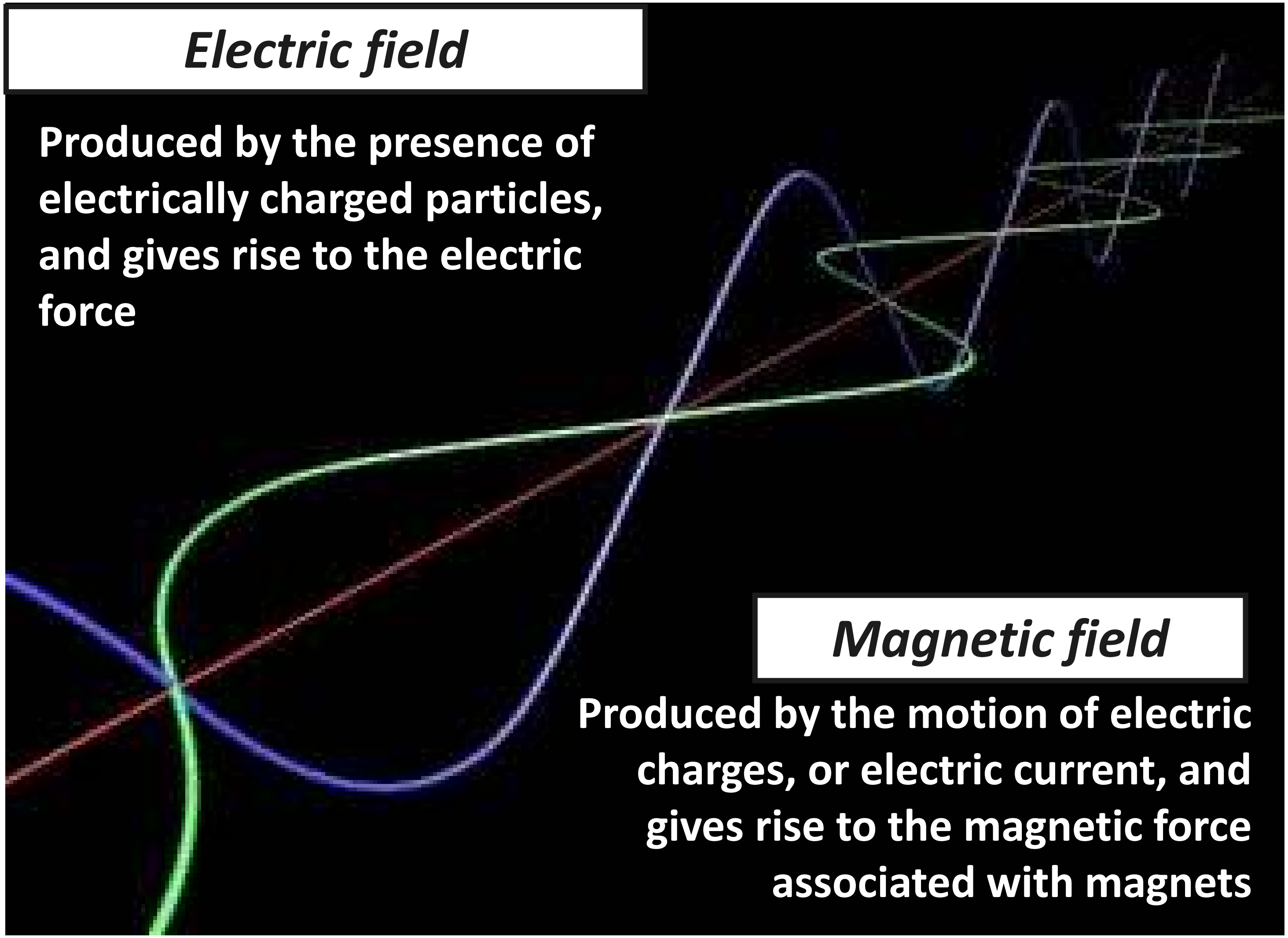
- Lecture notes and PPT
- Lab experiments

- Textbook:

- **Engineering Electromagnetics**

by William Hayt and John Buck, McGraw-Hill, 2012

What is Electromagnetism?



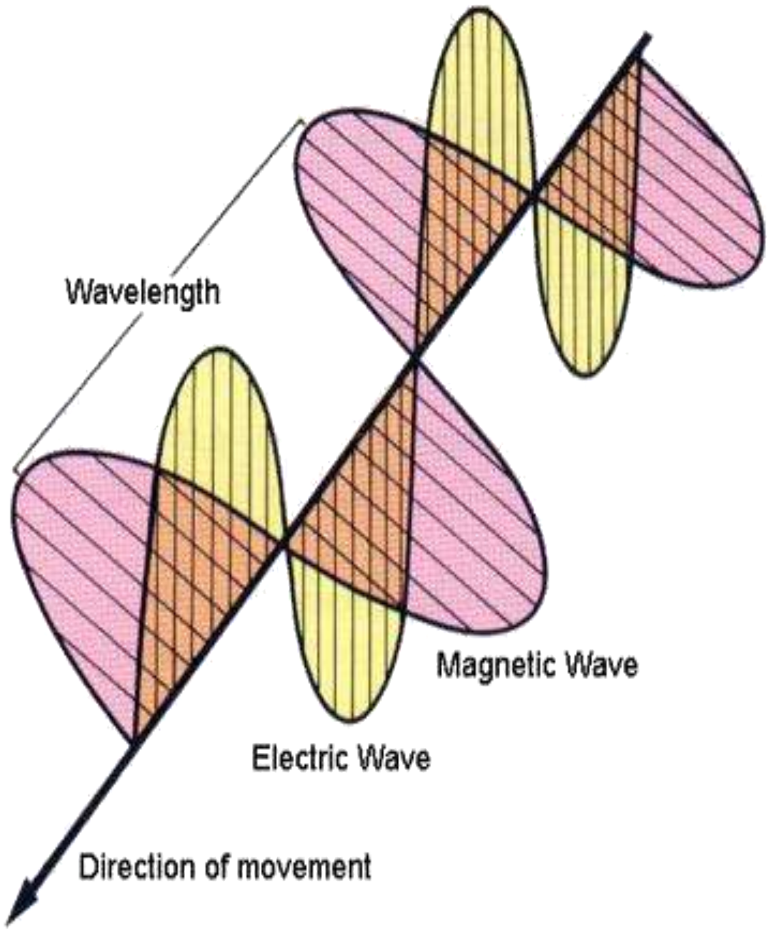
Electric field

Produced by the presence of electrically charged particles, and gives rise to the electric force

Magnetic field

Produced by the motion of electric charges, or electric current, and gives rise to the magnetic force associated with magnets

Electromagnetic Field



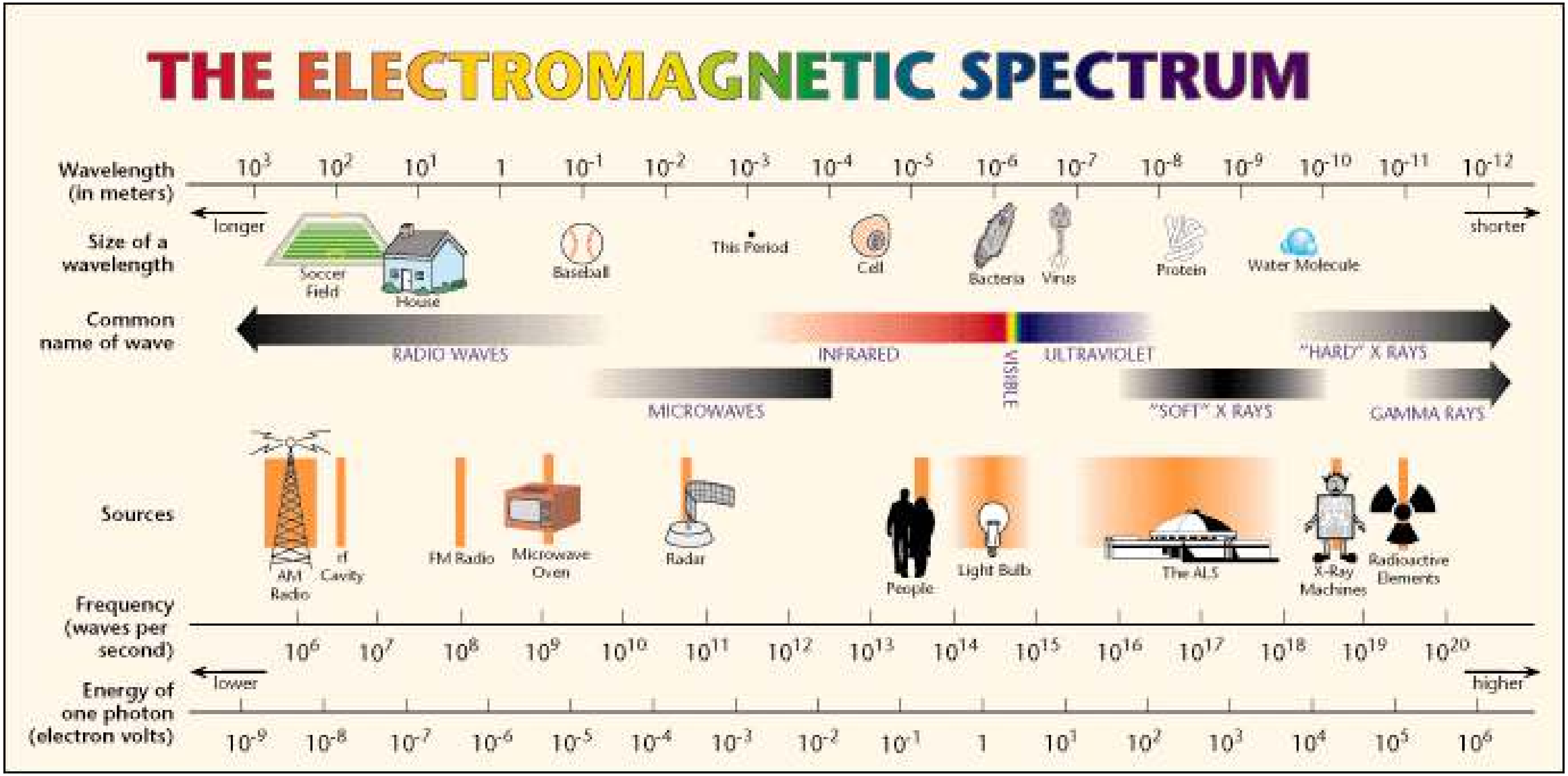
- An electromagnetic field is generated when charged particles, such as electrons, are *accelerated*.
- All electrically charged particles are surrounded by electric fields.
- Charged particles in motion produce magnetic fields.
- When the velocity of a charged particle changes, an electromagnetic field is produced.

Why do we learn Electromagnetism Fundamentals?

- It is the study of the underlying laws that govern the manipulation of electricity and magnetism, and how we use these laws to our advantage.
- It is the source of fundamental principles behind many branches of electrical engineering, and indirectly impacts many other branches.
- EM fields and forces are the basis of modern electrical systems. It represents an essential and fundamental background that underlies future advances in modern communications, computer systems, digital electronics, signal processing, and energy systems.

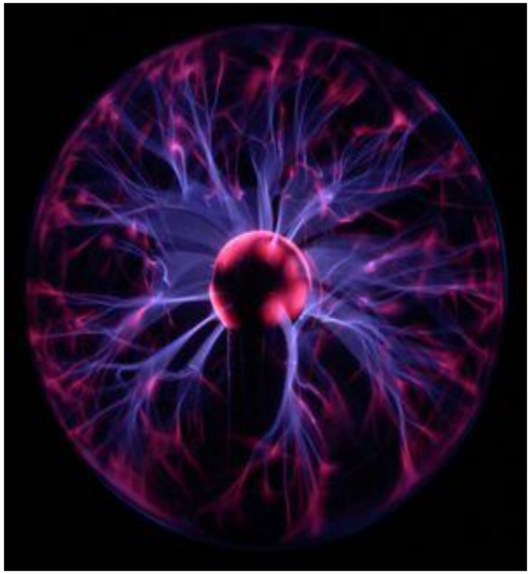
Why do we learn Electromagnetism Fundamentals?

- Electric and magnetic field exist nearly everywhere.



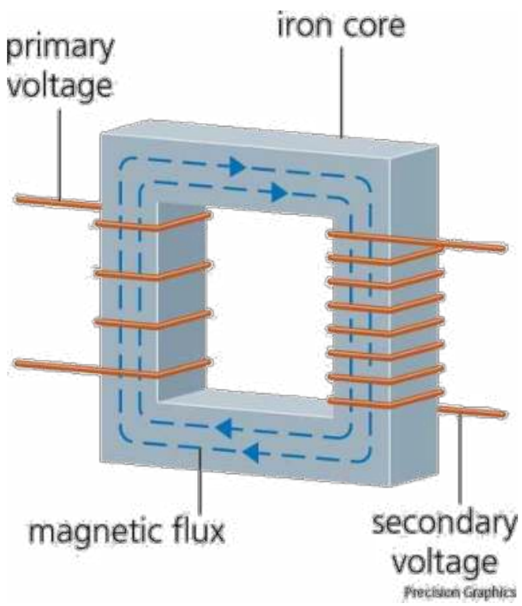
Applications

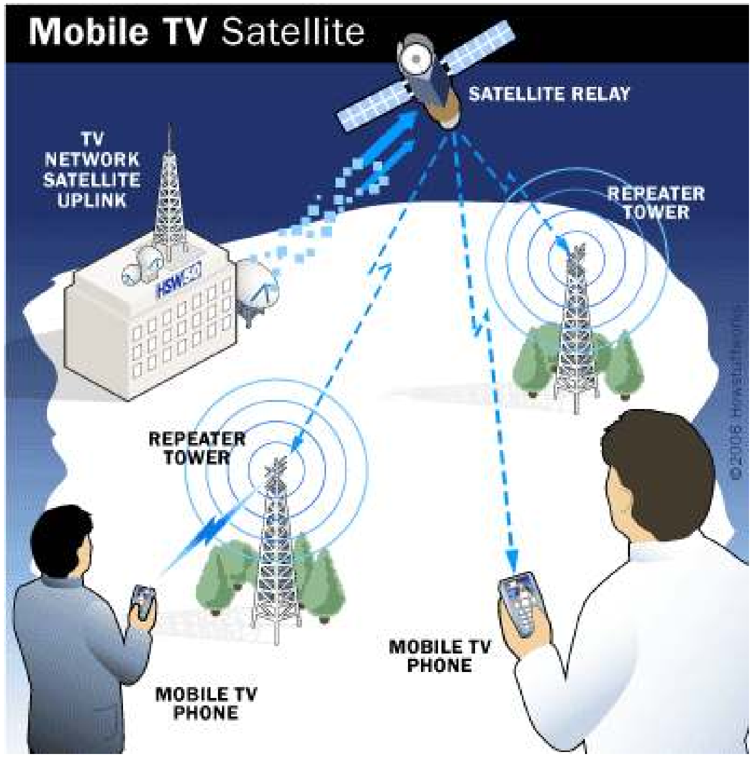
- Electromagnetic principles find application in various disciplines such as microwaves, x-rays, antennas, electric machines, plasmas, etc.



Applications

- Electromagnetic fields are used in induction heaters for melting, forging, annealing, surface hardening, and soldering operation.
- Electromagnetic devices include transformers, radio, television, mobile phones, radars, lasers, etc.

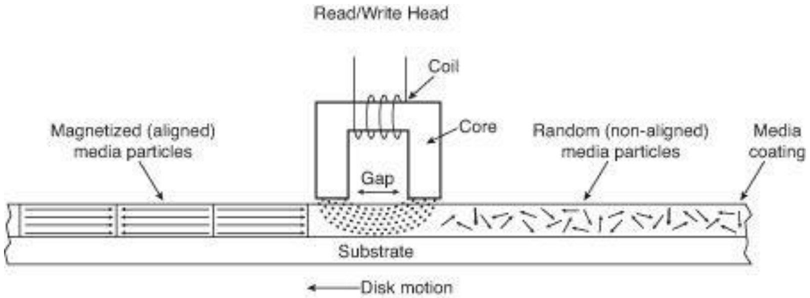




Applications

- A hard drive reads and writes data by using electromagnetism.
- The basic principle:
  (Write) If electric current (in form of digital signal) flows through a conductor, then a magnetic field is generated around the conductor.
  (Read) If a conductor pass through a magnetic field, then electric current (in form of digital signal) is generated.
- The disk constitutes the actual storage medium, consists of some form of substrate material (such as aluminium) on which a layer of magnetizable material has been deposited.
- As the field passes through the medium directly under the gap, it polarizes the magnetic particles so they are aligned with the field.

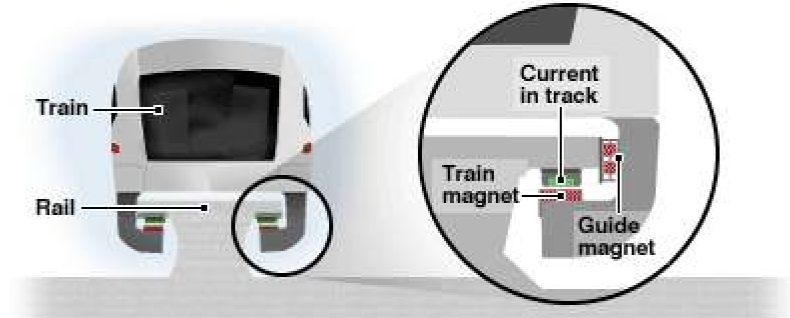
Hard drive: Magnetic Data Storage

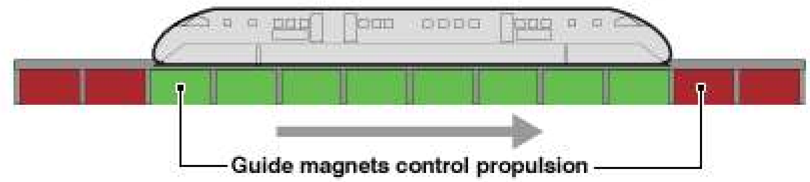


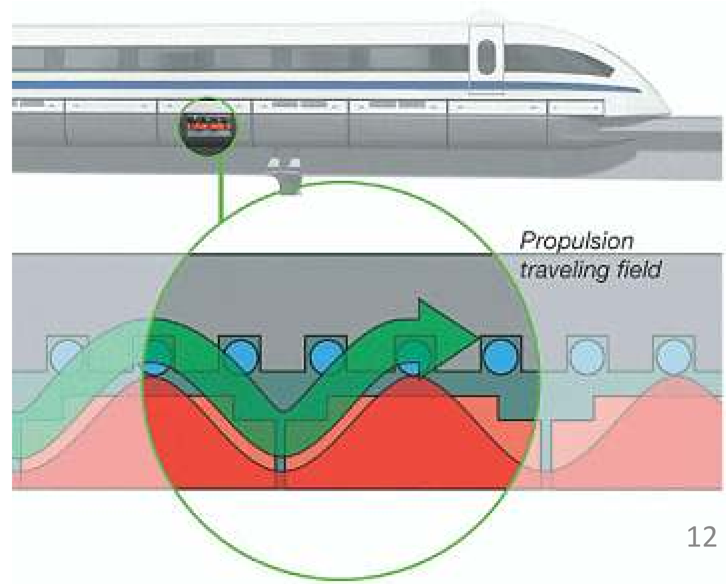
Applications

Transrapid Train

- A magnetic traveling field moves the vehicle without contact.
- The speed can be continuously regulated by varying the frequency of the alternating current.



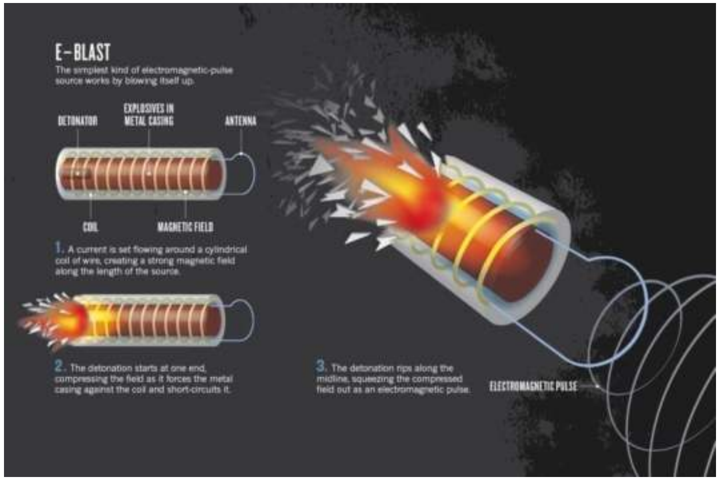




Applications

AGM-88E Anti-Radiation Guided Missile

- Able to guide itself to destroy a radar using the signal transmitted by the radar.
- Destroy radar and intimidate its operators, creates hole in enemy defense.
- Unit cost US$ 284,000 – US$ 870,000.



E-bomb (Electromagnetic-pulse bomb)

- Designed to attack people's dependency on electricity.
- Instead of cutting off power in an area, an e-bomb would destroy most machines that use electricity.
- Generators, cars, telecommunications would be non operable.

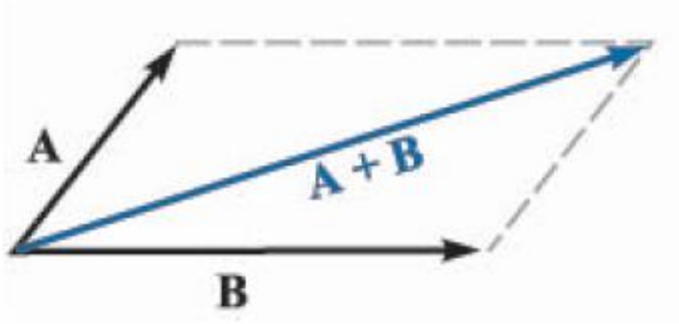
Chapter 1

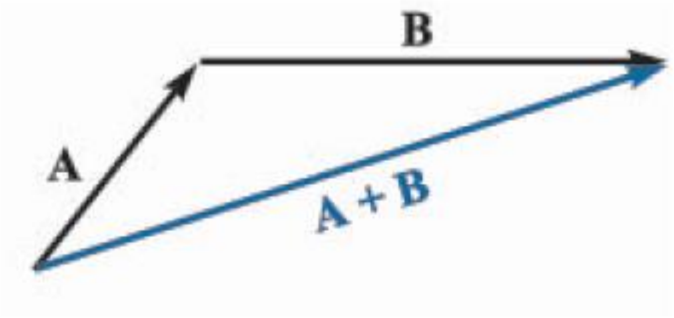
Vector Analysis

Scalars and Vectors

- Scalar refers to a quantity whose value may be represented by a single (positive or negative) real number.
- Some examples include distance, temperature, mass, density, pressure, volume, and time.
- A vector quantity has both a magnitude and a direction in space. We especially concerned with two- and three-dimensional spaces only.
- Displacement, velocity, acceleration, and force are examples of vectors.
    - Scalar notation: A or A (*italic* or plain)
    - Vector notation: **A** or $\vec{A}$ (bold or plain with arrow)

Vector Algebra





$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

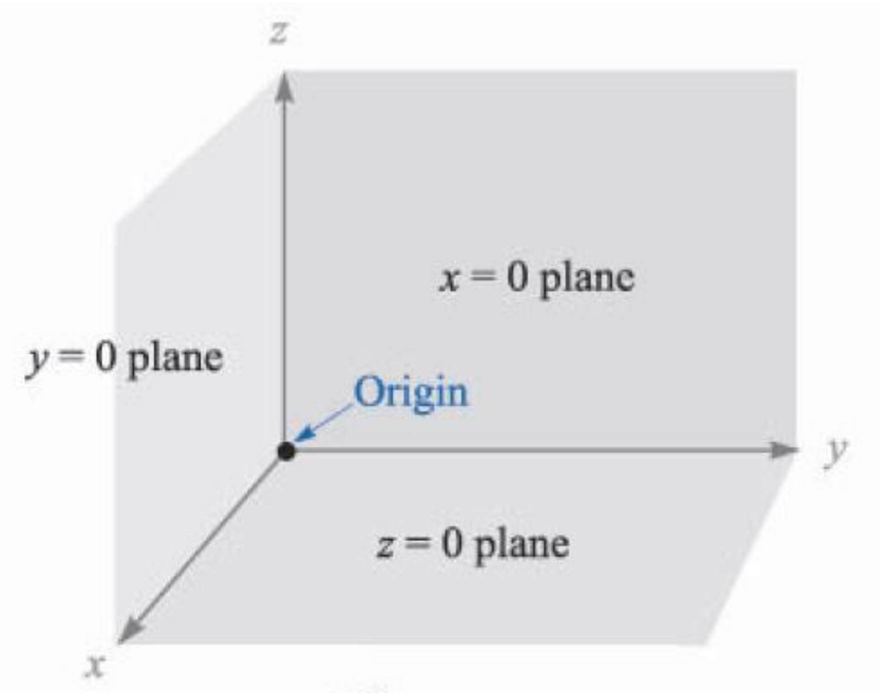
$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$$

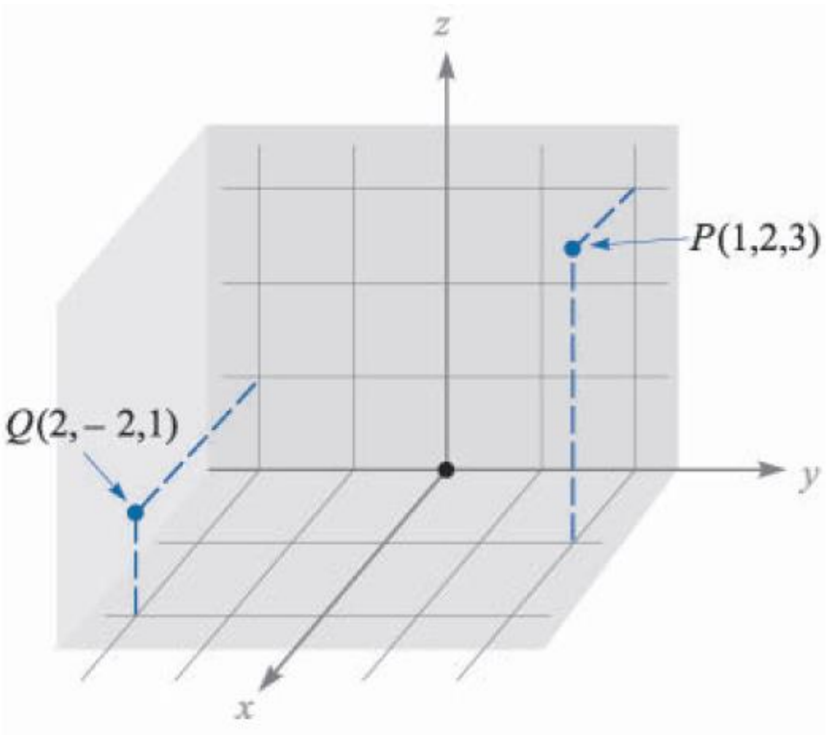
$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

$$\frac{\mathbf{A}}{n} = \frac{1}{n}\mathbf{A}$$

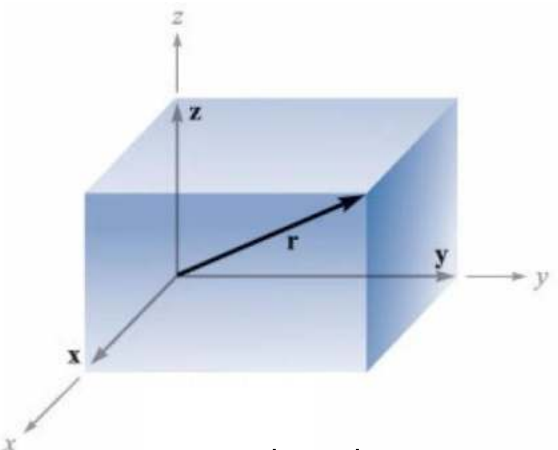
$$\mathbf{A} - \mathbf{B} = 0 \rightarrow \mathbf{A} = \mathbf{B}$$

Rectangular Coordinate System





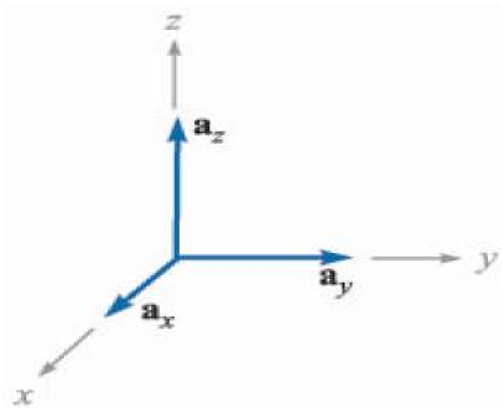
Vector Components and Unit Vectors

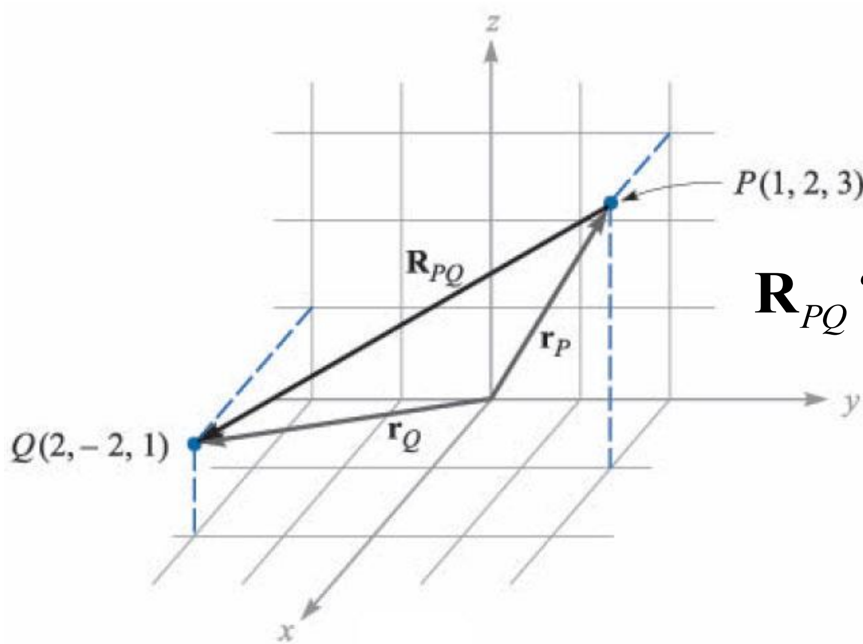


$$\mathbf{r} = \mathbf{x} + \mathbf{y} + \mathbf{z}$$

$$\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$$

$\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z$: unit vectors





$\mathbf{R}_{PQ}$?

$$\mathbf{R}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P$$

$$= (2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z) - (1\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)$$

$$= \mathbf{a}_x - 4\mathbf{a}_y - 2\mathbf{a}_z$$

Vector Components and Unit Vectors

- For any vector **B**, $\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$

$$|\mathbf{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2} = B$$

Magnitude of B

$$\mathbf{a}_B = \frac{\mathbf{B}}{\sqrt{B_x^2 + B_y^2 + B_z^2}} = \frac{\mathbf{B}}{|\mathbf{B}|}$$

Unit vector in the direction of B

- **Example**

Given points *M*(–1,2,1) and *N*(3,–3,0), find the vector $\mathbf{R}_{MN}$ and the unit vector $\mathbf{a}_{MN}$

$$\mathbf{R}_{MN} = (3\mathbf{a}_x - 3\mathbf{a}_y + 0\mathbf{a}_z) - (-1\mathbf{a}_x + 2\mathbf{a}_y + 1\mathbf{a}_z) = \underline{\underline{4\mathbf{a}_x - 5\mathbf{a}_y - \mathbf{a}_z}}$$

$$\mathbf{a}_{MN} = \frac{\mathbf{R}_{MN}}{|\mathbf{R}_{MN}|} = \frac{4\mathbf{a}_x - 5\mathbf{a}_y - 1\mathbf{a}_z}{\sqrt{4^2 + (-5)^2 + (-1)^2}} = \underline{\underline{0.617\mathbf{a}_x - 0.772\mathbf{a}_y - 0.154\mathbf{a}_z}}$$

The Dot Product

- Given two vectors **A** and **B**, the ***dot product***, or ***scalar product***, is defined as the product of the magnitude of **A**, the magnitude of **B**, and the cosine of the smaller angle between them:

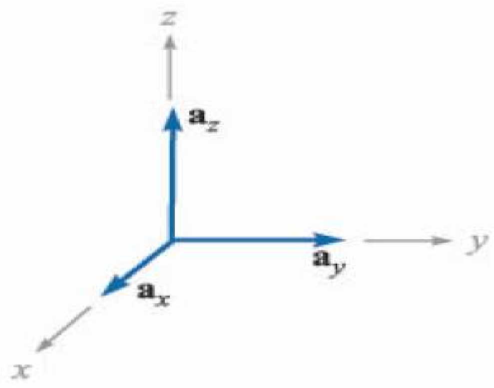
$$\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta_{AB}$$

- The dot product is a scalar, and it obeys the commutative law:

$$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$$

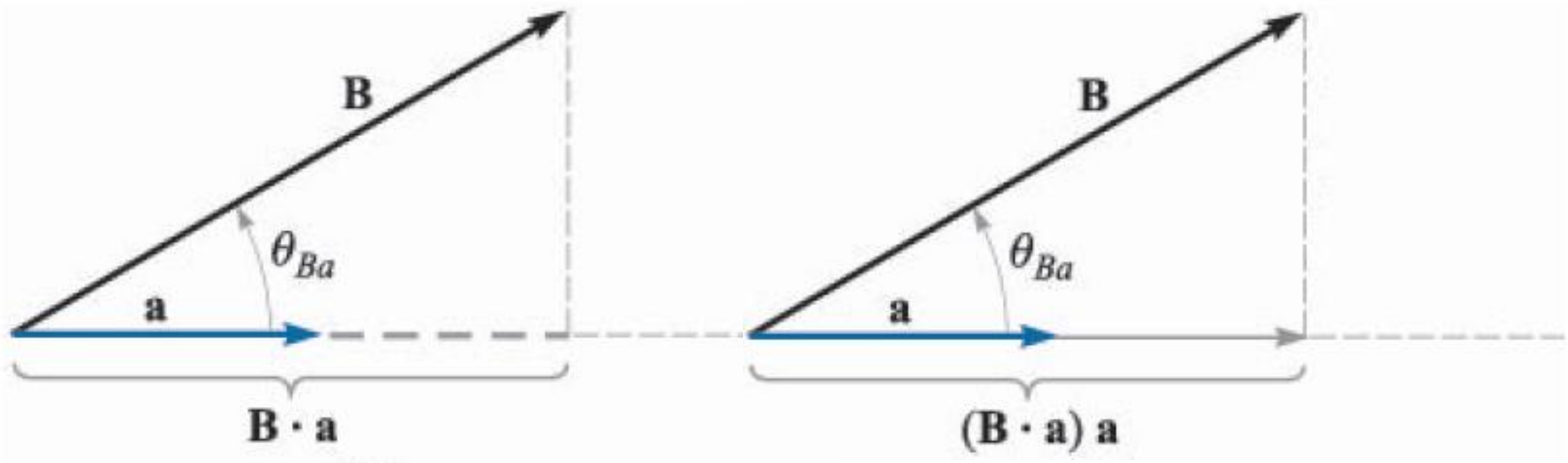
- For any vector $\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$ and $\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$$



The Dot Product

- One of the most important applications of the dot product is that of finding the component of a vector in a given direction.



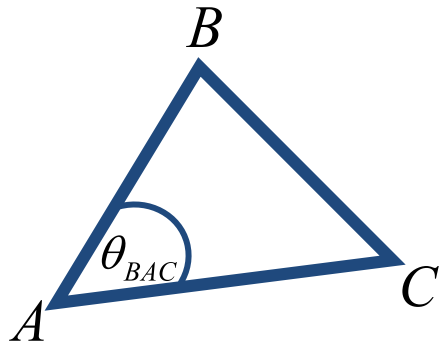
- The scalar component of **B** in the direction of the unit vector **a** is **B**·**a**
- The vector component of **B** in the direction of the unit vector **a** is (**B**·**a**)**a**

$$\mathbf{B}\cdot\mathbf{a} = |\mathbf{B}||\mathbf{a}|\cos\theta_{Ba} = |\mathbf{B}|\cos\theta_{Ba}$$

The Dot Product

- <u>Example</u>

The three vertices of a triangle are located at A(6,–1,2), B(–2,3,–4), and C(–3,1,5). Find: (a) $\mathbf{R}_{AB}$; (b) $\mathbf{R}_{AC}$; (c) the angle θ_{BAC} at vertex A; (d) the vector projection of $\mathbf{R}_{AB}$ on $\mathbf{R}_{AC}$.



$$\mathbf{R}_{AB} = (-2\mathbf{a}_x + 3\mathbf{a}_y - 4\mathbf{a}_z) - (6\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z) = \underline{\underline{-8\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z}}$$

$$\mathbf{R}_{AC} = (-3\mathbf{a}_x + 1\mathbf{a}_y + 5\mathbf{a}_z) - (6\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z) = \underline{\underline{-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z}}$$

$$\mathbf{R}_{AB} \cdot \mathbf{R}_{AC} = |\mathbf{R}_{AB}||\mathbf{R}_{AC}|\cos\theta_{BAC}$$

$$\Rightarrow \cos\theta_{BAC} = \frac{\mathbf{R}_{AB} \cdot \mathbf{R}_{AC}}{|\mathbf{R}_{AB}||\mathbf{R}_{AC}|} = \frac{(-8\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z) \cdot (-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)}{\left|\sqrt{(-8)^2 + (4)^2 + (-6)^2}\right|\left|\sqrt{(-9)^2 + (2)^2 + (3)^2}\right|} = \frac{62}{\left|\sqrt{116}\right|\left|\sqrt{94}\right|} = 0.594$$

$$\Rightarrow \theta_{BAC} = \cos^{-1}(0.594) = \underline{\underline{53.56^\circ}}$$

The Dot Product

- Example

The three vertices of a triangle are located at $A(6,-1,2)$, $B(-2,3,-4)$, and $C(-3,1,5)$. Find: (*a*) $\mathbf{R}_{AB}$; (*b*) $\mathbf{R}_{AC}$; (*c*) the angle θ_{BAC} at vertex A; (*d*) the vector projection of $\mathbf{R}_{AB}$ on $\mathbf{R}_{AC}$.

$$\mathbf{R}_{AB} \text{ on } \mathbf{R}_{AC} = \left(\mathbf{R}_{AB} \cdot \mathbf{a}_{AC}\right)\mathbf{a}_{AC}$$

$$= \left((-8\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z) \frac{(-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)}{\left|\sqrt{(-9)^2 + (2)^2 + (3)^2}\right|} \right) \frac{(-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)}{\left|\sqrt{(-9)^2 + (2)^2 + (3)^2}\right|}$$

$$= \frac{62}{\sqrt{94}} \frac{(-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)}{\sqrt{94}}$$

$$= \underline{\underline{-5.963\mathbf{a}_x + 1.319\mathbf{a}_y + 1.979\mathbf{a}_z}}$$

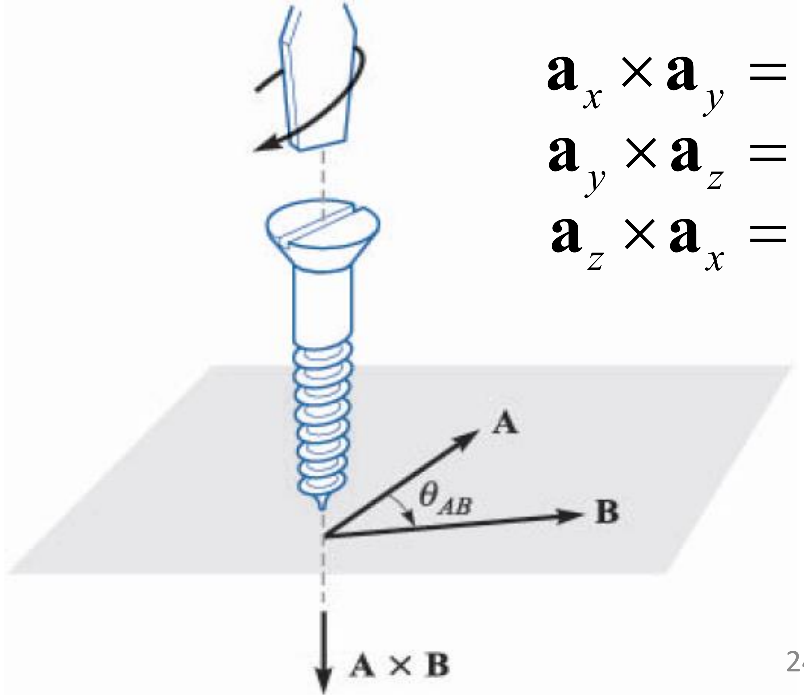
The Cross Product

- Given two vectors **A** and **B**, the magnitude of the ***cross product***, or ***vector product***, written as **A**×**B**, is defined as the product of the magnitude of **A**, the magnitude of **B**, and the sine of the smaller angle between them.
- The direction of **A**×**B** is perpendicular to the plane containing **A** and **B** and is in the direction of advance of a right-handed screw as **A** is turned into **B**.

$$\mathbf{A}\times\mathbf{B}=\mathbf{a}_N\left|\mathbf{A}\right|\left|\mathbf{B}\right|\sin\theta_{AB}$$

- The cross product is a vector, and it is not commutative:

$$(\mathbf{B}\times\mathbf{A})=-(\mathbf{A}\times\mathbf{B})$$



$$\mathbf{a}_x\times\mathbf{a}_y=\mathbf{a}_z$$
$$\mathbf{a}_y\times\mathbf{a}_z=\mathbf{a}_x$$
$$\mathbf{a}_z\times\mathbf{a}_x=\mathbf{a}_y$$

The Cross Product

- <u>Example</u>

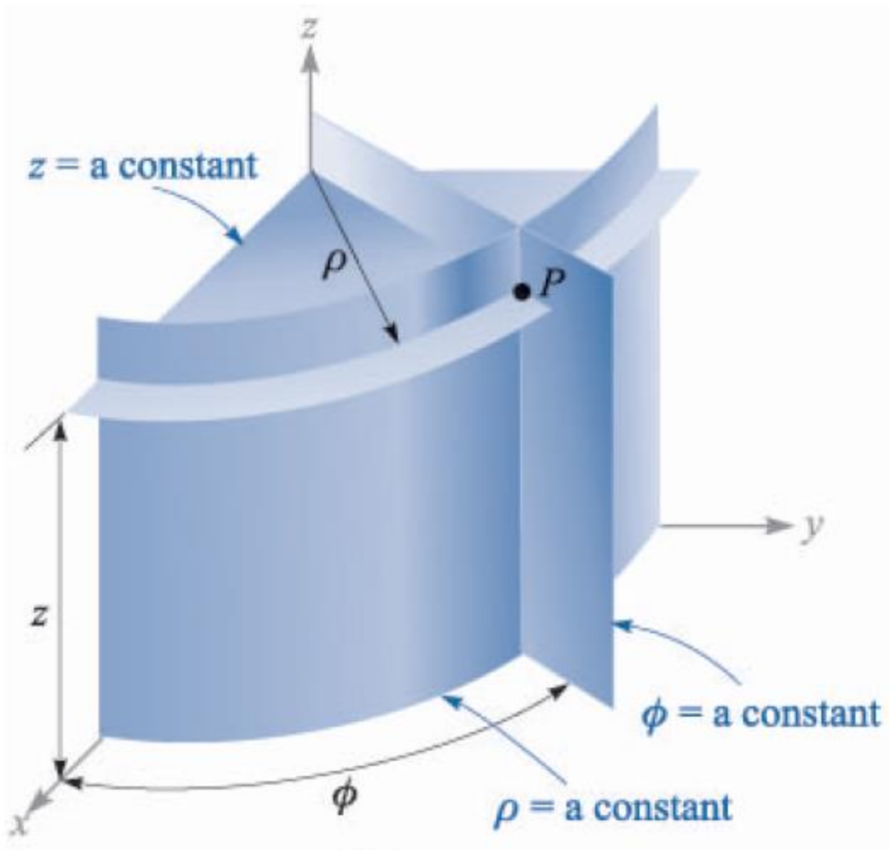
Given **A** = 2$\mathbf{a}_x$–**3**a_y+$\mathbf{a}_z$ and **B** = –4$\mathbf{a}_x$–2$\mathbf{a}_y$+5$\mathbf{a}_z$, find **A**×**B**.

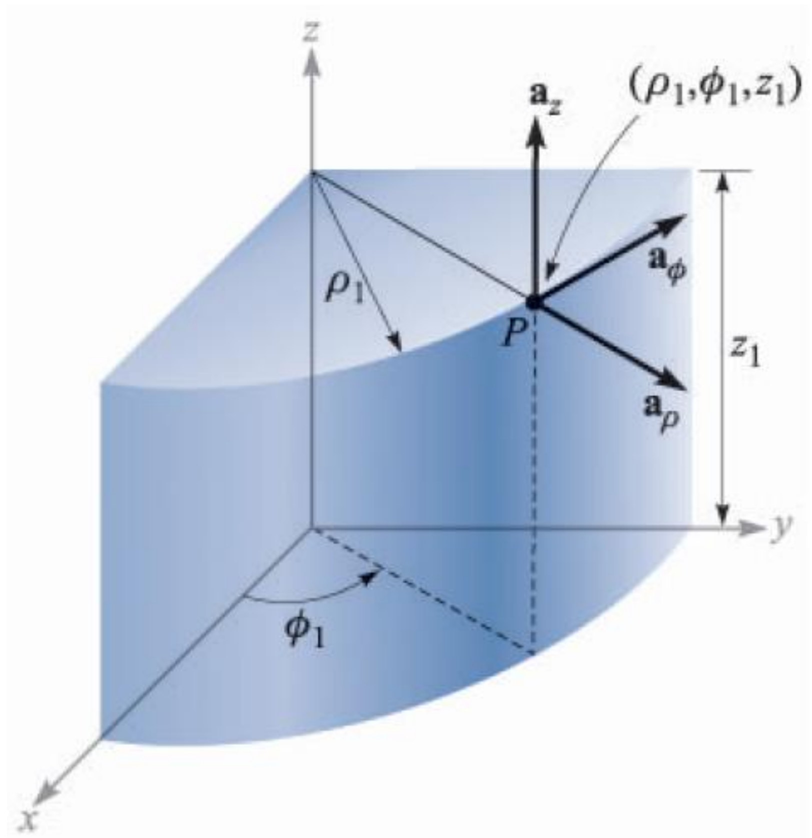
$$\mathbf{A}\times\mathbf{B} = (A_yB_z - A_zB_y)\mathbf{a}_x + (A_zB_x - A_xB_z)\mathbf{a}_y + (A_xB_y - A_yB_x)\mathbf{a}_z$$

$$= \big((-3)(5) - (1)(-2)\big)\mathbf{a}_x + \big((1)(-4) - (2)(5)\big)\mathbf{a}_y + \big((2)(-2) - (-3)(-4)\big)\mathbf{a}_z$$

$$= \underline{\underline{-13\mathbf{a}_x - 14\mathbf{a}_y - 16\mathbf{a}_z}}$$

The Cylindrical Coordinate System

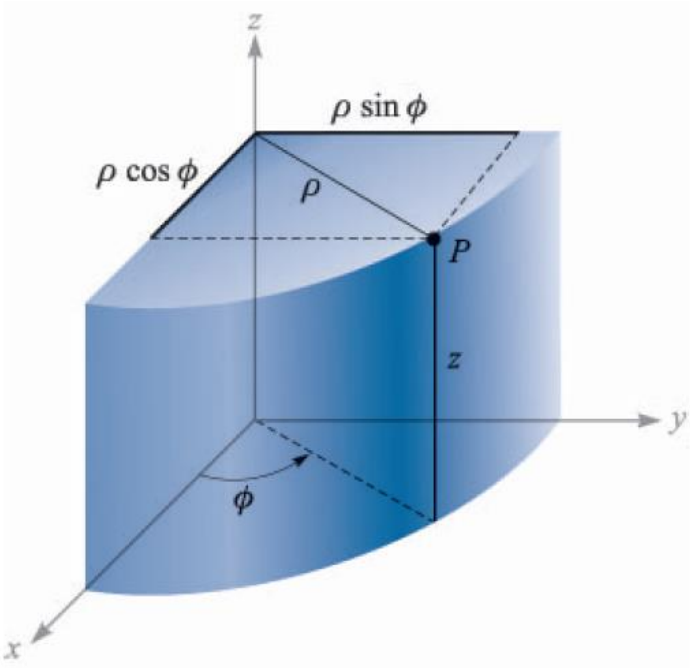




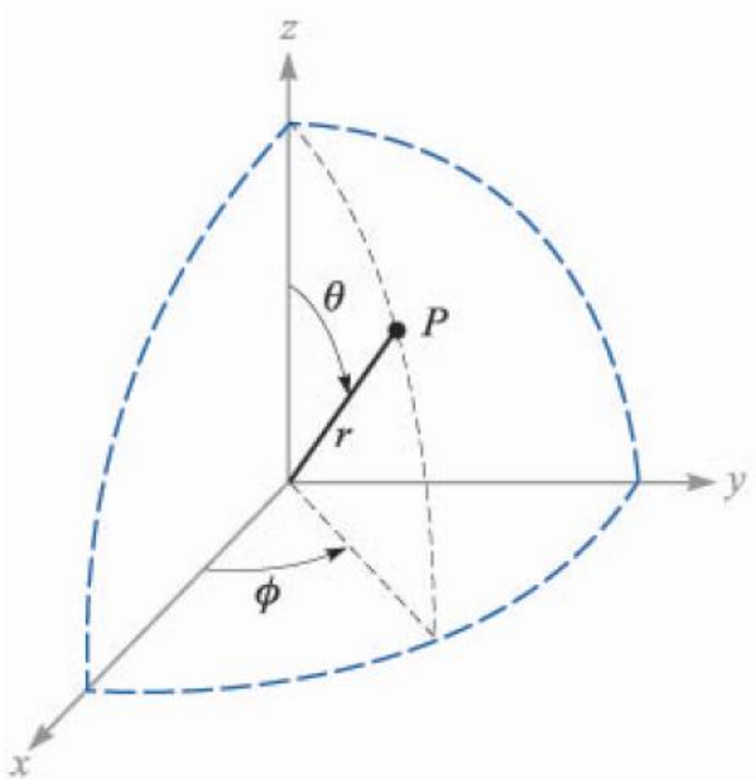
The Cylindrical Coordinate System

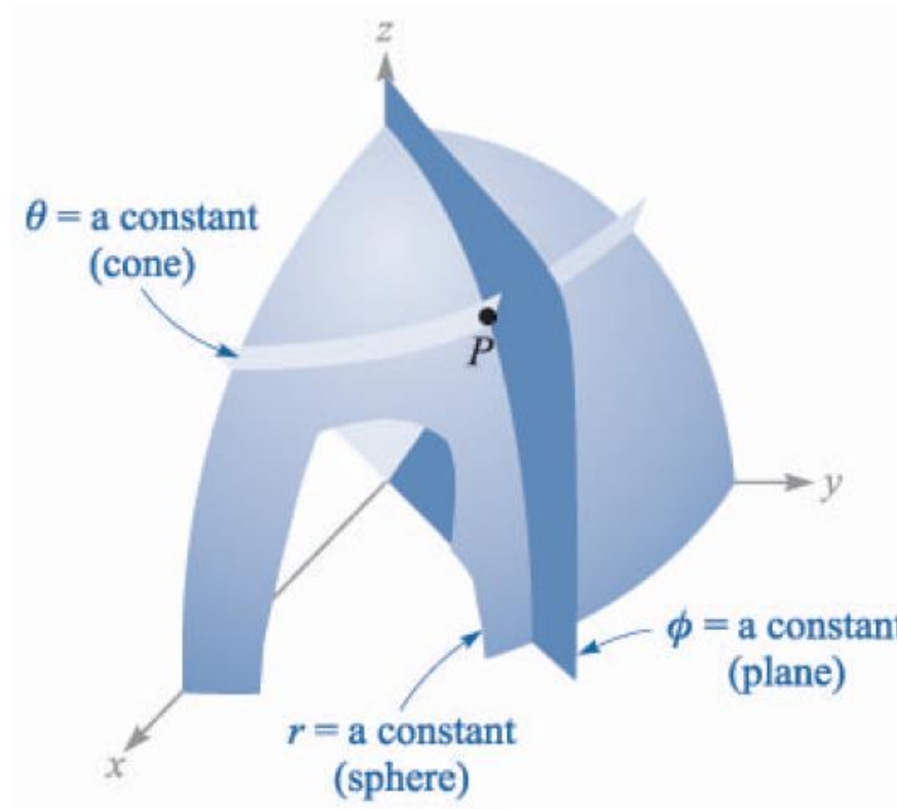
- Relation between the rectangular and the cylindrical coordinate systems

$x = \rho \cdot \cos\phi$	$\rho = \sqrt{x^2 + y^2}$
$y = \rho \cdot \sin\phi$	$\phi = \tan^{-1} \dfrac{y}{x}$
$z = z$	$z = z$



The Spherical Coordinate System

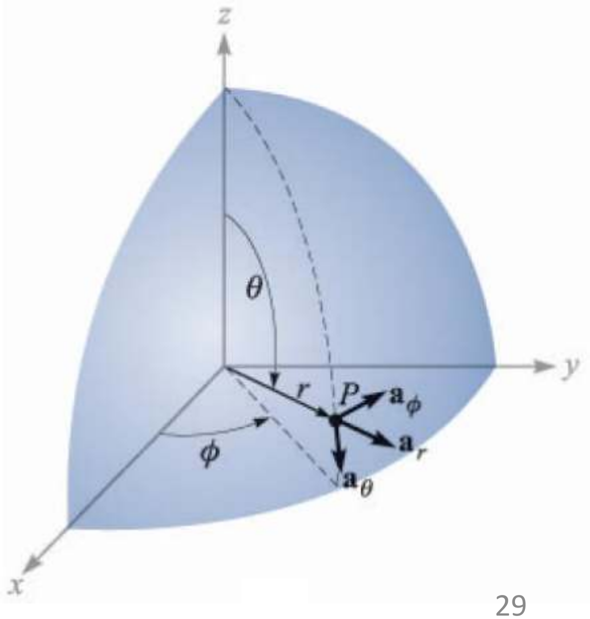




The Spherical Coordinate System

- Relation between the rectangular and the spherical coordinate systems

$x = r\sin\theta\cos\phi$	$r = \sqrt{x^2 + y^2 + z^2}, \quad r \geq 0$
$y = r\sin\theta\sin\phi$	$\theta = \cos^{-1}\dfrac{z}{\sqrt{x^2 + y^2 + z^2}}, \quad 0^\circ \leq \theta \leq 180^\circ$
$z = r\cos\theta$	$\phi = \tan^{-1}\dfrac{y}{x}$



The Spherical Coordinate System

- Example

Given the two points, *C*(–3,2,1) and *D*(*r* = 5, θ = 20°, Φ = –70°), find: (*a*) the spherical coordinates of *C*; (*b*) the rectangular coordinates of *D*.

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{(-3)^2 + (2)^2 + (1)^2} = 3.742$$

$$\theta = \cos^{-1}\frac{z}{\sqrt{x^2 + y^2 + z^2}} = \cos^{-1}\frac{1}{3.742} = 74.50°$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{2}{-3} = -33.69° + 180° = 146.31°$$

$$\therefore C(r = 3.742, \theta = 74.50°, \phi = 146.31°)$$

$$\therefore D(x = 0.585, y = -1.607, z = 4.698)$$

ACK
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Australian College of Kuwait

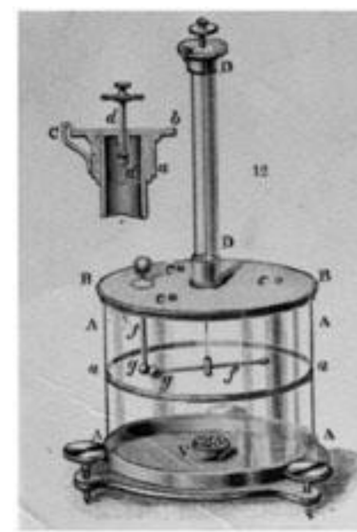
15FELE121
Electromagnetism Fundamentals

Chapter 2 – Coulomb's Law and Electric Field Intensity

Dr. Michel Nahas

The Experimental Law of Coulomb

- In 1600, Dr. Gilbert, a physician from England, published the first major classification of electric and non-electric materials.
- He stated that glass, sulfur, amber, and some other materials “not only draw to themselves straw, and chaff, but all metals, wood, leaves, stone, earths, even water and oil.”



- In the following century, a French Army Engineer, Colonel Charles Coulomb, performed an elaborate series of experiments using devices invented by himself.
- Coulomb could determine quantitatively the force exerted between two objects, each having a static charge of electricity.
- He wrote seven important treatises on electric and magnetism, developed a theory of attraction and repulsion between bodies of the opposite and the same electrical charge.

The Experimental Law of Coulomb

- Coulomb stated that the force between two very small objects separated in vacuum or free space by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the distance between them.

$$F = k\frac{Q_1 Q_2}{R^2}$$

- In SI Units, the quantity of charge *Q* is measured in coulombs (C), the separation *R* in meters (m) and the force *F* should be newtons (N)
- This will be achieved if the constant of proportionality *k* is written as:

$$k = \frac{1}{4\pi\varepsilon_0}$$

The Experimental Law of Coulomb

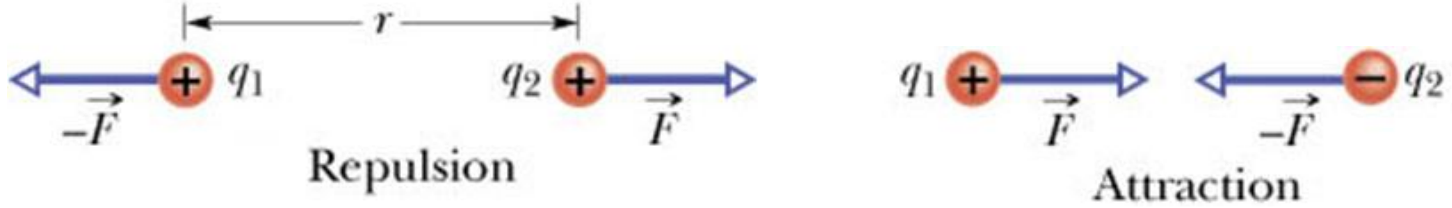
- The *permittivity of free space ε* is measured in farads per meter (F/m), and has the magnitude of:

$$\varepsilon_0 = 8.854 \times 10^{-12} = \frac{1}{36\pi} 10^{-9} \text{ F/m}$$

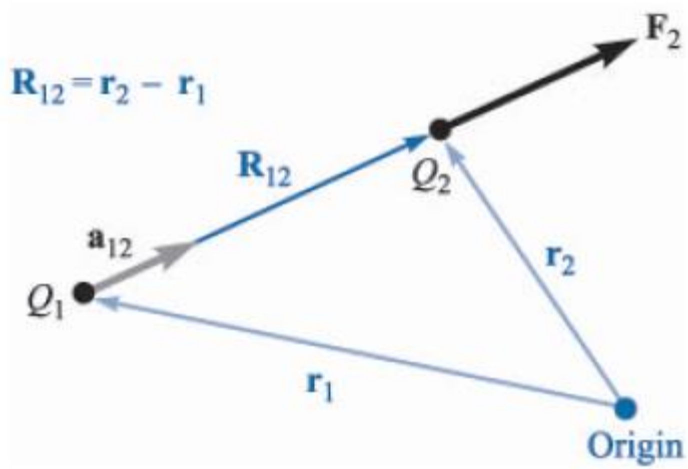
- The Coulomb's law is now:

$$F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R^2}$$

- The force *F* acts along the line joining the two charges. It is repulsive if the charges are alike in sign and attractive if they are of opposite sign.



The Experimental Law of Coulomb



$$\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

- In vector form, Coulomb's law is written as:

$$\mathbf{F}_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R_{12}^2} \mathbf{a}_{12}$$

- F_2 is the force on Q_2, for the case where Q_1 and Q_2 have the same sign, while a_{12} is the unit vector in the direction of R_{12}, the line segment from Q_1 to Q_2.

The Experimental Law of Coulomb

- Example

A charge $Q_1 = 3\times10^{-4}$ C at $M(1,2,3)$ and a charge of $Q_2 = -10^{-4}$ C at $N(2,0,5)$ are located in a vacuum. Determine the force exerted on Q_2 by Q_1.

$$\mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1$$

$$= (2\mathbf{a}_x + 0\mathbf{a}_y + 5\mathbf{a}_z) - (1\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)$$

$$= 1\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z$$

$$\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|}$$

$$= \frac{1}{3}(1\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z)$$

$$\mathbf{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R_{12}^2}\mathbf{a}_{12}$$

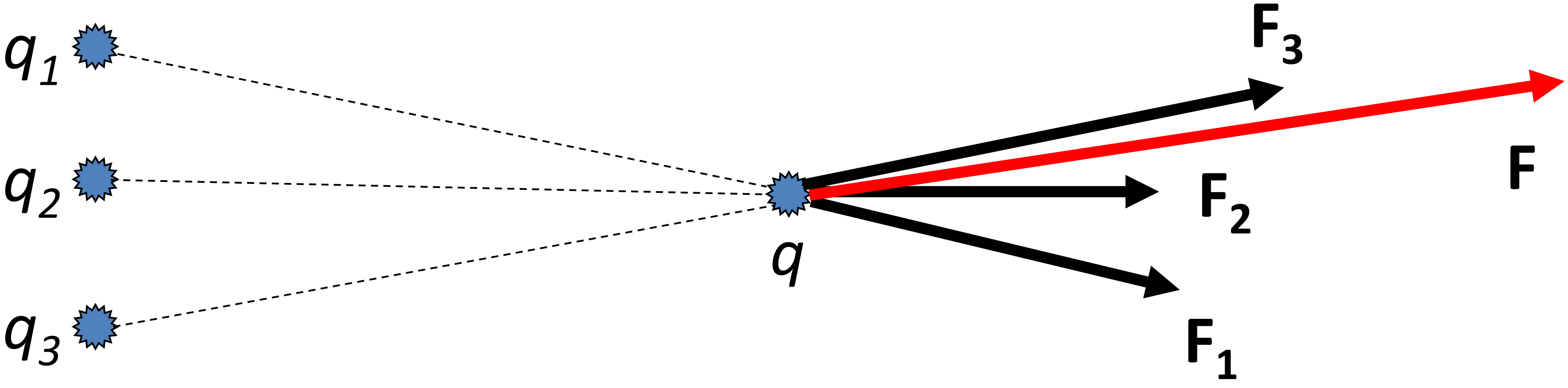
$$= \frac{1}{4\pi(8.854\times10^{-12})}\frac{(3\times10^{-4})(-10^{-4})}{3^2}\frac{1}{3}(1\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z)$$

$$= \underline{\underline{-10\mathbf{a}_x + 20\mathbf{a}_y - 20\mathbf{a}_z \ \text{N}}}$$

$$\boxed{\mathbf{F}_1 = -\mathbf{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R_{12}^2}\mathbf{a}_{21}}$$

Superposition of electric forces

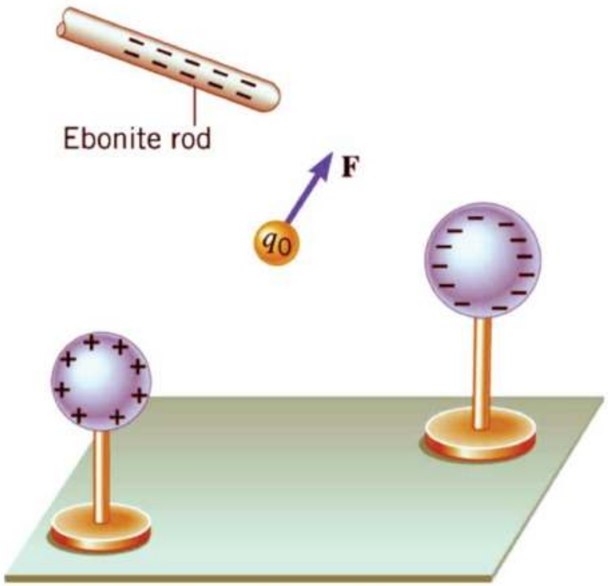
- Net force is the *vector sum* of forces from each charge



Net force on *q*: $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

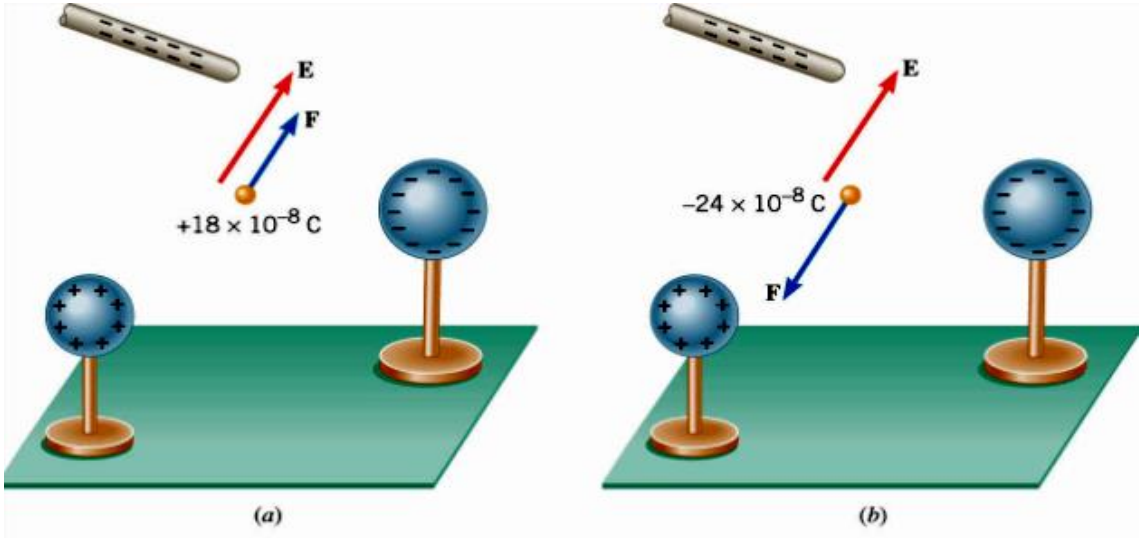
Electric Field

- Abstraction
- Separates cause and effect in Coulomb's law



$$\vec{E} = \frac{\vec{F}}{q_0}$$

Units: N/C



Electric Field Intensity

- Let us consider one charge, say Q_1, fixed in position in space.
- Now, imagine that we can introduce a second charge, Q_t, as a “unit test charge”, that we can move around.
- We know that there exists everywhere a force on this second charge →

  This second charge is displaying the existence of a force field.

- The force on it is given by Coulomb’s law as:

$$\mathbf{F}_t = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_t}{R_{1t}^2}\mathbf{a}_{1t}$$

- Writing this force as a “force per unit charge” gives:

$$\frac{\mathbf{F}_t}{Q_t} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_{1t}^2}\mathbf{a}_{1t}$$

Vector Field,
Electric Field Intensity

Electric Field Intensity

- We define the electric field intensity as the vector of force on a unit positive test charge.
- Electric field intensity, E, is measured by the unit newtons per coulomb (N/C) or volts per meter (V/m).

$$\mathbf{E} = \frac{\mathbf{F}_t}{Q_t} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_{1t}^2}\mathbf{a}_{1t}$$

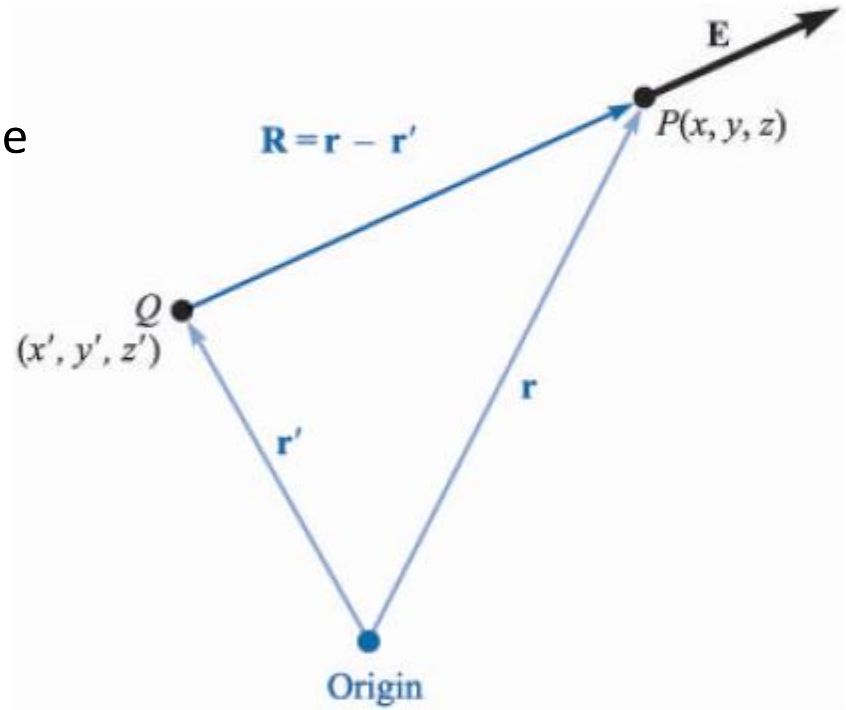
- The field of a single point charge can be written as:

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\mathbf{a}_R$$

- a_R is a unit vector in the direction from the point at which the point charge *Q* is located, to the point at which E is desired/measured.

Electric Field Intensity

- For a charge which is not at the origin of the coordinate, the electric field intensity is:



$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{|\mathbf{r}-\mathbf{r}'|^2} \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

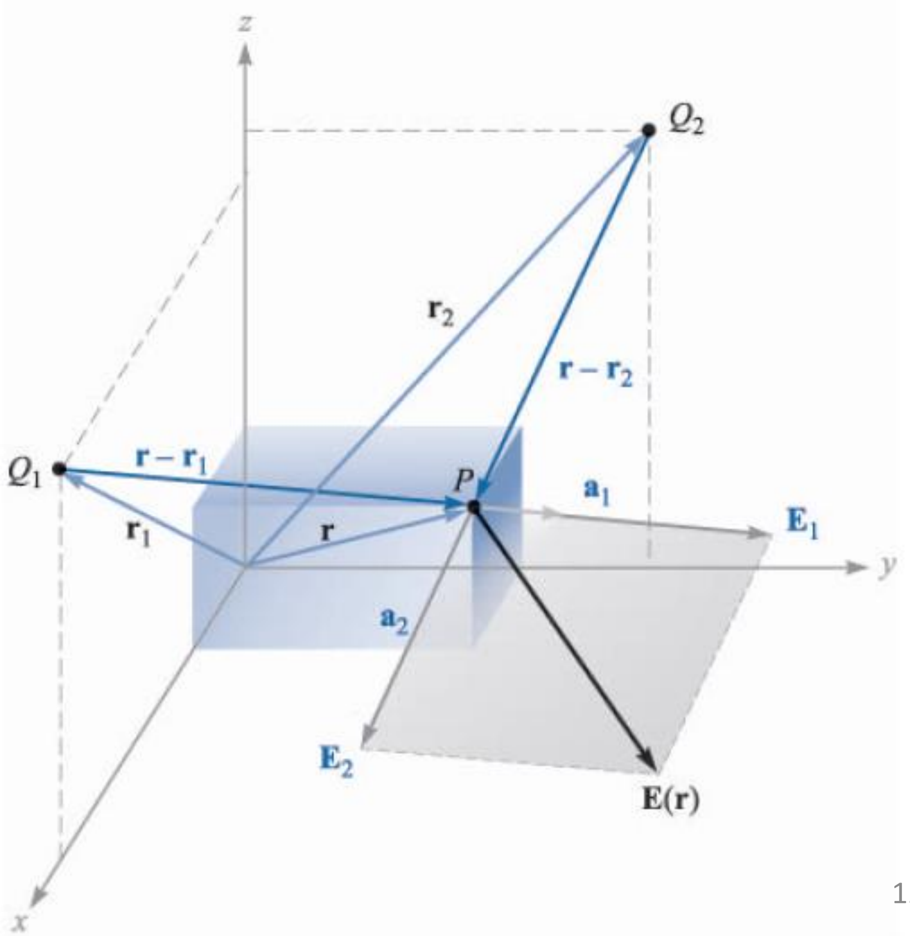
$$= \frac{1}{4\pi\varepsilon_0} \frac{Q(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q\left[(x-x')\mathbf{a}_x + (y-y')\mathbf{a}_y + (z-z')\mathbf{a}_z\right]}{\left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{3/2}}$$

Electric Field Intensity

- The electric field intensity due to two point charges, say Q_1 at r_1 and Q_2 at r_2, is the sum of the electric field intensity on Q_t caused by Q_1 and Q_2 acting alone (Superposition Principle).

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}-\mathbf{r}_1|^2}\mathbf{a}_1 + \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}-\mathbf{r}_2|^2}\mathbf{a}_2$$



Electric Field Intensity

- Example

A charge Q_1 of 2 μC is located at $P_1(0,0,0)$ and a second charge of 3 μC is at $P_2(-1,2,3)$. Find E at $M(3,-4,2)$.

$$\mathbf{r}-\mathbf{r}_1 = 3\mathbf{a}_x - 4\mathbf{a}_y + 2\mathbf{a}_z, \quad |\mathbf{r}-\mathbf{r}_1| = \sqrt{29}$$

$$\mathbf{r}-\mathbf{r}_2 = 4\mathbf{a}_x - 6\mathbf{a}_y - \mathbf{a}_z, \quad |\mathbf{r}-\mathbf{r}_2| = \sqrt{53}$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{|\mathbf{r}-\mathbf{r}_1|^2}\mathbf{a}_1 + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{|\mathbf{r}-\mathbf{r}_2|^2}\mathbf{a}_2$$

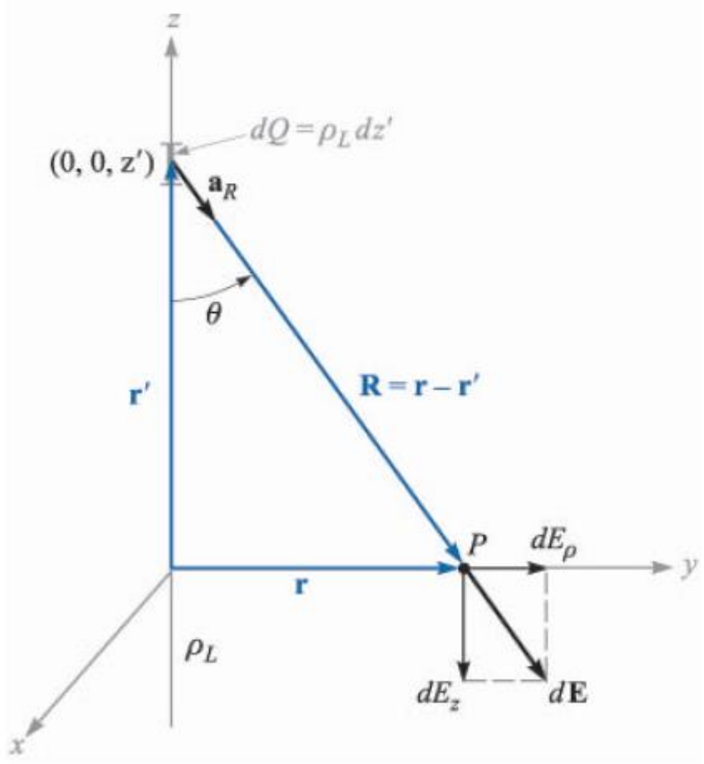
$$= \frac{1}{4\pi\varepsilon_0}\left\{\frac{Q_1(\mathbf{r}-\mathbf{r}_1)}{|\mathbf{r}-\mathbf{r}_1|^3} + \frac{Q_2(\mathbf{r}-\mathbf{r}_2)}{|\mathbf{r}-\mathbf{r}_2|^3}\right\}$$

$$= \frac{1}{4\pi\varepsilon_0}\left\{\frac{2\times10^{-6}(3\mathbf{a}_x - 4\mathbf{a}_y + 2\mathbf{a}_z)}{|\sqrt{29}|^3} + \frac{3\times10^{-6}(4\mathbf{a}_x - 6\mathbf{a}_y - \mathbf{a}_z)}{|\sqrt{53}|^3}\right\}$$

$$= \underline{\underline{623.7\mathbf{a}_x - 879.92\mathbf{a}_y + 160.17\mathbf{a}_z \ \text{V/m}}}$$

Field of a Line Charge

$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho} \mathbf{a}_\rho$$



- The field falls off <u>inversely</u> with the distance to the charged line, as compared with the point charge, where the field decreased <u>with the square</u> of the distance.

$$E_\rho = \frac{\rho_L}{2\pi\varepsilon_0 \rho}$$

Field of a Line Charge

- <u>Example:</u>
  Infinite uniform line charges of 5 nC/m lie along the (positive and negative) *x* and *y* axes in free space. Find E at: (*a*) $P_A(0,0,4)$; (*b*) $P_B(0,3,4)$.

$$\mathbf{E}(P_A) = \frac{\rho_{L_x}}{2\pi\varepsilon_0\rho_x}\mathbf{a}_{\rho_x} + \frac{\rho_{Ly}}{2\pi\varepsilon_0\rho_y}\mathbf{a}_{\rho_y}$$

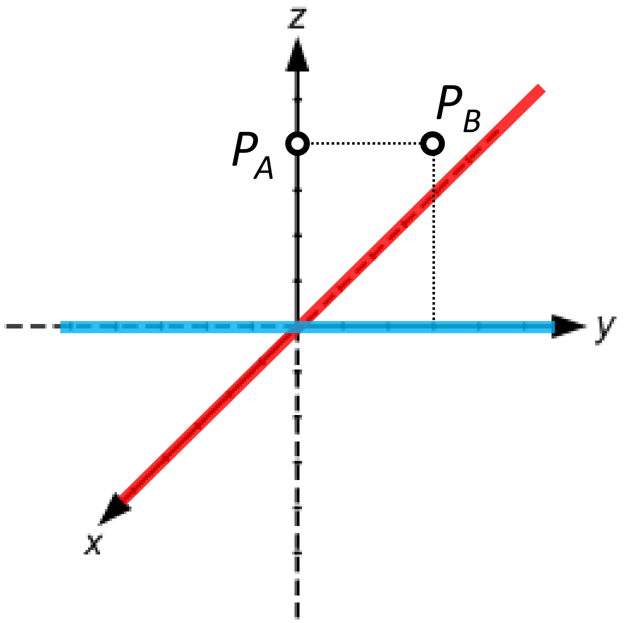
$$= \frac{5\times10^{-9}}{2\pi\varepsilon_0(4)}\mathbf{a}_z + \frac{5\times10^{-9}}{2\pi\varepsilon_0(4)}\mathbf{a}_z$$

$$= \underline{\underline{44.939\mathbf{a}_z \text{ V/m}}}$$

$$\mathbf{E}(P_B) = \frac{\rho_{L_x}}{2\pi\varepsilon_0\rho_x}\mathbf{a}_{\rho_x} + \frac{\rho_{Ly}}{2\pi\varepsilon_0\rho_y}\mathbf{a}_{\rho_y}$$

$$= \frac{5\times10^{-9}}{2\pi\varepsilon_0(5)}(0.6\mathbf{a}_y + 0.8\mathbf{a}_z) + \frac{5\times10^{-9}}{2\pi\varepsilon_0(4)}\mathbf{a}_z$$

$$= \underline{\underline{10.785\mathbf{a}_y + 36.850\mathbf{a}_z \text{ V/m}}}$$

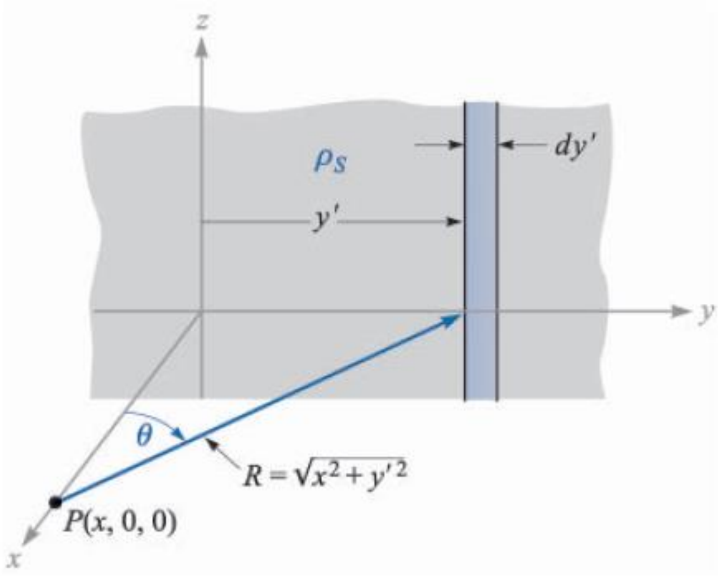


- ρ is the shortest distance between an observation point and the line charge

Field of a Sheet of Charge

- Fact: The electric field is always directed away from the positive charge, into the negative charge.
- We now introduce a unit vector a_N, which is normal to the sheet and directed away from it.

$$\mathbf{E} = \frac{\rho_s}{2\varepsilon_0}\mathbf{a}_N$$



- The field of a sheet of charge is constant in magnitude and direction. It is not a function of distance.

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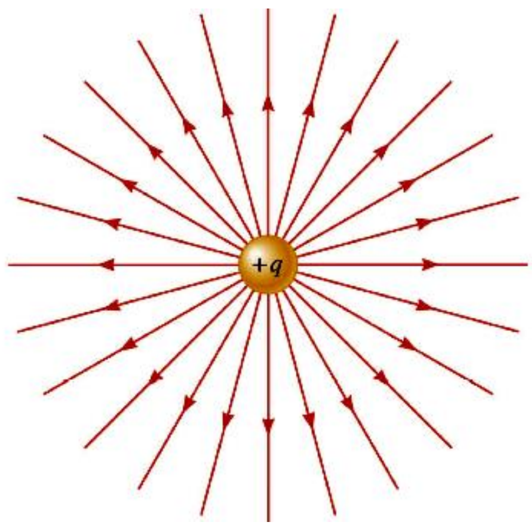
Electromagnetism Fundamentals

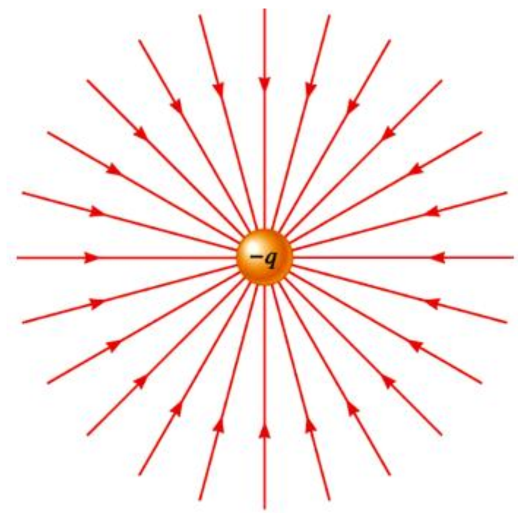
Chapter 3 – Electric Field Lines, Electric Flux Density and Electric Potential

Dr. Michel Nahas

Electric Field Lines

- Electric field lines are also known as lines of force

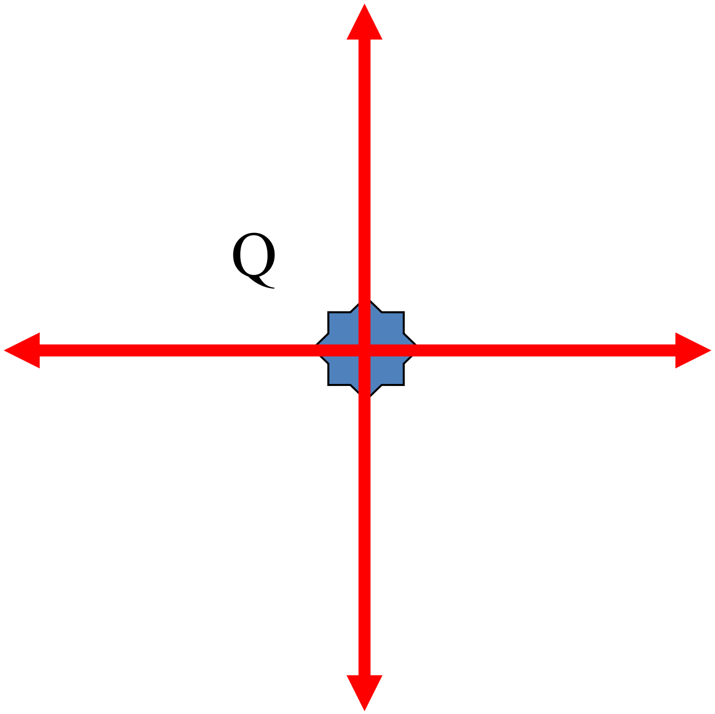


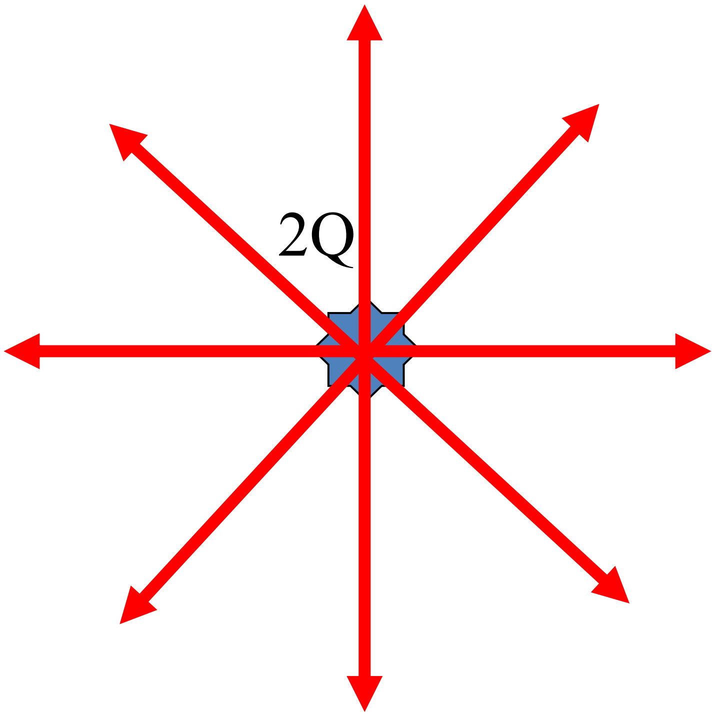


- Radial for point charges
  - Out for positive (begin)
  - In for negative (end)

Electric Field Lines

- The number of field lines is proportional to the charge
- They begin and end only on charges → they never cross



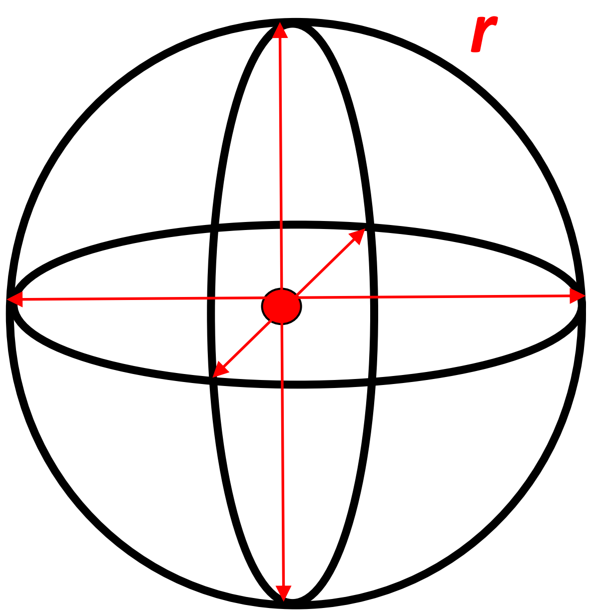


Line density

- Line density is proportional to field strength

- ***Line density at radius r:***

$$\frac{\text{Number of lines}}{\text{area of sphere}} = \frac{N}{4\pi r^2} \propto \frac{1}{r^2}$$

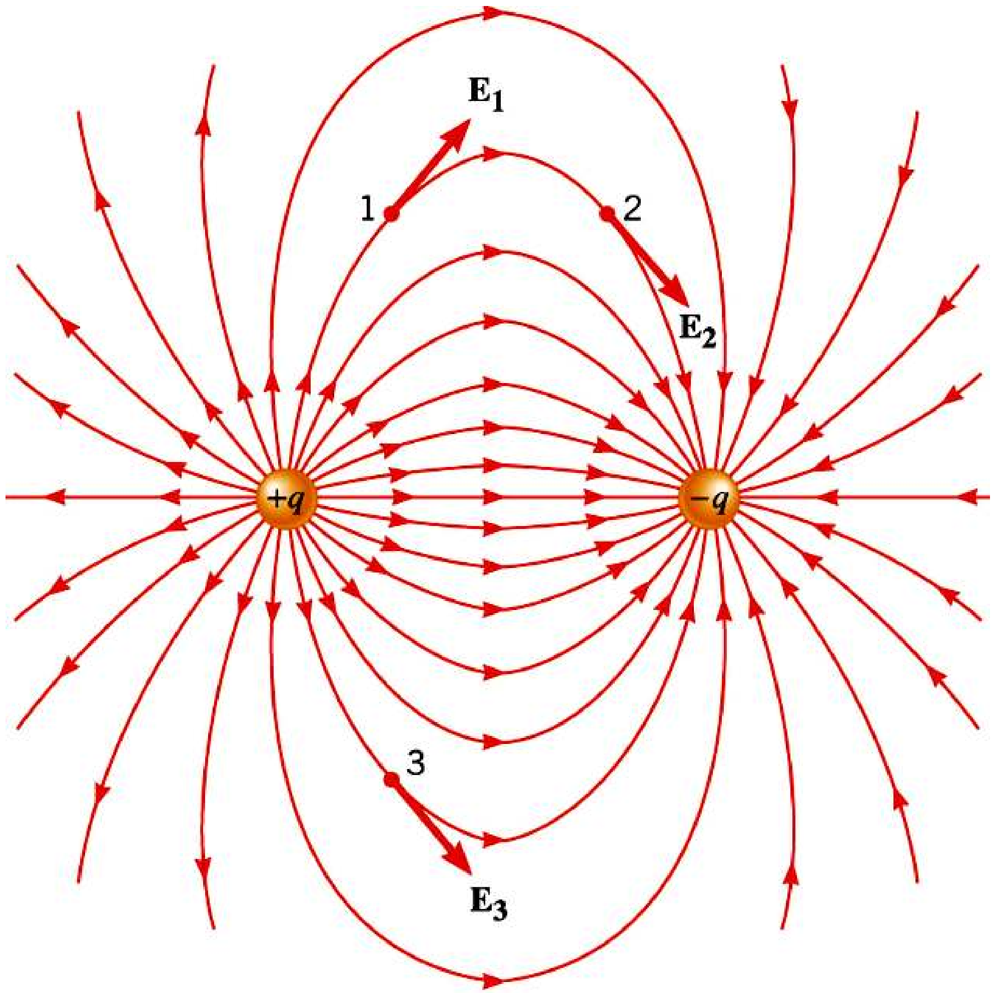


Lines of force model <==> inverse-square law

Lines-of-force models

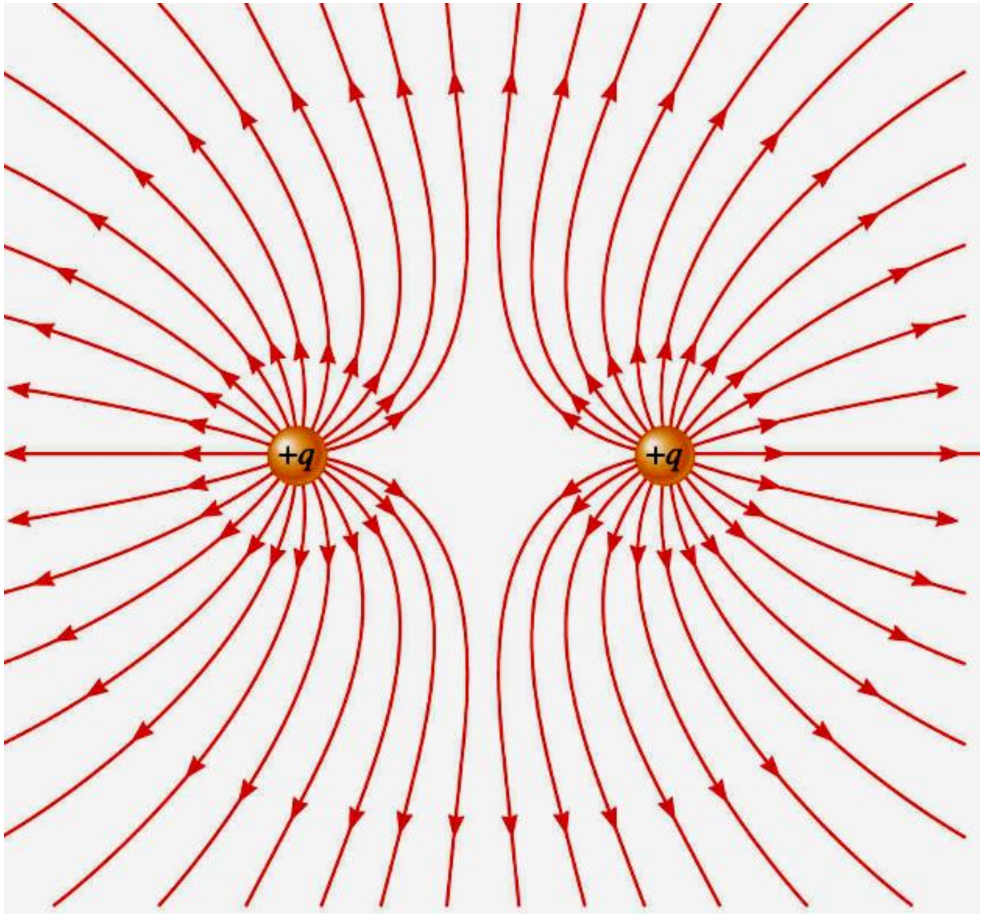
- ***Two opposite charges***

  (Dipole)



Lines-of-force models

- ***<u>Two positive charges</u>***



Work

- You do work when you push an object up a hill
- The longer the hill the more work you do: more distance
- The steeper the hill the more work you do: more force

$$W = F_{||} d$$

The **work** W done on an object by an agent exerting a constant force is the product of the component of the force in the direction of the displacement and the magnitude of the displacement

Energy

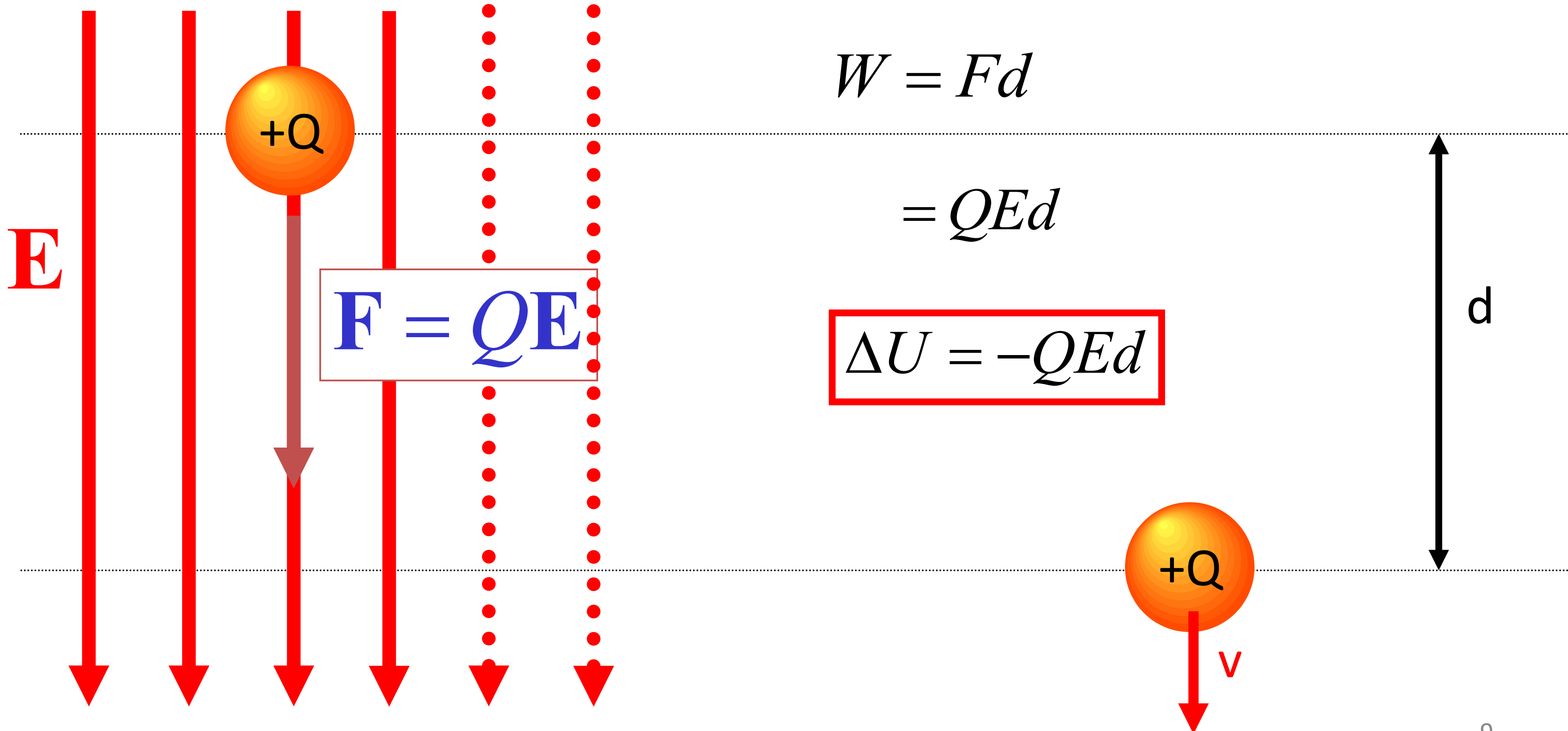
- Energy is the capacity to do work
- Energy cannot be created or destroyed
- Example: Kinetic Energy

$$K = \frac{1}{2}mv^2$$

  - m: mass
  - v: velocity
- Energy can be converted into other forms of energy
- When we do work on any object we transfer energy to it

Electric Potential Energy

- Charges have electrical potential energy: ΔU



$$W = Fd$$

$$= QEd$$

$$\Delta U = -QEd$$

Electric Potential Energy

- Work done by electric field on charged particle is:

  Q.E.d

- Particle has gained Kinetic Energy: **Q.E.d**

- Particle must therefore have lost Potential Energy

  $$\Delta U = -Q.E.d$$

Electric Potential

- The electric potential energy depends on the charge present
- Change in potential is change in potential energy for a test charge per unit charge
- We can define the electric potential V which does not depend on charge by using a “test” charge Q_0

$$\Delta V = \frac{\Delta U}{Q_0}$$

- For uniform field:

$$\Delta U = -Q_0 E d$$

$$\Delta V = \frac{\Delta U}{Q_0} = -Ed$$

Electric Potential

- Compare with the Electric Field and Coulomb Force

$$\Delta V = \frac{\Delta U}{Q_0}$$

$$\mathbf{E} = \frac{\mathbf{F}}{Q_0}$$

$$\Delta U = Q\Delta V$$

$$\mathbf{F} = Q\mathbf{E}$$

- If we know the potential field, this allows us to calculate changes in potential energy for any charge introduced

Electric Potential

Electric Potential is a scalar field

- Electric potential
  - It is defined everywhere
  - It doesn’t depend on a charge being there
  - But it does not have any direction

Electric Potential

- SI Units of Electric Potential

$$\Delta V = \frac{\Delta U}{Q_0}$$

Units are J/C

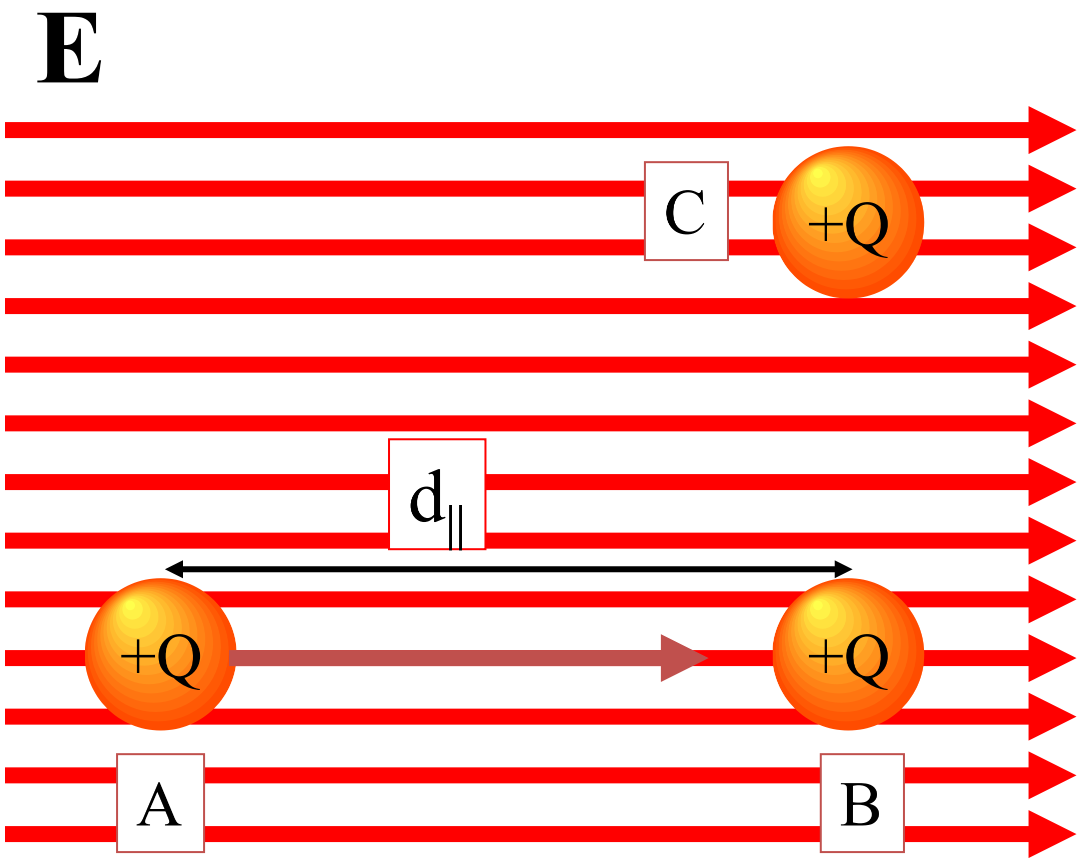
- Alternatively called Volts (V)

- We have seen $\Delta V = -Ed$

$$E = -\Delta V / d$$

- Thus E also has units of V/m

Potential in Uniform field



$$W_{BC} = F_{||}d = 0$$

$$W_{AB} = F_{||}d = QEd_{||}$$

$$W_{AC} = W_{AB} + W_{BC}$$
$$= QEd_{||}$$

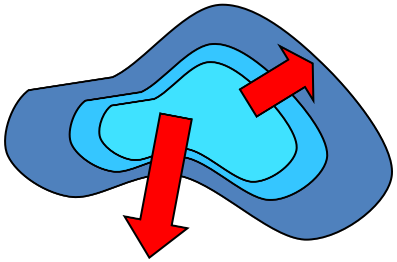
$$\Delta U_{AC} = -QEd_{||}$$

$$\Delta V_{AC} = -Ed_{||}$$

Equi-potential Lines

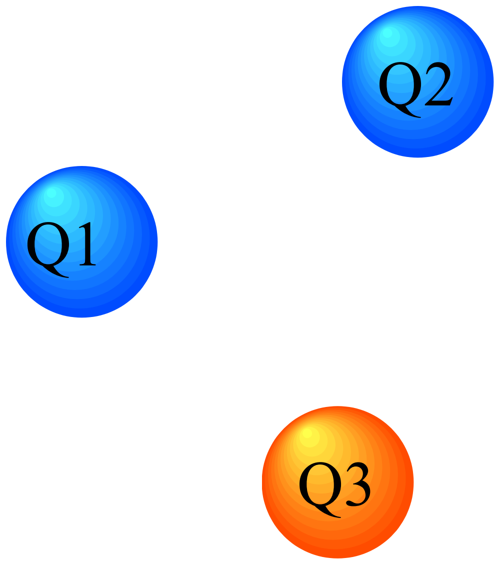
- Like elevation, potential can be displayed as contours
- Like elevation, potential requires a zero point, potential V=0 at r=∞
- Like slope & elevation, we can obtain the Electric Field from the potential field

$$E = \frac{\Delta V}{\Delta r}$$



A contour diagram

Potential Energy in 3 charges



$$U_{12} = Q_2 V = Q_2 \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_{12}}$$

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{Q}{r}$$

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r_{12}}$$

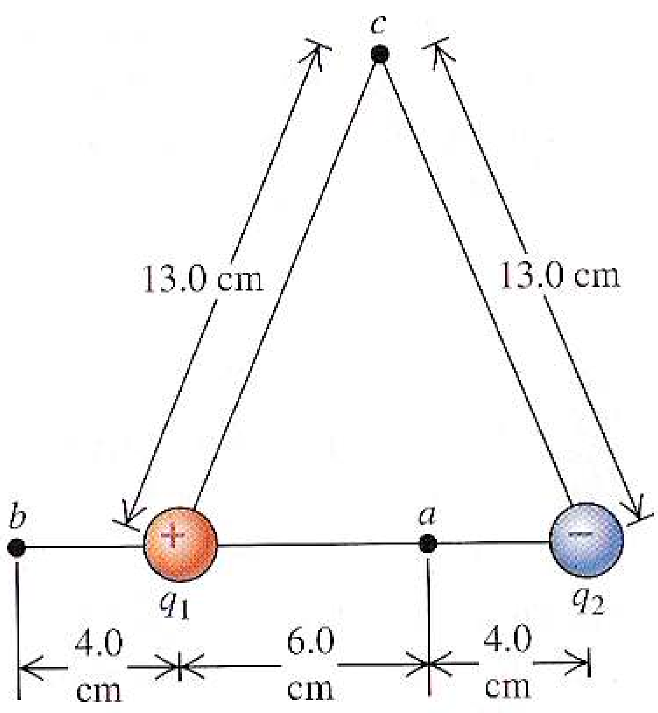
$$U = U_{12} + Q_3 V_3 = U_{12} + Q_3 \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{r_{13}} + \frac{Q_2}{r_{23}}\right]$$

$$U = U_{12} + U_{13} + U_{23}$$

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1 Q_2}{r_{12}} + \frac{Q_1 Q_3}{r_{13}} + \frac{Q_2 Q_3}{r_{23}}\right]$$

Example: Potential

- An electric dipole consists of two charges q_1 = +12 nC and q_2 = -12 nC, placed 10 cm apart as shown in the figure. Compute the potential at points a, b and c.

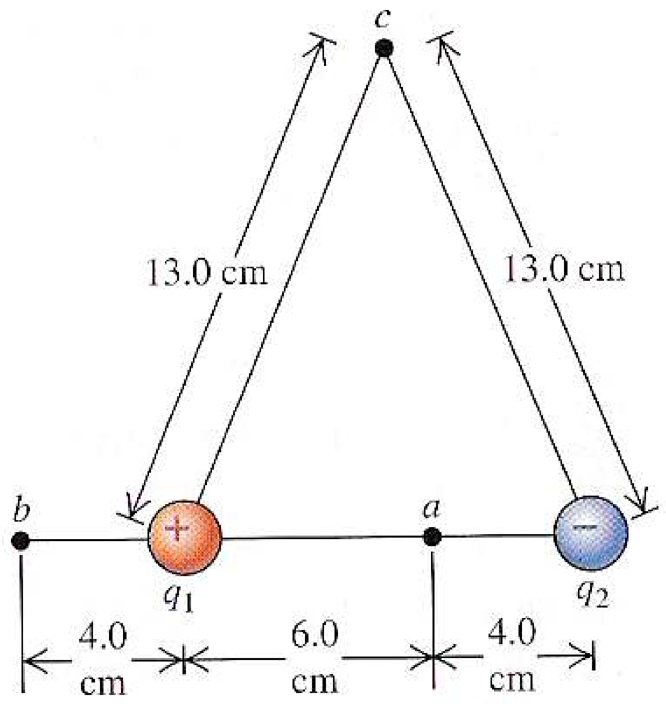


$$V_a = k\sum\left(\frac{q_1}{r_{1a}} + \frac{q_2}{r_{2a}}\right)$$

$$V_a = 8.99x10^9\left(\frac{12x10^{-9}}{0.06} + \frac{-12x10^{-9}}{0.04}\right)$$

$V_a =$ **- 899 V**

Example: Potential



$$V_b = k\sum(\frac{q_1}{r_{1b}} + \frac{q_2}{r_{2b}})$$

$$V_b = 8.99x10^9(\frac{12x10^{-9}}{0.04} + \frac{-12x10^{-9}}{0.14})$$

$V_b =$ **1926.4 V**

$V_c =$ **0 V**

Since direction isn't important, the electric potential at "c" is zero. The electric field however is NOT. The electric field would point to the right.

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Electromagnetism Fundamentals

Chapter 4 – Conductors, Dielectrics and Capacitance

Dr. Michel Nahas

Current and Current Density

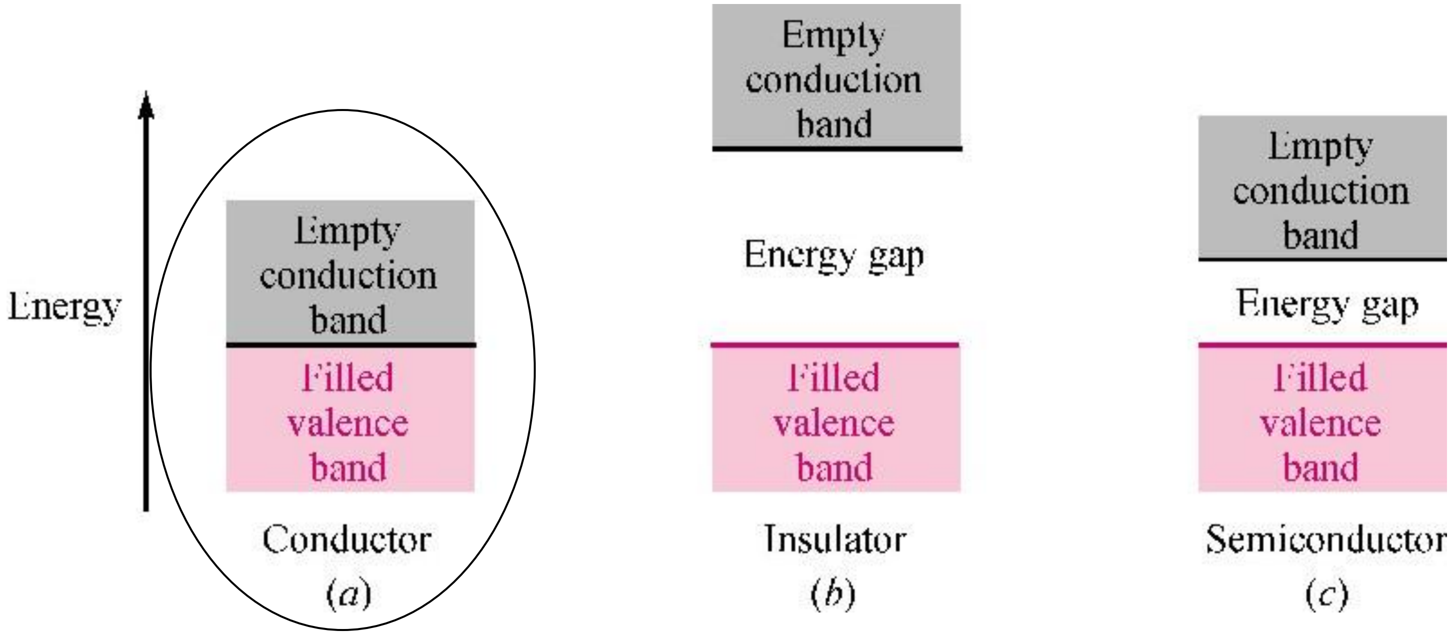
- Current I is the motion of charges Q

$$I = \frac{dQ}{dt}$$

- Current density is a vector represented by **J**

$$\Delta I = \mathbf{J} \cdot \Delta \mathbf{S}$$

Conductors, Dielectrics and Semiconductors



The Energy Band Structure in Three Different Types of Materials at 0 Kelvin

a) The conductor exhibits no energy gap between the valence and conduction bands
b) The Insulator shows a large energy gap
c) The semiconductor has only a small energy gap

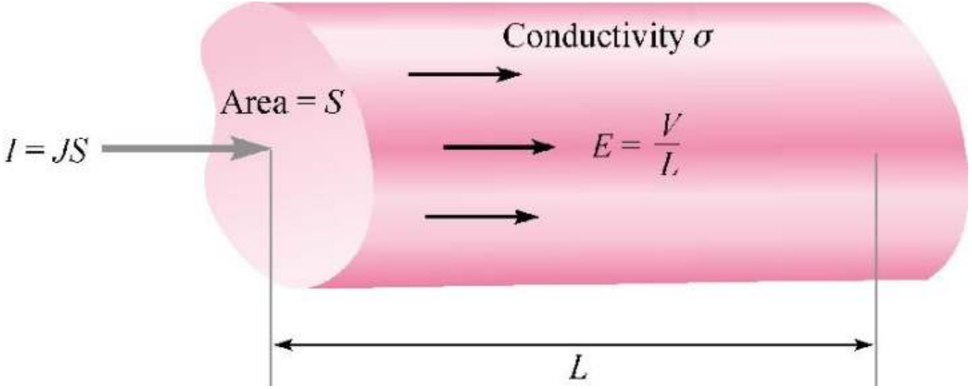
Electrical Classification of Solid Materials

Materials	Resistivity (Ω-cm)
Insulators	$10^5 < r < \infty$
Semiconductors	$10^{-3} < r < 10^5$
Conductors	$r < 10^{-3}$

Metallic Conductors

Force in experienced by an electron in a E field

$$F = -e \cdot E$$



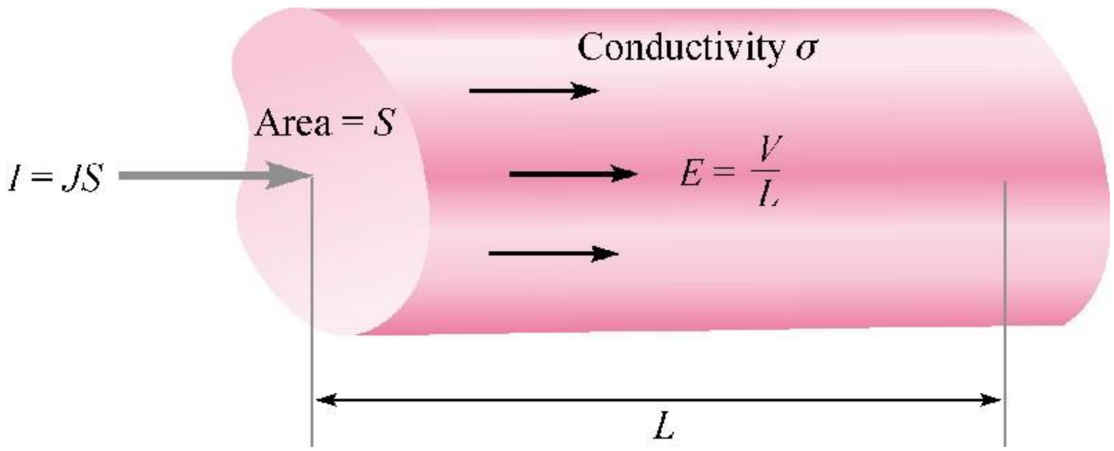
Drift velocity

$v_d = \mu_e \cdot E$	μ_e	mobility of an electron
$J = -\rho_e \cdot \mu_e \cdot E$	ρ_e	free electron charge density (negative value)
$J = \sigma \cdot E$	σ	conductivity (sigma) measured in siemens / m

Metallic Conductors

- In a conductor, electric current can flow freely, in an insulator it cannot. Metals such as copper typify conductors, while most non-metallic solids are said to be good insulators, having extremely high resistance to the flow of charge through them
- "Conductor" implies that the outer electrons of the atoms are loosely bound and free to move through the material
- Most atoms hold on to their electrons tightly and are insulators. In copper, the valence electrons are essentially free and strongly repel each other. Any external influence which moves one of them will cause a repulsion of other electrons which propagates, "domino fashion" through the conductor

Metallic Conductors



Assume that J and E are uniform

$$J = \frac{I}{S} = \sigma \cdot E = \sigma \cdot \frac{V}{L}$$

$$I = J \cdot S$$

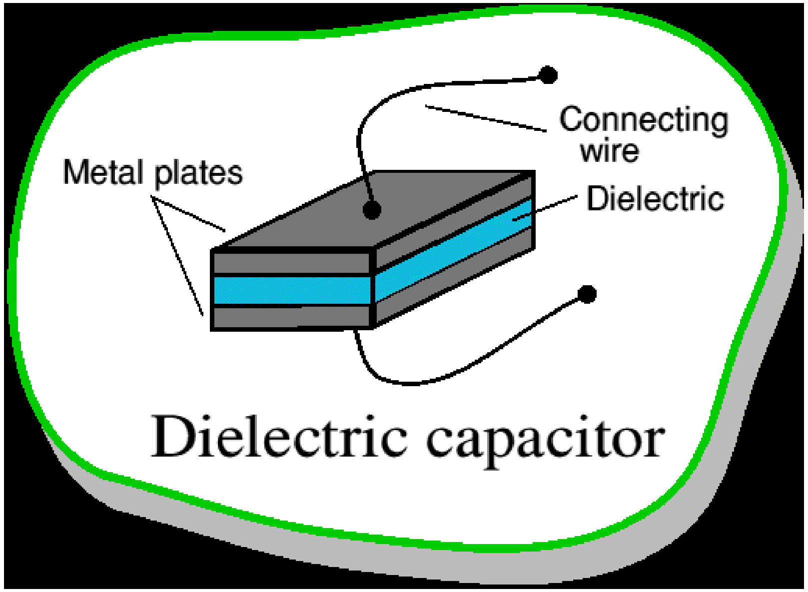
$$V = \frac{L}{\sigma \cdot S} \cdot I \qquad V = I \cdot R$$

$$V = E \cdot L$$

$$R = \frac{L}{\sigma \cdot S} \qquad R = \frac{V_{ab}}{I}$$

Applications of Electric Potential

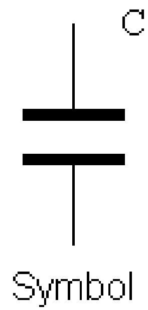
- We can use a set of plates with an electric field that is called a Parallel Plate Capacitor and store charges between the plates



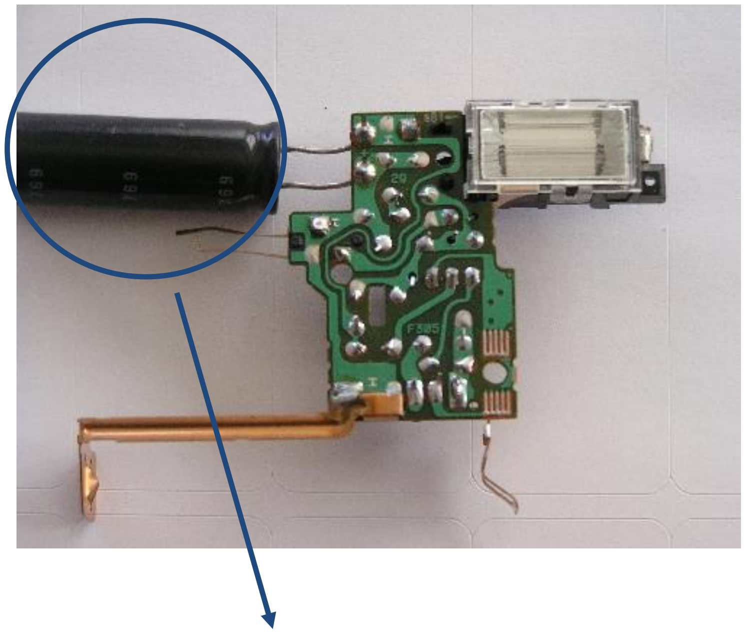
Storing Charges - Capacitors

- A capacitor consists of 2 conductors of any shape placed near one another without touching
- It is common; to fill up the region between these 2 conductors with an insulating material called a dielectric
- We charge these plates with opposing charges to set up an electric field

Capacitors



- Capacitors can be easily purchased at a local electronics shop and are commonly found in disposable cameras.
- When a voltage is applied to an empty capacitor, current flows through the capacitor and each side of the capacitor becomes charged.
- The two sides have equal and opposite charges.
- When the capacitor is fully charged, the current stops flowing. The collected charge is then ready to be discharged and when you press the flash it discharges very quickly released it in the form of light.

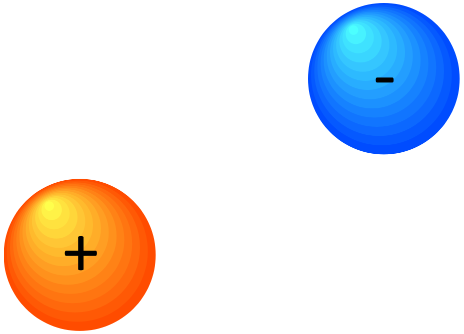


Cylindrical Capacitor

Capacitors

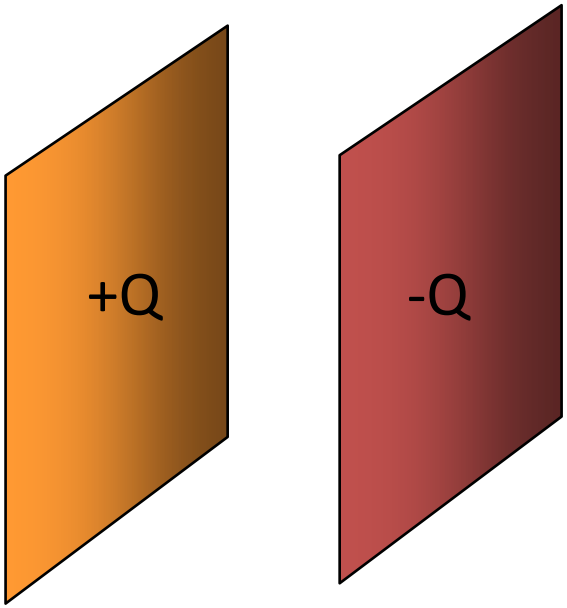
- A system of two conductors, each carrying equal charge is known as a capacitor

Example 1: two metal spheres

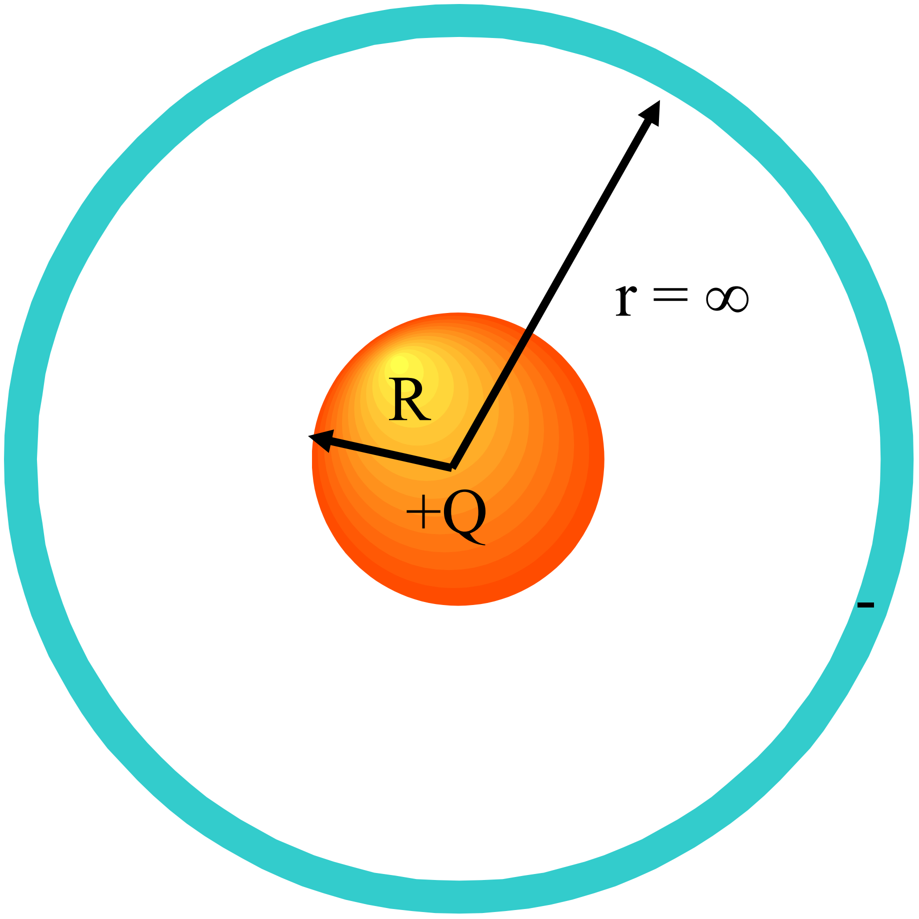


Each conductor is called a plate

Example 2: two parallel sheets



Capacitance of charged sphere



$$C = \frac{Q}{\Delta V}$$ definition

$$V = +\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$ potential due to isolated charge

Capacitance

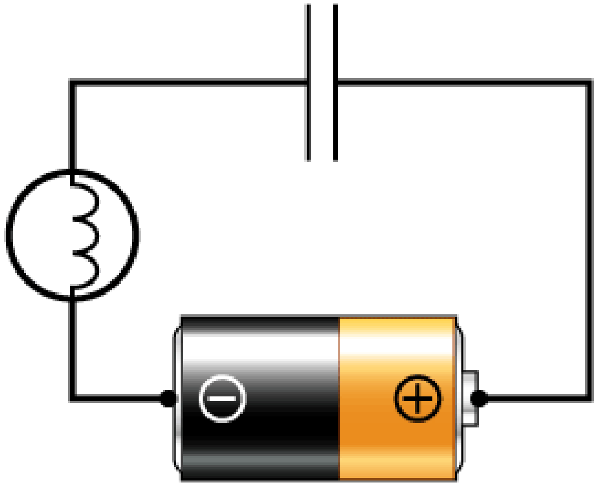
- Capacitance is a measure of the amount of charge a capacitor can store (its “capacity”)
- Experiments show that the charge in a capacitor is proportional to the electric potential difference (voltage) between the plates

Capacitance

- In the picture below, the capacitor is symbolized by a set of parallel lines. Once it's charged, the capacitor has the same voltage as the battery (1.5 volts on the battery means 1.5 volts on the capacitor)

- The difference between a capacitor and a battery is that a capacitor can dump its entire charge in a tiny fraction of a second, where a battery would take minutes to completely discharge itself

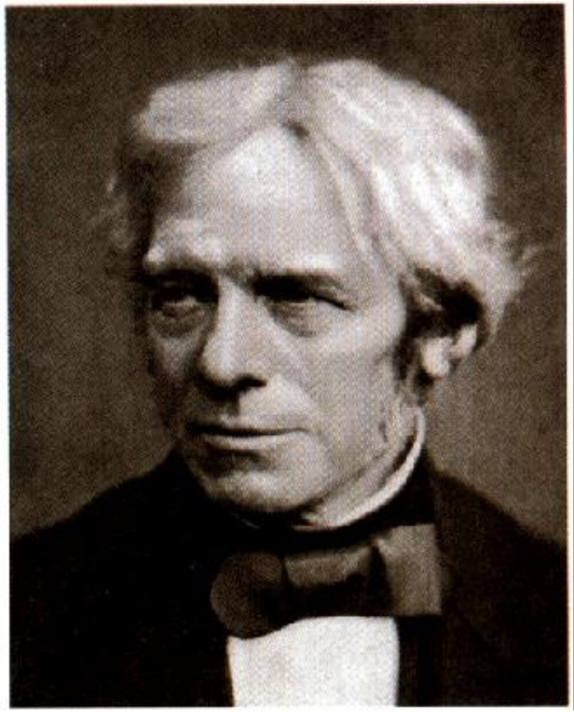
- That's why the electronic flash on a camera uses a capacitor -- the battery charges up the flash's capacitor over several seconds, and then the capacitor dumps the full charge into the flash tube almost instantly



Units

$$C = \frac{Q}{\Delta V}$$

Thus SI units of capacitance are:



C/V

Remember that V is also J/C so unit is also C^2J^{-1}

This unit is also known as the **farad** after Michael Faraday

1F = 1C / V

Capacitance

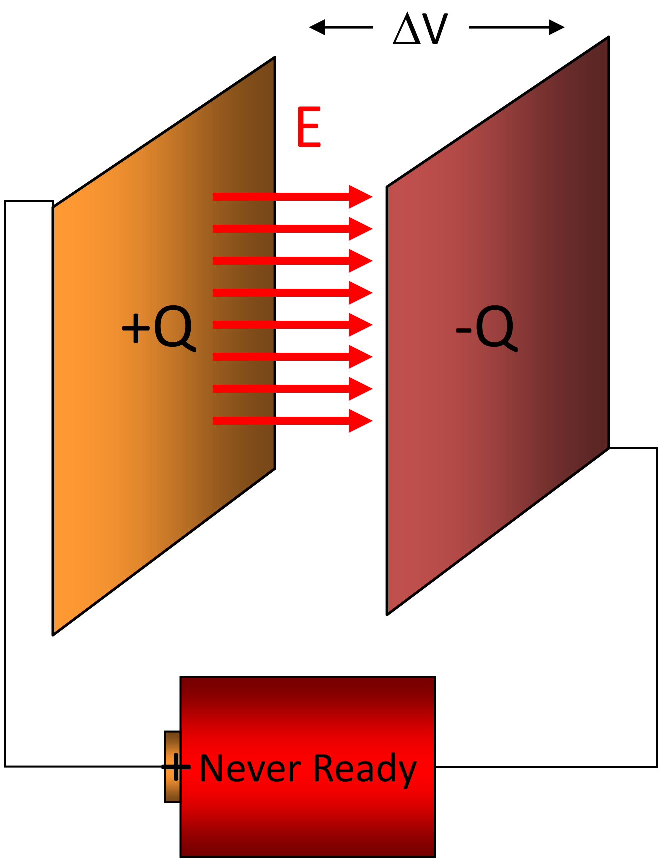
- Experiments show that the charge in a capacitor is proportional to the electric potential difference (voltage) between the plates.
- The constant of proportionality C is the capacitance which is a property of the conductor

$$Q \propto \Delta V$$

$$Q = C\Delta V$$

$$C = \frac{Q}{\Delta V}$$

Capacitance of parallel plates



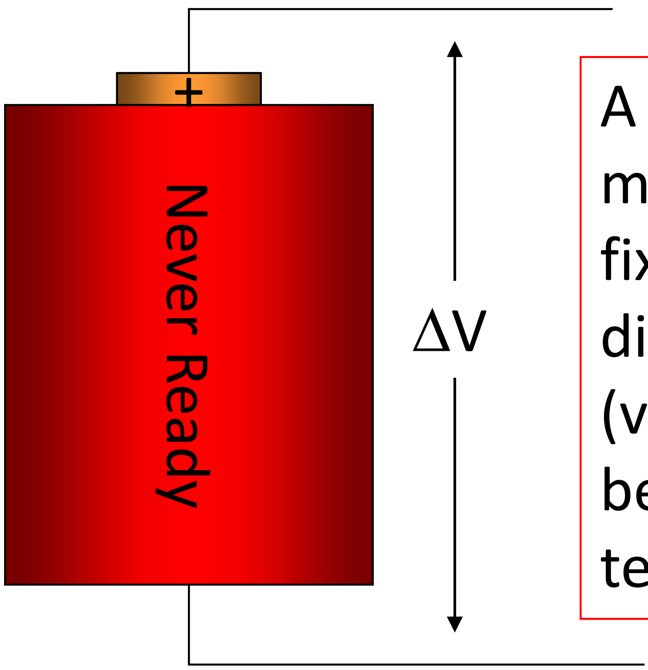
Intuitively

The bigger the plates the more surface area over which the capacitor can store charge $C \propto A$

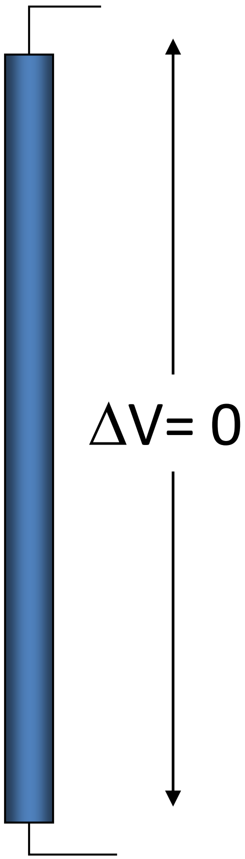
Moving plates together

Initially E is constant (no charges moving) thus $\Delta V = Ed$ decreases charges flows from battery to increase $\Delta V \Rightarrow C \propto 1/d$

Batteries, Conductors & Potential



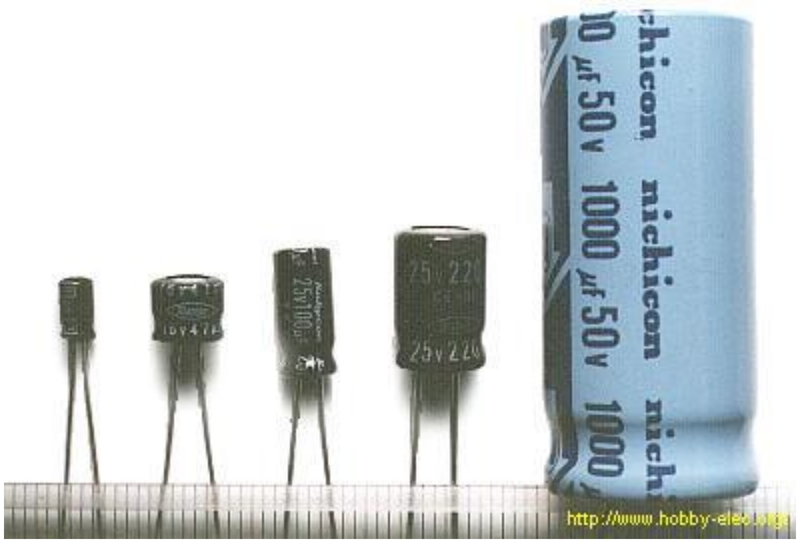
A battery maintains a fixed potential difference (voltage) between its terminals



A conductor has E=0 within and thus $\Delta V = Ed = 0$

Capacitor Geometry

- The capacitance of a capacitor depends on HOW you make it.



$$C \alpha A \quad C \alpha \frac{1}{\mathrm{d}}$$

$$A = \text{area of plate}$$

$$\mathrm{d} = \text{distance beteween plates}$$

$$C \alpha \frac{A}{d}$$

$$\varepsilon_o = \text{constant of proportionality}$$

$$\varepsilon_\mathrm{o} = \text{vacuum permittivity constant}$$

$$\varepsilon_\mathrm{o} = 8.85x10^{-12} \frac{C^2}{Nm^2}$$

$$C = \frac{\varepsilon_o A}{d}$$

Capacitor Problems

- What is the AREA of a 1F capacitor that has a plate separation of 1 mm?

$$C = \varepsilon_o \frac{A}{D}$$

$$1 = 8.85x10^{-12} \frac{A}{0.001}$$

$A =$ **1.13x10^{8} m^{2}**

$Sides =$ **10629 m**

Is this a practical capacitor to build?

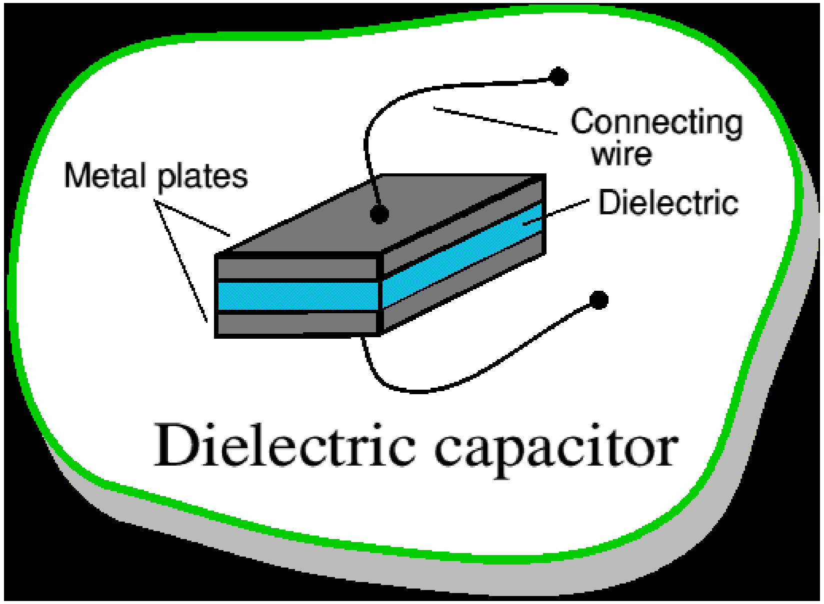
NO! – How can you build this then?

The answer lies in REDUCING the AREA. But you must have a CAPACITANCE of 1 F. **How can you keep the capacitance at 1 F and reduce the Area at the same time?**

Add a DIELECTRIC!!!

Dielectric

- Remember, the dielectric is an insulating material placed between the conductors to help store the charge. In the previous example we assumed there was NO dielectric and thus a vacuum between the plates.



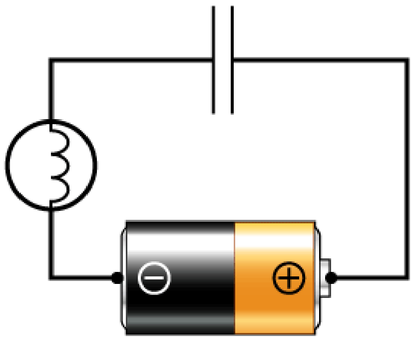
$$C = k\varepsilon_o \frac{A}{d}$$

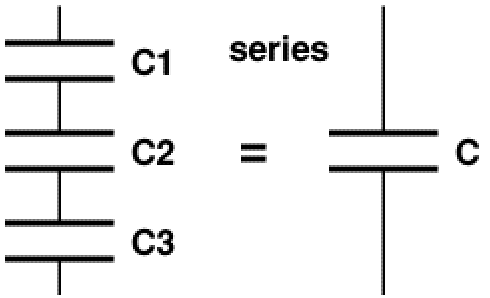
$$k = \text{Dielectric}$$

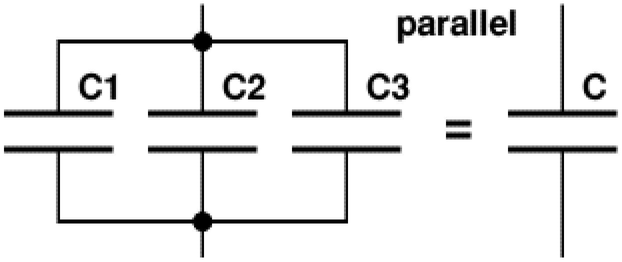
All insulating materials have a dielectric constant associated with it. Here now you can reduce the AREA and use a LARGE dielectric to establish the capacitance at 1 F.

Using MORE than 1 capacitor

- Let's say you decide that 1 capacitor will not be enough to build what you need to build. You may need to use more than 1. There are 2 basic ways to assemble them together
  - **Series:** One after another
  - **Parallel:** between a set of junctions and parallel to each other.

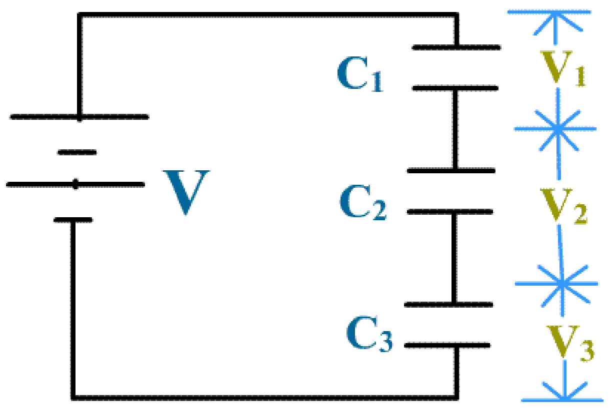






Capacitors in Series

- Capacitors in series each charge each other by INDUCTION. So they each have the **SAME charge**. The electric potential on the other hand is divided up amongst them. In other words, the sum of the individual voltages will equal the total voltage of the battery or power source.



Capacitors in series

$$V_{Total} = V_1 + V_2 + V_3 \ldots$$

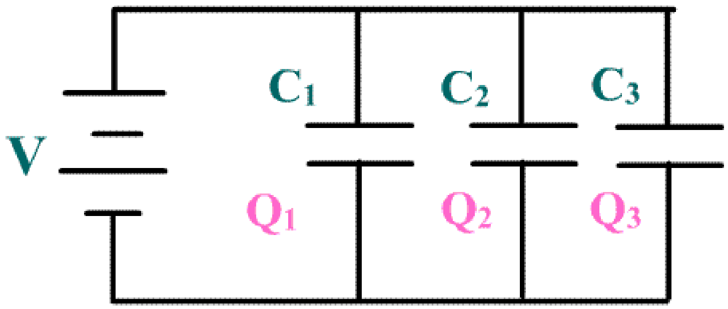
$$V = \frac{Q}{C}$$

$$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

$$\frac{1}{C_{Series}} = \Sigma \frac{1}{C_i}$$

Capacitors in Parallel

- In a parallel configuration, the **voltage is the same** because ALL THREE capacitors touch BOTH ends of the battery. As a result, they split up the charge amongst them.



Capacitors in parallel

$$Q_{Total} = Q_1 + Q_2 + Q_3 \ldots$$

$$Q = CV$$

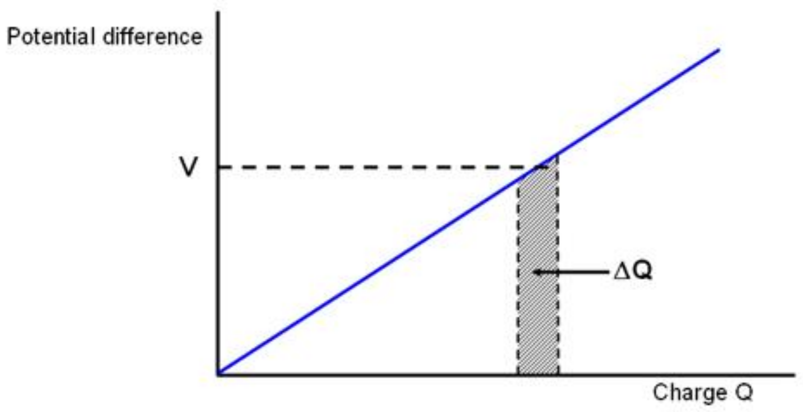
$$C_T V_T = C_1 V_1 + C_2 V_2 + C_3 V_3$$

$$V_T = V_1 = V_2 = V_3$$

$$C_{Parallel} = \Sigma C_i$$

Capacitors "STORE" energy

- Anytime you have a situation where energy is "STORED" it is called POTENTIAL. In this case we have capacitor potential energy, U_c



Suppose we plot a V vs. Q graph. If we wanted to find the AREA we would MULTIPLY the 2 variables according to the equation for Area.

A = bh

When we do this we get Area = VQ

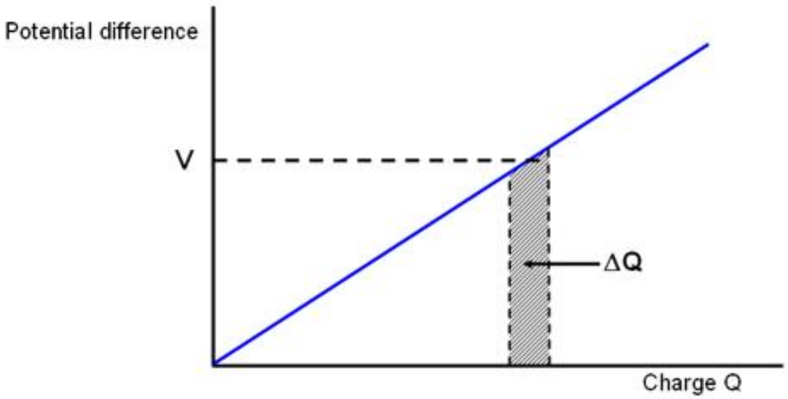
Let's do a unit check!

Voltage = Joules/Coulomb

Charge = Coulombs

Area = **ENERGY**

Potential Energy of a Capacitor



- Since the AREA under the line is a triangle, the ENERGY(area) = 1/2VQ

$$U_C = \frac{1}{2} VQ \quad C = \frac{Q}{V}$$

$$U_C = \frac{1}{2} V(VC) \rightarrow \frac{1}{2} CV^2$$

$$U_C = \frac{1}{2} \left(\frac{Q}{C}\right) Q \rightarrow \frac{Q^2}{2C}$$

most common form

- This energy or area is referred as the **potential energy stored inside a capacitor**.

- Note: The slope of the line is the inverse of the capacitance.

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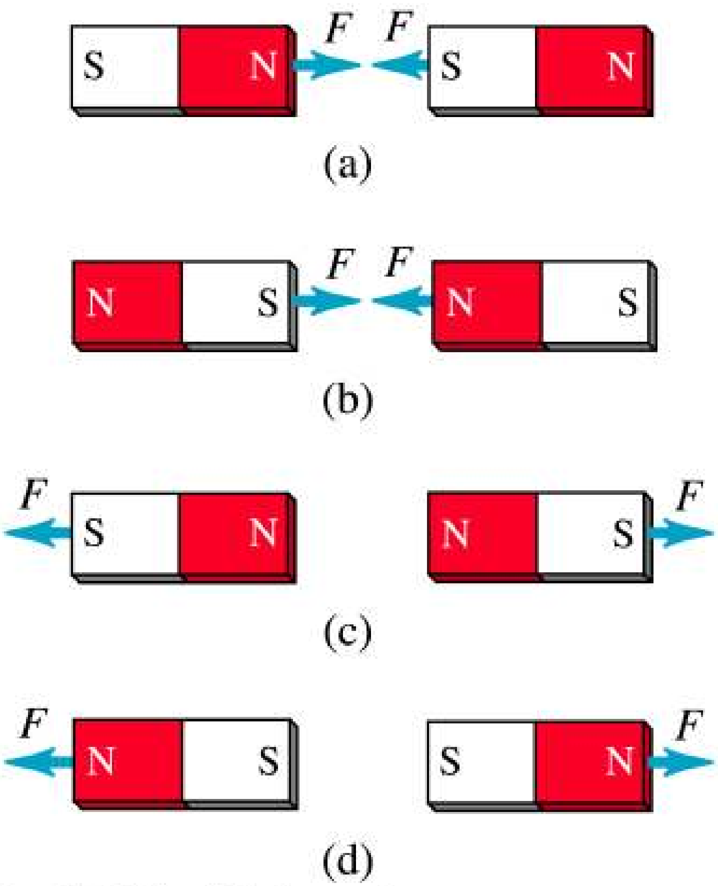
Electromagnetism Fundamentals

Chapter 5 – Magnetic Field and Magnetic Force

Dr. Michel Nahas

Facts about Magnetism

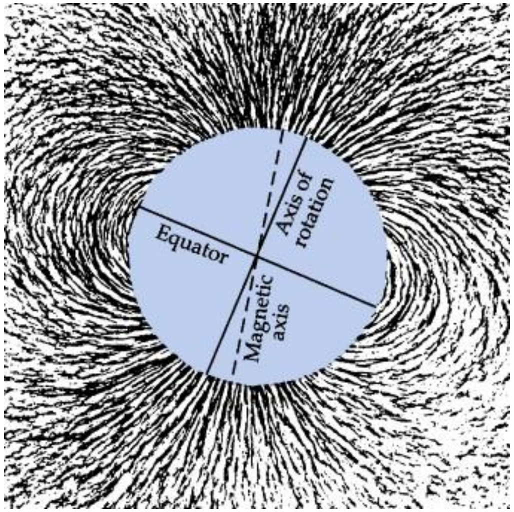


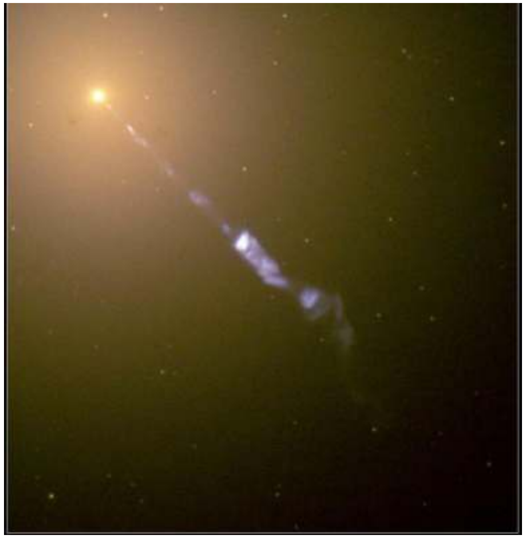
- Magnets have 2 poles (north and south)
- Poles cannot be isolated. They occur only in pairs, as dipoles.
- Like poles repel
- Unlike poles attract
- Magnets create a MAGNETIC FIELD around them

Importance of Magnetic Fields

- Practical Uses
  - Electric motors, Loud speakers, Navigation (Earth's magnetic field)
- In Experimental Physics
  - Mass spectrometers, Particle accelerators, Plasma confinement
- In the Universe
  - Stars (e.g. the Sun), Interstellar space, Intergalactic structure, Jets

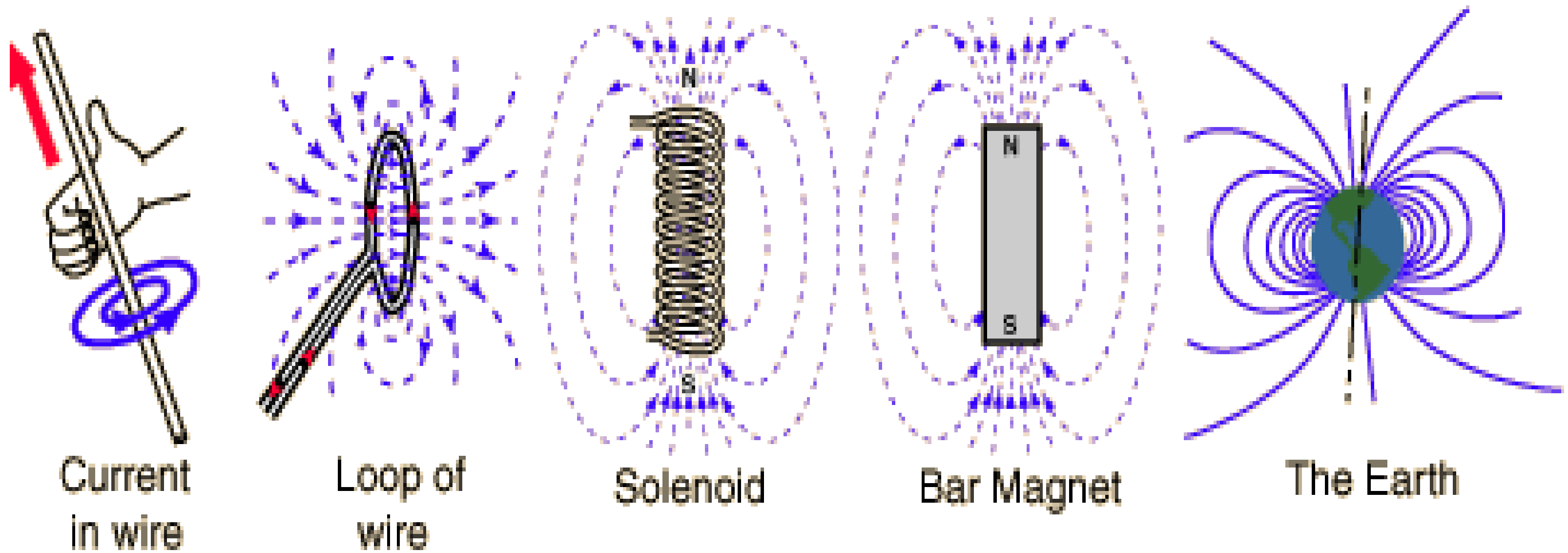
Importance of Magnetic Fields





Magnetic Field Sources

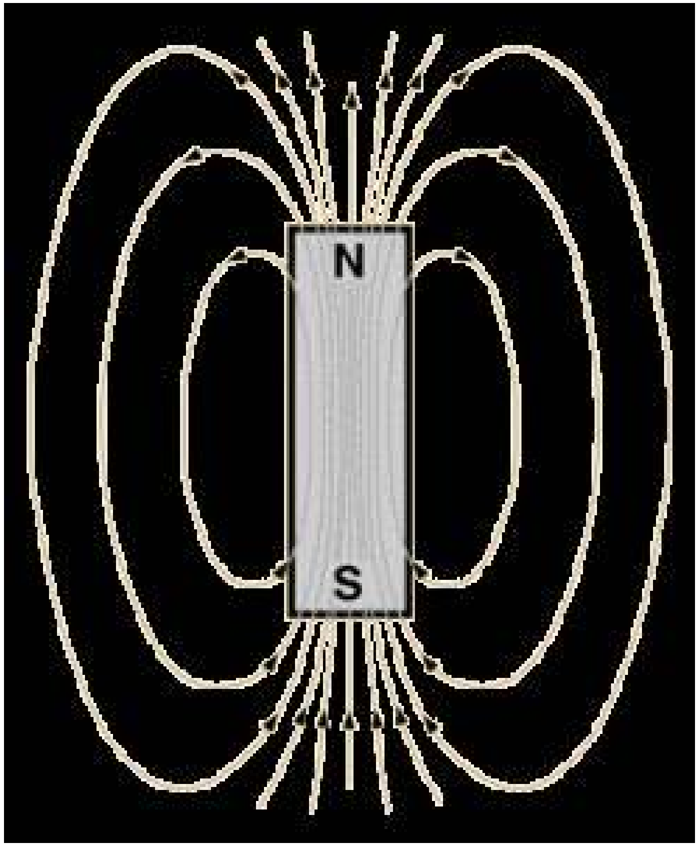
- Magnetic fields are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits.



Magnetic Field Sources

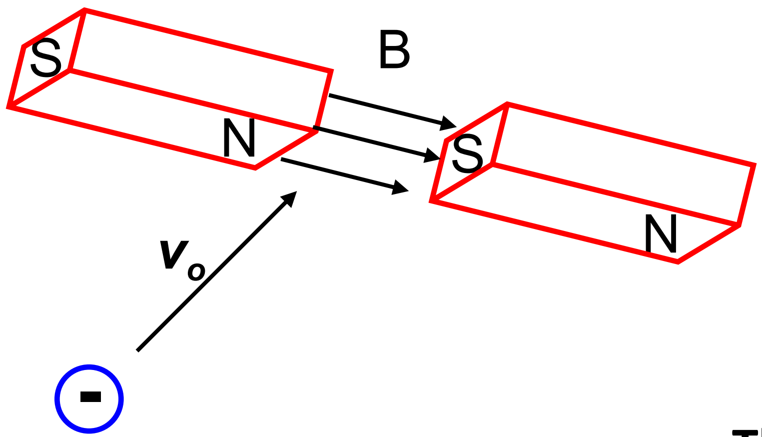
https://www.youtube.com/watch?v=zbTrHWW3xvU

Magnetic Field



- A bar magnet has a magnetic field around it. This field is 3D in nature and often represented by lines LEAVING north and ENTERING south
- To define a magnetic field you need to understand the MAGNITUDE and DIRECTION
- We sometimes call the magnetic field a B-Field as the letter "B" is the SYMBOL for a magnetic field with the TESLA (T) as the unit

Magnetic Force on a moving charge



- If a MOVING CHARGE moves into a magnetic field it will experience a MAGNETIC FORCE. This deflection is 3D in nature.

$$F_B = qv \otimes B$$

$$F_B = qvB\sin\theta$$

The conditions for the force are:

- Must have a magnetic field present
- Charge must be moving
- Charge must be positive or negative
- Charge must be moving **PERPENDICULAR** to the field.

https://www.youtube.com/watch?v=tbCXaER0w-s

Example

- A proton moves with a speed of $1.0x10^5$ m/s through the Earth's magnetic field, which has a value of 55 µT at a particular location. When the proton moves eastward, the magnetic force is a maximum, and when it moves northward, no magnetic force acts upon it. **What is the magnitude and direction of the magnetic force acting on the proton?**

$$F_B = qvB, \theta = 90, \sin 90 = 1$$

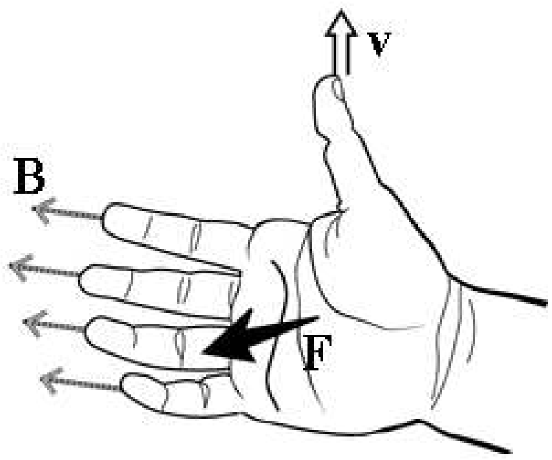
$$F_B = (1.6x10^{-19})(1.0x10^{5})(55x10^{-6})$$

$$F_B = \mathbf{8.8x10^{-19}\ N}$$

The direction cannot be determined precisely by the given information. Since no force acts on the proton when it moves northward (meaning the angle is equal to ZERO), we can infer that the magnetic field must either go northward or southward.

Direction of the magnetic force

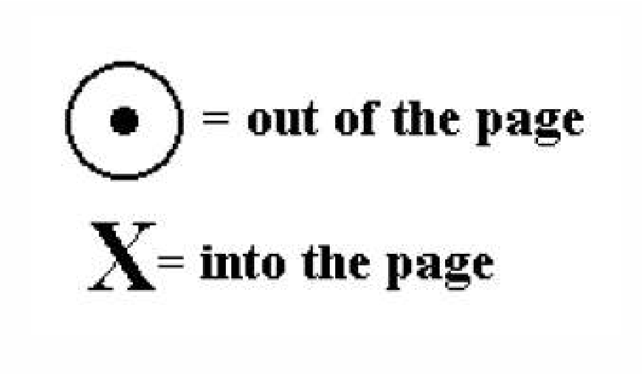
Right Hand Rule:



Basically you hold your right hand flat with your thumb perpendicular to the rest of your fingers

For **NEGATIVE** charges use left hand!

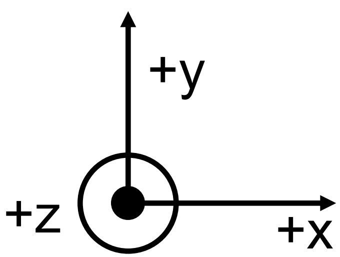
- To determine the DIRECTION of the force on a **POSITIVE** charge we use a special technique that helps us understand the 3D/perpendicular nature of magnetic fields.

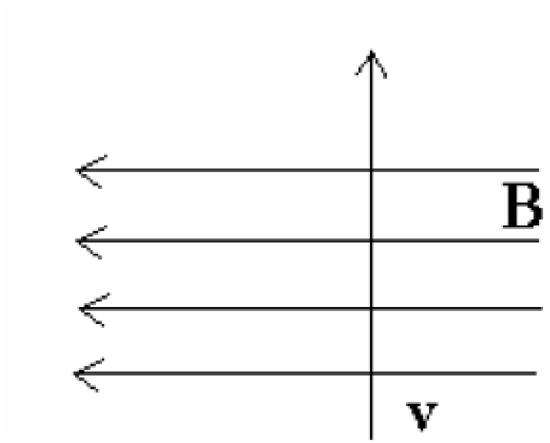


- The Fingers = Direction B-Field
- The Thumb = Direction of velocity
- The Palm = Direction of the Force

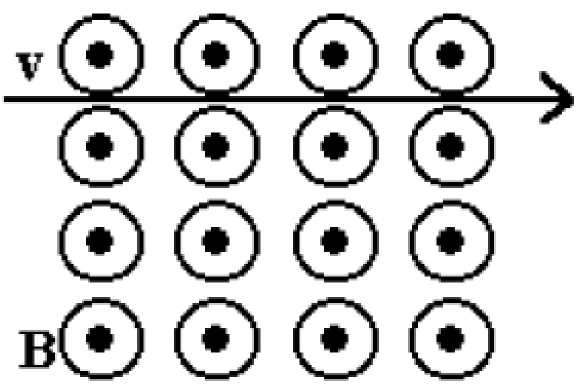
Example

- Determine the direction of the unknown variable for a **<u>proton</u>** moving in the field using the coordinate axis given

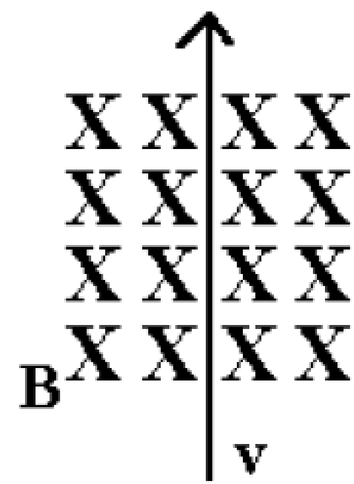




B = -x
v = +y
F = **+z**



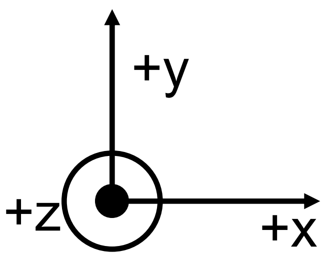
B =+Z
v = +x
F = **-y**

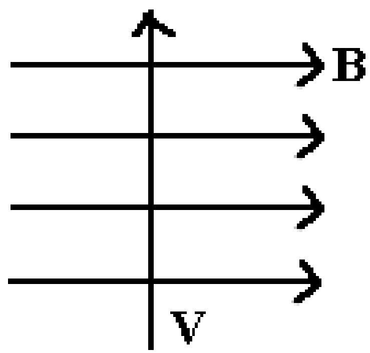


B = -z
v = +y
F = **-x**

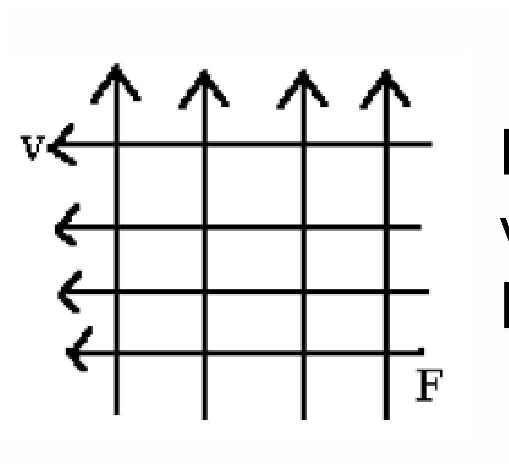
Example

- Determine the direction of the unknown variable for an **<u>electron</u>** using the coordinate axis given.

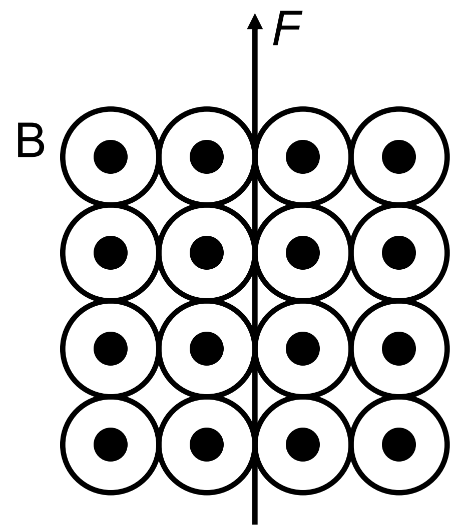




B = +x
v = +y
F = **+z**

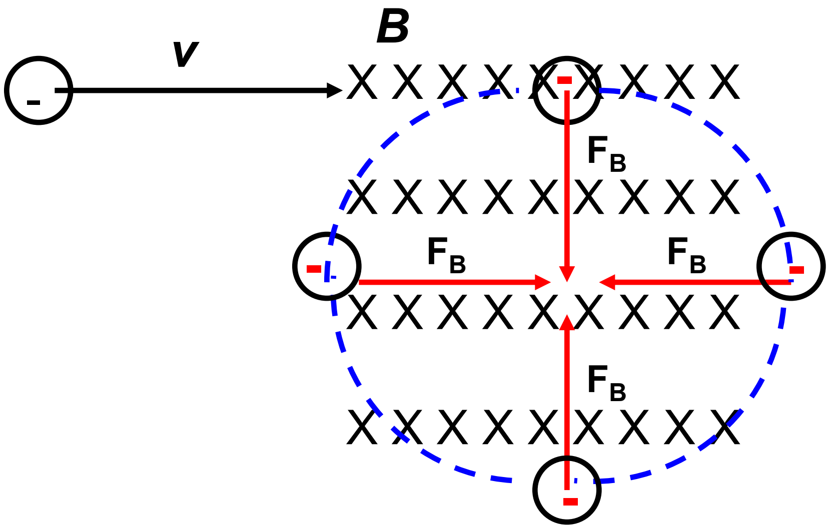


B = **-z**
v = - x
F = +y

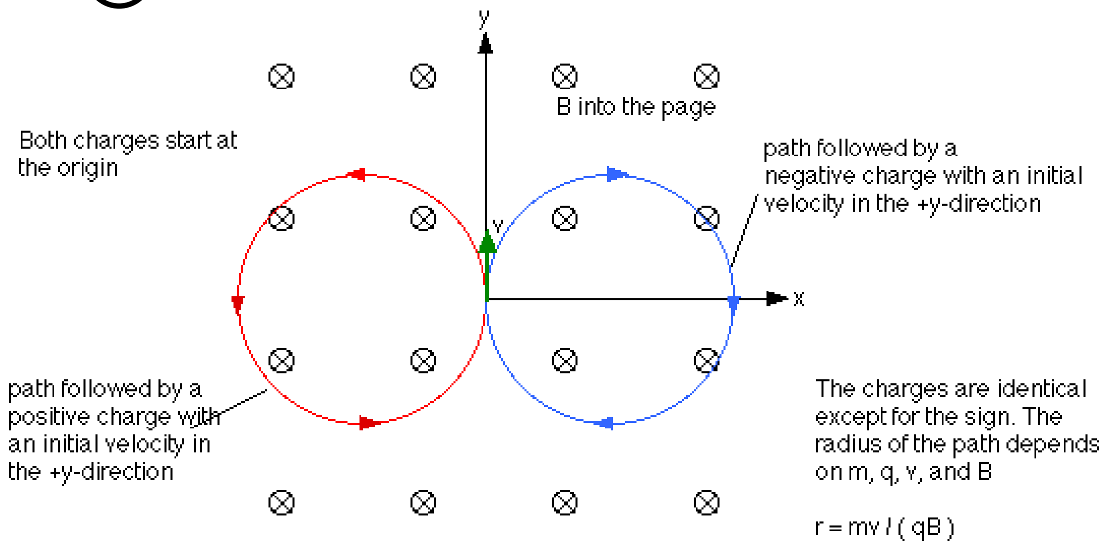


B = +z
v = **+x**
F = +y

Magnetic Force and Circular Motion



- Suppose we have an electron traveling at a velocity , ***v***, entering a magnetic field, ***B***, directed into the page. What happens after the initial force acts on the charge?



Magnetic Force and Circular Motion

$$F_B = qvB,\ F_c = \frac{mv^2}{r},\ F_B = F_c$$

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

- The magnetic force is equal to the centripetal force and thus can be used to solve for the circular path.
- Or, if the radius is known, could be used to solve for the MASS of the ion. This could be used to determine the material of the object.

There are many "other" types of forces that can be set equal to the magnetic force.

$$F_B = qvB$$

$$mg = qvB$$

$$ma = qvB$$

Example

- A singly charged positive ion has a mass of 2.5 x 10^{-26} kg. After being accelerated through a potential difference of 250 V, the ion enters a magnetic field of 0.5 T, in a direction perpendicular to the field. **Calculate the radius of the path of the ion in the field.**

$$q = 1.6x10^{-19}\,C$$
$$m = 2.5x10^{-26}\,kg$$
$$\Delta V = 250V$$
$$B = 0.5T$$
$$r = ?$$

$$F_B = F_c \quad qvB = \frac{mv^2}{r} \quad r = \frac{mv}{qB}$$

We need to solve for the velocity!

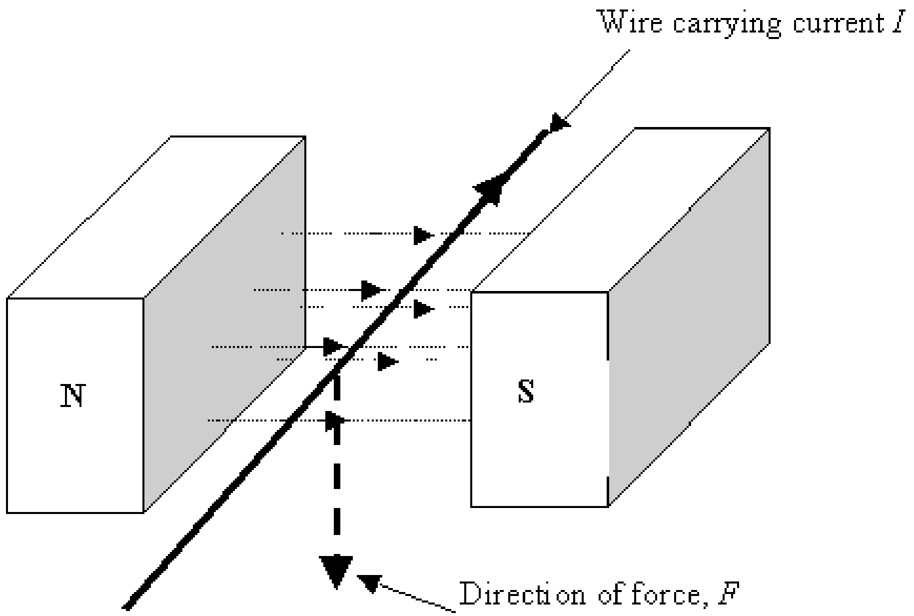
$$r = \frac{(2.5x10^{-26})(56{,}568)}{(1.6x10^{-19})(0.5)} =$$ **0.0177 m**

$$\Delta V = \frac{W}{q} = \frac{\Delta K}{q} = \frac{\frac{1}{2}mv^2}{q}$$

$$v = \sqrt{\frac{2\Delta V q}{m}} = \sqrt{\frac{2(250)(1.6x10^{-19})}{2.5x10^{-26}}} =$$ **56568 m/s**

Charges moving in a wire

- Up to this point we have focused our attention on PARTICLES or CHARGES only. The charges could be moving together in a wire. Thus, if the wire had a CURRENT (moving charges), it too will experience a force when placed in a magnetic field.



You simply used the RIGHT HAND ONLY and the thumb will represent the direction of the CURRENT instead of the velocity.

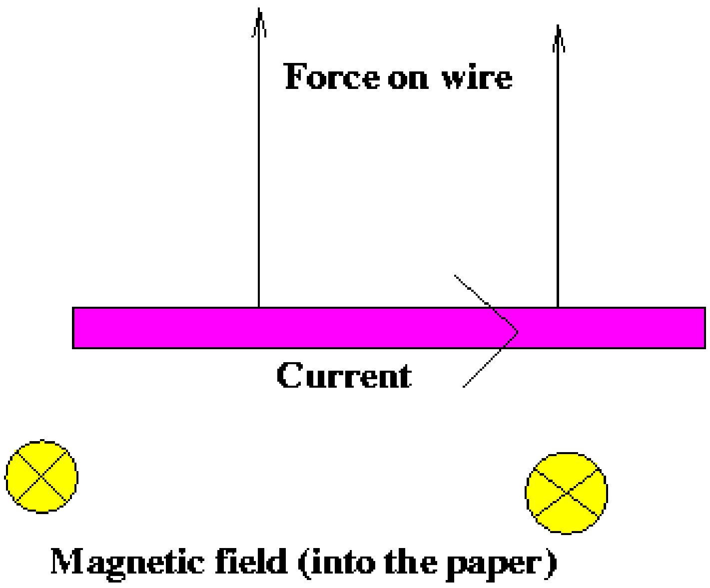
Charges moving in a wire

$$F_B = qvB\sin\theta \times \frac{t}{t}$$

$$F_B = (\frac{q}{t})(vt)B\sin\theta$$

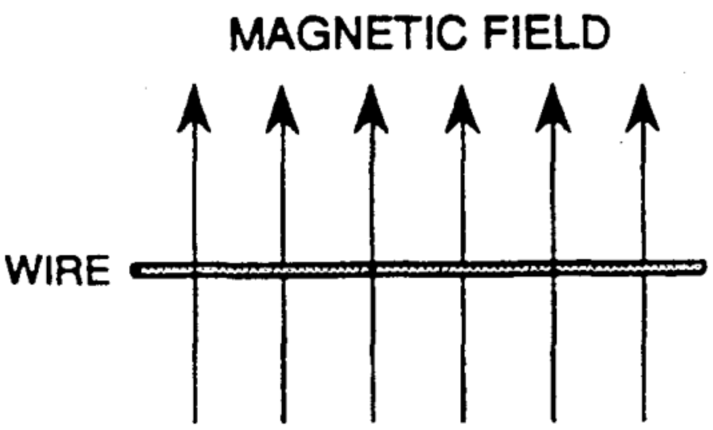
$$F_B = ILB\sin\theta$$

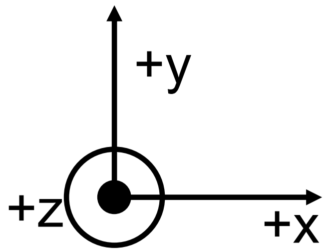
At this point it is VERY important that you understand that the MAGNETIC FIELD is being produced by some **EXTERNAL AGENT**



Example

- A 36-m length wire carries a current of 22 A running from right to left. **Calculate the magnitude and direction of the magnetic force acting on the wire if it is placed in a magnetic field with a magnitude of 0.50x10^{-4} T and directed up the page**.





$$F_B = ILB\sin\theta$$

$$F_B = (22)(36)(0.50x10^{-4})\sin 90$$

$F_B =$ **0.0396 N**

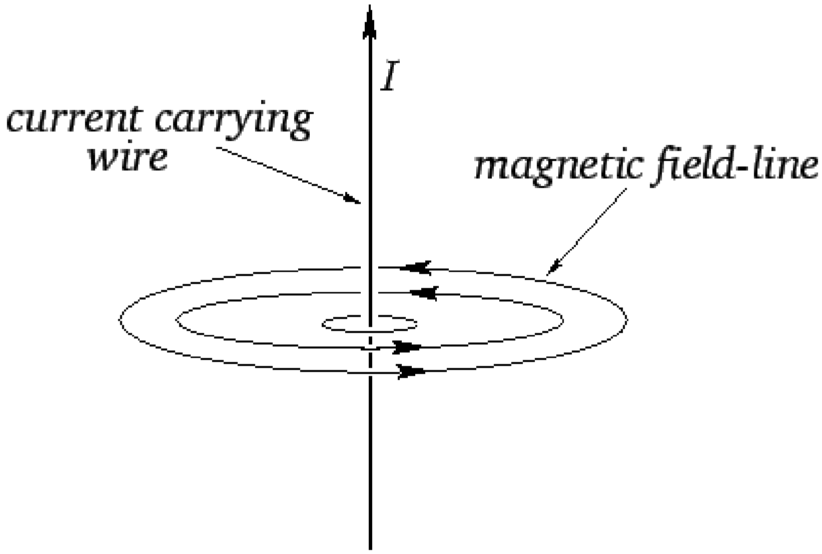
B = +y

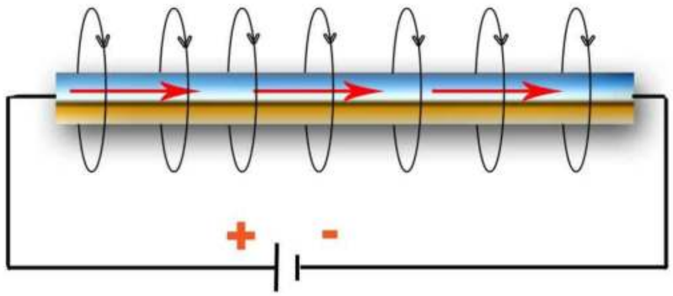
I = -x

F = **-z, into the page**

Why does the wire move?

- The real question is WHY does the wire move? It is easy to say the EXTERNAL field moved it. But how can an external magnetic field FORCE the wire to move in a certain direction?
- **THE WIRE ITSELF MUST BE MAGNETIC**!!! In other words the wire has its own INTERNAL MAGNETIC FIELD that is attracted or repulsed by the EXTERNAL FIELD.

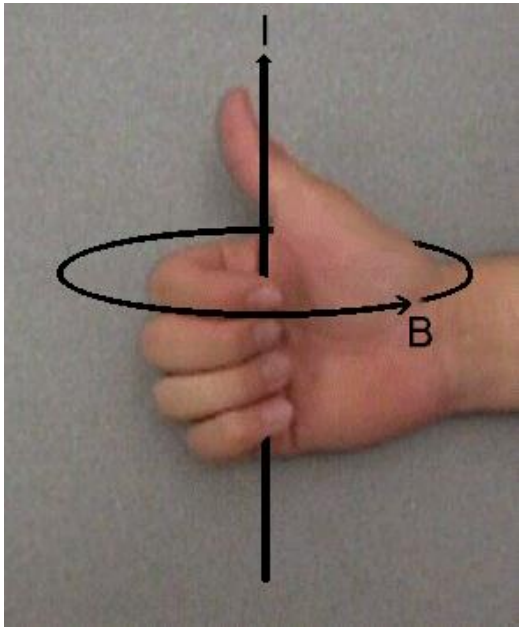


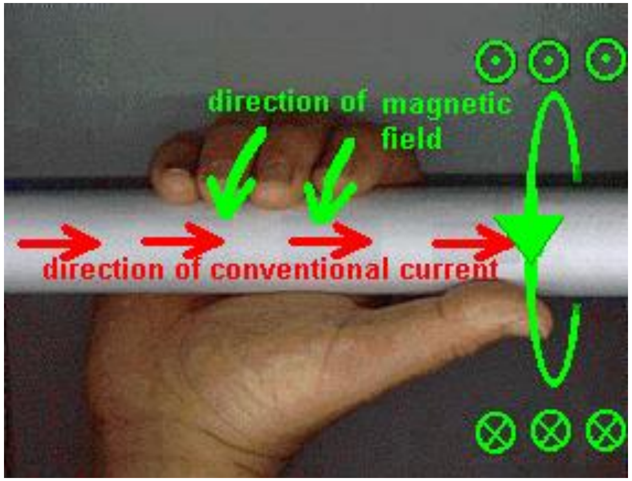


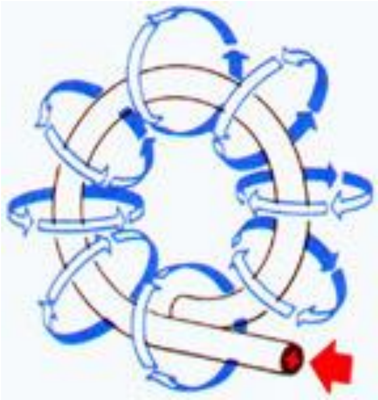
As it turns out, the wire's OWN internal magnetic field makes concentric circles round the wire.

A current carrying wire's INTERNAL magnetic field

- To figure out the DIRECTION of this INTERNAL field you use the right hand rule. You point your thumb in the direction of the current then CURL your fingers. Your fingers will point in the direction of the magnetic field







The MAGNITUDE of the internal field

- The magnetic field, B, is directly proportional to the current, I, and inversely proportional to the circumference.

$$B \alpha\, I \qquad B \alpha \frac{1}{2\pi r}$$

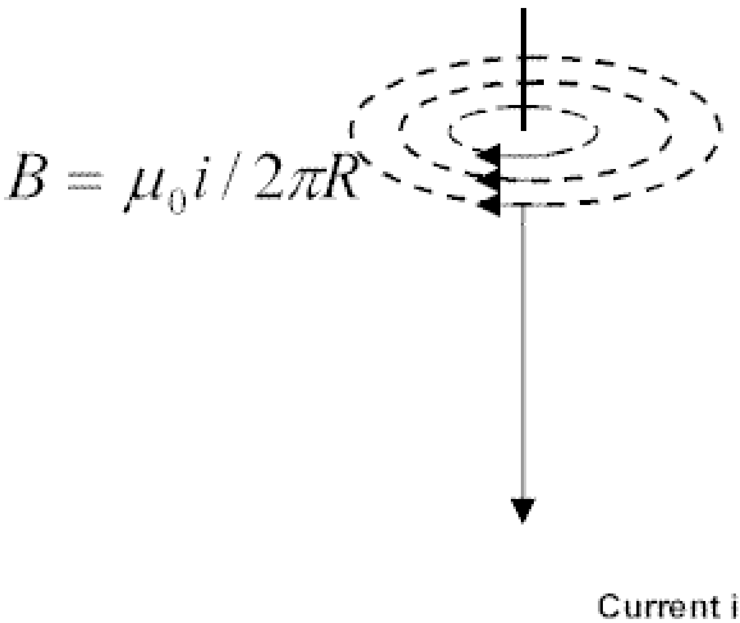
$$B \alpha \frac{I}{2\pi r}$$

$$\mu_o = \text{constant of proportionality}$$

$$\mu_o = \text{vacuum permeability constant}$$

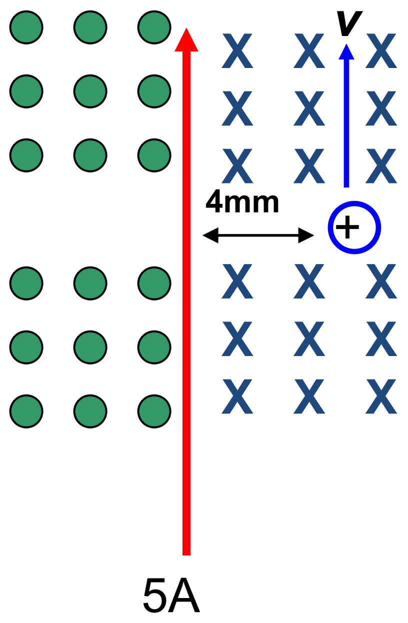
$$\mu_o = 4\pi\, x10^{-7} (1.26x10^{-6}) \frac{Tm}{A}$$

$$B_{\text{internal}} = \frac{\mu_o I}{2\pi r}$$



Example

- A long, straight wires carries a current of 5.00 A. At one instant, a proton, 4 mm from the wire travels at 1500 m/s parallel to the wire and in the same direction as the current. Find the **magnitude** <u>and</u> **<u>direction</u>** of the magnetic force acting on the proton due to the field caused by the current carrying wire.



$$F_B = qvB_{EX} \qquad B_{IN} = \frac{\mu_o I}{2\pi r}$$

$$B_{IN} = \frac{(1.26x10^{-6})(5)}{2(3.14)(0.004)} = $$ **2.51 x 10^{-4} T**

B = +z
v = +y
F = **-x**

$$F_B = (1.6x10^{-19})(1500)(B_{wire}) =$$

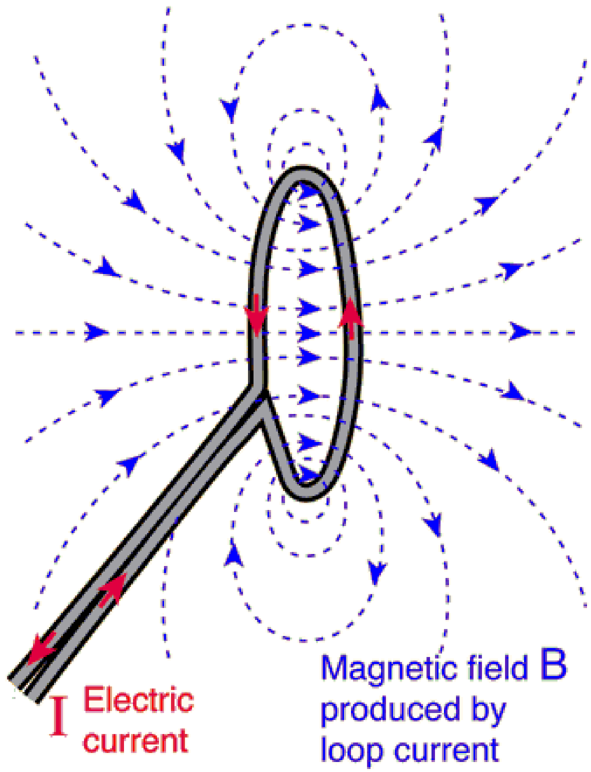
6.02 x 10^{-20} N

Magnetic Field of Current Loop

- Examining the direction of the magnetic field produced by a current-carrying segment of wire shows that all parts of the loop contribute magnetic field in the same direction inside the loop.

- Electric current in a circular loop creates a magnetic field which is more concentrated in the center of the loop than outside the loop.

- Stacking multiple loops concentrates the field even more into what is called a solenoid.



Field at Center of Current Loop

- For a loop with 1 turn (N=1) of radius r, the magnetic field at the center of the loop can be calculated as:

$$B = \frac{\mu_0 I}{2r}$$

- For a coil with N-turns of radius r, the magnetic field at the center of the coil can be calculated as:

$$B = N \frac{\mu_0 I}{2r}$$

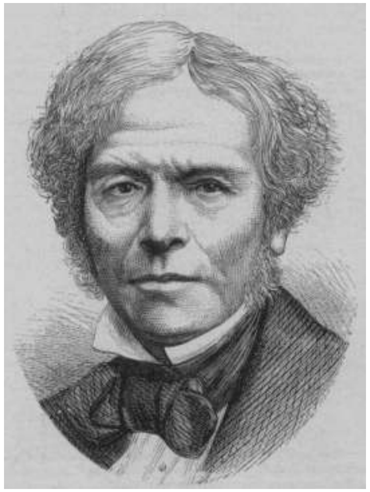
15FELE121
Electromagnetism Fundamentals

Chapter 6 – Magnetic Induction, Flux and Laws

Dr. Michel Nahas

What is E/M Induction?

- Electromagnetic Induction is the process of using magnetic fields to produce voltage, and in a complete circuit, a current.

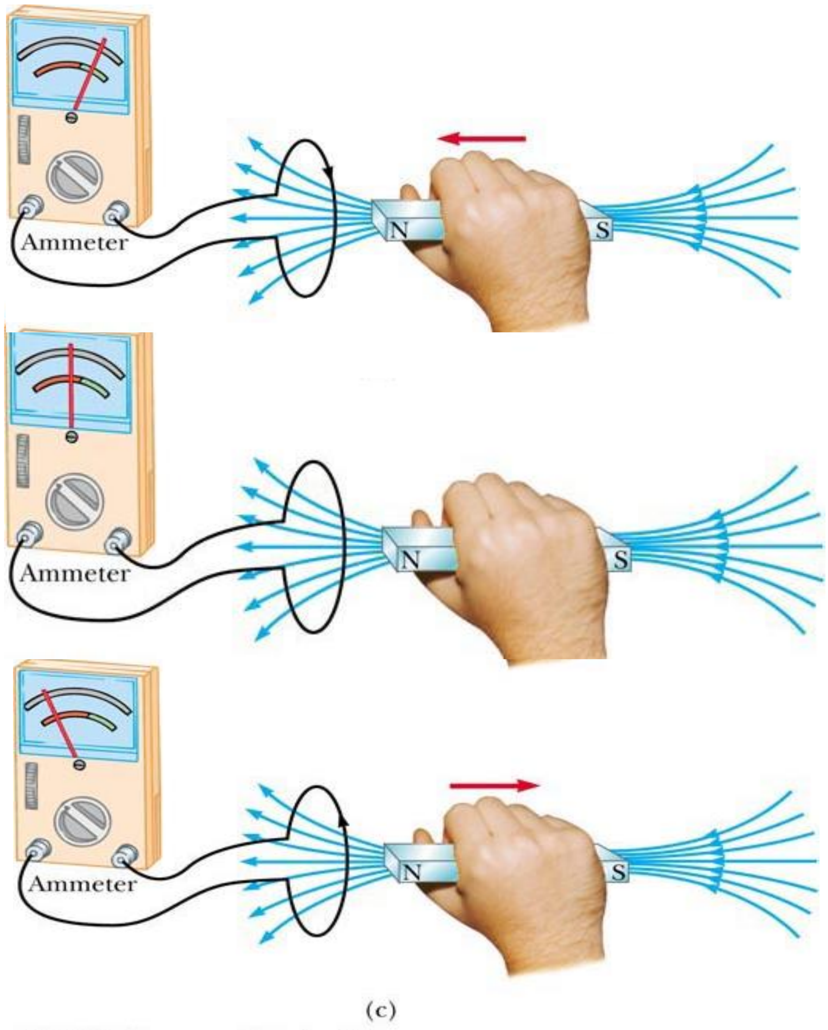


- **Michael Faraday** first discovered it, using some of the works of Hans Christian Oersted. His work started at first using different combinations of wires and magnetic strengths and currents, but it wasn't until he tried moving the wires that he got any success.
- It turns out that electromagnetic induction is created by just that - the moving of a conductive substance through a magnetic field.

https://www.youtube.com/watch?v=tC6E9J925pY

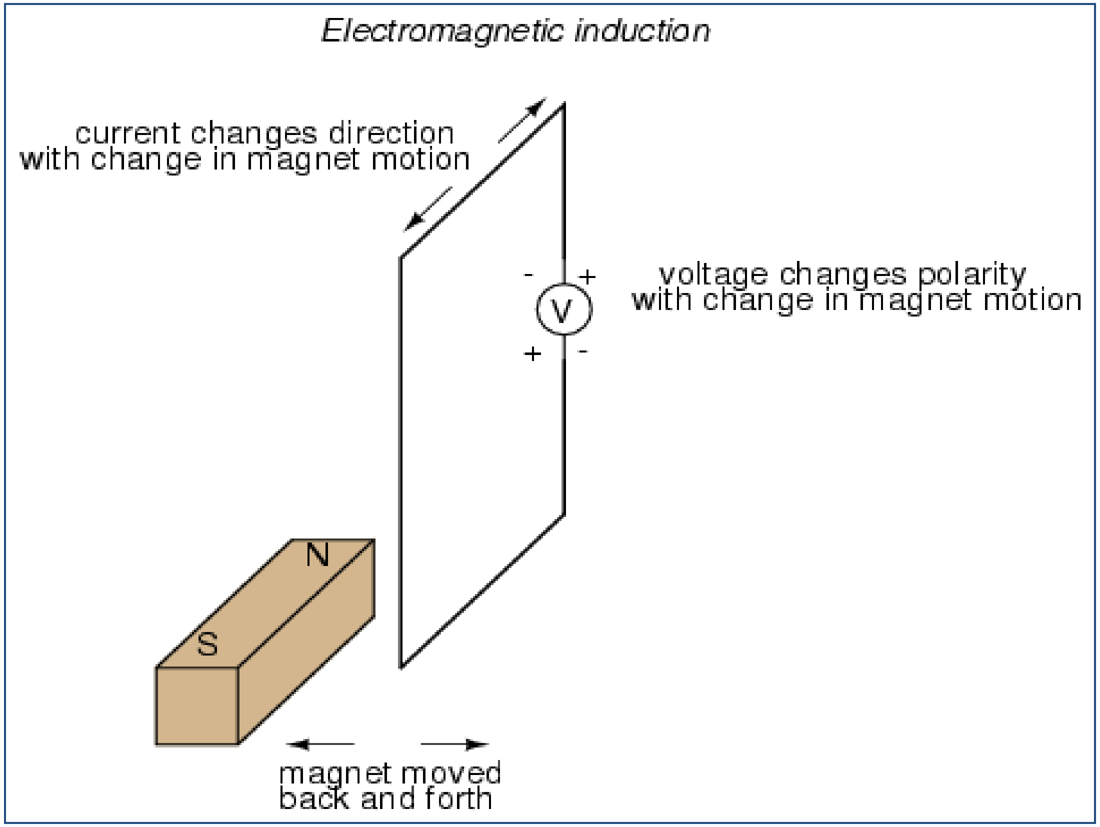
Faraday’s Observations

- When a magnet moves toward a loop of wire, the ammeter shows the presence of a current
- **When the magnet is held stationary, there is no current**
- When the magnet moves away from the loop, the ammeter shows a current in the opposite direction (c)
- **If the loop is moved instead of the magnet, a current is also detected**



Magnetic Induction

- As the magnet **moves** back and forth a current is said to be INDUCED in the wire.



Experimental Conclusions

- **A current is set up in the circuit as long as there is *relative motion* between the magnet and the loop**
  - The same experimental results are found whether the loop moves or the magnet moves
- **The current is called an *induced current* since there is no power source.**
- **An EMF is actually induced by a change in the *magnetic flux.***

Faraday's Law & Electromagnetic Induction

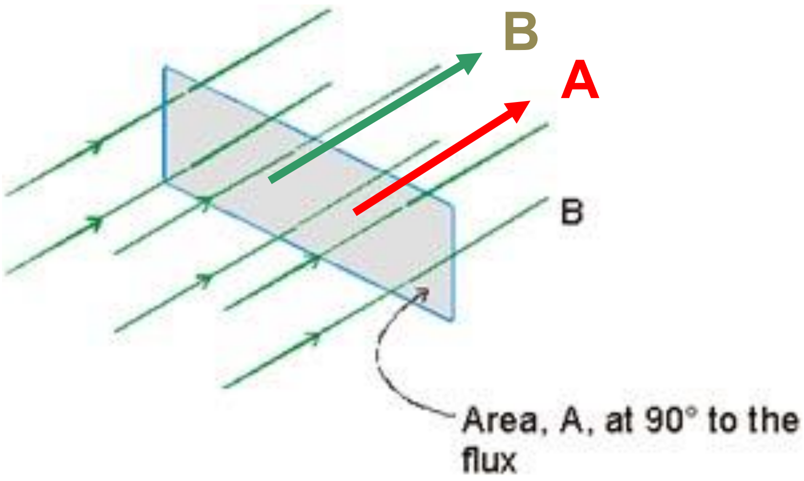
- The instantaneous emf induced in a circuit equals the time rate of change of magnetic flux through the circuit.

- EM induction refers to electricity deriving from magnetism whereas electromagnetism is the opposite.

Magnetic Flux

- The first step to understanding the complex nature of electromagnetic induction is to understand the idea of magnetic flux.

- Flux is a general term associated with a FIELD that is bound by a certain AREA. So MAGNETIC FLUX is any AREA that has a MAGNETIC FIELD passing through it.



- We generally define an AREA vector as one that is perpendicular to the surface of the material. Therefore, you can see in the figure that the AREA vector and the Magnetic Field vector are **PARALLEL**. This then produces a DOT PRODUCT between the 2 variables that then define flux.

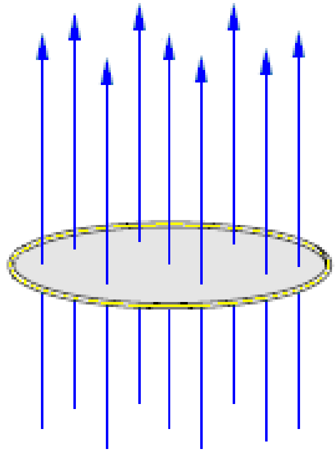
https://www.youtube.com/watch?v=H0177eUFzkM

Magnetic Flux – The DOT product

$$\Phi_B = B \bullet A$$

$$\Phi_B = BA\cos\theta$$

$$\text{Unit : Tm}^2 \text{ or Weber(Wb)}$$



How could we CHANGE the flux over a period of time?

- We could move the magnet away or towards (or the wire)
- We could increase or decrease the area
- We could ROTATE the wire along an axis that is PERPENDICULAR to the field thus changing the angle between the area and magnetic field vectors.

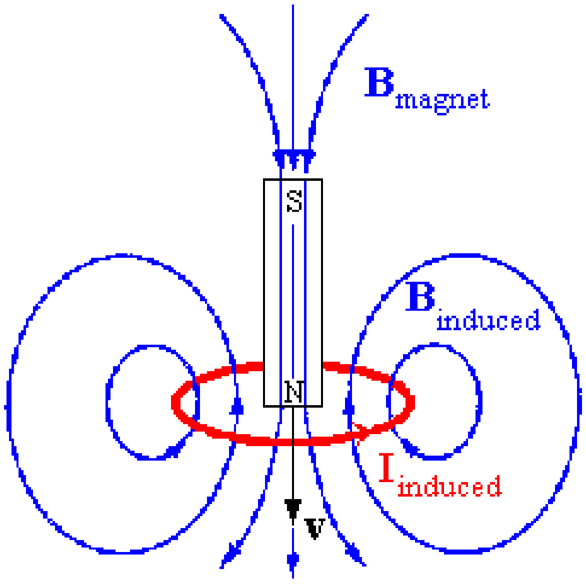
Faraday's Law

- Faraday learned that if you change any part of the flux over time you could induce a current in a conductor and thus create a source of EMF (voltage, potential difference).
- Since we are dealing with time here were a talking about the **RATE of CHANGE of FLUX,** which is called **Faraday's Law.**

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

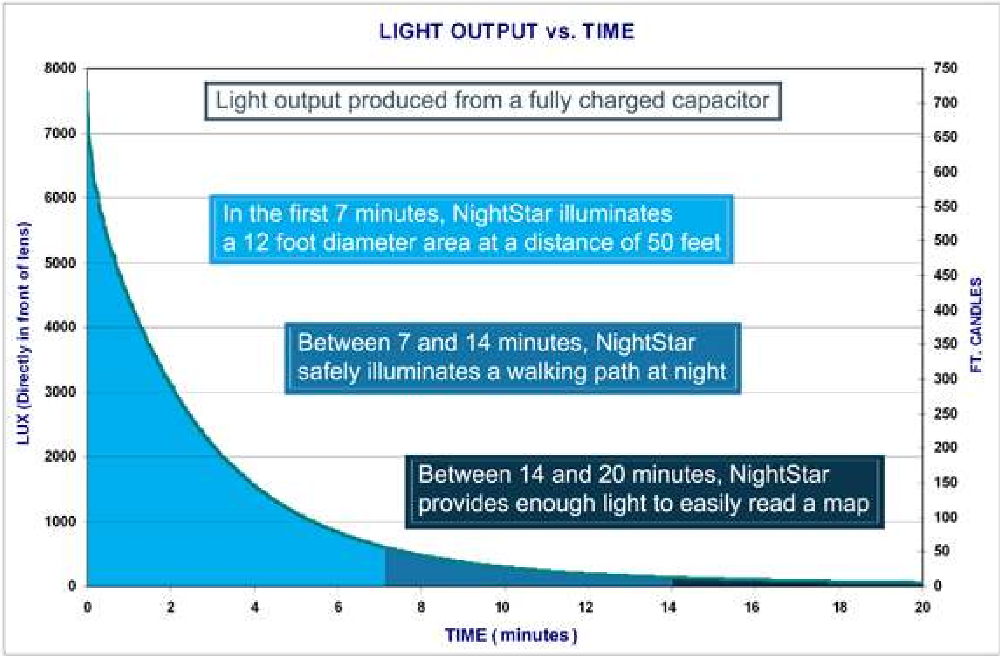
$N = \#$ turns of wire

$$\varepsilon = -\frac{d\Phi_B}{dt} \qquad \Phi_B = -\int \varepsilon \, dt$$



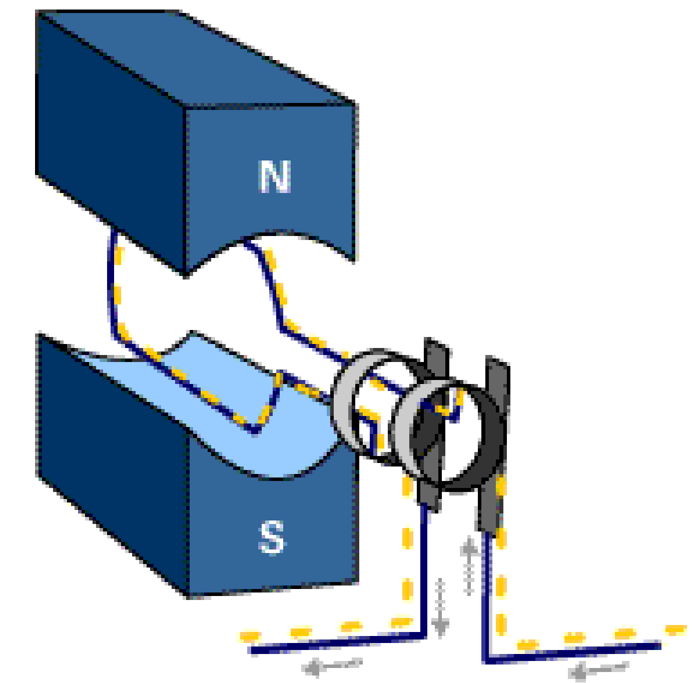
Useful Applications

- The Forever Flashlight uses the Faraday Principle of Electromagnetic Energy to eliminate the need for batteries.
- The Faraday Principle states that if an electric conductor, like copper wire, is moved through a magnetic field, electric current will be generated and flow into the conductor.



Useful Applications

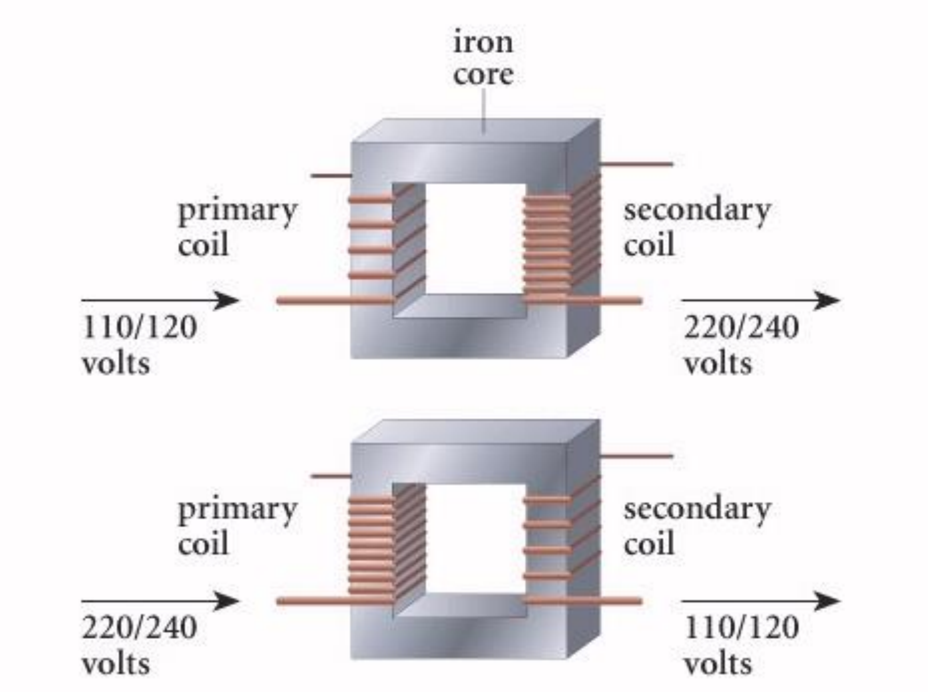
- AC Generators use Faraday's law to produce rotation and thus convert electrical and magnetic energy into rotational kinetic energy.
- This idea can be used to run all kinds of motors.
- Since the current in the coil is AC, it is turning on and off thus creating a CHANGING magnetic field of its own. Its own magnetic field interferes with the shown magnetic field to produce rotation.



Transformers

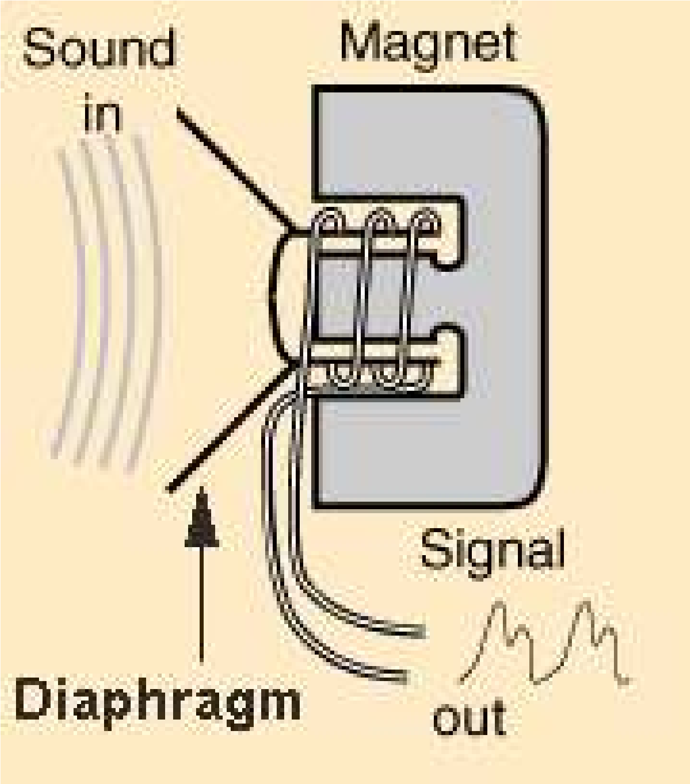
- Probably one of the greatest inventions of all time is the transformer. AC Current from the primary coil moves quickly BACK and FORTH (thus the idea of changing!) across the secondary coil. The moving magnetic field caused by the changing field (flux) induces a current in the secondary coil.

- If the secondary coil has MORE turns than the primary you can step up the voltage and runs devices that would normally need MORE voltage than what you have coming in. We call this a **STEP UP** transformer.
- We can use this idea in reverse as well to create a **STEP DOWN** transformer.



Microphones

- A microphone works when sound waves enter the filter of a microphone.
- Inside the filter, a diaphragm is vibrated by the sound waves which in turn moves a coil of wire wrapped around a magnet.
- The movement of the wire in the magnetic field induces a current in the wire.
- Thus sound waves can be turned into electronic signals and then amplified through a speaker.



Example

- A coil with 200 turns of wire is wrapped on an 18.0 cm square frame. Each turn has the same area, equal to that of the frame, and the total resistance of the coil is 2.0Ω . A uniform magnetic field is applied perpendicularly to the plane of the coil. If the field changes uniformly from 0 to 0.500 T in 0.80 s, **find the magnitude of the induced emf in the coil while the field has changed as well as the magnitude of the induced current.**

$$|\varepsilon| = N\frac{\Delta\Phi_B}{\Delta t} = N\frac{\Delta BA\cos\theta}{\Delta t}$$

$$|\varepsilon| = 200\frac{(0.500-0)(0.18x0.18)\cos 0}{0.80}$$

$|\varepsilon| =$ **4.05 V**

- **Why did you find the ABSOLUTE VALUE of the EMF?**
- **What happened to the "–" that was there originally?**

$$\varepsilon = IR = I(2)$$

$I =$ **2.025 A**

Lenz's Law

- **Lenz's law** gives the **direction** of the induced emf and current resulting from electromagnetic induction.
- The law provides a physical interpretation of the **choice of sign** in Faraday's law of induction, indicating that the induced emf and the change in flux have **opposite signs.**

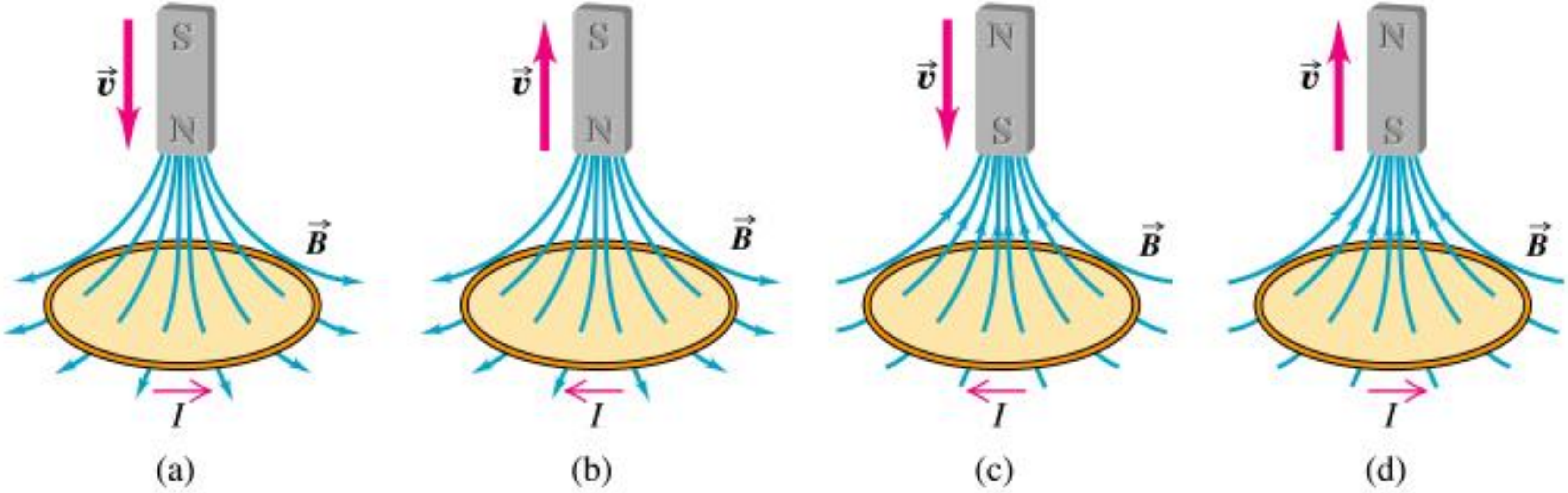
$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

Lenz's Law (the minus sign)

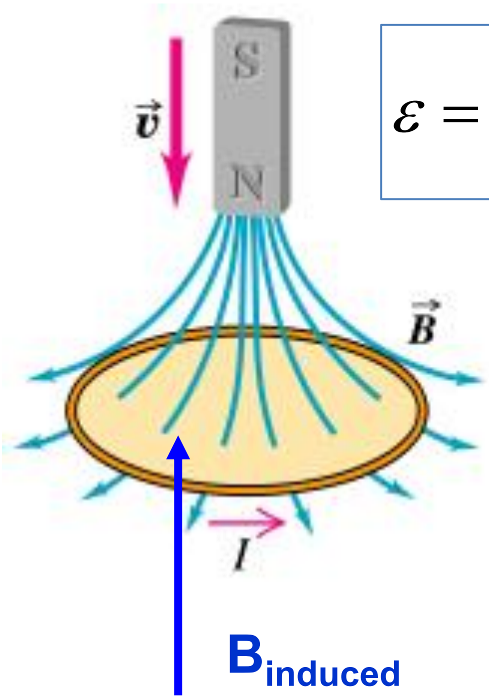
https://www.youtube.com/watch?v=2_M83gNOOEg

Lenz's Law

- In the figure below, we see that the direction of the current changes.
- Lenz's Law helps us determine the **DIRECTION** of that current.



Lenz's Law & Faraday's Law



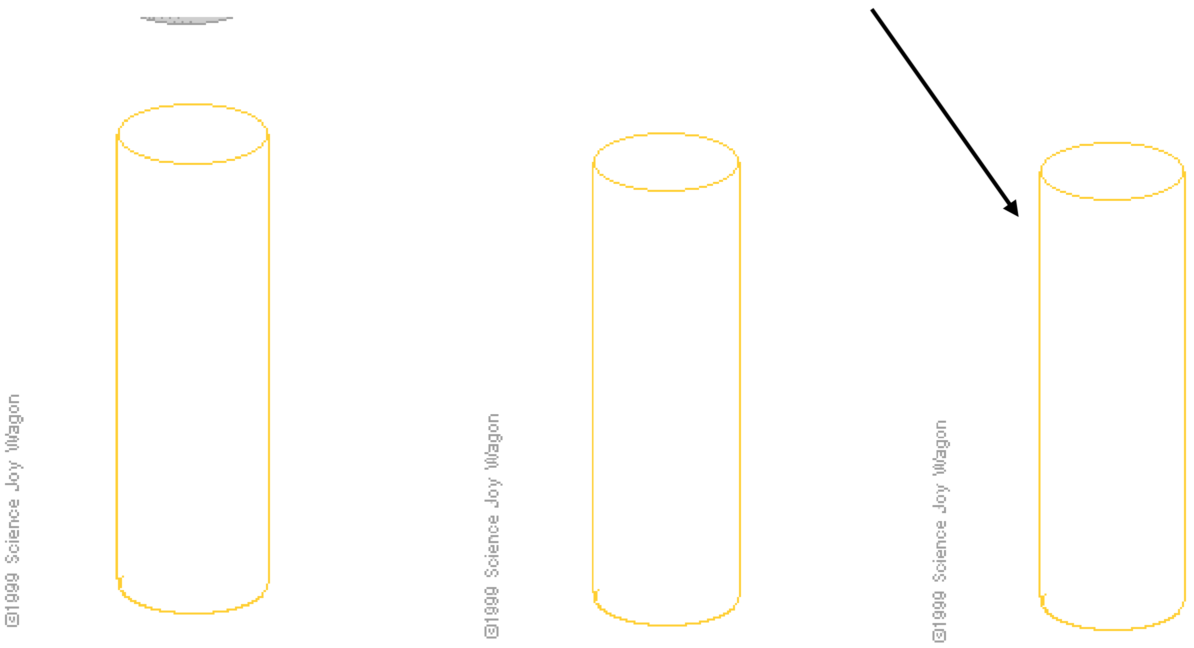
$$\varepsilon = -N \frac{\Delta \Phi_B}{\Delta t}$$

- Let's consider a magnet with its north pole moving TOWARDS a conducting loop.
- DOES THE FLUX CHANGE? **Yes!**
- DOES THE FLUX INCREASE OR DECREASE? **Increase**
- WHAT SIGN DOES THE "Δ" GIVE YOU IN FARADAY'S LAW? **Positive**
- DOES LENZ'S LAW CANCEL OUT? **NO**
- **What does this mean?**

- This means that the INDUCED MAGNETIC FIELD around the WIRE caused by the moving magnet OPPOSES the original magnetic field. Since the original B field is downward, the induced field is upward! We then use the curling right hand rule to determine the direction of the current.

Lenz's Law

- The INDUCED current creates an INDUCED magnetic field of its own inside the conductor that opposes the original magnetic field.



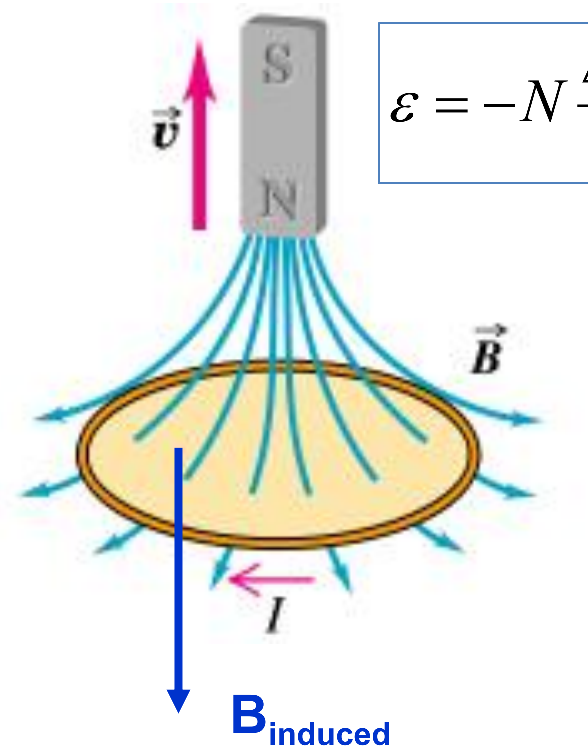
Since the induced field opposes the direction of the original it attracts the magnet upward slowing the motion caused by gravity downward.

A magnet is dropped down a conducting tube.

The magnet INDUCES a current above and below the magnet as it moves.

If the motion of the magnet were NOT slowed this would violate conservation of energy!

Lenz's Law



$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

- Let's consider a magnet with its north pole moving AWAY from a conducting loop.
- DOES THE FLUX CHANGE? **Yes!**
- DOES THE FLUX INCREASE OR DECREASE? **Decreases**
- WHAT SIGN DOES THE "Δ" GIVE YOU IN FARADAY'S LAW? **Negative**
- DOES LENZ'S LAW CANCEL OUT? **Yes**
- **What does this mean?**

In this case, the induced field DOES NOT oppose the original and points in the same direction. Once again use your curled right hand rule to determine the DIRECTION of the current.

In summary

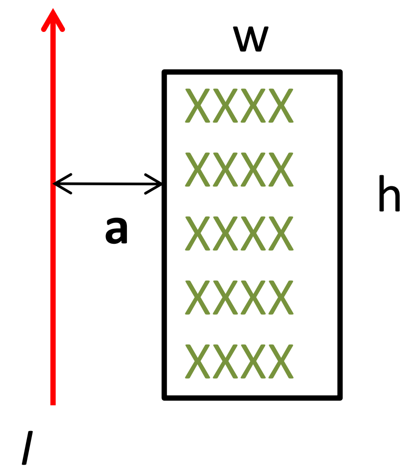
- **Faraday's Law** is basically used to find the MAGNITUDE of the induced EMF. The magnitude of the current can then be found using Ohm's Law provided we know the conductor's resistance.

- **Lenz's Law** is part of Faraday's Law and can help you determine the direction of the current provided you know HOW the flux is changing

Example

- **A long, straight wire carrying a current, I is placed near a loop of dimensions w and h as shown. Calculate the magnetic flux for this loop.**

What is the direction of the magnetic field inside the loop due to the current carrying wire? **Into the page**



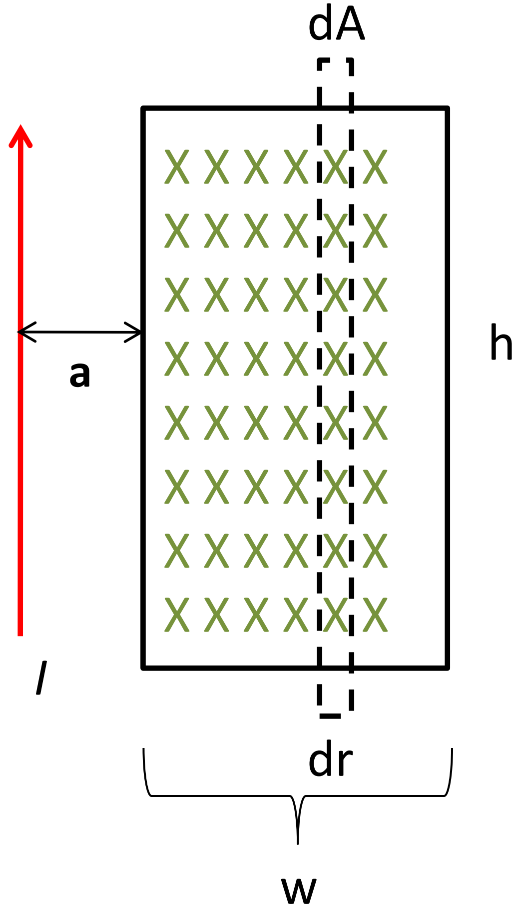
$$\Phi = BA\cos\theta \quad B_{wire} = \frac{\mu_o I}{2\pi a} \quad A = wh$$

$$\Phi = \frac{\mu_o I w h}{2\pi a}$$

BUT…here is the problem. The spacial uniformity IS NOT the same as you move away from the wire. The magnetic field CHANGES, or in this case decreases, as you move away from the wire the FLUX changes.
So the formula above does NOT illustrate the correct function for the flux.

Example

- You begin by taking a slice of the area. In other words, begin with a differential amount of AREA, ***dA***, that is a differential amount of distant wide, which we will call, ***dr***.
- Then we must think about our limits. We need to SUM all of the area starting at "a" and going to "w+a".



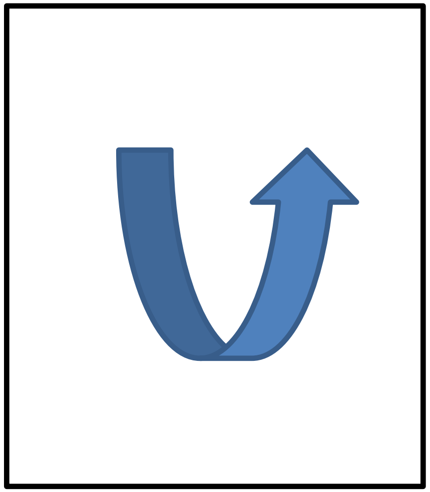
$$\Phi = BA\cos\theta \quad B_{wire} = \frac{\mu_o I}{2\pi r} \quad \Phi = \frac{\mu_o I}{2\pi r} dA \quad dA = hdr$$

$$\Phi = \int_a^{w+a} \frac{\mu_o I h}{2\pi r} dr \rightarrow \frac{\mu_o I h}{2\pi} \int_a^{w+a} \frac{1}{r} dr$$

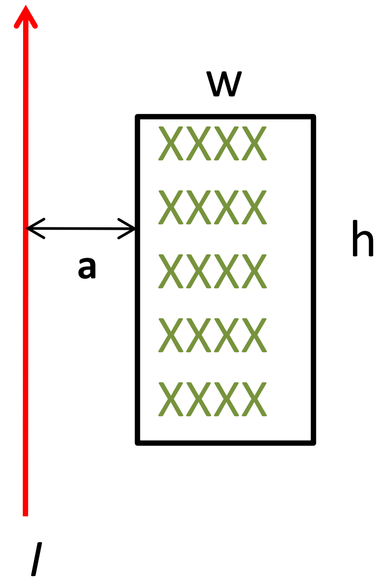
$$\Phi = \frac{\mu_o I h}{2\pi} \ln(\frac{w+a}{a})$$

Example

- **If the loop is moving TOWARDS the wire, what is the direction of the "induced" current around the loop?**



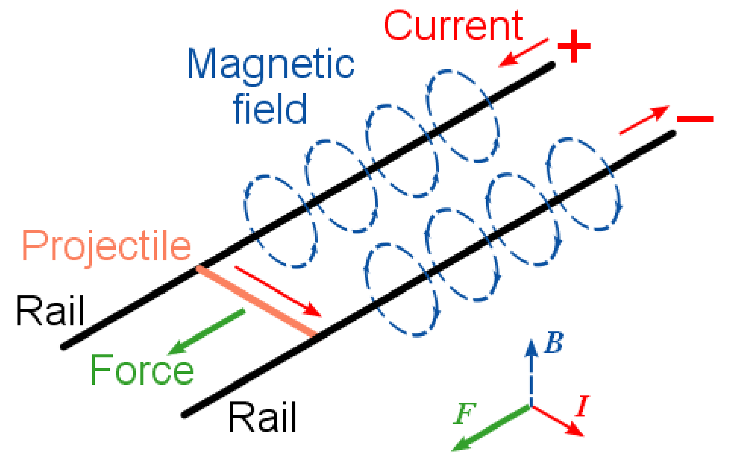
Since the original field is INTO THE PAGE and the FLUX INCREASES, the negative sign (Lenz's Law) in Faraday's Law remains and therefore the induced field is in the opposite direction to oppose the change, which is OUT OF THE PAGE. This produces a current which is ***counter-clockwise*** around the loop



Motional EMF – The Rail Gun

- A railgun consists of two parallel metal rails (hence the name) connected to an electrical power supply. When a conductive projectile is inserted between the rails (from the end connected to the power supply), it completes the circuit. Electrons flow from the negative terminal of the power supply up the negative rail, across the projectile, and down the positive rail, back to the power supply.

- In accordance with the right-hand rule, the magnetic field circulates around each conductor. Since the current is in opposite direction along each rail, the net magnetic field between the rails (**B**) is directed vertically. In combination with the current (**I**) across the projectile, this produces a magnetic force which accelerates the projectile along the rails. There are also forces acting on the rails attempting to push them apart, but since the rails are firmly mounted, they cannot move. The projectile slides up the rails away from the end with the power supply.



Motional Emf

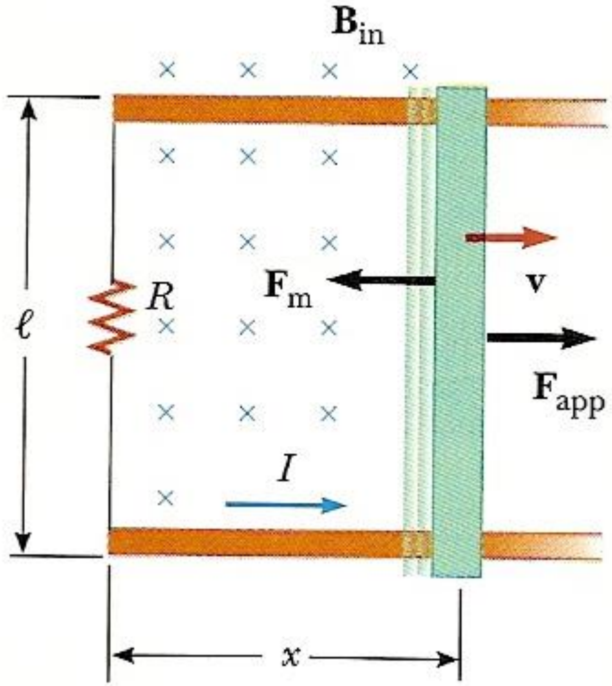
- There are many situations where motional EMF can occur that are different from the rail gun. Suppose a bar of length, **ℓ**, is pulled to right at a speed, **v**, in a magnetic field, ***B***, directed into the page. The conducting rod itself completes a circuit across a set of parallel conducting rails with a resistor mounted between them.

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

$$\varepsilon = \frac{BA}{t} \rightarrow \frac{Blx}{t}; \quad \boxed{\varepsilon = Blv}$$

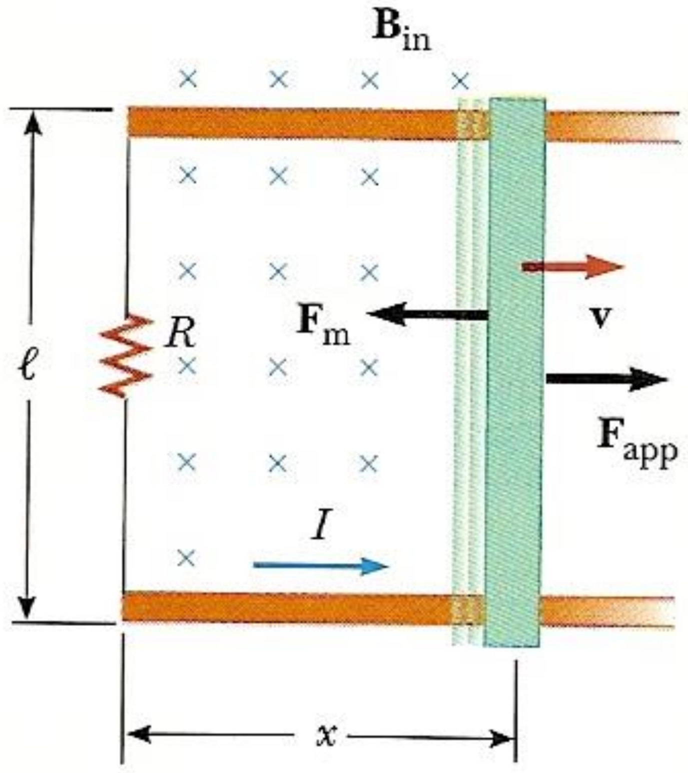
$$\varepsilon = IR$$

$$I = \frac{Blv}{R}$$



Motional EMF

- In the figure, we are applying a force this time to the rod.
- Due to Lenz's Law the magnetic force opposes the applied force.
- Since we know that the magnetic force acts to the left and the magnetic field acts into the page, we can use the RHR to determine the direction of the current around the loop and the resistor.



Example

- An airplane with a wing span of 30.0 m flies parallel to the Earth's surface at a location where the downward component of the Earth's magnetic field is 0.60 x10^{-4} T. **Find the difference in potential between the wing tips if the speed of the plane is 250 m/s.**

$$\varepsilon = Blv$$

$$\varepsilon = 0.60x10^{-4}(30)(250)$$

$\varepsilon =$ **0.45 V**

In 1996, NASA conducted an experiment with a 20,000-meter conducting tether. When the tether was fully deployed during this test, the orbiting tether generated a potential of 3,500 volts. This conducting single-line tether was severed after five hours of deployment. It is believed that the failure was caused by an electric arc generated by the conductive tether's movement through the Earth's magnetic field.

Inductance

- After investigating with Faraday's Law, we see that the magnetic flux is DIRECTLY related to the current. The proportionality constant in this case is called INDUCTANCE, L, which is a type of magnetic resistance.
- The unit of inductance is the **HENRY, H**.

$$\Phi_B \; \alpha \; I$$

$$L = \text{constant of proportionality}$$

$$\Phi_B = LI$$

If you divide both side by time we get:

$$\frac{\Phi_B}{t} = L\frac{I}{t} \qquad \varepsilon = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

$$\varepsilon = -L\frac{dI}{dt} \quad \Rightarrow \quad L = -\frac{\varepsilon}{dI/dt}$$

Inductance

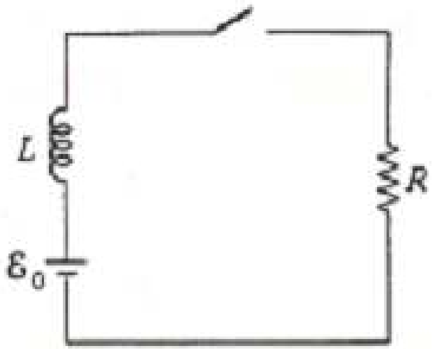
$$L = \frac{\Phi_B}{I} = \text{self-inductance}$$

$$\varepsilon = -L\frac{dI}{dt} = \text{Back EMF}$$

- So what happens when we hook up a giant coil of wire to a circuit? We throw the switch and the current flows. The circuit will try to resist the change in flux as a result of the current.
- This is called BACK EMF! Usually the back EMF is very small so we don't need to worry about it.
- BUT, if there is a coil of wire the effect is VERY STRONG! If a current creates a magnetic flux in any circuit element we define this as SELF INDUCTANCE, L. The unit for inductance is Henry.

What this tells us is HOW LARGE an INDUCED EMF we can expect across the coils of an inductor per change in current per unit time.

Inductors in a circuit



Using Kirchhoff's voltage law we have:

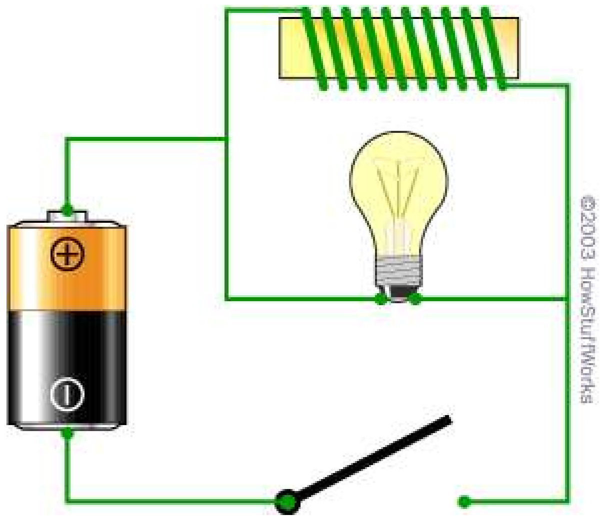
$$\varepsilon_o - L\frac{dI}{dt} - IR = 0$$

Multiply by I

$$\varepsilon_o I - LI\frac{dI}{dt} - I^2 R = 0,\ P = \frac{dU}{dt}$$

$$U_B = \int_0^t LI\frac{dI}{dt}\,dt = \int_0^I LI\,dI = L\int_0^I I\,dI$$

$$U_B = \frac{1}{2}LI^2$$

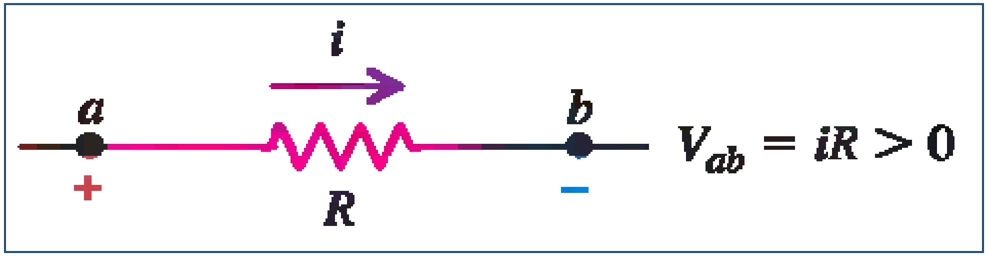


What we have now is MAGNETIC ENERGY stored in an INDUCTOR!

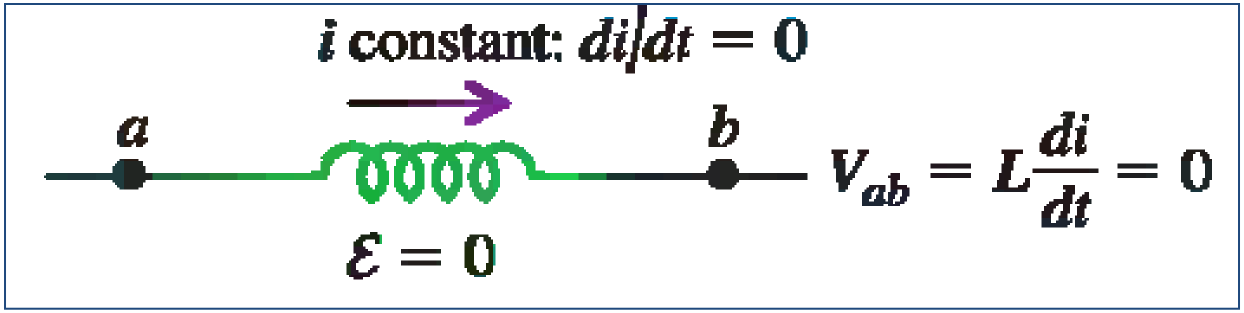
This is very similar to a capacitor storing charge and producing electrical potential energy.

Potential across an Inductor

- The potential difference across a resistor depends on the current.
  - Resistor with current i flowing from a to b; potential drops from a to b.

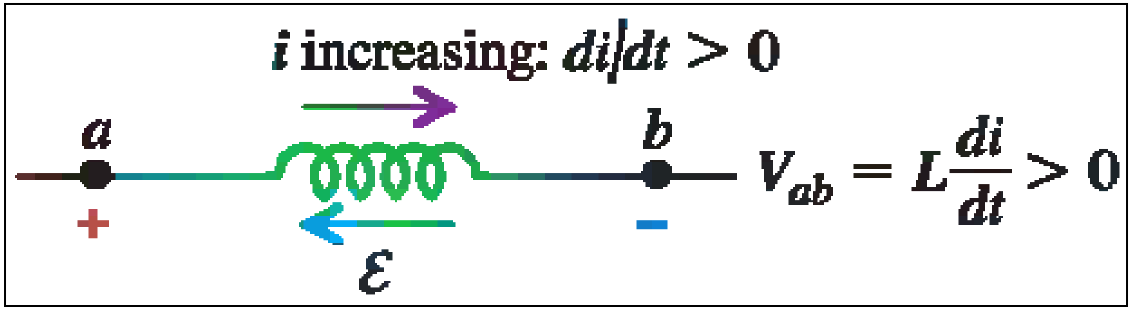


- The potential difference across an inductor depends on the rate of change of the current.
  - Inductor with constant current i flowing from a to b; no potential difference.

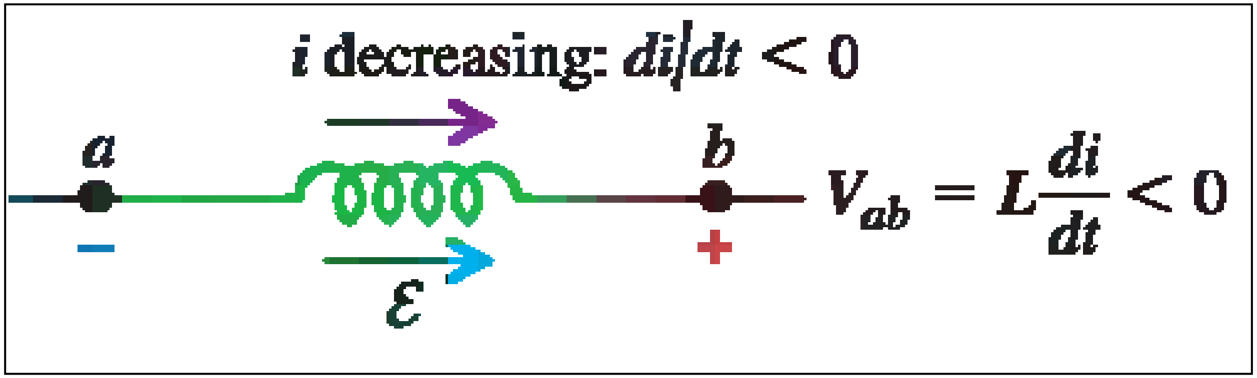


Potential across an Inductor

– Inductor with increasing current I flowing from a to b; potential drops from a to b.



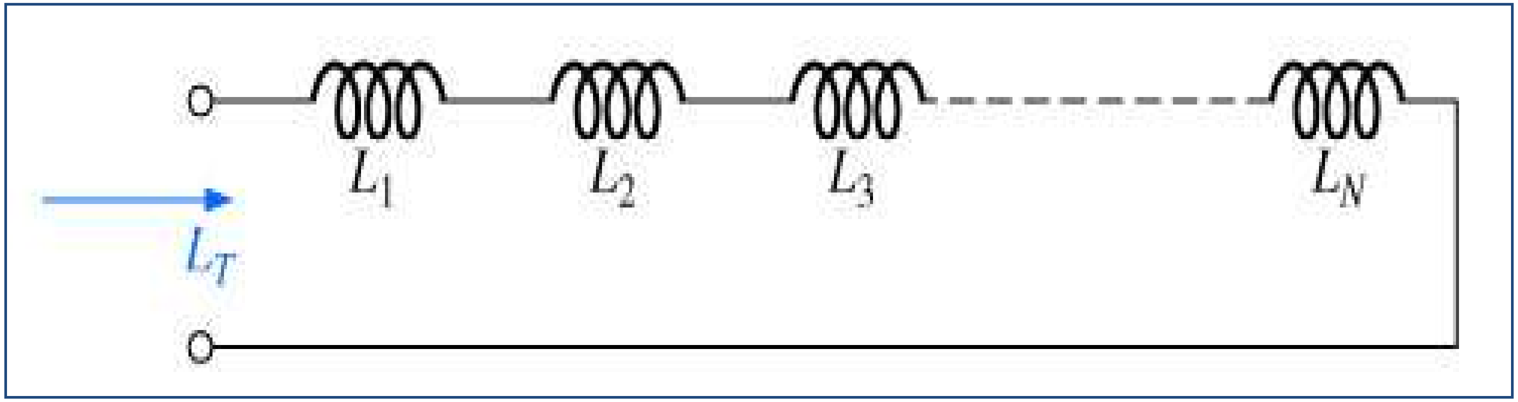
– Inductor with decreasing current I flowing from a to b; potential increases from a to b.



Inductors in Series

- Inductors, like resistors and capacitors, can be placed in series.
  - Total inductance can be increased by placing inductors in series.

$$L_T = L_1 + L_2 + L_3 + . . . + L_N$$

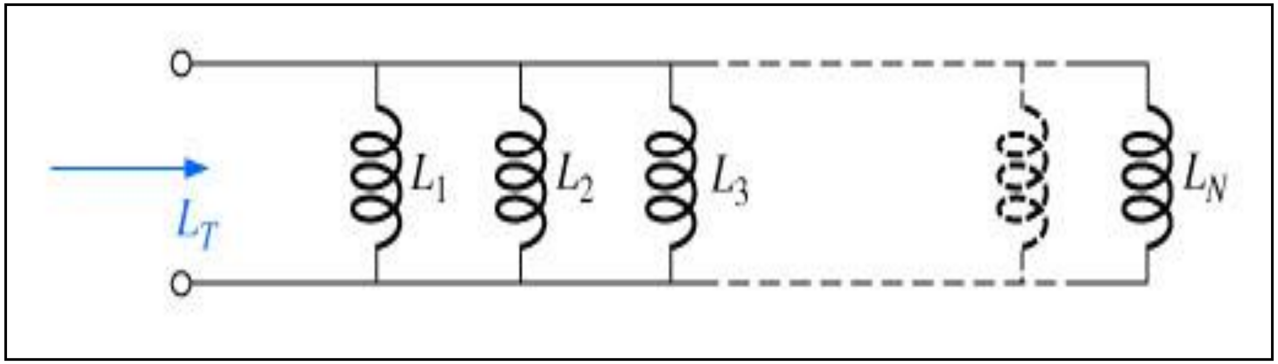


Inductors in Parallel

- Inductors, like resistors and capacitors, can be placed in parallel.
  - Total inductance can be decreased by placing inductors in parallel.

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots + \frac{1}{L_N}$$

$$L_T = \left(L_1^{-1} + L_2^{-1} + L_3^{-1} + \ldots + L_N^{-1}\right)^{-1}$$



15FELE121
Electromagnetism Fundamentals

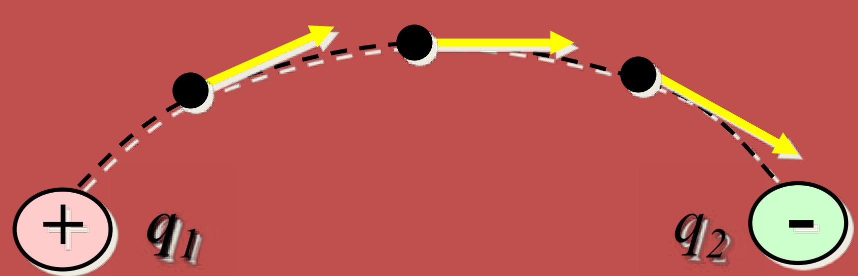
Chapter 7 – Electromagnetic Wave Propagation

Dr. Michel Nahas

Maxwell's Theory

Electromagnetic theory developed by James Maxwell (1831 – 1879) is based on four concepts:

1. Electric fields E begin on positive charges and end on negative charges and Coulomb's law can be used to find the field E and the force on a given charge.

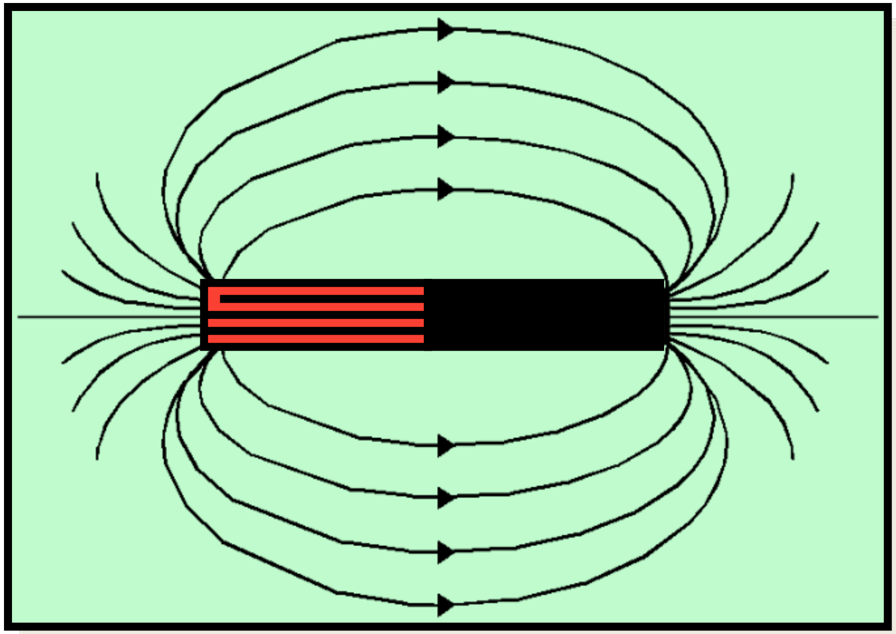


$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$F = qE$$

Maxwell's Theory (Cont.)

2. Magnetic field lines Φ do not begin or end, but rather consist of entirely closed loops.



$$B = \frac{\Phi}{A \sin\theta}$$

$$B = \frac{F}{qv \sin\theta}$$

Maxwell's Theory (Cont.)

3. A changing magnetic field ΔB induces an emf and therefore an electric field E (Faraday's Law).

Faraday's Law:

$$\mathcal{E} = -N \frac{\Delta \Phi}{\Delta t}$$

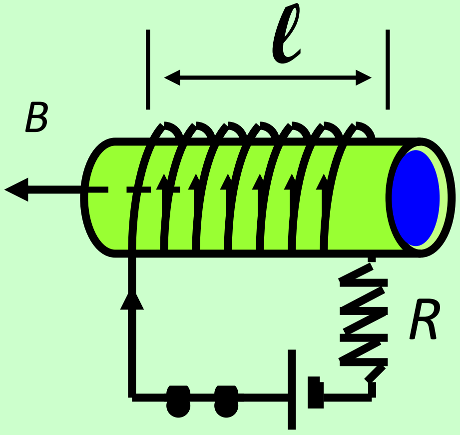
A change in flux $\Delta\Phi$ can occur by a change in area or by a change in the B-field:

$\Delta\Phi = B\,\Delta A$ $\quad\quad$ $\Delta\Phi = A\,\Delta B$

Maxwell's Theory (Cont.)

4. Moving charges (or an electric current) induce a magnetic field B.

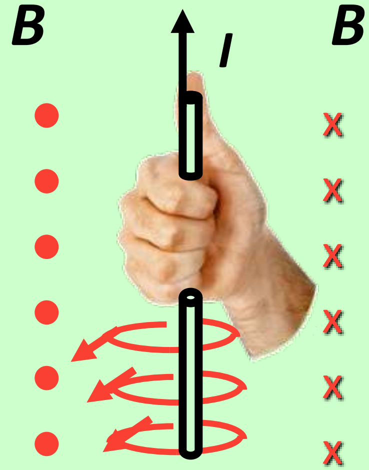
Solenoid



Inductance *L*

Current *I* induces *B* field

$$B = \frac{\mu_0 N I}{\ell}$$



Lenz's law

Electromagnetic Wave

- In general, **waves** are means of transporting energy or information
- Typical examples of EM waves include radio waves, TV signals, radar beams, and light rays
- All forms of EM energy share three fundamental characteristics:
  - They all travel at high velocity
  - In traveling, they assume the properties of waves
  - They radiate outward from a source
- Wave motion occurs when a disturbance at point A, at time t_o, is related to what happens at point *B*, at time $t > t_0$

https://www.youtube.com/watch?v=fZnYE3kvhhA

Electric Wave

- An electric wave can be expressed as following:

$$E = A \sin(\omega t - \beta z)$$

- Note the following characteristics of the wave:

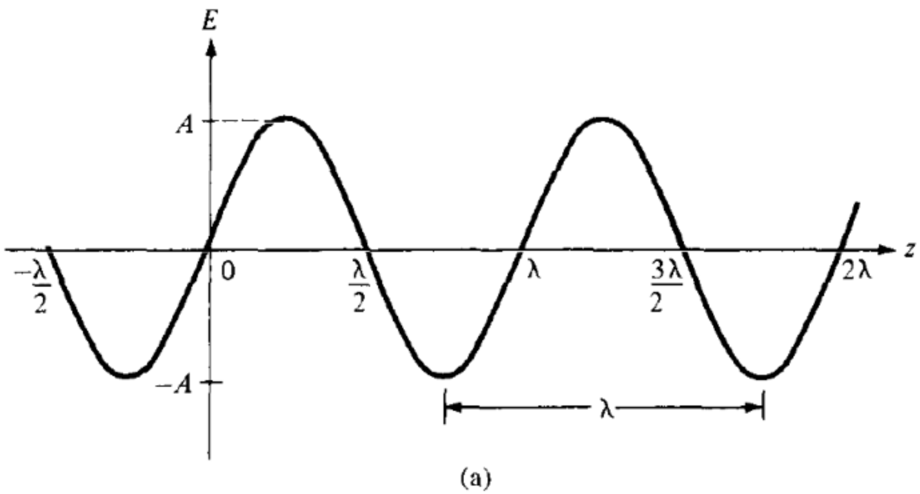
1. It is time harmonic and travels at the speed *u*
2. *A* is called the *amplitude* of the wave and has the same units as *E*
3. *(wt -* βz) is the *phase* (in radians) of the wave; it depends on time *t* and space variable *z*
4. w is the *angular frequency* (in radians/second); β is the *phase constant* or *wave number* (in radians/meter)

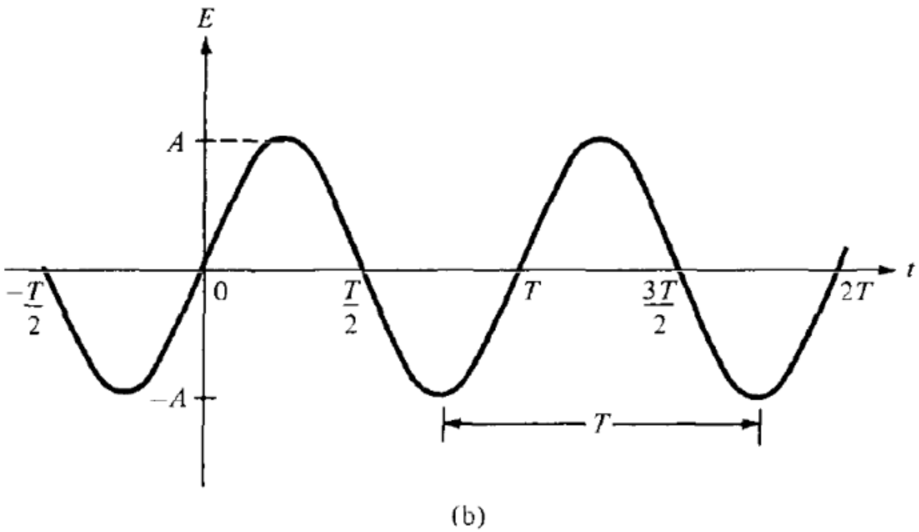
$$\omega = 2\pi f$$

$$\beta = \frac{\omega}{u}$$

$$u = \frac{\omega}{\beta}$$

Electric Wave





- Plot of *E(z, t)* with constant *t:*
- The wave takes distance λ to repeat itself
- λ is called the *wavelength* (in meters)

- Plot of *E(z, t)* with constant *z:*
- The wave takes time *T* to repeat itself
- *T* is known as the *period* (in seconds)

Electric Wave

- Since it takes time T for the wave to travel distance λ at the speed u, we get:

$$\lambda = uT$$

- But T = 1/f, where f is the frequency (the number of cycles per second) of the wave in Hertz (Hz):

$$u = f\lambda \quad \text{and} \quad \beta = \frac{2\pi}{\lambda}$$

- Every wavelength of distance traveled, a wave undergoes a phase change of 2π radians

Electric Wave

$$E = A \sin(\omega t - \beta z)$$

- The negative sign in *(wt - βz)* is associated with a wave propagating in the *+z* direction (forward traveling or positive-going wave)
- Whereas, a positive sign in *(wt + βz)* indicates that a wave is traveling in the *-z* direction (backward traveling or negative going wave)
- In free space, the wave travels at the speed of light c:

$$u = c$$

Example: Electric Wave

The electric field in free space is given by

$$\mathbf{E} = 50 \cos(10^8 t + \beta x)\, \mathbf{a}_y \text{ V/m}$$

(a) Find the direction of wave propagation.

(b) Calculate β and the time it takes to travel a distance of $\lambda/2$.

(c) Sketch the wave at $t = 0$, $T/4$, and $T/2$.

Solution:

(a) From the positive sign in $(\omega t + \beta x)$, we infer that the wave is propagating along $-\mathbf{a}_x$. This will be confirmed in part (c) of this example.

Example: Electric Wave

Solution:

(b) In free space, $u = c$.

$$\beta = \frac{\omega}{c} = \frac{10^8}{3 \times 10^8} = \frac{1}{3}$$

or

$$\beta = 0.3333 \text{ rad/m}$$

If T is the period of the wave, it takes T seconds to travel a distance λ at speed c. Hence to travel a distance of $\lambda/2$ will take

$$t_1 = \frac{T}{2} = \frac{1}{2}\frac{2\pi}{\omega} = \frac{\pi}{10^8} = 31.42 \text{ ns}$$

Example: Electric Wave

Solution:

(c) At $t = 0, \quad E_y = 50 \cos \beta x$

At $t = T/4, E_y = 50 \cos\left(\omega \cdot \frac{2\pi}{4\omega} + \beta x\right) = 50 \cos(\beta x + \pi/2)$

$= -50 \sin \beta x$

At $t = T/2, E_y = 50 \cos\left(\omega \cdot \frac{2\pi}{2\omega} + \beta x\right) = 50 \cos(\beta x + \pi)$

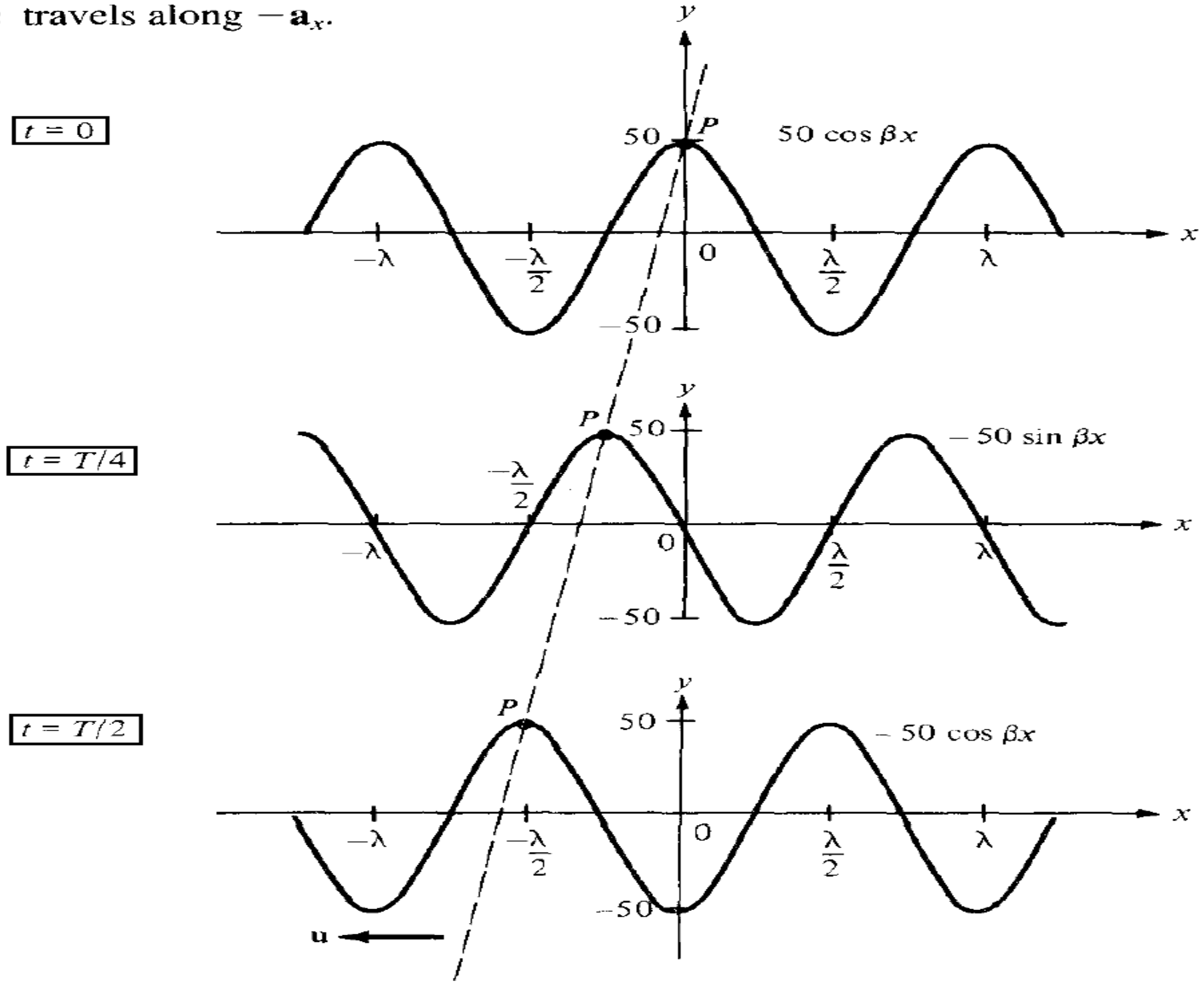
$= -50 \cos \beta x$

E_y at $t = 0, T/4, T/2$ is plotted against x as shown in Figure 10.3. Notice that a point P (arbitrarily selected) on the wave moves along $-\mathbf{a}_x$ as t increases with time. This shows that the wave travels along $-\mathbf{a}_x$.

Example: Electric Wave

Solution:

(c) wave travels along $-\mathbf{a}_x$.



EM Wave

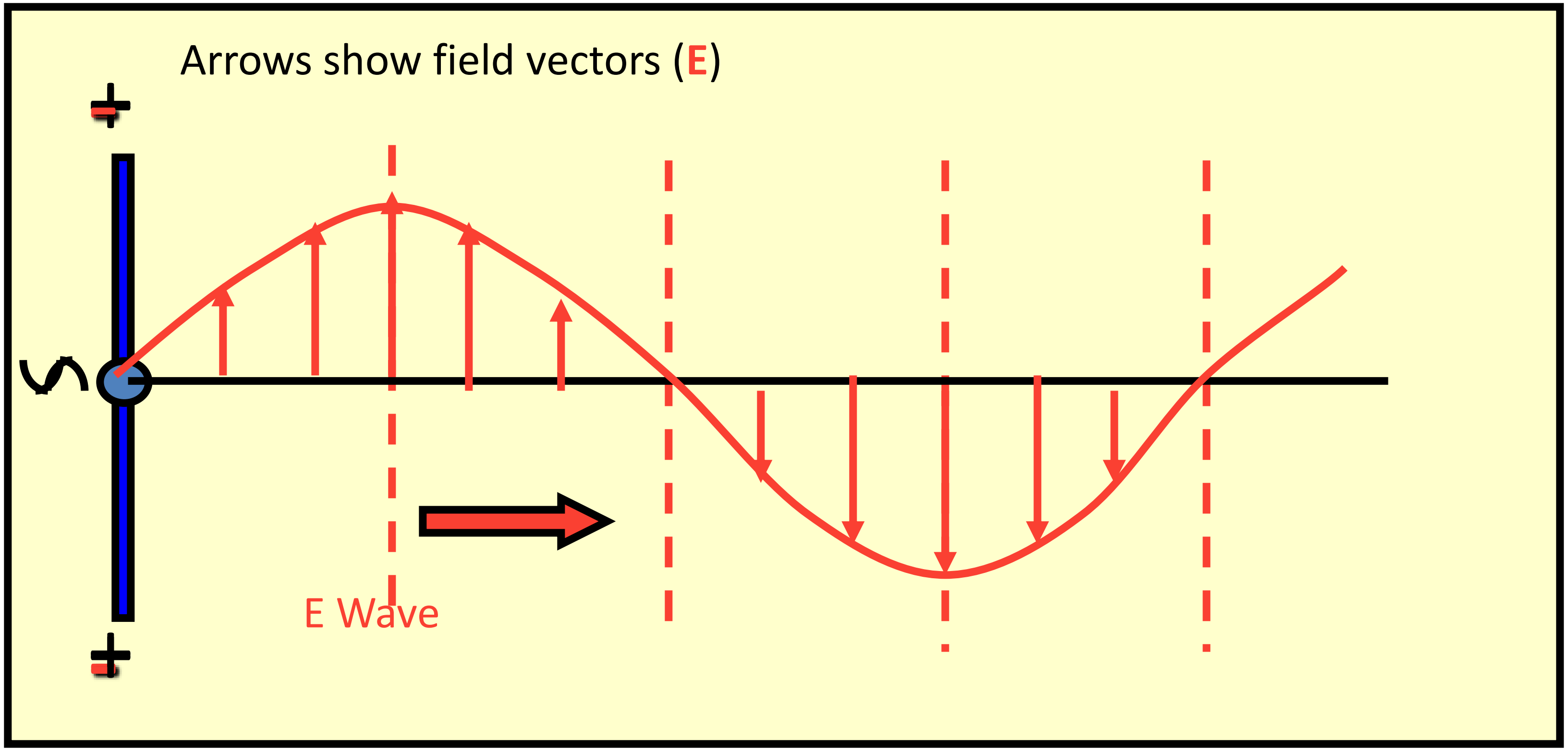
- EM waves may propagate in the following media:

1. Free space ($\sigma = 0, \varepsilon = \varepsilon_o, \mu = \mu_o$)
2. Lossless dielectrics ($\sigma = 0, \varepsilon = \varepsilon_r\varepsilon_o, \mu = \mu_r\mu_o$, or $\sigma \ll \omega\varepsilon$)
3. Lossy dielectrics ($\sigma \neq 0, \varepsilon = \varepsilon_r\varepsilon_o, \mu = \mu_r\mu_o$)
4. Good conductors ($\sigma \simeq \infty, \varepsilon = \varepsilon_o, \mu = \mu_r\mu_o$, or $\sigma \gg \omega\varepsilon$)

- A **lossy dielectric** is a medium in which an EM wave loses power as it propagates due to poor conduction.

Production of an Electric Wave

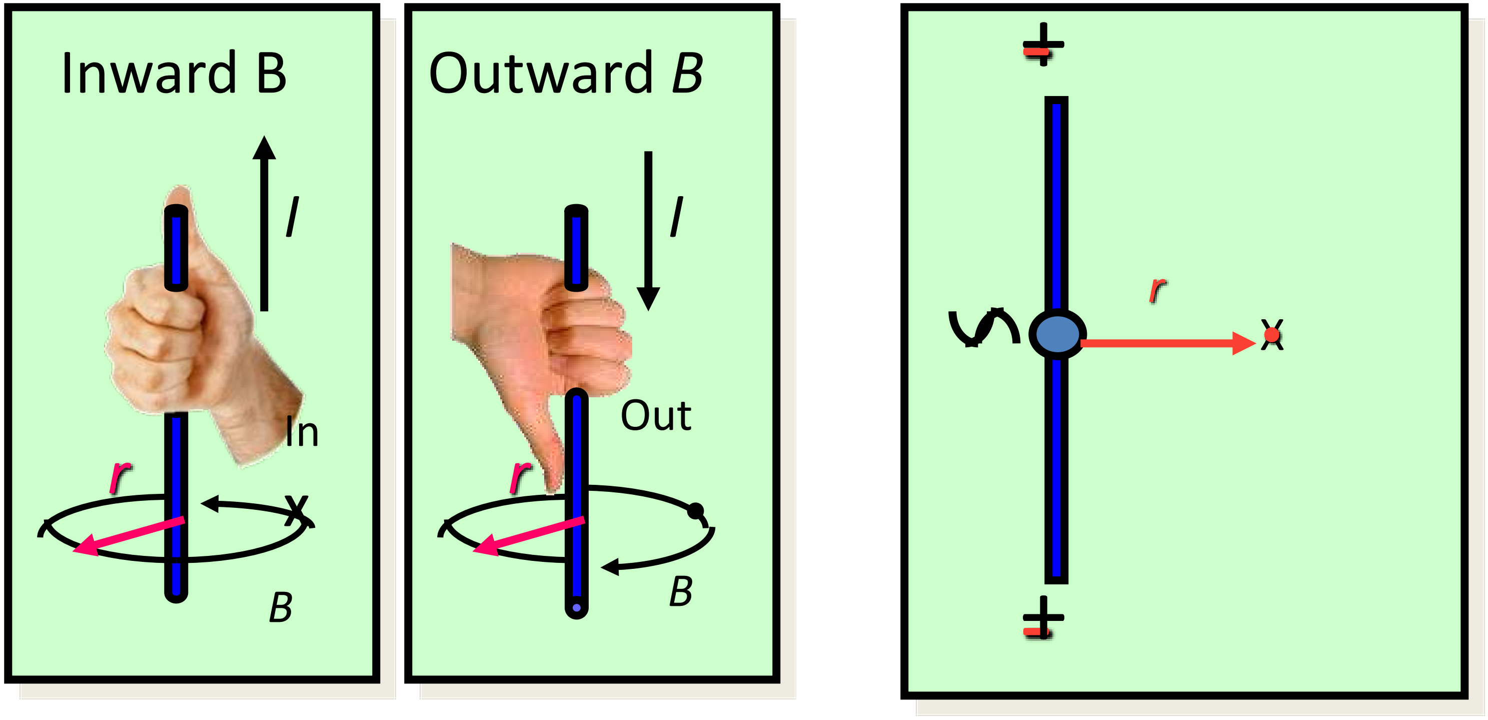
- Consider two metal rods connected to an ac source with sinusoidal current and voltage.



Vertical transverse sinusoidal E-waves.

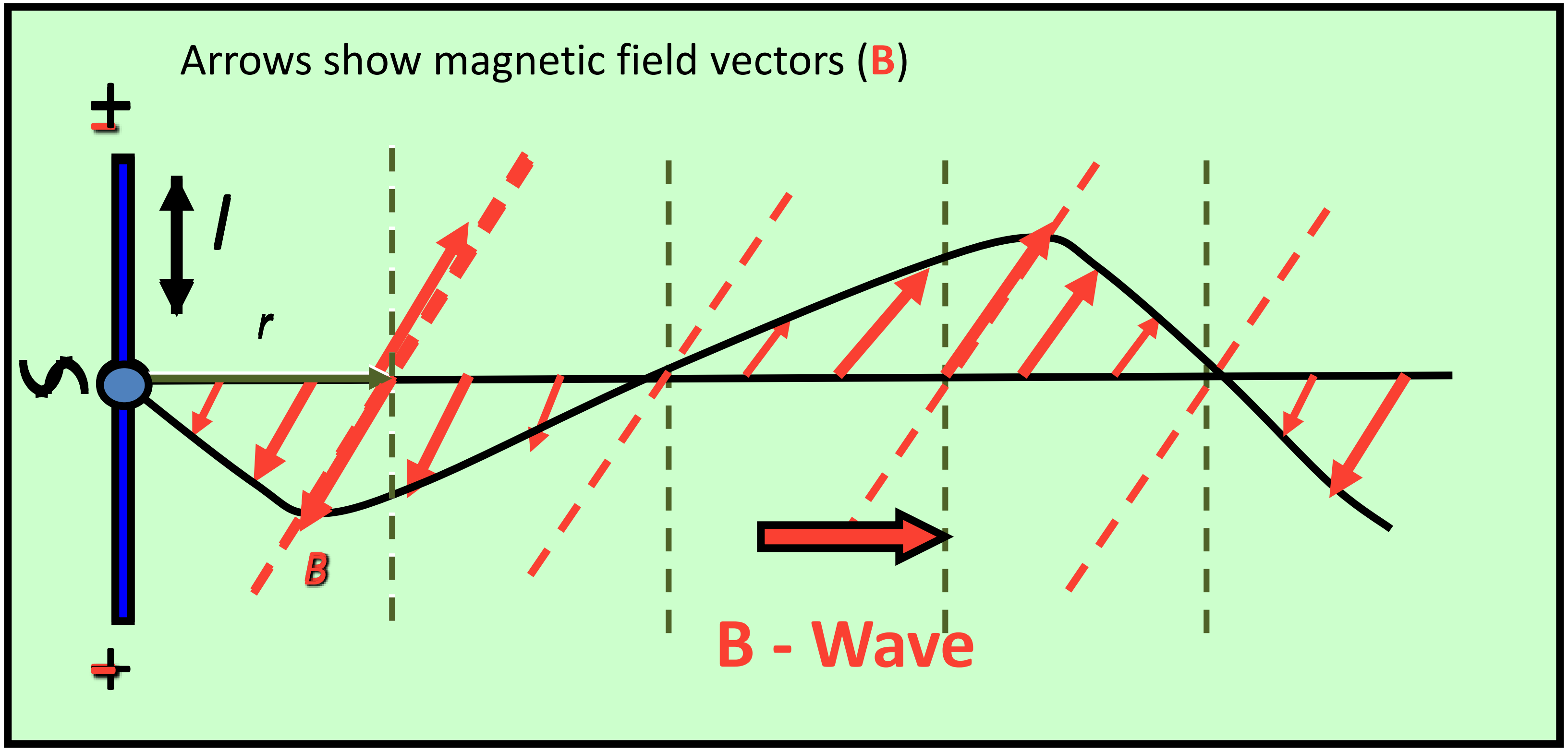
An Alternating Magnetic Field

- The ac sinusoidal current also generates a magnetic wave alternating in and out of paper.



A Magnetic Wave Generation

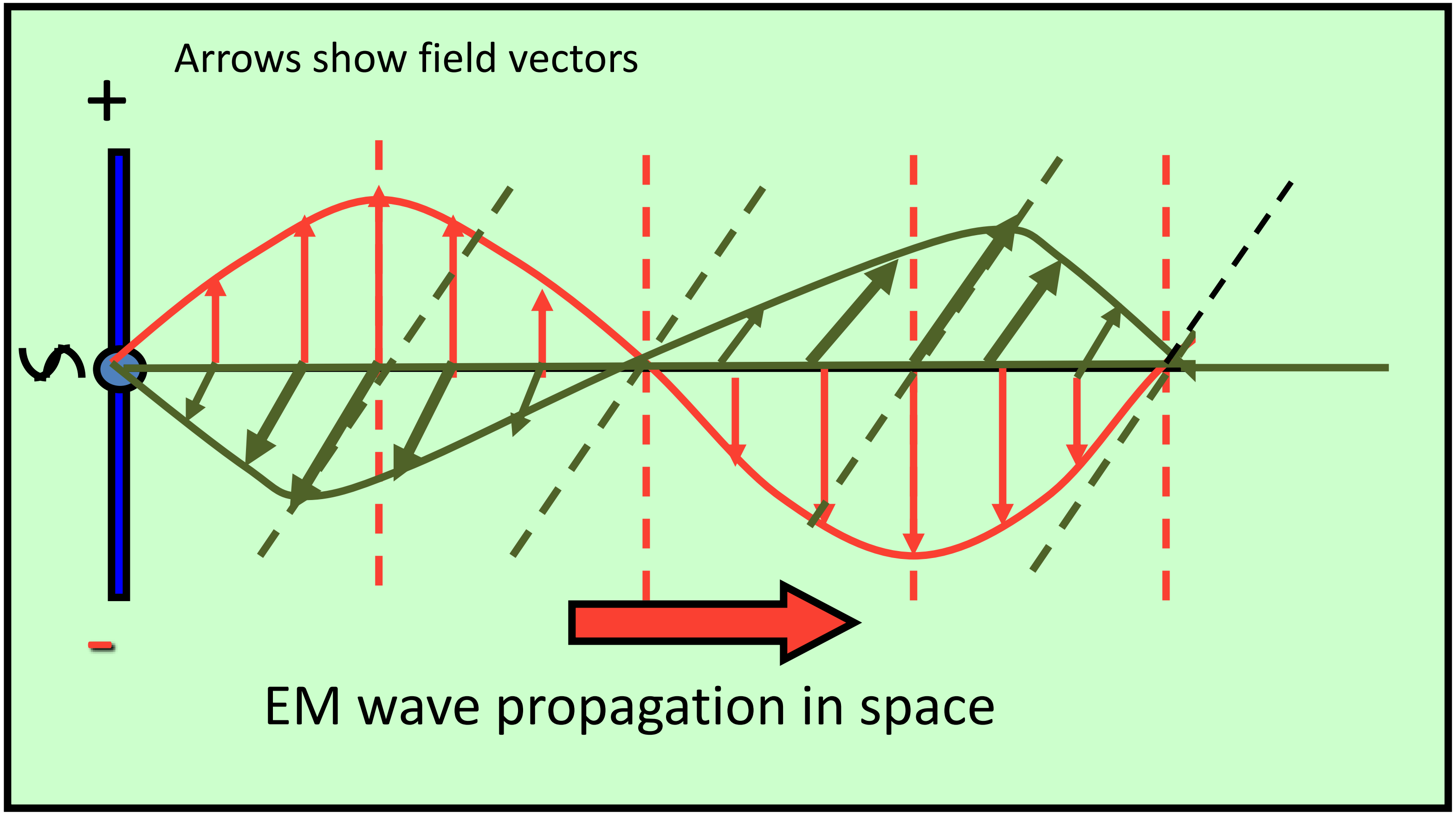
- The generation of a magnetic wave due to an oscillating ac current.



Horizontal transverse sinusoidal B-waves.

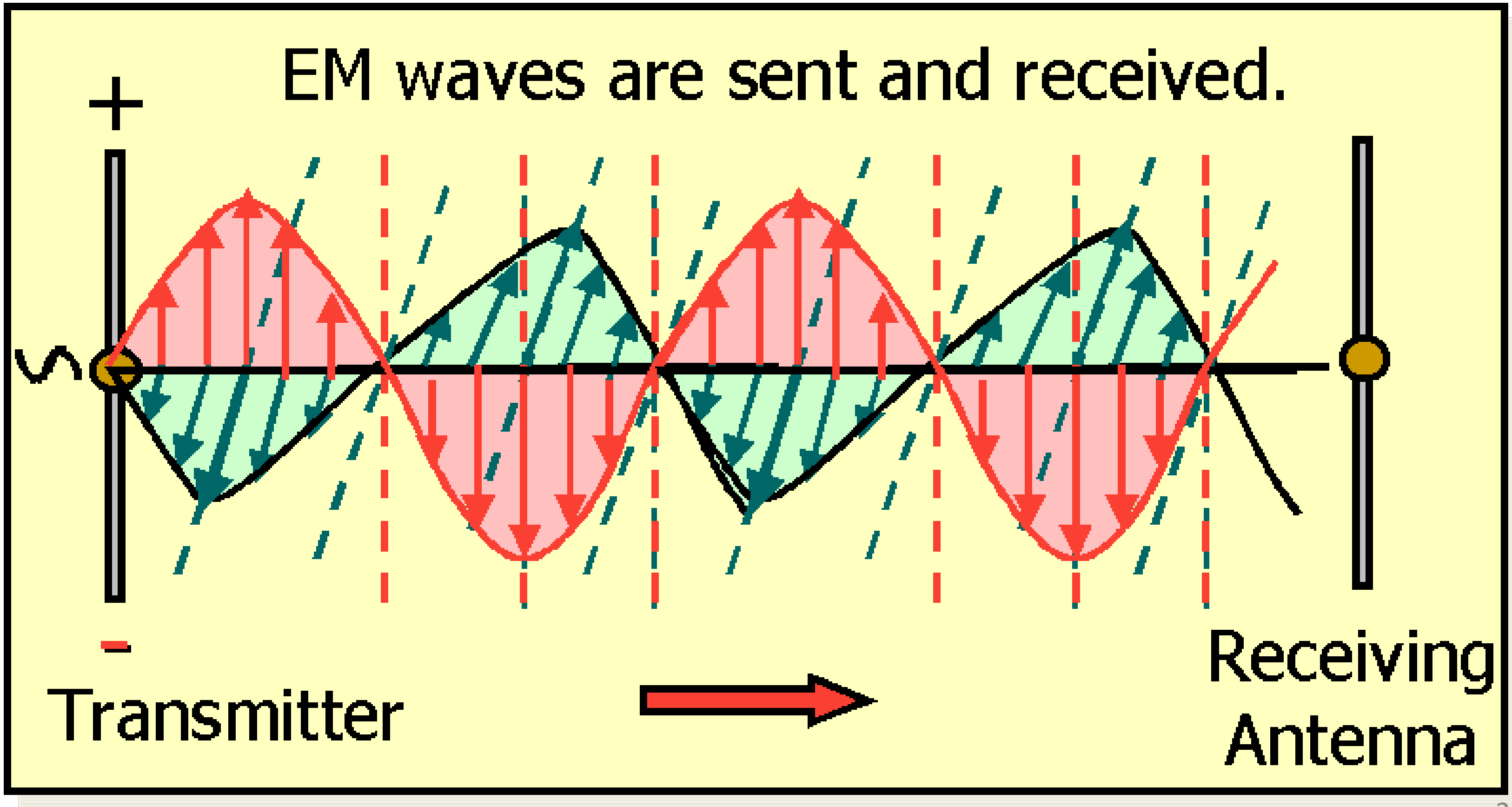
An Electromagnetic Wave

- An electromagnetic wave consists of combination of a transverse electric field and a transverse magnetic field perpendicular to each other.



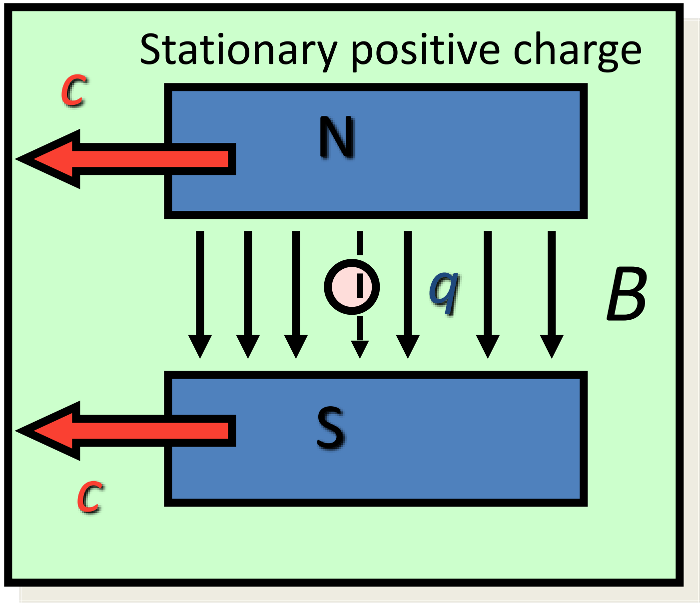
Transmitting and Receiving

- An ac current generates an EM wave which then generates an ac signal at receiving antenna.



A B-field Moves Past a Charge

- Relativity tells us that there is no preferred frame of reference. Consider that a magnetic field B moves at the speed of light c past a stationary charge q:



Charge *q* experiences a magnetic force F

$$F = qcB \quad \text{or} \quad \frac{F}{q} = cB$$

But electric field *E = F/q*:

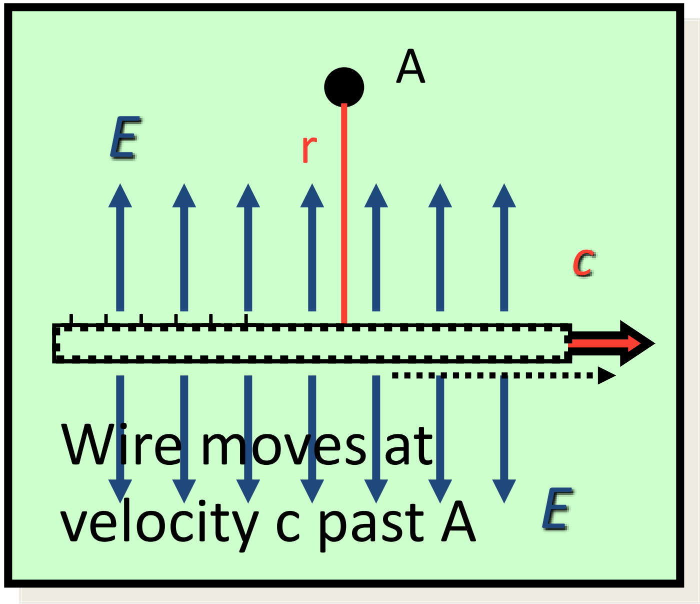
Substitution shows:

$$E = cB$$

$$c = \frac{E}{B}$$

An E-field Moves Past a Point

A length of wire ℓ moves at velocity c past point A:



A current I is simulated.

In time t, a length of wire ℓ = ct passes point A

Charge density: $$\lambda = \frac{q}{\ell} = \frac{q}{ct}$$

In time t: $q = \lambda ct$

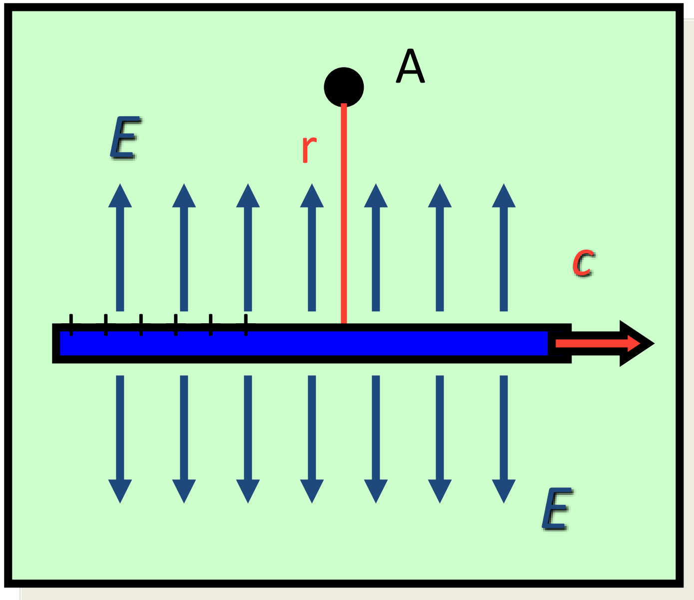
Thus, the current I is:

$$I = \frac{q}{t} = \frac{\lambda ct}{t} = \lambda c$$

Simulated current I:

$$I = \lambda c$$

Moving E-field (Cont.)



A B-field is created by the simulated current: $I = \lambda c$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 \lambda c}{2\pi r}$$

Recall from Gauss' law:

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Eliminating λ from these two equations gives:

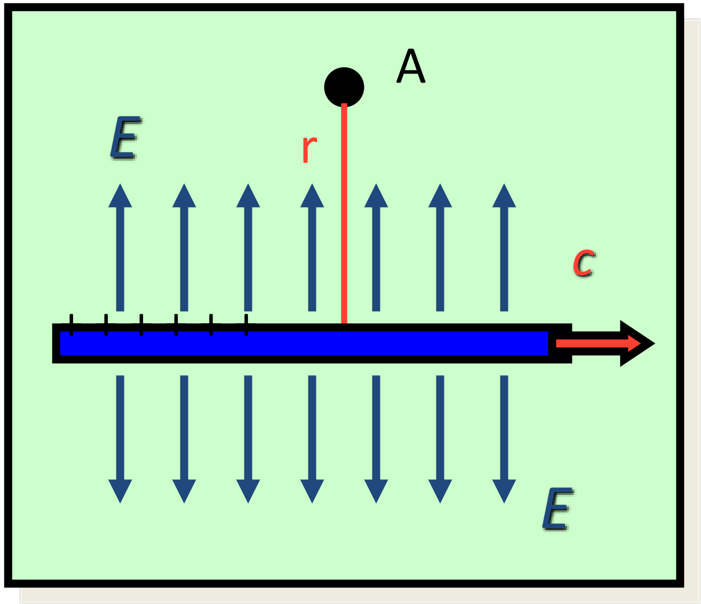
$$B = \varepsilon_0 \mu_0 c E$$

The Speed of an EM Wave

For EM waves, we have seen:

$$c = \frac{E}{B}$$

$$B = \varepsilon_0 \mu_0 c E$$



Substituting $E = cB$ into latter equation gives:

$$B = \varepsilon_0 \mu_0 c (cB)$$

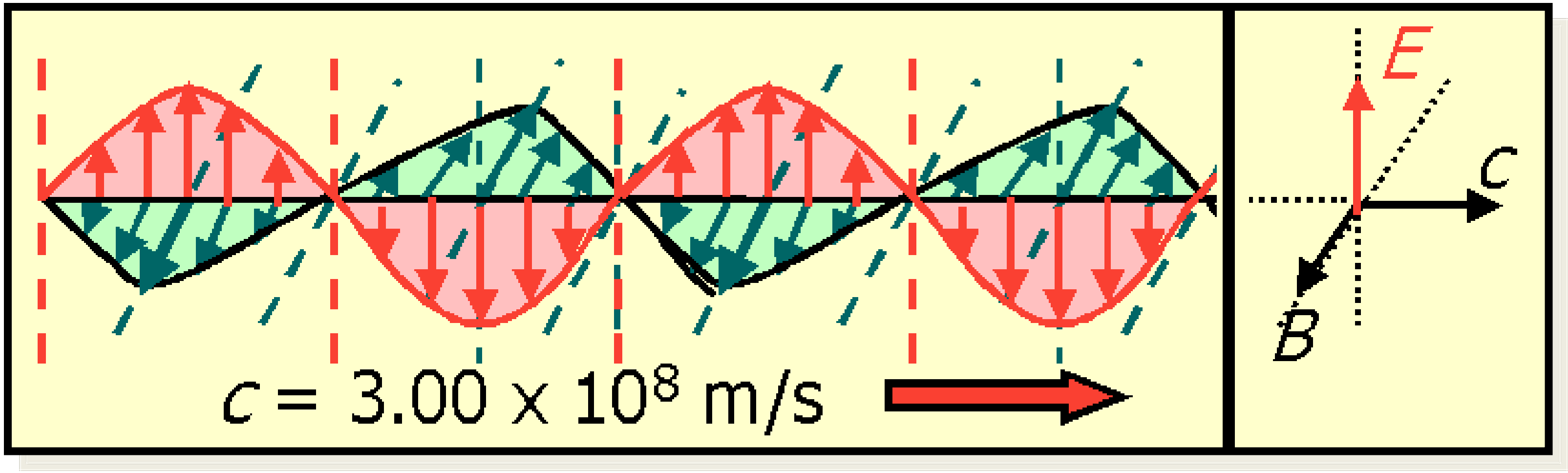
$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

EM-waves travel at the speed of light, which is:

$$c = 3.00 \times 10^8 \text{ m/s}$$

Important Properties for All Electromagnetic Waves

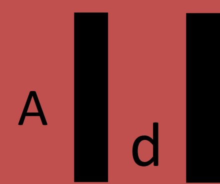
- EM waves are transverse waves. Both E and B are perpendicular to wave velocity *c*.
- The ratio of the E-field to the B-field is constant and equal to the velocity *c*.



Energy Density for an E-field

- Energy density u is the energy per unit volume (J/m^3) carried by an EM wave.
- Consider u for the electric field E of a capacitor as given below:

Energy density u for an E-field:



$$u = \frac{U}{Vol.} = \frac{U}{Ad}$$

Recall $C = \frac{\varepsilon_0 A}{d}$ and $V = Ed$:

$$U = \tfrac{1}{2} C V^2 = \tfrac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2$$

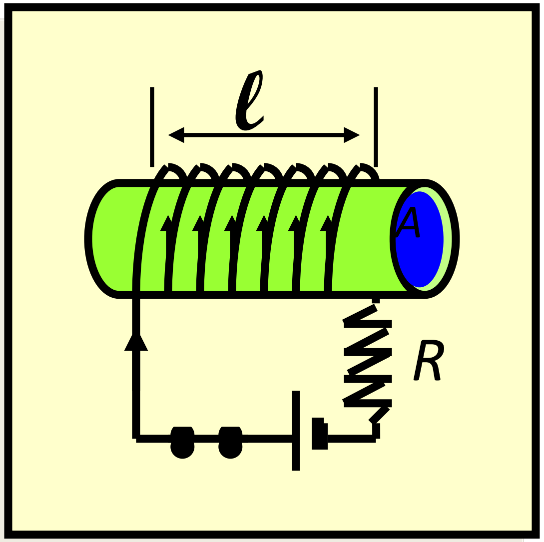
$$u = \frac{U}{Ad} = \frac{\tfrac{1}{2}\varepsilon_0 A d E^2}{Ad}$$

Energy density u:

$$u = \tfrac{1}{2}\varepsilon_0 E^2$$

Energy Density for a B-field

- The energy density u for a B-field using a solenoid of inductance L:



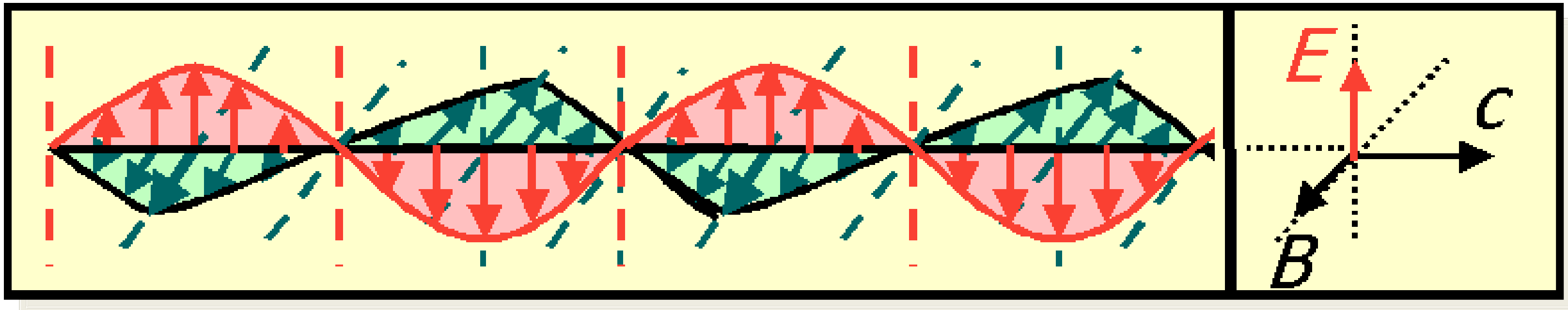
$$L = \frac{\mu_0 N^2 A}{\ell}; \quad U = \tfrac{1}{2} L I^2; \quad V = A\ell$$

$$B = \frac{\mu_0 N I}{\ell} \quad \rightarrow \quad \frac{NI}{\ell} = \frac{B}{\mu_0}$$

$$u = \frac{U}{A\ell} = \frac{\mu_0 N^2 I^2}{2\ell^2}$$

Energy density for B-field: $$u = \frac{B^2}{2\mu_0}$$

Energy Density for EM Wave



- The energy of an EM wave is shared equally by the electric and magnetic fields, so that the total energy density of the wave is given by:

Total energy density: $$u = \tfrac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$$

Or, since energy is shared equally: $$u = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

Average Energy Density

- The E and B-fields fluctuate between their maximum values E_m and B_m.
- An average value of the energy density can be found from the root-mean-square values of the fields:

$$E_{rms} = \frac{E_m}{\sqrt{2}} \quad \text{and} \quad B_{rms} = \frac{B_m}{\sqrt{2}}$$

The average energy density u_{avg} is therefore:

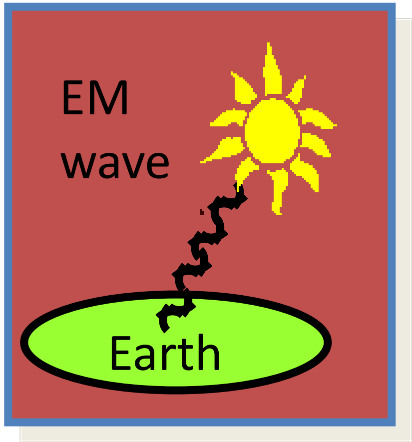
$$u_{avg} = \tfrac{1}{2}\varepsilon_0 E_m^2 \quad \text{or} \quad u_{avg} = \varepsilon_0 E_{rms}^2$$

Example 1

The maximum amplitude of an E-field from sunlight is 1010 V/m. What is the root-mean-square value of the B-field?

$$B_m = \frac{E_m}{c} = \frac{1010 \text{ V/m}}{3 \text{ x } 10^8 \text{m/s}} = 3.37 \ \mu\text{T}$$

$$B_{rms} = \frac{B_m}{\sqrt{2}} = \frac{3.37 \ \mu\text{T}}{1.414}; \quad B_{rms} = 2.38 \ \mu\text{T}$$



What is the average energy density of the wave?

$$u_{avg} = \tfrac{1}{2}\varepsilon_0 E_m^2 = \tfrac{1}{2}(8.85 \text{ x } 10^{-12} \tfrac{\text{Nm}^2}{\text{C}^2})(1010 \text{ V/m})^2$$

$$u_{avg} = 4.51 \text{ x } 10^{-6} \frac{\text{J}}{\text{m}^3}$$

Note that the total energy density is twice this value.

Wave Intensity *I*

- The intensity of an EM wave is defined as the power per unit area (W/m^2).

- EM wave moves distance *ct* through area A as shown below:

Total energy = density x volume

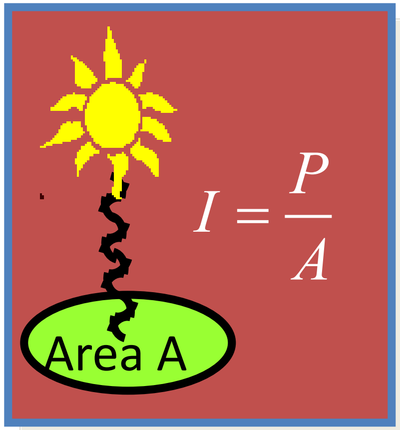
Total energy = u(ctA)

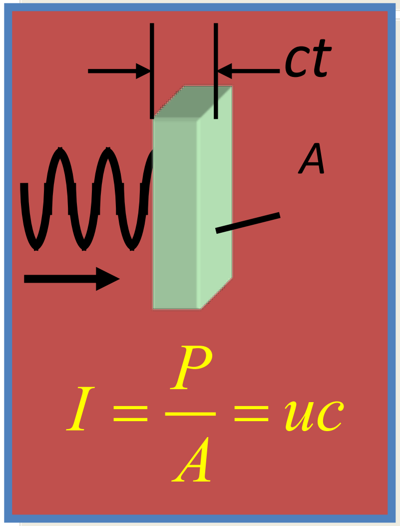
$$I = \frac{P}{A} = \frac{\text{Total E}}{\text{Time} \cdot \text{Area}} = \frac{uctA}{tA} = uc$$

- And Since $u = \varepsilon_o E^2$

Total intensity:

$$I = c\varepsilon_0 E_m^2$$



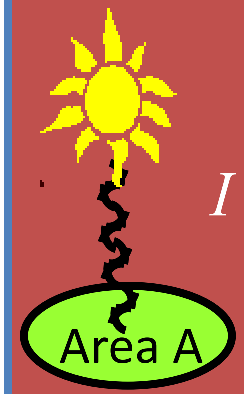


Calculating Intensity of Wave

- In calculating intensity, you must distinguish between average values and total values:

$$I_T = c\varepsilon_0 E_m^2 = 2c\varepsilon_0 E_{rms}^2$$

$$I_{avg} = \tfrac{1}{2} c\varepsilon_0 E_m^2 = c\varepsilon_0 E_{rms}^2$$



$$I = \frac{P}{A}$$

$$I_{avg} = \tfrac{1}{2} c\varepsilon_0 E_m^2$$

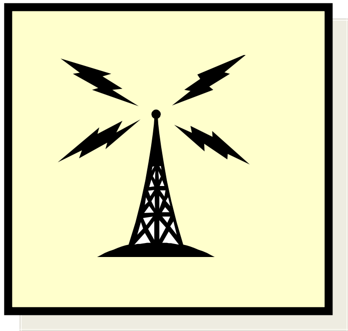
Since $E = cB$, we can also express I in terms of B:

$$I_T = \frac{c}{\mu_0} B_m^2 = \frac{2c}{\mu_0} B_{rms}^2$$

$$I_{avg} = \frac{c}{2\mu_0} B_m^2 = \frac{c}{\mu_0} B_{rms}^2$$

Example 2

A signal received from a radio station has E_m = 0.0180 V/m. What is the average intensity at that point?



The average intensity is:

$$I_{avg} = \tfrac{1}{2} c\varepsilon_0 E_m^2$$

$$I_{avg} = \tfrac{1}{2}(3 \text{ x } 10^8 \text{m/s})(8.85 \text{ x } 10^{-12} \tfrac{\text{Nm}^2}{\text{C}^2})(0.018 \text{ V/m})^2$$

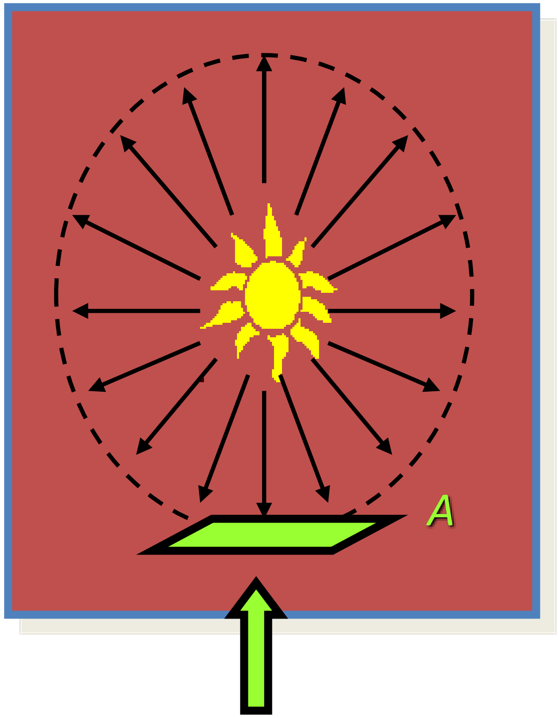
$$I_{avg} = 4.30 \text{ x } 10^{-7} \text{ W/m}^2$$

Note that intensity is power per unit area. The power of the source remains constant, but the intensity decreases with the square of distance.

Wave Intensity and Distance

- The intensity *I* at a distance r from an isotropic source:

$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$



- The average power of the source can be found from the intensity at a distance *r* :

- For isotropic conditions:

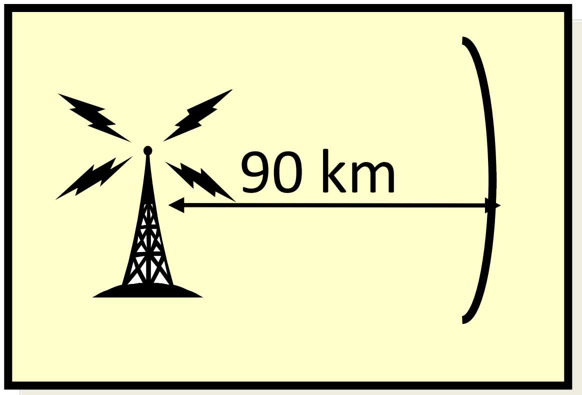
$$P = AI_{avg} = (4\pi r^2)I_{avg}$$

For power falling on surface of area A:

$$P = I_{avg} A$$

Example 3

In Example 2, an average intensity of 4.30 x 10^{-7} W/m^2 was observed at a point. If the location is 90 km (r = 90,000 m) from the isotropic radio source, what is the average power emitted by the source?



$$I_{avg} = \frac{P}{4\pi r^2} = 4.30 \text{ x } 10^{-7}\,\text{W/m}^2$$

$$P = (4\pi r^2)(4.30 \text{ x } 10^{-7}\,\text{W/m}^2)$$

$$P = 4\pi(90{,}000\text{ m})^2(4.30 \text{ x } 10^{-7}\,\text{W/m}^2)$$

Average power of transmitter: P = 43.8 kW

This assumes isotropic propagation, which is not likely.

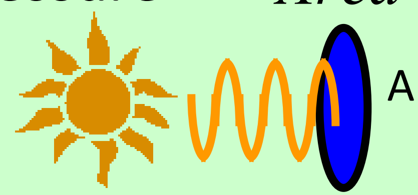
Radiation Pressure

- EM-waves not only carry energy, but also carry momentum and exert pressure when absorbed or reflected from objects.

- Recall that Power = F v

$$I = \frac{P}{A} = \frac{Fc}{A} \quad \text{or} \quad \frac{\mathrm{F}}{\mathrm{A}} = \frac{I}{c}$$

Radiation Pressure $= \dfrac{Force}{Area}$



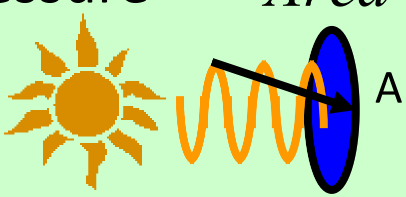
- The pressure is due to the transfer of momentum. The above relation gives the pressure for a completely absorbing surface.

Radiation Pressure (Cont.)

- The change in momentum for a fully reflected wave is twice that for an absorbed wave, so that the radiation pressures are as follows:

Absorbed wave:

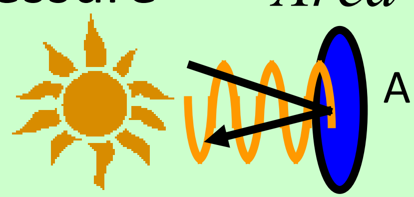
Radiation Pressure $\frac{Force}{Area}$



$$\frac{F}{A} = \frac{I}{c}$$

Reflected wave:

Radiation Pressure $\frac{Force}{Area}$



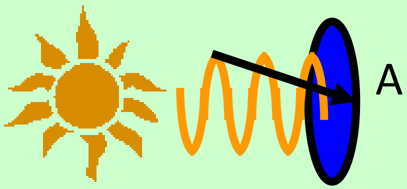
$$\frac{F}{A} = \frac{2I}{c}$$

Example 4

The average intensity of direct sunlight is around 1400 W/m². What is the average force on a fully absorbing surface of area 2.00 m²?

Absorbed wave:

$$\text{Radiation Pressure} = \frac{Force}{Area}$$



For absorbing surface:

$$\frac{F}{A} = \frac{I}{c}$$

$$F = \frac{IA}{c}$$

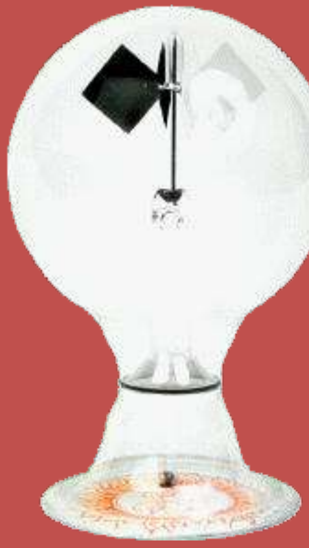
$$F = \frac{(1400\ \text{W/m}^2)(2.00\ \text{m}^2)}{3 \times 10^8\ \text{m/s}}$$

$F = 9.33 \times 10^{-6}$ N

The Radiometer

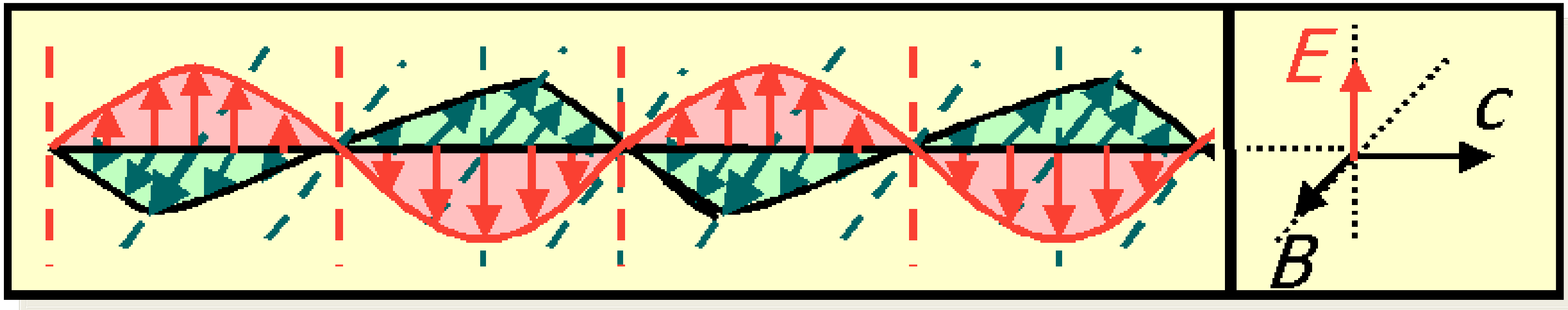
A radiometer is a device which demonstrates the existence of radiation pressure:

One side of the panels is black (totally absorbing) and the other white (totally reflecting). The panels spin under light due to the pressure differences.



Radiometer

Summary



- EM waves are transverse waves. Both E and B are perpendicular to wave velocity *c*.
- The ratio of the E-field to the B-field is constant and equal to the velocity *c*.
- Electromagnetic waves carry both energy and momentum and can exert pressure on surfaces.

Summary (Cont.)

EM-waves travel at the speed of light, which is:

$c = 3.00 \times 10^8$ m/s

$$c = \frac{E}{B}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Total Energy Density: $$u = \tfrac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$$

$$E_{rms} = \frac{E_m}{\sqrt{2}} \quad \text{and} \quad B_{rms} = \frac{B_m}{\sqrt{2}}$$

Summary (Cont.)

The average energy density:

$$u_{avg} = \tfrac{1}{2}\varepsilon_0 E_m^2 \quad \text{or} \quad u_{avg} = \varepsilon_0 E_{rms}^2$$

$$I_{avg} = \tfrac{1}{2} c\varepsilon_0 E_m^2 = c\varepsilon_0 E_{rms}^2$$

Intensity and Distance	Totally Absorbing	Totally Reflecting
$I = \frac{P}{A} = \frac{P}{4\pi r^2}$	$\frac{F}{A} = \frac{I}{c}$	$\frac{F}{A} = \frac{2I}{c}$