Meter:

The distance traveled by light in vacuum during a time of 1/299 792 458 second.

Kilogram (kg):

The mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France.

Second:

9 192 631 770 times the period of vibration of radiation from the cesium-133 atom.

Chapter 5

Motion in two Dimensions (Projectile Motion)

أهداف المحاضرة:

١. التعرف على المقذوف و كيفية حركته و مساره
٢. تطبيق معادلات الحركة التي درسناها بالفصل الثالث على حركة المقذوف السينية و الصادية
٣. معرفة أقصى مدى و أقصى ارتفاع يصل له المقذوف
٤. حل بعض المسائل الفيزيائية على حركة المقذوف

Definition:

Projectile : An object that moves through the air only under the influence of gravity after an initial thrust. Its vertical (y) and horizontal (x) motions can be considered independent

- تعريف المقذوف: جسم يتحرك بالهواء بعد تعرضة لقوة دفع ابتدائية، حيث أننا نستطيع التعامل مع الحركة الصادية و السينية بطريقة منفصلة

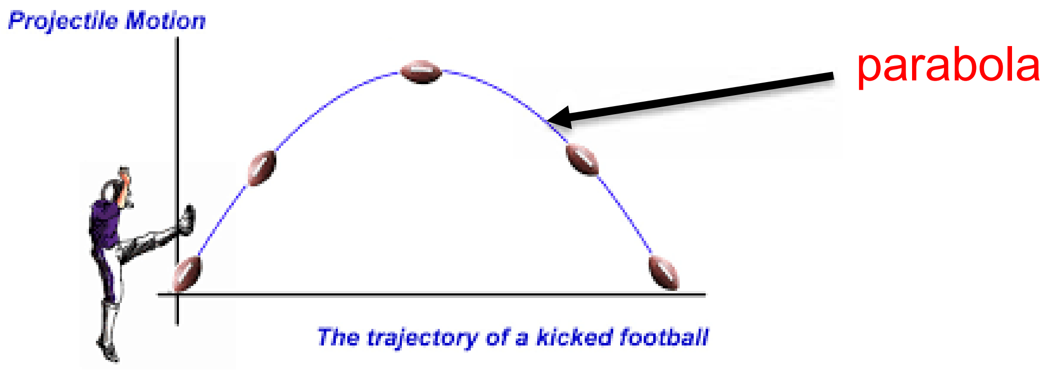
projectile motion:

The projectile motion is two-dimensional motion

حركة المقذوف هي حركة في بعدين مثل لاعب يركل الكرة بزاوية لمسافة بعيدة أو مدفع يطلق قذيفة

the path of a projectile, which is called trajectory(المسار) is always parabola.

- مسار المقذوف ياخذ شكل (parabola)، انظر الشكل التالي



Did you understand? If yes, answer the following questions:

1. **What is a projectile? Give an example.**
2. **How does the trajectory of the projectile look like? Draw it.**

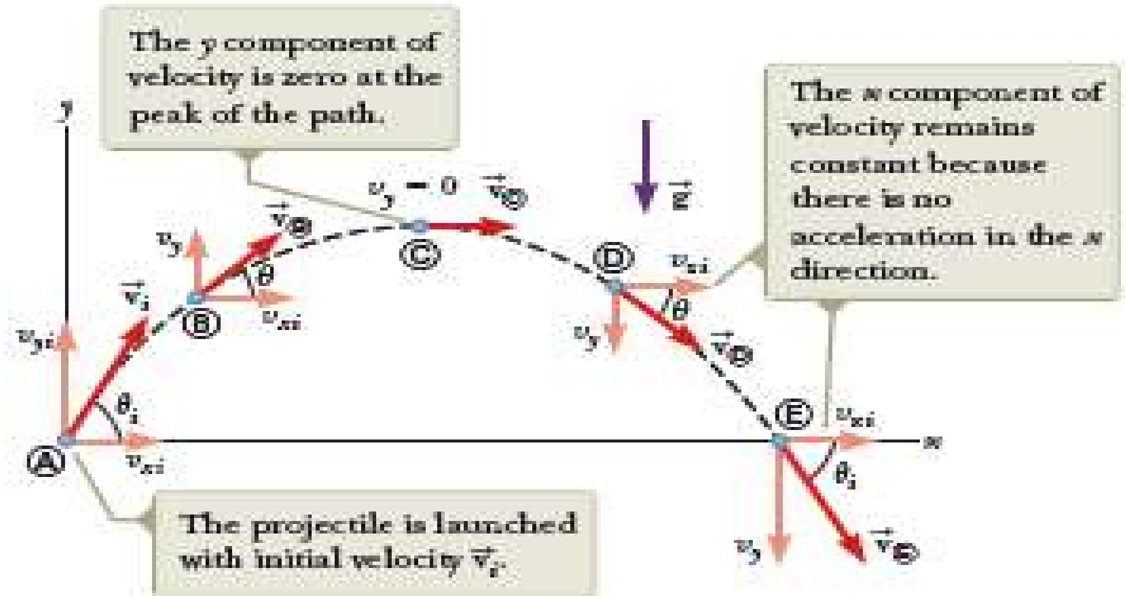
سنقوم الان بتفصيل حركة المقذوف بالاتجاهين السيني و الصادي

Assumptions of projectile motion:

The horizontal and vertical components of a projectile's motion are completely independent of each other and can be handled separately, with time t as the common variable for both components.

يمكننا أن نتعامل مع معادلات حركة المقذوف كحركة في الاتجاه السيني و نجد معادلاتها و حركة في الاتجاه الصادي و نجد معادلاتها (تذكر أن الحركتين لا يعتمدان على بعضهما). المتغير الوحيد المشترك بالحركتين هو الزمن

أنظر الشكل التالي:



ينطلق المقذوف بسرعة ابتدائية مقدارها (v_i). حسب ما تعلمنا بالمتجهات فان هذه السرعة لها مركبتين:

مركبة سينية $v_{xi} = v_i \cos\theta$: (x-component)

مركبة صادية $v_y = v_i \cos\theta$ =:(y-component)

سنأخذ المدخلات التالية بعين الاعتبار:

1. The velocity in the x-direction is always constant, therefore the acceleration in x-direction is always zero over the range of motion,

السرعة بالاتجاه السيني دائما ثابته و مقدارها $(v_x = v_{xi} = v_i \cos\theta)$، لذلك التسارع في الاتجاه السيني دائما صفر طيلة حركة المقذوف

2. The velocity in the y-direction varies throughout the projectile motion. The acceleration in the y-direction is the free-fall acceleration (-g) which is constant throughout the whole motion and is directed downward.

السرعة بالاتجاه الصادي متغيرة والتسارع في الاتجاه الصادي هو تسارع السقوط الحر طيلة حركة المقذوف

3. The time t as the common variable for the motion in the x & y-directions

الزمن هو العامل المشترك بين الحركة السينية و الصادية

4. The effect of air resistance is negligible.

نهمل مقاومة الهواء

Did you understand? If yes, answer the following questions:

1. **What is the acceleration in the x-direction?**
2. **What is the acceleration in the y-direction?**
3. **Is the time of fly of the projectile in the x- & y- directions different?**

Let us recall the kinematic equations given in Chapter 3:

$$v_{xf} = v_{xi} + at$$

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$

Repeating with the y component and using $y_i = 0$ and $a_y = -g$, we obtain

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2 = (v_i \sin \theta_i)t - \frac{1}{2}gt^2 \tag{4.12}$$

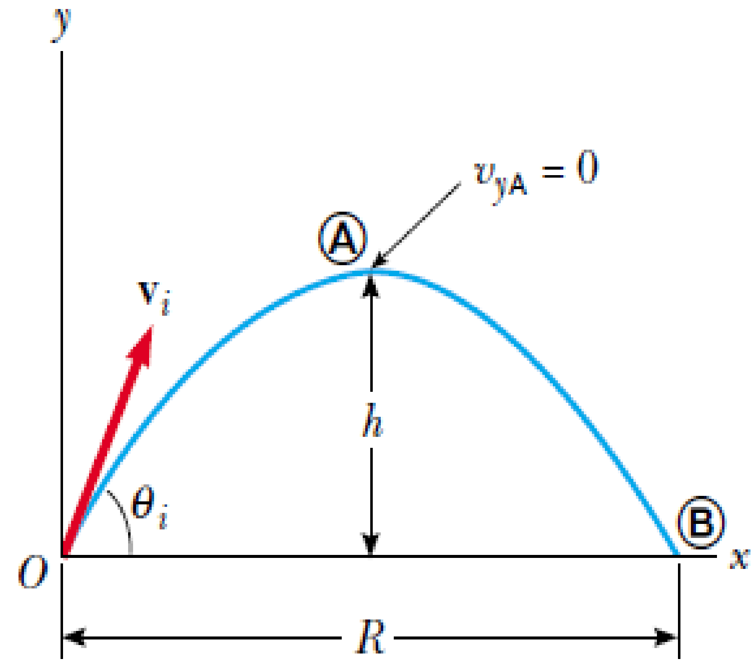
Next, from Equation 4.11 we find $t = x_f/(v_i \cos \theta_i)$ and substitute this expression for t into Equation 4.12; this gives

$$y = (\tan \theta_i)x - \left(\frac{g}{2{v_i}^2 \cos^2 \theta_i}\right)x^2$$

Horizontal Range and Maximum Height of a Projectile:

$$v_{yf} = v_{yi} - gt \rightarrow 0 = v_i \sin\theta_i - gt_{A}$$

$$t_{A} = \frac{v_i \sin\theta_i}{g}$$



Substituting this expression for t_A *into the y part* and replacing $y = y_A$ *with h :*

$$y_f = y_i + v_{yi}t - \tfrac{1}{2}gt^2 \rightarrow h = (v_i \sin\theta_i)\frac{v_i \sin\theta_i}{g} - \tfrac{1}{2}g\left(\frac{v_i \sin\theta_i}{g}\right)^2$$

$$\boxed{h = \frac{v_i^2 \sin^2\theta_i}{2g}}$$

The range R is the horizontal position of the projectile at a time that is twice the time at which it reaches its peak, that is, at time $t_B = 2t_A$.

$$R = v_{xi}t_B = (v_i \cos\theta_i)2t_A$$

$$= (v_i \cos\theta_i)\frac{2v_i \sin\theta_i}{g} = \frac{2v_i^2 \sin\theta_i \cos\theta_i}{g}$$

Using the identity $\sin 2\theta = 2\sin\theta\cos\theta$

$$\boxed{R = \frac{v_i^2 \sin 2\theta_i}{g}}$$

The maximum value of *R is* $R_{max} = v_i^2/g$*. This result follows* from the fact that the maximum value of sinθ_i *is 1, which occurs when* θ_i*=90°.* Therefore, *R is a maximum when* θ_i *= 45°.*

Example 4.2 the long jump page 87

A long jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.

(a) How far does he jump in the horizontal direction?

(b) What is the maximum height reached?

(a)

$$R = \frac{V_o^2 \sin 2\theta}{g} = \frac{(11)^2 \sin 2x20}{9.8}$$

$$R = \frac{121 \sin 40}{9.8} = \frac{121x0.64}{9.8} = 7.9m$$

(b)

$$h = \frac{V_o^2 \sin^2 \theta}{2g} = \frac{121x \sin^2 20}{2x9.8} =$$

$$h = \frac{121x(0.34)^2}{19.6} = \frac{121x0.1169}{19.6} = 0.72m$$

Example 4.5 The End of the Ski Jump (Page 90)

A ski jumper leaves the ski track moving in the horizontal direction with a speed of 25.0 m/s as shown in Figure 4.14. The landing incline below her falls off with a slope of 35.0°. Where does she land on the incline?

We categorize the problem as one of a particle in projectile motion. As with other projectile motion problems, we use the particle under ***constant velocity model for the horizontal motion*** and the particle under ***constant acceleration model for the vertical motion.***

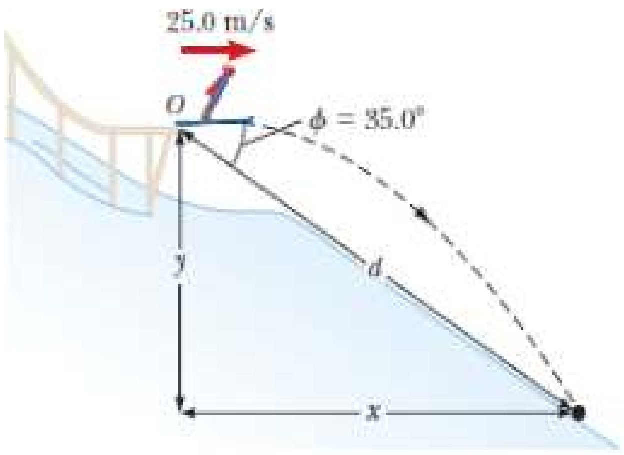


Figure 4.14 (Example 4.5) A ski jumper leaves the track moving in a horizontal direction.

It is convenient to select ***the beginning of the jump as the origin***. The initial velocity components are v_{xi} = 25.0 m/s and (v_{yi} = 0). From the right triangle in Figure 4.14, we see that the jumper's x and y coordinates at the landing point are given by: $x_f = d \cos \Theta$ and $y_f = 2d \sin \Theta$.

(1) $x_f = v_{xi} t$

(2) $y_f = v_{yi} t + (1/2) g t^2$

(3) $d \cos \Theta = v_{xi} t$

(4) $2d \sin \Theta = - (1/2) g t^2$

Solve Equation (3) for t and substitute the result into Equation (4):

$$-d \sin \phi = -\tfrac{1}{2} g \left(\frac{d \cos \phi}{v_{xi}} \right)^2$$

Solve for *d* and substitute numerical values:

$$d = \frac{2 v_{xi}^2 \sin \phi}{g \cos^2 \phi} = \frac{2(25.0 \text{ m/s})^2 \sin 35.0^\circ}{(9.80 \text{ m/s}^2) \cos^2 35.0^\circ} = 109 \text{ m}$$

Evaluate the x and y coordinates of the point at which the skier lands:

$$x_f = d \cos \phi = (109 \text{ m}) \cos 35.0^\circ = 89.3 \text{ m}$$

$$y_f = -d \sin \phi = -(109 \text{ m}) \sin 35.0^\circ = -62.5 \text{ m}$$

Homework:
15