American University Of The Middle East

CVL203: Principles and Practice of Geomatics

Lectures 15-16: Vertical Curves

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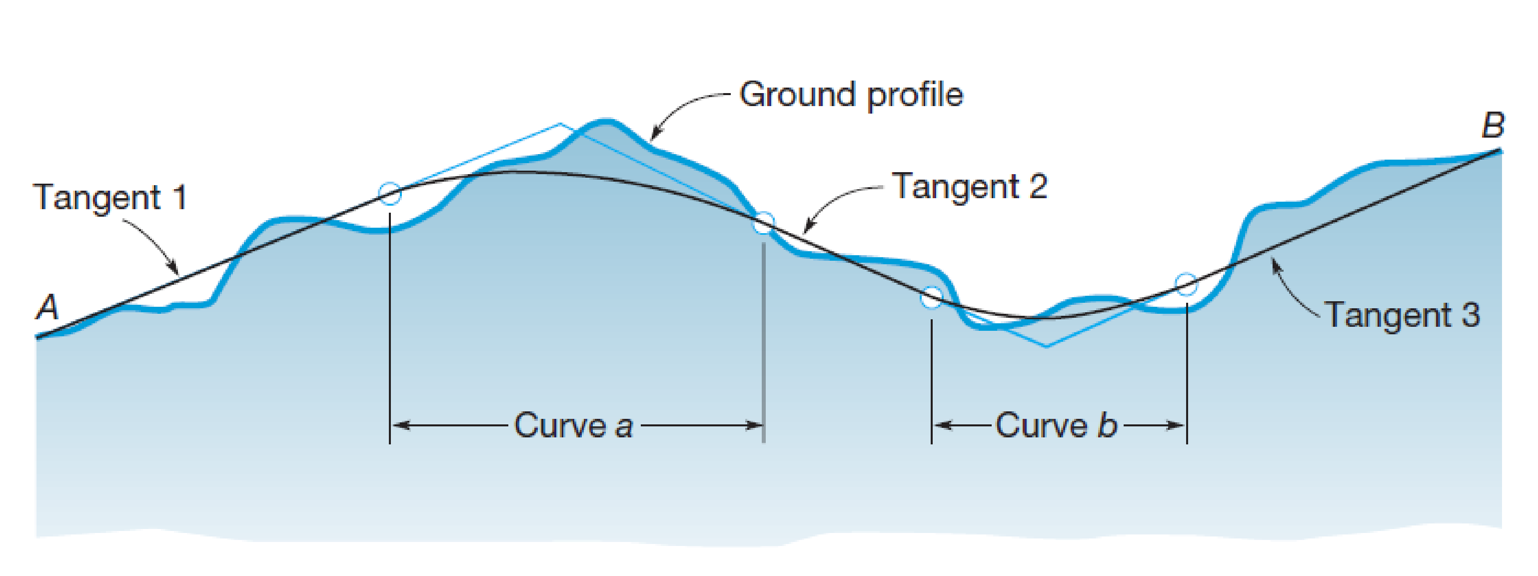
Introduction

Vertical Curves:

Curves needed to provide smooth transitions between straight segments (tangents) of grade lines for highways and railroads.



Crest (curve a)	Sag (curve b)
The change in grade is negative; the curve turns downward.	The change in grade is positive ; the curve turns upward

Introduction

There are several factors that must be taken into account when designing a grade line of tangents and curves on any highway or railroad project:

1) providing a good fit with the existing ground profile, thereby minimizing the depths of cuts and fills,
2) balancing the volume of cut material against fill,
3) maintaining adequate drainage,
4) not exceeding maximum specified grades, and
5) meeting fixed elevations such as intersections with other roads.

In addition, the curves must be designed to

1) fit the grade lines they connect,
2) have lengths sufficient to meet specifications covering a maximum rate of change of grade (which affects the comfort of vehicle occupants), and
3) provide sufficient sight distance for safe vehicle operation.

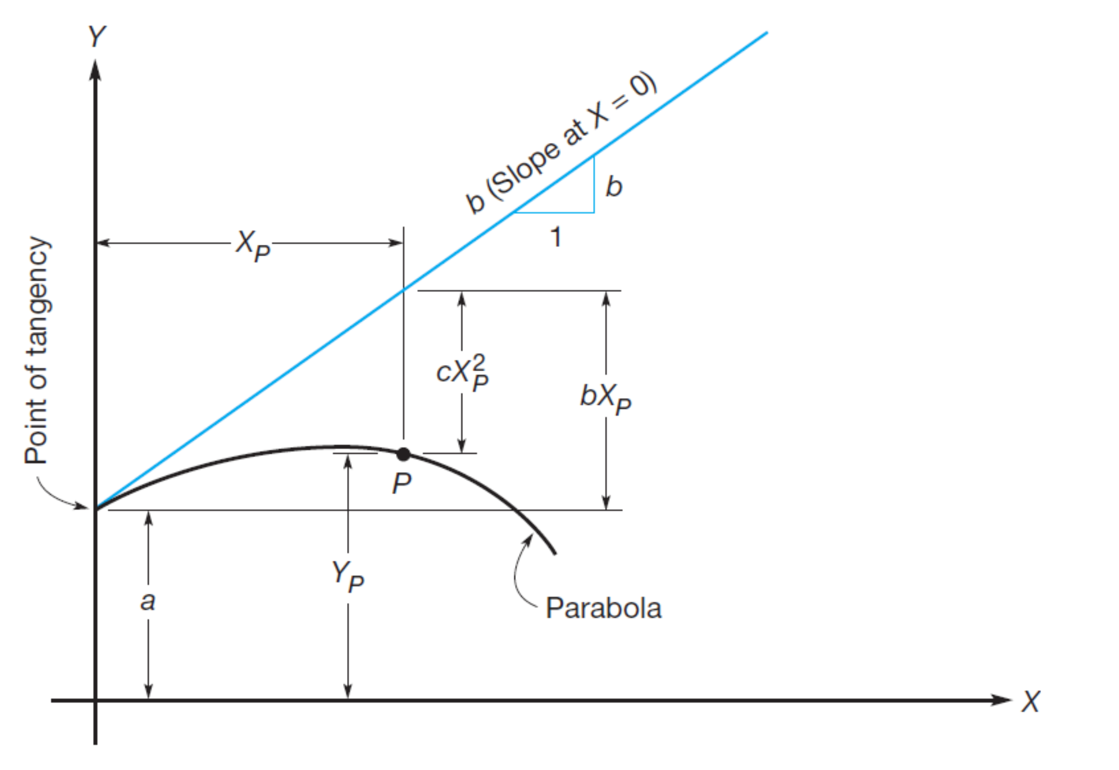
General Equation of a Vertical Parabolic Curve

The general mathematical expression of a parabola, with respect to an *XY* rectangular coordinate system, is given by:

$$Y_p = a + bX_p + cX_p{}^2$$

Where Y_p is the ordinate at any point p on the parabola located at distance X_p from the origin of the curve, and a, b, and c are constants.

- a is the ordinate at the beginning of the curve where $X = 0$,
- b is the slope of a tangent to the curve at $X = 0$,
- bX_p is the change in the ordinate along the tangent over distance X_p, and
- $cX_p{}^2$ is the parabola's departure from the tangent (tangent offset) in distance X_p



For a crest curve, b is positive and c is negative.

Equation of an Equal Tangent Vertical Parabolic Curve

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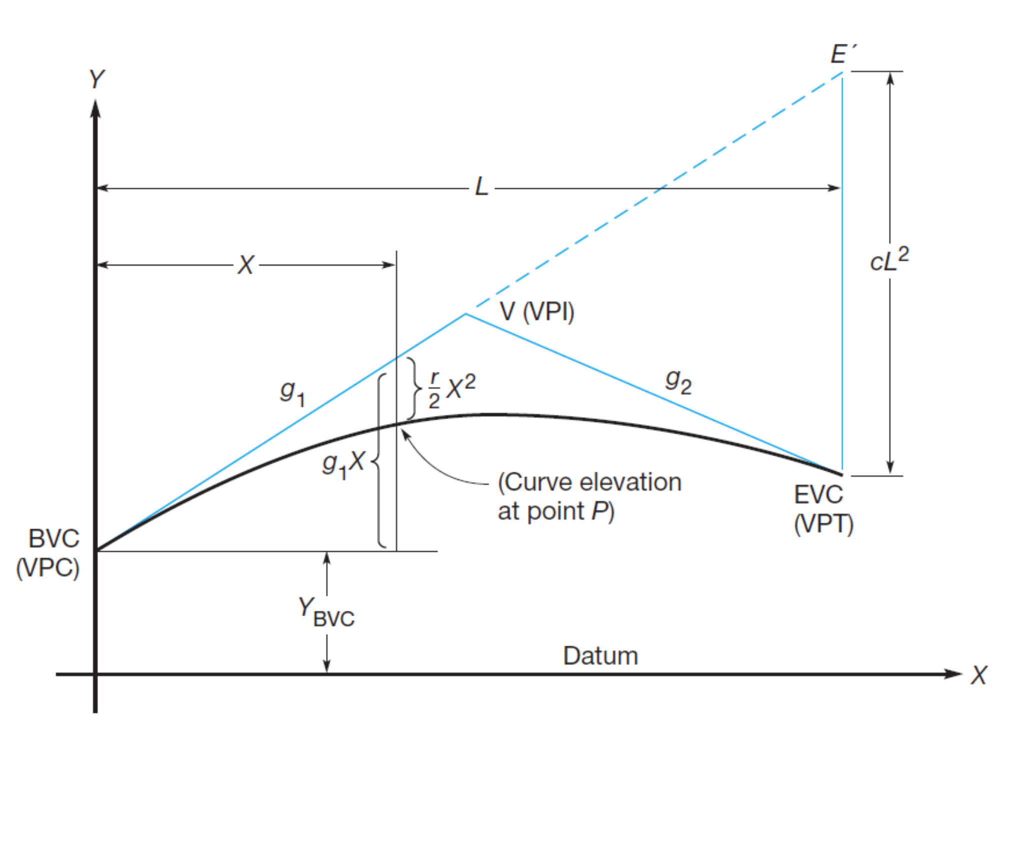
- BVC: Beginning of vertical curve, also called VPC (vertical point of curvature).
- V: Vertex, also called the VPI (vertical point of intersection).
- EVC: End of vertical curve, also called VPT (vertical point of tangency).
- g_1: The percent grade of the back tangent (straight segment preceding *V*).
- g_2: The percent grade of the forward tangent (straight segment following *V*).
- L: the curve length in the horizontal distance (in stations) from the BVC to the EVC.
  - Note that on the XY axis system, X values are horizontal distances measured from the BVC, and Y values are elevations measured from the vertical datum of reference.



This curve is called equal tangent because the horizontal distances from the *BVC* to *V* and from *V* to the *EVC* are equal, each being *L*/2.

Equation of an Equal Tangent Vertical Parabolic Curve

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Substituting this surveying terminology into the general equation of a parabola:

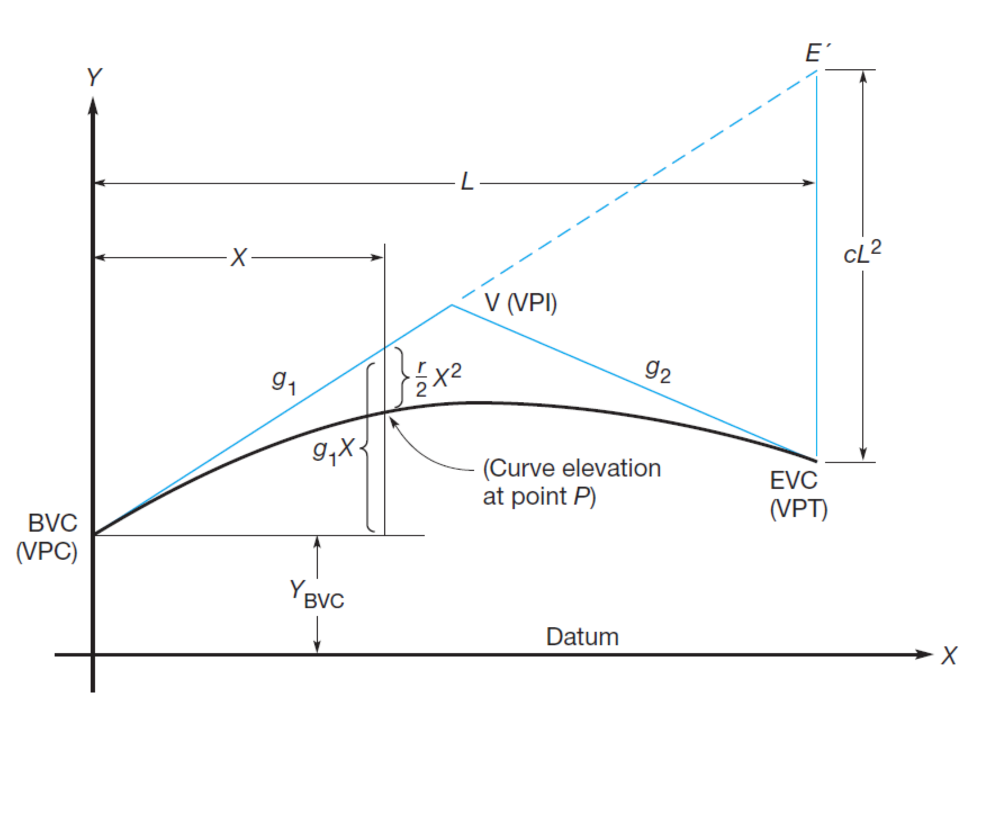
$$Y = Y_{BVC} + g_1 X + cX^2$$

Where Y is in meters, g_1 is in percent grade, X must be in units of 100 m or 1/10th stations, where full stations are 1 km apart.

Determine constant c:

$$cL^2 = g_1\left(\frac{L}{2}\right) + g_2\left(\frac{L}{2}\right) - g_1 L$$

$$c = \frac{g_2 - g_1}{2L}$$



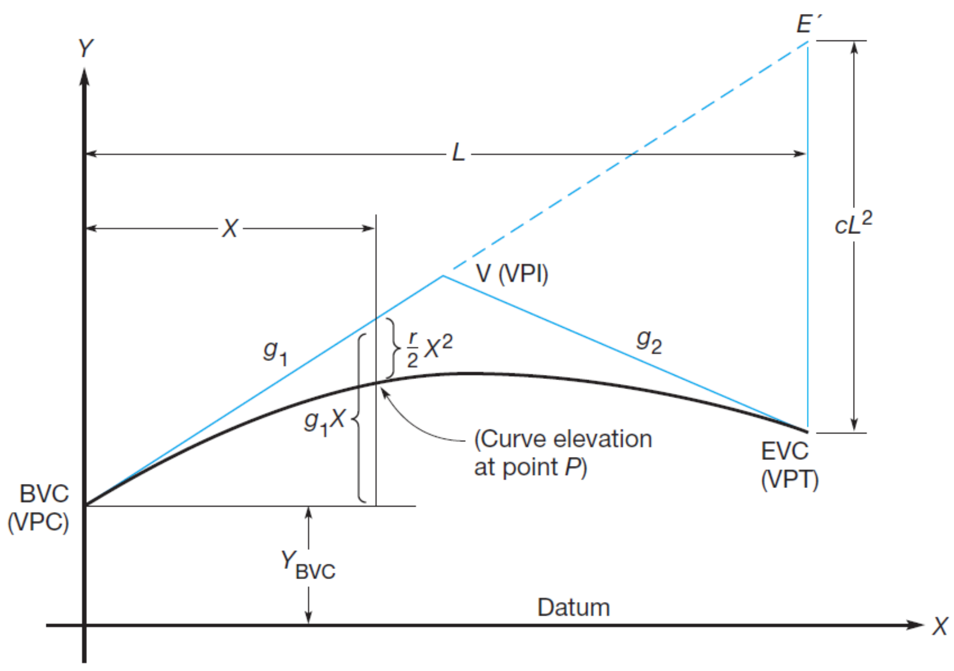
$$Y = Y_{BVC} + g_1 X + \left(\frac{g_2 - g_1}{2L}\right)X^2$$

Equation of an Equal Tangent Vertical Parabolic Curve

The rate of change of grade, *r*, for an equal-tangent parabolic curve equals the total grade change from BVC to EVC divided by length L (in stations for the English system, or 1/10th stations for metric units), over which the change occurs:

$$r = \frac{g_2 - g_1}{L}$$

The value *r* (which is negative for a crest curve and positive for a sag type) is an important design parameter because it controls the rate of curvature, and hence rider comfort.



Tangent Offset Equation:

$$Y = Y_{BVC} + g_1 X + \left(\frac{r}{2}\right) X^2$$

High or Low Point on a Vertical Curve

It may be necessary to determine the elevation and location of the low (or high) point on a vertical curve to investigate drainage conditions, clearance beneath overhead structures, cover over pipes, and sight distance.

At the low or high point, a tangent to the curve will be horizontal and its slope equal to zero. Based on this fact, by taking the derivative of *tangent offset equation* and setting it equal to zero, we get:

$$X = \frac{g_1 L}{g_1 - g_2}$$

Where X is the distance from the BVC to the high or low point of the curve (in stations in the English system of units, and in 1/10th stations in the metric system), g_1 the tangent grade through the BVC, g_2 the tangent grade through the EVC, and L the curve length (in stations or 1/10th stations).

Substitute $r = \frac{g_2 - g_1}{L}$ in $X = \frac{g_1 L}{g_1 - g_2}$ gives $X = \frac{-g_1}{r}$

VC Computations Using the Tangent Offset Equation

In designing grade lines, the locations and individual grades of the tangents are normally selected first. This produces a series of intersection points *V*, each defined by its station and elevation.

- A curve is then chosen to join each pair of intersecting tangents.
- The parameter selected in vertical-curve design is length *L*.
- Having chosen it, the station of the *BVC* is obtained by subtracting *L*/2 from the vertex station.
- Adding *L* to the *BVC* station then determines the *EVC* station.

Stationing for the points on curve that are computed and staked are those that are *evenly divisible by the selected staking increment.* In the metric system, if 40m is the staking increment, then 2+400, 2+440, 2+480, 2+520, and so on would be staked.

Example:

A grade g_1 of −3.629% intersects grade g_2 of 0.151% at a vertex whose station and elevation are 5 + 265.000 and 350.520 m, respectively. An equal-tangent parabolic curve of 240 m length will be used to join the tangents. Compute and tabulate the curve for staking at 40 m increments.

Calculate the position and the elevation of the low point.

VC Computations Using the Tangent Offset Equation

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VC Computations Using the Tangent Offset Equation

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VC Computations Using the Tangent Offset Equation

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Example:

By Equation $r = \dfrac{g_2 - g_1}{L}$

$$r = \frac{0.151 + 3.629}{2.4} = 1.575$$

[Note that L used in Equation (25.4) is in units of m/100, or 1/10th stations.]

Stationing

$$\begin{aligned} VPI \text{ Station} &= 5 + 265 \\ -L/2 &= \quad\;\; 120 \\ BVC \text{ Station} &= 5 + 145 \\ +L &= \quad\;\; 240 \\ EVC \text{ Station} &= 5 + 385 \end{aligned}$$

$$\text{Elev}_{BVC} = 350.520 + 3.629(120/100) = 354.875$$

VC Computations Using the Tangent Offset Equation

Example:

The remaining calculations employ the equation $X = \frac{g_1 L}{g_1 - g_2}$

Station	$X\left(\frac{m}{100}\right)$	g_1X	$\frac{rX^2}{2}$	Curve Elevation (m)	First Difference	Second Difference
5 + 385.000 (EVC)	2.400	−8.710	4.536	350.701		
5 + 360.000	2.150	−7.802	3.640	350.713	−0.223	0.252
5 + 320.000	1.750	−6.351	2.412	350.936	−0.475	0.252
5 + 280.000	1.350	−4.899	1.435	351.411	−0.727	0.252
5 + 240.000	0.950	−3.448	0.711	352.138	−0.979	0.252
5 + 200.000	0.550	−1.996	0.238	353.117	−1.232	
5 + 160.000	0.150	−0.544	0.018	354.348		
5 + 145.000 (BVC)	0.000	−0.000	0.000	354.875		

Check: $EVC = ELEV_V - g_2\left(\frac{L}{2}\right) = 350.520 + (0.151 \times 120/100) = 350.701$

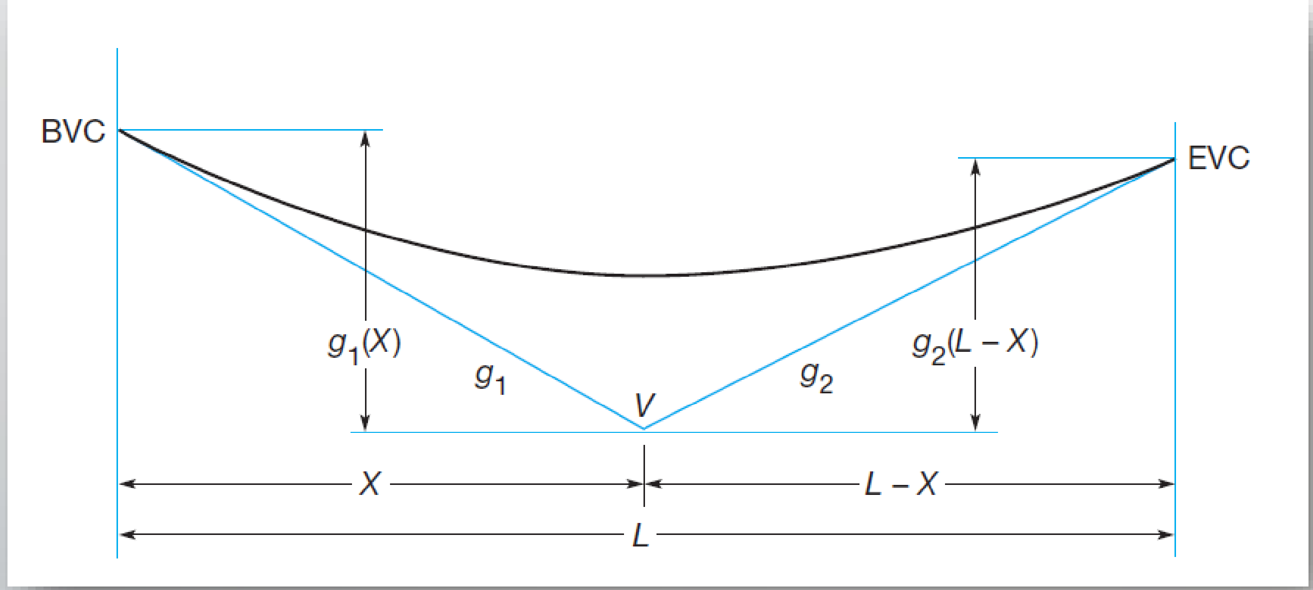
Example:

The low point is located at (5+375.413) and an elevation of 350.694 m

Equal Tangent Property of a Parabola

The curve defined in the figure has been called an equal-tangent parabolic curve, which means the vertex occurs at a distance $X = L2$ from the BVC.

In the figure blow, assume the horizontal distance from BVC to V is an unknown value X; thus, the remaining distance from V to EVC is $L – X$: two expressions can be written for the elevation of the *EVC.*



Equal Tangent Property of a Parabola

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First equation:
Use the equation $Y = Y_{BVC} + g_1 X + \left(\frac{g_2 - g_1}{2L}\right) X^2$ and take $X = L$

Then: $$Y_{EVC} = Y_{BVC} + g_1 L + \left(\frac{g_2 - g_1}{2L}\right) L^2$$

Second equation:
Use changes in elevation that occur along the tangents where *X* is the distance between *V* and EVC, which gives:

$$Y_{EVC} = Y_{BVC} + g_1 X + g_2 (L - X)$$

Equating both equations and solving, gives X = L/2. Thus, distances BVC to V and V to EVC are equal—hence the term *equal-tangent parabolic curve*.

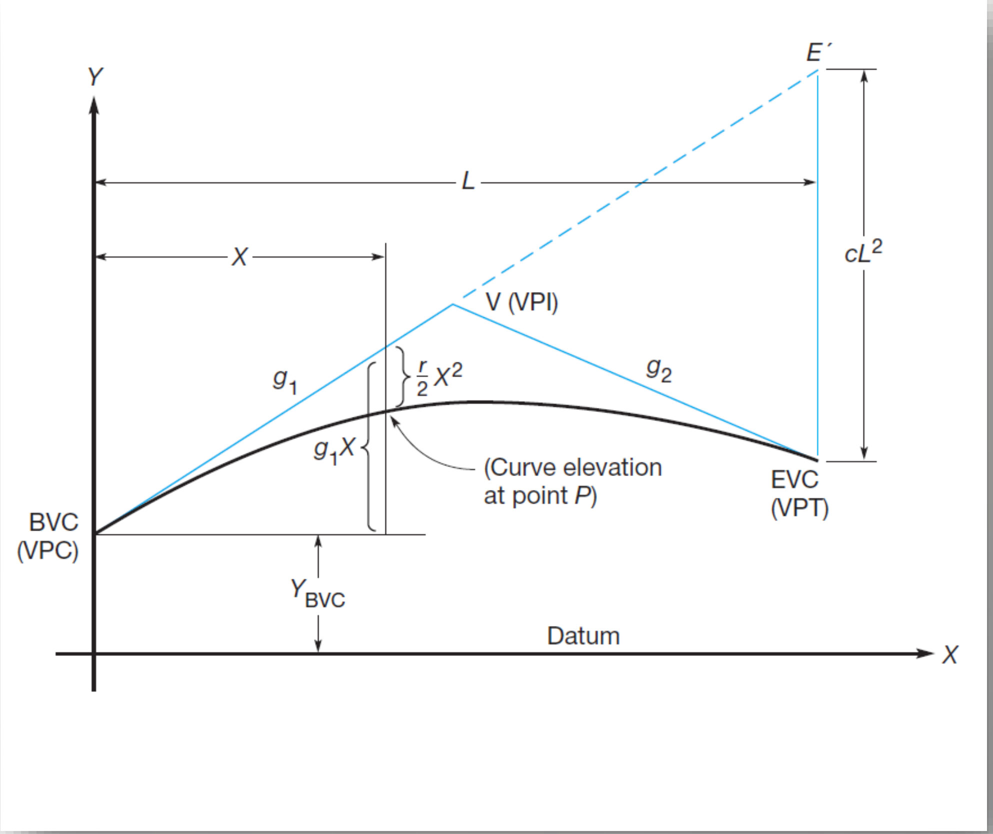
Curve Computations by Proportion

Property of a Parabola:
Offsets from a tangent to a parabola are proportional to the squares of the distances from the point of tangency.

Calculating the offset *E* at the midpoint of the curve, and then computing offsets at any other distance *X* from the *BVC* by proportion according to the following formula conveniently utilize this property.

$$\text{offset}_X = E\left(\frac{X}{L/2}\right)^2$$

The value of *E* in the equation is simply the difference in elevation from the curve's midpoint to the VPI. (X is in stations)



Designing a Curve to Pass Through a Fixed Point

The problem of designing a parabolic curve to pass through a point of fixed station and elevation is frequently encountered in practice.

- For example, it occurs where a new grade line must meet an existing railroad or highway crossings or when a minimum vertical distance must be maintained between the grade line and underground utilities or drainage structures.

$$Y = Y_{BVC} + g_1 X + \left(\frac{g_2 - g_1}{2L}\right) X^2$$

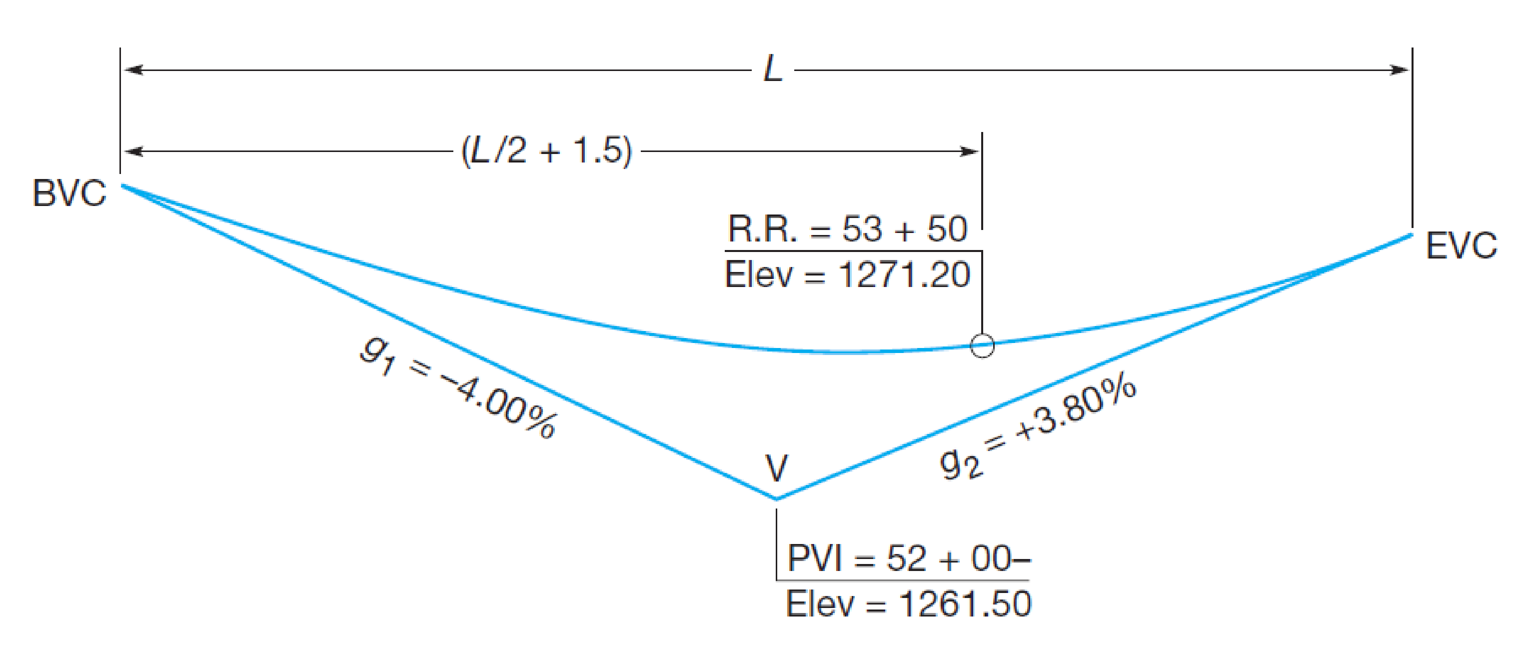
Given the station and elevation of the VPI, and grades and of the back and forward tangents, respectively, the problem consists of calculating the curve length required to meet the fixed condition.

- It is solved by substituting known quantities into the equation of and reducing the equation to its quadratic form containing only *L* as an unknown.
- Two values will satisfy the quadratic equation, but the correct one will be obvious.

Designing a Curve to Pass Through a Fixed Point

Example:

In Figure 25.8, grades $g_1 = -4.00\%$ and $g_2 = +3.80\%$ meet at VPI station 52 + 00 and elevation 1261.50. Design a parabolic curve to meet a railroad crossing, which exists at station 53 + 50 and elevation 1271.20.



Designing a Curve to Pass Through a Fixed Point

Example:

Designing a Curve to Pass Through a Fixed Point

Example:

Designing a Curve to Pass Through a Fixed Point

Example:

In referring to Figure 25.8 and substituting known quantities into Equation the following equation is obtained:

$$1271.20 = \left[1261.50 + 4.00\left(\frac{L}{2}\right)\right] + \left[-4.00\left(\frac{L}{2} + 1.5\right)\right]$$

$$+ \left[\frac{3.80 + 4.00}{2L}\left(\frac{L}{2} + 1.5\right)^2\right]$$

In this expression, the value of X for the railroad crossing is $L/2 + 1.5$ and the terms within successive brackets are Y_{BVC}, g_1X, and $(r/2)X^2$, respectively. Reducing the equation to quadratic form gives

$$0.975L^2 - 9.85L + 8.775 = 0$$

Solving by use of Equation (11.3) for L gives 9.1152 stations. To check the solution, $L = 9.1152$ stations and $X = [9.1152/2) + 1.5]$ stations are used in Equation (25.3) to calculate the elevation at station 53 + 50. A value of 1271.20 checks the computations.

Sight Distance

The vertical alignments of highways should provide ample sight distance for safe vehicular operation. Two types of sight distances are involved:

1) *stopping sight distance* (the distance required, for a given “design speed,” to safely stop a vehicle thus avoiding a collision with an unexpected stationary object in the roadway ahead)

2) *passing sight distance* (the distance required for a given design speed, on two-lane two-way highways to safely overtake a slower moving vehicle, pass it, and return to the proper lane of travel leaving suitable clearance for an oncoming vehicle in the opposing lane).

TABLE 25.4 MINIMUM SIGHT DISTANCES FOR VARYING DESIGN SPEEDS ON LEVEL SECTIONS

Design Speed (mph)	Stopping Sight Distance (ft)	Passing Sight Distance (ft)
30	200	1090
40	305	1470
50	425	1835
60	570	2135
70	730	2480

Sight Distance

- Given the grades of two intersecting tangent sections, the length of vertical curve used to provide a transition from one to the other fixes the sight distance. A longer curve provides a greater sight distance.

- The formula for length of curve *L* necessary to provide sight distance *S* on a *crest vertical curve*, ***where S is less than L*** is:

$$L = \frac{S^2(g_1 - g_2)}{2(\sqrt{h_1} + \sqrt{h_2})^2}$$

- ***When S is greater than L***:

  If the vehicle is off the curve but on the tangent leading to it and *S is greater than L*, the applicable sight distance formula is:

$$L = 2S - \frac{2(\sqrt{h_1} + \sqrt{h_2})^2}{g_1 - g_2}$$

Sight Distance

The units of *S* and *L* are one-tenth stations in the metric system. Also the units of h_1 (the height of the driver's eye) and h_2 (the height of an object sighted on the roadway ahead) are in meters for the metric system.

(For design, AASHTO recommends 3.5 ft (1080 mm) for h_1. Recommended values for h_2 are 2.0 ft (600 mm) for stopping and 4.25 ft (1.300 m) for passing. The lower value for h_2 represents the size of an object that would damage a vehicle and the higher value represents the height of an oncoming car.)

Sight Distance

- *For a Sag curve*:

(a) S less than L

$$L = \frac{S^2(g_2 - g_1)}{4 + 3.5S}$$

(b) S greater than L

$$L = 2S - \frac{4 + 3.5S}{g_1 - g_2}$$

Sources of errors and Mistakes

Some sources of error in staking out parabolic curves are:

1) Making errors in measuring distances and angles when staking the centreline.
2) Not holding the level rod plumb when setting blue tops.
3) Using a leveling instrument that is out of adjustment.

Mistakes:

1) Arithmetic mistakes.
2) Failure to properly account for the algebraic signs of g_1 and g_2.
3) Subtracting offsets from tangents for a sag curve or adding them for a crest curve.
4) Failure to make the second-difference check.
5) Not completing the level circuit back to a benchmark after setting blue tops.

End of Lecture 15-16: Vertical Curves