Name:______________________________ **Student ID**______________________

Due Date: The assignment is due **Thursday 24/09/2020**.
Directions: This is a group assignment of maximum two people. Try to work out your solutions independently. Plagiarism will be dealt with very seriously. Write well-organized, detailed solutions. Use MATLAB Live Script to generate a report that includes the solutions and all related commands, scripts and function. The report should include also test runs of the functions and all the requested plots.

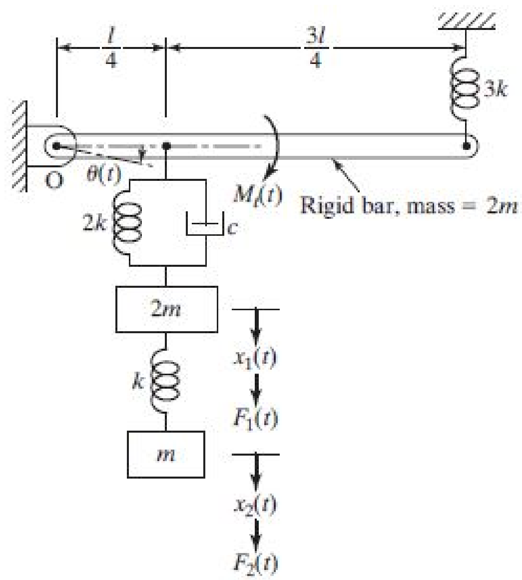
Question:	1	2	3	4	5	Total
Points:	10	20	25	35	10	100
Score:						

1. (10 points) Plot the forced-vibration response of a viscously damped three-degree-of-freedom system for 15 seconds with equations of motion given as

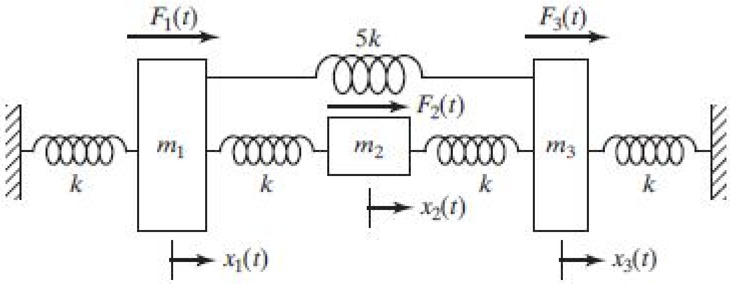
$$10\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}\begin{Bmatrix}\ddot{x}_1\\\ddot{x}_2\\\ddot{x}_3\end{Bmatrix}+5\begin{bmatrix}3&-1&0\\-1&4&-3\\0&-3&3\end{bmatrix}\begin{Bmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\end{Bmatrix}+20\begin{bmatrix}7&-3&0\\-3&5&-2\\0&-2&2\end{bmatrix}\begin{Bmatrix}x_1\\x_2\\x_3\end{Bmatrix}=\begin{Bmatrix}5\cos 2t\\2\sin 5t\\0\end{Bmatrix}$$

and the system is subjected to zero initial conditions.

2. (20 points) Determine the equations of motion of a rigid bar connected to springs, masses and a damper as shown in the figure below and write the equations in a matrix form. Assume the mass m is 1 kg, the bar has a length of 3 m, the stiffness constant k is 80 N/m and the damping constant c is $16\,\mathrm{N\cdot s/m}$. Find the undamped natural frequencies of the system and the corresponding mode shapes. Can the response of the system be determined using Modal Analysis? Explain your answer and suggest a way to solve it.



3. (25 points) Determine the equations of motion of the system shown in the figure below and write the equations in a matrix form. Find the natural frequencies, modal matrix, mass and stiffness matrices in the principal coordinates $\mathbf{q}(t)$ and their corresponding initial conditions using MATLAB. Find the response using Modal Analysis by utilizing Symbolic package for integrating the second-order differential equations and plot the response for 5 seconds. Assume $m_1 = 2\,\mathrm{kg}$, $m_2 = 1\,\mathrm{kg}$, $m_3 = 2\,\mathrm{kg}$, $k = 1\,\mathrm{N/m}$ $F_1(t) = F_2(t) = F_3(t) = 0$, and initial conditions of $x_1(0) = 0, \dot{x}_1(0) = 1, x_2(0) = -1, \dot{x}_2(0) = 0, x_3(0) = 1$, and $\dot{x}_3(0) = -1$.



4. (35 points) Develop a general-purpose MATLAB program `[x,omega,Phi] = modal(M,C,K,F,x0,x0dot)` that finds the total response $x_i(t)$ of a viscously damped n-degree-of-freedom system under a general applied force $F_i(t)$ where $i = 1, 2, \ldots, n$. The function requires the following input data:

   a) `M, C, K` = mass, damping, and stiffness matrices of size $n \times n$ of a viscously damped n-degree-of-freedom system.
   b) `F` = function handle of dimension $n \times 1$, containing the known expressions of $F_i(t)$.
   c) `x0` = array of dimension $n \times 1$, containing the initial displacement values of $x_i(t)$.
   d) `x0dot` = array of dimension $n \times 1$, containing the initial velocity values of $x_i(t)$.

   The output of the function is

   a) `x` = array of dimension $n \times 1$, containing the determined expressions of the total response $x_i(t)$.
   b) `omega` = array of dimension $n \times 1$, containing the undamped natural frequencies of the system ω_i.
   c) `Phi` = array of dimension $n \times n$, containing the corresponding mode shapes, ϕ_i, for each of the undamped natural frequency ω_i.

   The function should report an error if the matrices $\mathbf{M}, \mathbf{C}, \mathbf{K}$ and the vector $\mathbf{F}$ do not have the same size. The program can run without passing either $\mathbf{C}$ or $\mathbf{F}$.

5. (10 points) Test the developed program by plotting the total response for 10 seconds of the following systems,

   (a) Free-undamped system

$$\mathbf{M} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{K} = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 5 & -3 \\ 0 & -3 & 7 \end{bmatrix}, \qquad \mathbf{x}_0 = \begin{Bmatrix} 0 \\ -1 \\ 1 \end{Bmatrix}, \qquad \dot{\mathbf{x}}_0 = \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix}$$

   (b) Free-damped system

$$\mathbf{M} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} 6 & -2 \\ -2 & 2 \end{bmatrix}, \qquad \mathbf{K} = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}, \qquad \mathbf{x}_0 = \begin{Bmatrix} 1 \\ -0.5 \end{Bmatrix}, \qquad \dot{\mathbf{x}}_0 = \begin{Bmatrix} 0.3 \\ 1.5 \end{Bmatrix}$$

   (c) Forced-undamped system

$$\mathbf{M} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \mathbf{K} = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}, \qquad \mathbf{F} = \begin{Bmatrix} 3\sin(4t) \\ 0 \end{Bmatrix}, \qquad \mathbf{x}_0 = \begin{Bmatrix} 1 \\ -0.5 \end{Bmatrix}, \qquad \dot{\mathbf{x}}_0 = \begin{Bmatrix} 0.3 \\ 1.5 \end{Bmatrix}$$

   (d) Forced-damped system

$$\mathbf{M} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} 5.1 & -1.7 & 0 \\ -1.7 & 5.1 & -3.4 \\ 0 & -3.4 & 3.4 \end{bmatrix}, \qquad \mathbf{K} = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 2 \end{bmatrix},$$

$$\mathbf{F} = \begin{Bmatrix} 0 \\ 2\sin(4t) \\ 0 \end{Bmatrix}, \qquad \mathbf{x}_0 = \begin{Bmatrix} 1 \\ -0.5 \\ 0.1 \end{Bmatrix}, \qquad \dot{\mathbf{x}}_0 = \begin{Bmatrix} 0.3 \\ 1.5 \\ 0 \end{Bmatrix}$$