Chapter 3

Particle Kinematics and dynamics

1. A particle's position:

Is the location of the particle with respect to a chosen reference point that we can consider to be the origin of a coordinate system.

هو موقع الجسم نسبة الي هدف ثابت

2. The Distance:

The Distance is the length of a path followed by a particle. For example, the basketball players.

المسافة هو طول المسار الذي يقطعه الجسم

Distance is a scalar quantity.

Displacement الازاحة

It is the change in the object's position over time interval

هو تغير موضع جسم خلال فترة زمنية

Or

It is difference between the final and initial position of an object

هو الفرق بين الموضع النهائي والابتدائي للجسم

$$\Delta x = x_f - x_i \; m$$

It is a vector quantity and its dimension is L

Displacement has positive (+) and negative (-) signs to indicate vector direction:

(+) displacement $\Delta x > 0$

(-) displacement, so that $\Delta x < 0$.

Velocity: It is a vector quantity, its units: m/s and its dimensions: LT^{-1}

Speed: It is a scalar quantity

Average velocity

It is the displacement divided by time interval هو الازاحة مقسومة علي الزمن الكلي

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \; m/s$$

Average speed

It is the total distance divided by total time interval هو خارج قسمة المسافة الكلية علي الزمن الكلي

$$= \frac{Total\ distance}{Total\ time} \; m/s$$

It is not the magnitude of the average velocity

Example 2.1: page 60 & 61 in the book

Calculating the Average Velocity and Speed?

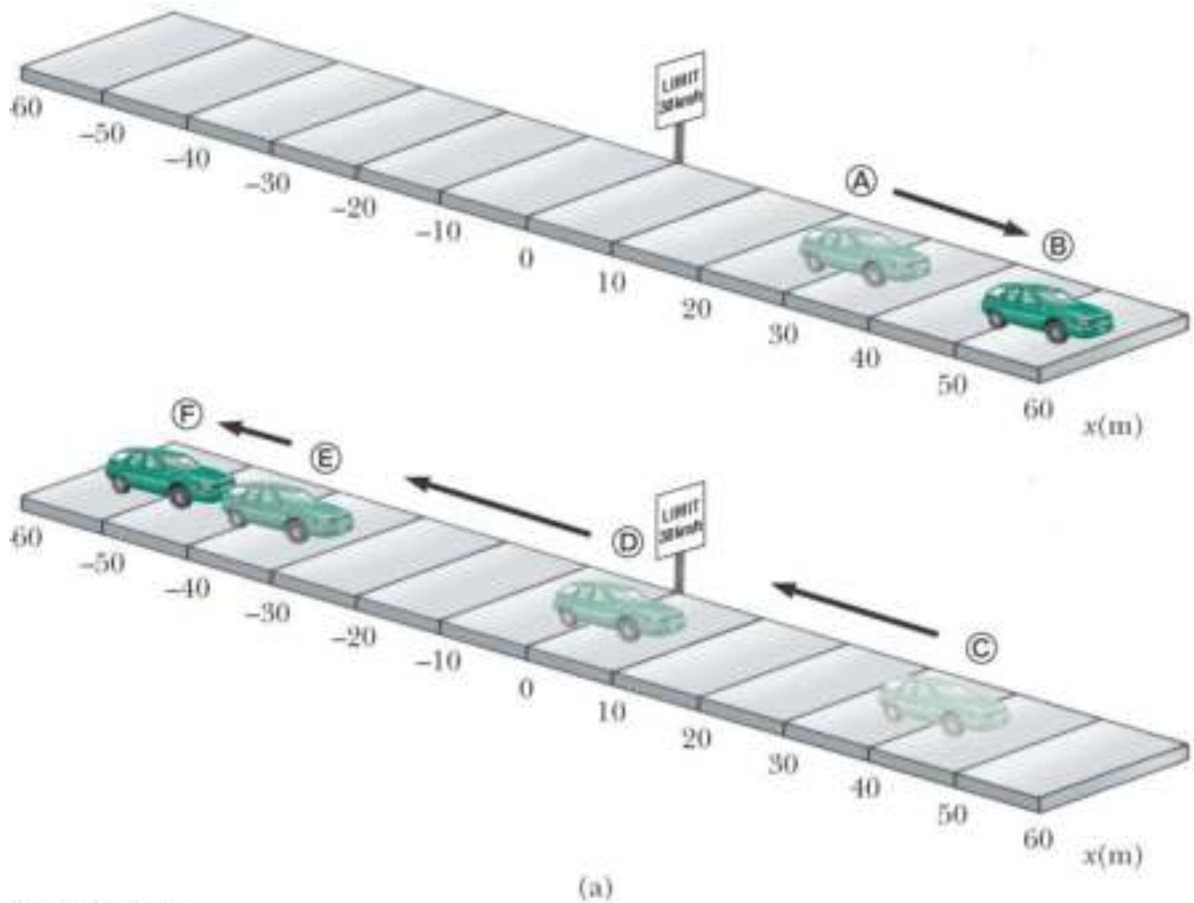
Find the displacement, average velocity, and average speed of the car in Figure 2.1a between positions (A) and (F).

احسب الازاحة والسرعة المتوسطة للسيارة الموضحة بالشكل التالي

Table 2.1

Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53



(a)

$x_A = 30$ m at $t_A = 0$s $x_F = -53$ m at $t_F = 50$ s.

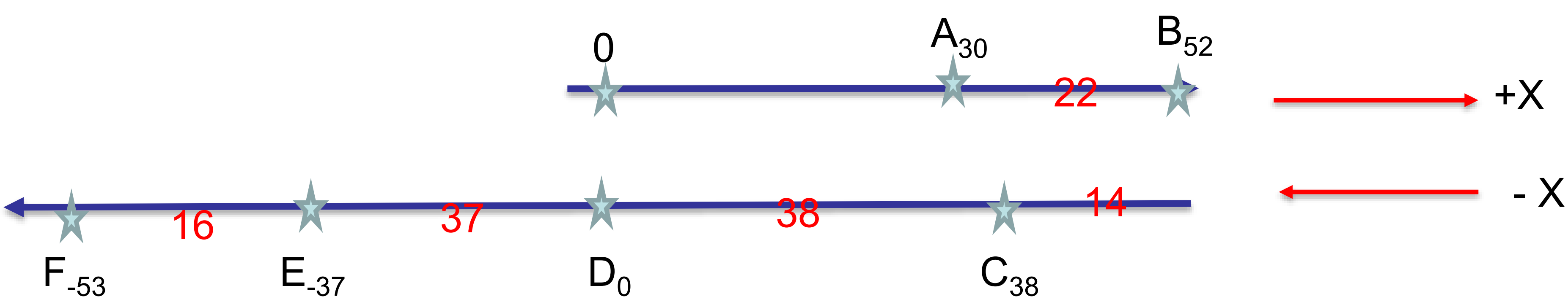
$$\Delta x = x_F - x_A = -53\text{ m} - 30\text{ m} = \boxed{-83\text{ m}}$$

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_F - x_A}{t_F - t_A}$$

$$= \frac{-53\text{ m} - 30\text{ m}}{50\text{ s} - 0\text{ s}} = \frac{-83\text{ m}}{50\text{ s}}$$

$$= \boxed{-1.7\text{ m/s}}$$

$$\text{Average speed} = \frac{127\text{ m}}{50\text{ s}} = \boxed{2.5\text{ m/s}}$$



Instantaneous velocity

It is the limit of average velocity as the time interval Δt approaches zero

$$v_{inst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \; m/s$$

Instantaneous speed

It is the magnitude of the instantaneous velocity

Example 2.3

page 26 in the book

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$ m.

Determine the displacement and average velocity of the particle in the time interval $t = 0$ s to $t = 1$ s and from $t = 1$ s to $t = 3$ s

Answer

At time interval $t = 0$ s to $t = 1$ s:

Displacement: $\Delta x = x_{f\,(t=1)} - x_{(t=0)} = (-4+2) - (0) = -2\,m$

Average velocity: $v_{av} = \frac{\Delta x}{\Delta t} = \frac{-2}{1-0} = -2\,m/s$

At time interval $t = 1$ s to $t = 3$ s

Displacement: $\Delta x = x_{f\,(t=3)} - x_{(t=1)} = (-12+18) - (-4+2) = 6+2 = +8\,m$

Average velocity: $v_{av} = \frac{\Delta x}{\Delta t} = \frac{8}{3-1} = +4\,m/s$

Average acceleration التسارع المتوسط

It is the change in velocity divided by time interval.

هو التغير في السرعة خلال فترة زمنية معينة

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \; m/s^2$$

Instantaneous acceleration التسارع اللحظي

It is the limit of average acceleration as the time interval Δt approaches 0

هو تغير التسارع المتوسط بين نقطتي البداية والنهاية

$$a_{inst} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \; m/s^2$$

Example 2.6

page 34 & 35 in the book

The velocity of a particle moving along the x-axis varies in time according to the expression $v(t) = 40 - 5t^2$ (m/s)

(a) Find the average acceleration in the time interval t = 0 s to t = 2 s

(b) Determine the acceleration at t = 2 s

Answer

(a) At time interval $t = 0$ s to $t = 2$ s

$$v_{xA} = 40 - 5t_A^2 = 40 - 5(0)^2 = +40 \text{ m/s}$$

$$v_{xB} = 40 - 5t_B^2 = 40 - 5(2.0)^2 = +20 \text{ m/s}$$

$$a_{x,avg} = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{20 \text{ m/s} - 40 \text{ m/s}}{2.0 \text{ s} - 0 \text{ s}}$$

$$= -10 \text{ m/s}^2$$

(b) At time $t = 2$ s

$$a_{inst} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$\Delta v = v_{f(t=t+\Delta t)} - v_{(t=t)} = (40 - 5t^2)_{t=t+\Delta t} - (40 - 5t^2)_{t=t}$$

$$\therefore \Delta v = [40 - 5(t + \Delta t)^2] - (40 - 5t^2)$$

$$\therefore \Delta v = [40 - 5(t^2 + 2t\Delta t + \Delta t^2)] - 40 + 5t^2$$

$$\therefore \Delta v = 40 - 5t^2 - 10t\Delta t - 5\Delta t^2 - 40 + 5t^2$$

$$\therefore \Delta v = -10t\Delta t - 5\Delta t^2$$

$$a_{inst} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{-10t\Delta t - 5\Delta t^2}{\Delta t}$$

$$\therefore a_{inst} = \lim_{\Delta t \to 0} (-10t - 5\Delta t) = -10t$$

$$\therefore a_{inst(t=2s)} = -(10 \times 2) = -20\ m/s^2$$

حل آخر للفقرة B

(b) At time $t = 2$ s   Instantaneous acceleration

$$a_{inst} = \frac{dv}{dt} = \frac{d}{dt}(40 - 5t^2) = -10t$$

$$\therefore a_{inst(t=2s)} = (-10 \times 2) = -20\ m/s^2$$

Analysis Model: Particle Under Constant Acceleration

Equation 1

$$\because a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0} \quad \therefore a = \frac{v - v_0}{t} \quad \therefore v - v_0 = at$$

$$\therefore v = v_0 + at$$

Equation 2

The average velocity $v_{av} = \frac{v + v_0}{2}$ Also $v_{av} = \frac{x}{t}$

$$\therefore \frac{x}{t} = \frac{v + v_0}{2} \qquad \therefore x = \frac{(v + v_0)t}{2}$$

$$\therefore x = \frac{1}{2}(v + v_0)t = \frac{1}{2}[(v_0 + at) + v_0]t = \frac{1}{2}[2v_0 + at]t$$

$$\therefore x = \frac{1}{2}[2v_0 t + at^2]$$

$$\therefore x = v_0 t + \frac{1}{2}at^2$$

Equation 3

$$\because v = v_0 + at \qquad \therefore v - v_0 = at \qquad \therefore t = \frac{v - v_0}{a}$$

$$\because v_{av} = \frac{v + v_0}{2} = \frac{x}{t} \qquad \therefore x = \left(\frac{v + v_0}{2}\right)t$$

$$\therefore x = \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) \quad \therefore x = \frac{v^2 - v_0^2}{2a} \qquad \therefore v^2 - v_0^2 = 2ax$$

$$\therefore v^2 = v_0^2 + 2ax$$

The three equations are

$$\therefore v = v_0 + at$$

$$\therefore x = \mathrm{x_i} + v_o t + \frac{1}{2}at^2$$

$$\therefore v^2 = v_0^2 + 2ax$$

Example 2.7: Page 38 in the book

A jet lands on the aircraft carrier at 63 m/s.
(A) What its acceleration if it stops in 2 s?

طائرة تهبط على حاملة طائرات بسرعة 63 م / ث. ما تسارعه إذا توقف خلال 2 ثانية؟

Answer

$v_0 = 63$ m/s , $a = ?$, $v = 0$, $t = 2$ s

$\because v = v_0 + at \qquad \therefore 0 = 63 + 2a \qquad \therefore -63 = 2a$

$$\therefore a = \frac{-63}{2} = -31.5\ m/s^2$$

(B) If the jet touches down at position $xi_t = 0$, what is its final position?

إذا لامست الطائرة سطح حاملة الطائرات في الموضع X_i في زمن t=0 ، فما هو موضعها النهائي؟

$$X_f = X_i + \tfrac{1}{2}(v_{xi} + v_{xf})t = 0 + \tfrac{1}{2}(63\ m/s + 0)(2s) = 63\ \text{m}$$

Homework:
21, 28