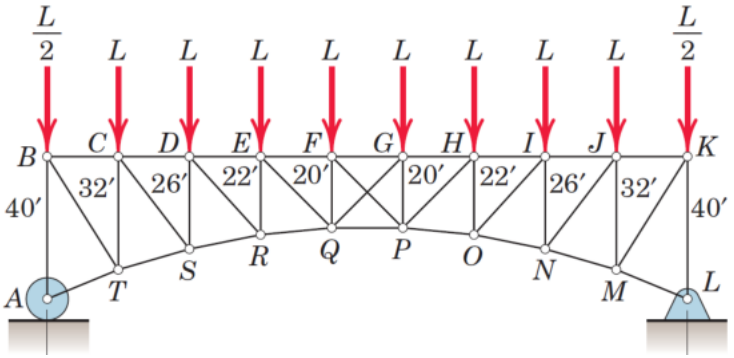

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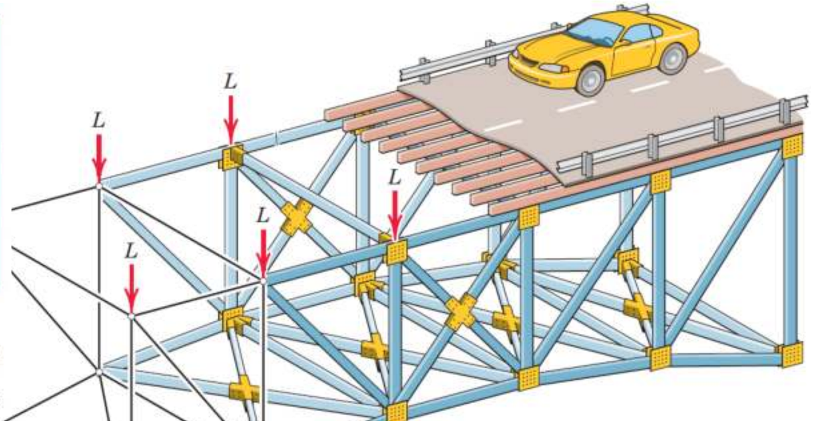
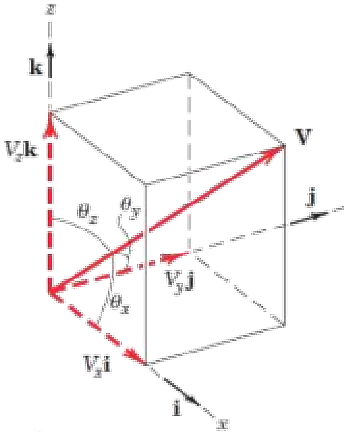


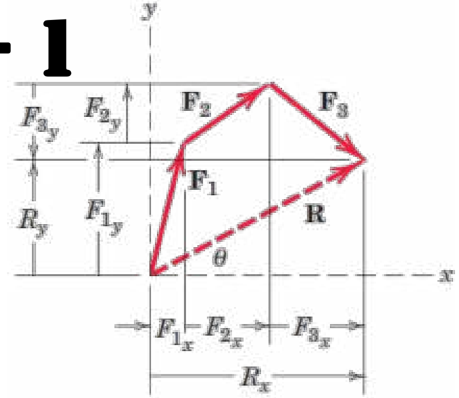


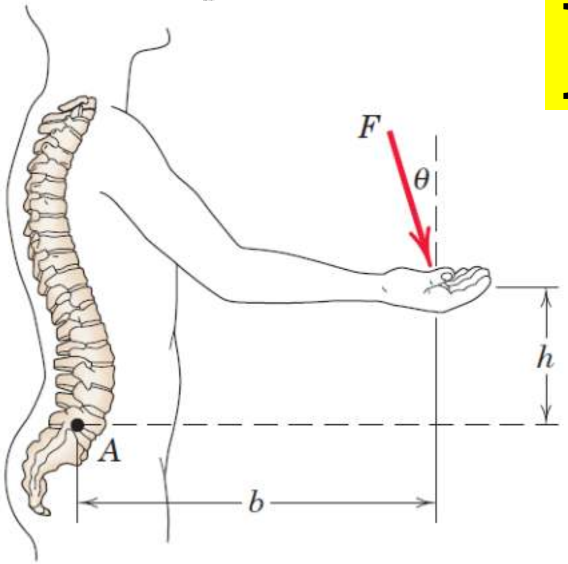
ENGINEERING MECHANICS – 1

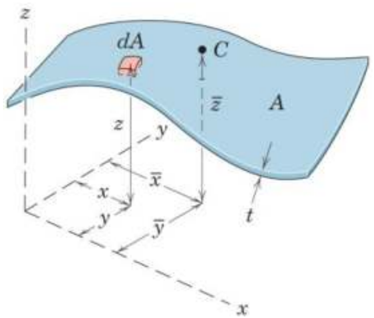
ENG 203

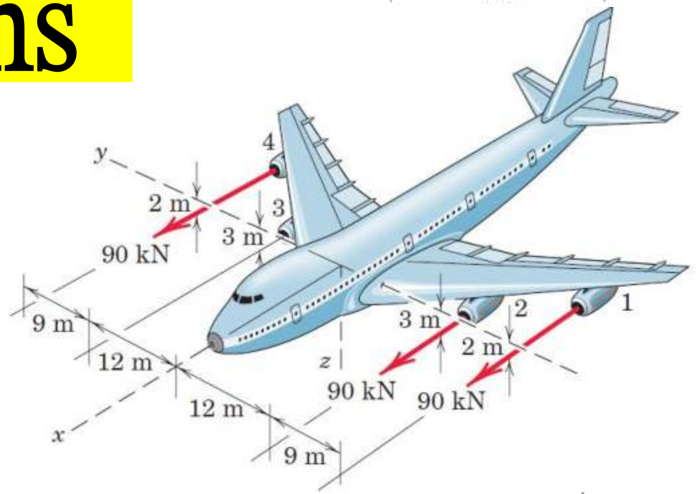
CHAPTER - 2

Force Systems





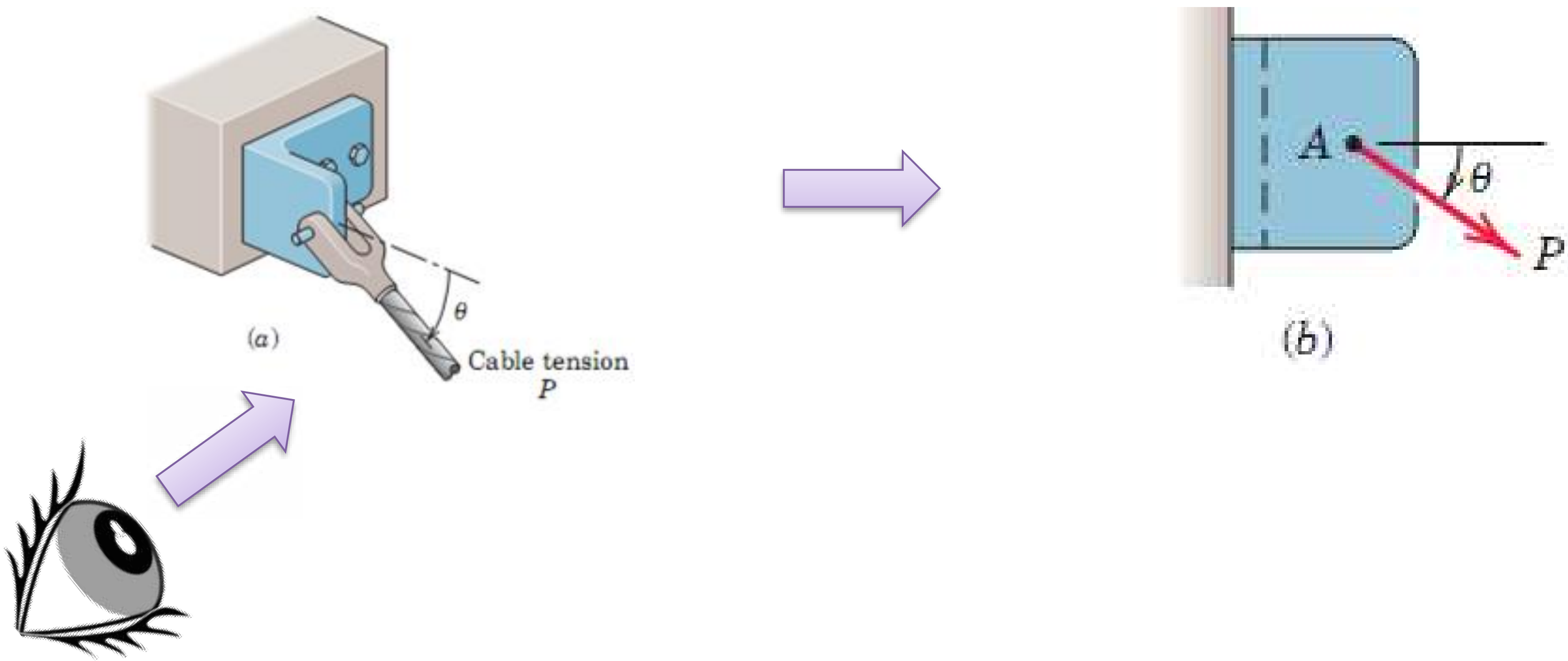




CHAPTER OUTLINE

- ✓ *Resultant & Components of Vectors*
- ✓ *Moment*
- ✓ *Couple*
- ✓ *Force - Couple System*
- ✓ *Extra Exercises*

Representation of the Forces

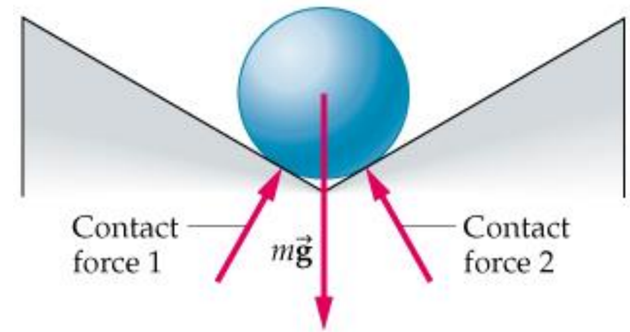


Complete specification of the action of a force must include its ***magnitude, direction***, *and* ***point of application***, and therefore we must treat it as a ***fixed vector.***

Force Classification

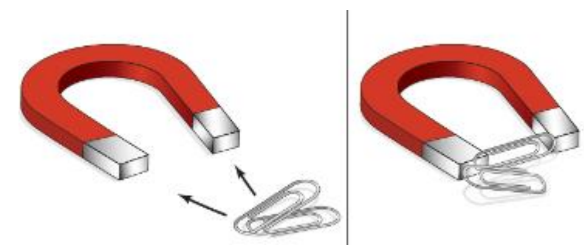
Forces are classified as either **contact** or **body forces**.

- A ***contact force:*** is produced by direct physical contact.



Contact Force

- **A *body force:*** *e.g.* <u>gravitational, electric, or magnetic field</u>. An example of a body force is your <u>**weight**</u>.

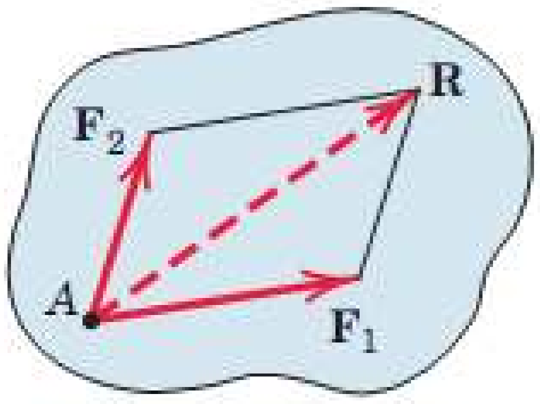


Magnetic Field

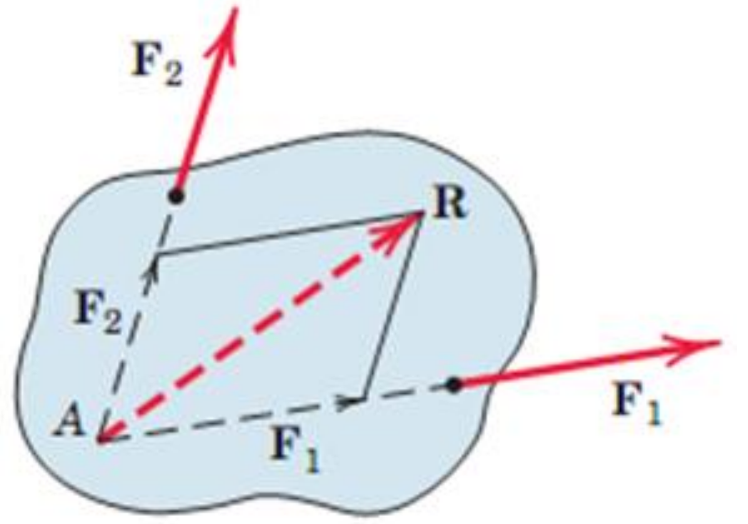
- Forces may be further classified as either ***concentrated*** or ***distributed***.

<u>Resultant</u> of Concurrent Forces

- ✓ *By **the <u>principle of transmissibility</u>**: → **R (Resultant)** at the point of concurrency **A**,* as shown in Fig **b**.



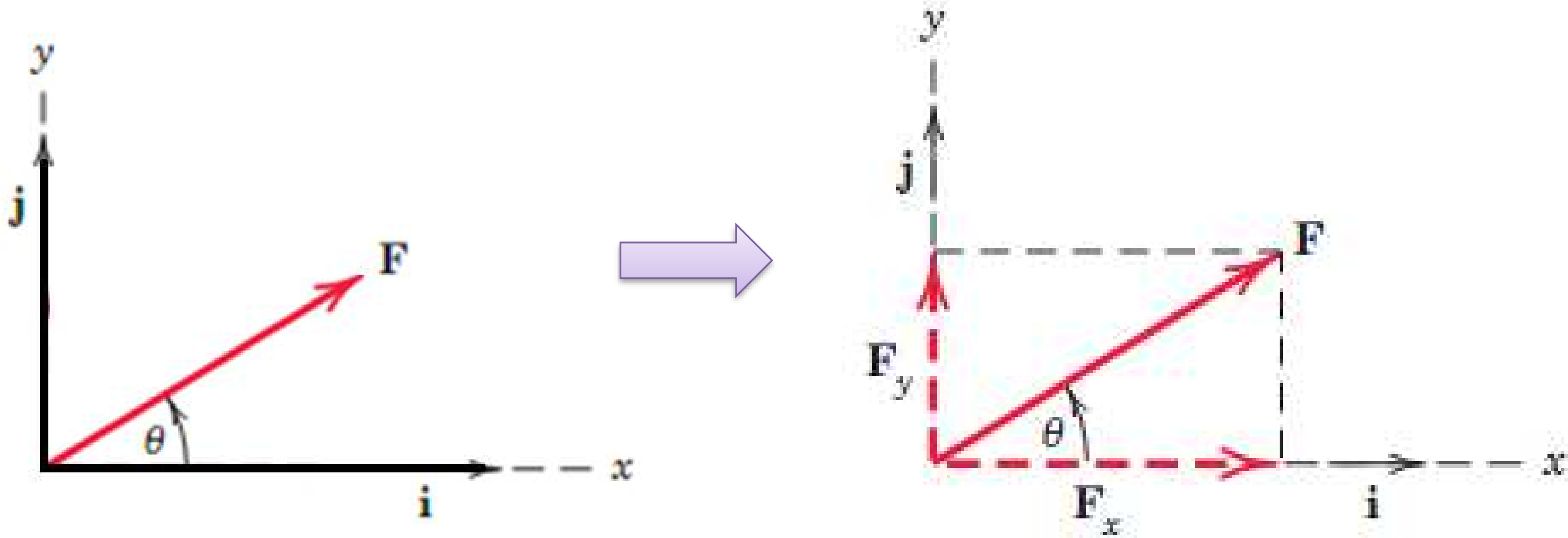
(a)



(b)

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$$

Rectangular **Components**



$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

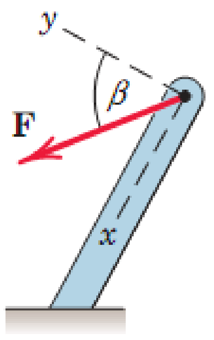
where:

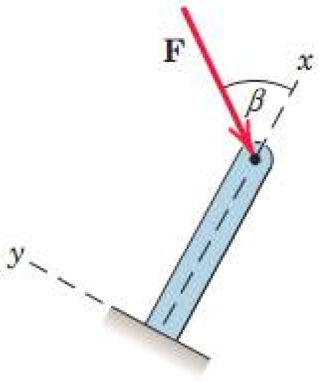
$\boldsymbol{F_x}$ and $\boldsymbol{F_y}$ are *vector* ***components*** *of* $\boldsymbol{F}$ *in the* $\boldsymbol{x}$ *− and* $\boldsymbol{y}$ *− directions.*

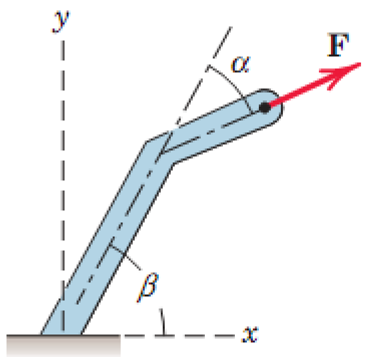
$$F_x = F\cos\theta \qquad F = \sqrt{F_x{}^2 + F_y{}^2}$$

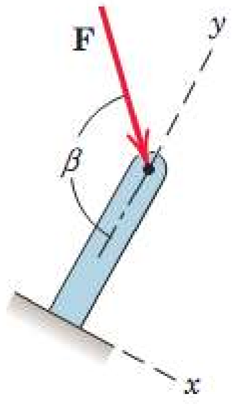
$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

Rectangular Components

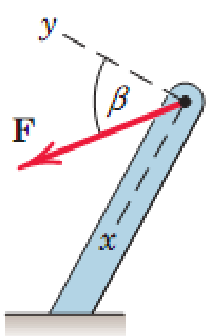


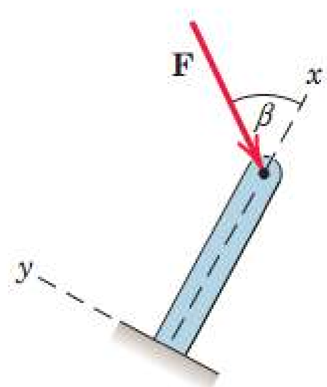


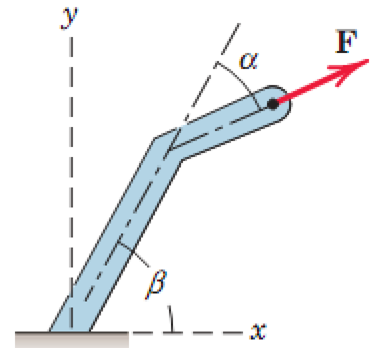


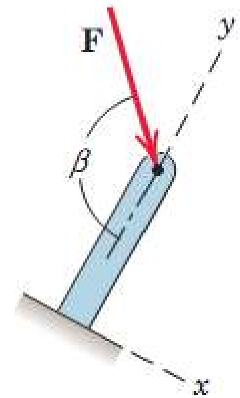


Rectangular Components









$$F_x = F \sin \beta$$
$$F_y = F \cos \beta$$

$$F_x = -F \cos \beta$$
$$F_y = -F \sin \beta$$

$$F_x = F \cos(\beta - \alpha)$$
$$F_y = F \sin(\beta - \alpha)$$

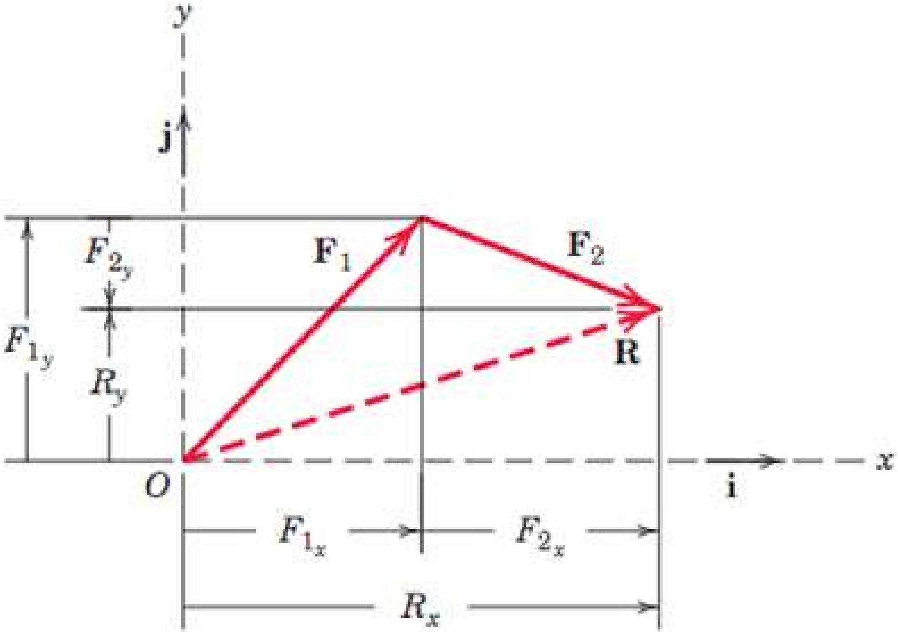
$$F_x = F \sin(\pi - \beta)$$
$$F_y = -F \cos(\pi - \beta)$$

Resultant of Two Forces

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (F_{1_x}\mathbf{i} + F_{1_y}\mathbf{j}) + (F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j})$$

or

$$R_x\mathbf{i} + R_y\mathbf{j} = (F_{1_x} + F_{2_x})\mathbf{i} + (F_{1_y} + F_{2_y})\,\mathbf{j}$$

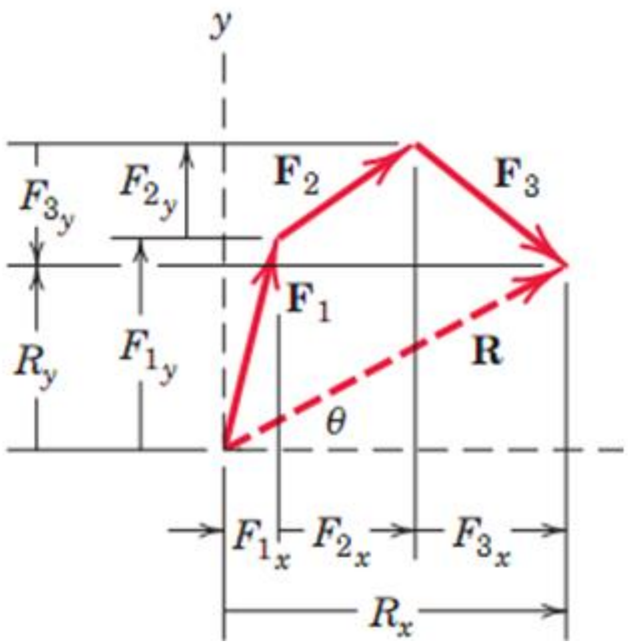


$$R_x = F_{1_x} + F_{2_x} = \Sigma F_x$$

$$R_y = F_{1_y} - F_{2_y} = \Sigma F_y$$

Negative!

Resultant of a System of Forces



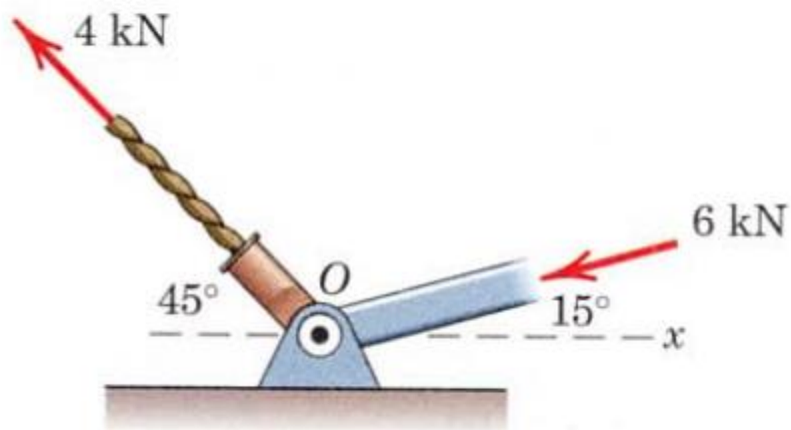
$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

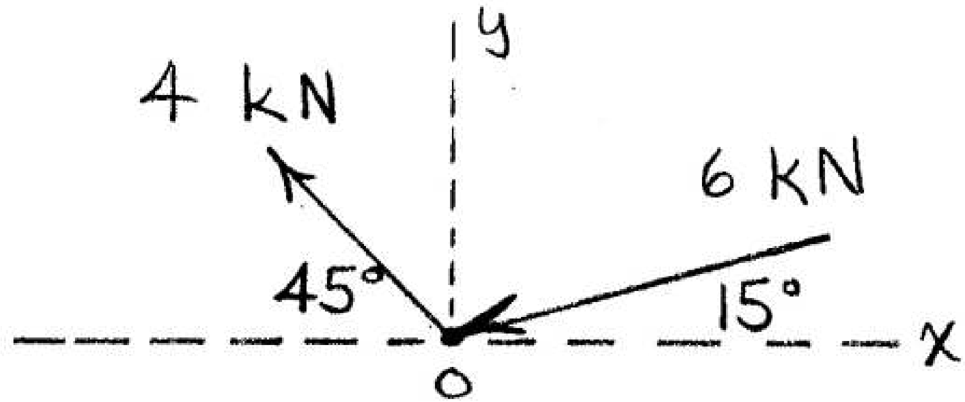
$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

EXERCISE - 1

The two structural members, one of which is in tension and the other in compression, exert the indicated forces on joint O. Determine the magnitude of the resultant $\mathbf{R}$ of the two forces and the angle θ which $\mathbf{R}$ makes with the positive x-axis.



Solution – Exercise - 1



$$R_x = \Sigma F_x = -4\cos 45° - 6\cos 15° = -8.62 \text{ kN}$$

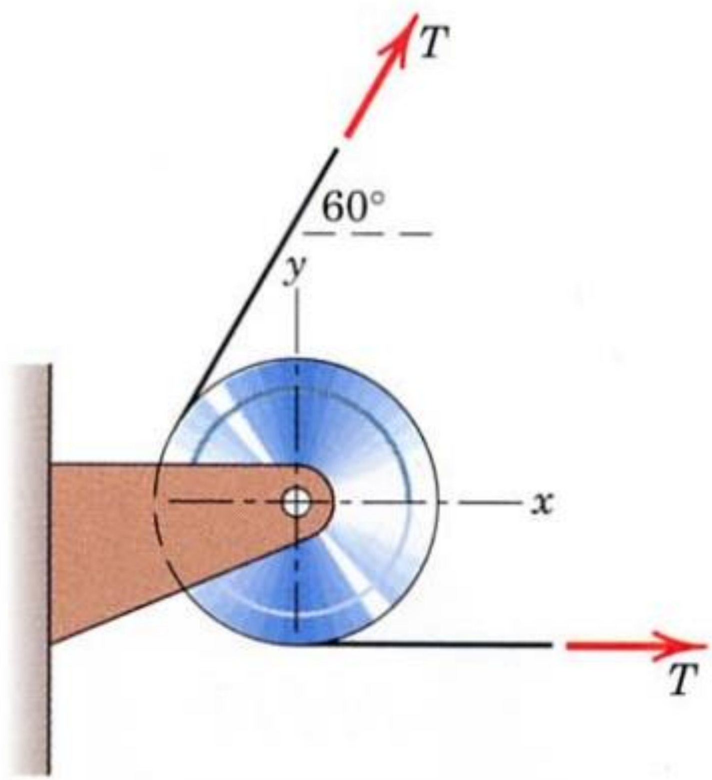
$$R_y = \Sigma F_y = 4\sin 45° - 6\sin 15° = 1.276 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{8.72 \text{ kN}}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{1.276}{-8.62}\right) = \underline{171.6°}$$

EXERCISE - 2

If the equal tensions T in the pulley cable are 400 N, express in vector notation the force **R** exerted on the pulley by the two tensions. Determine the magnitude of **R**.



Solution – Exercise - 2

$$R_x = \Sigma F_x = 400 + 400 \cos 60^\circ = 600 \text{ N}$$

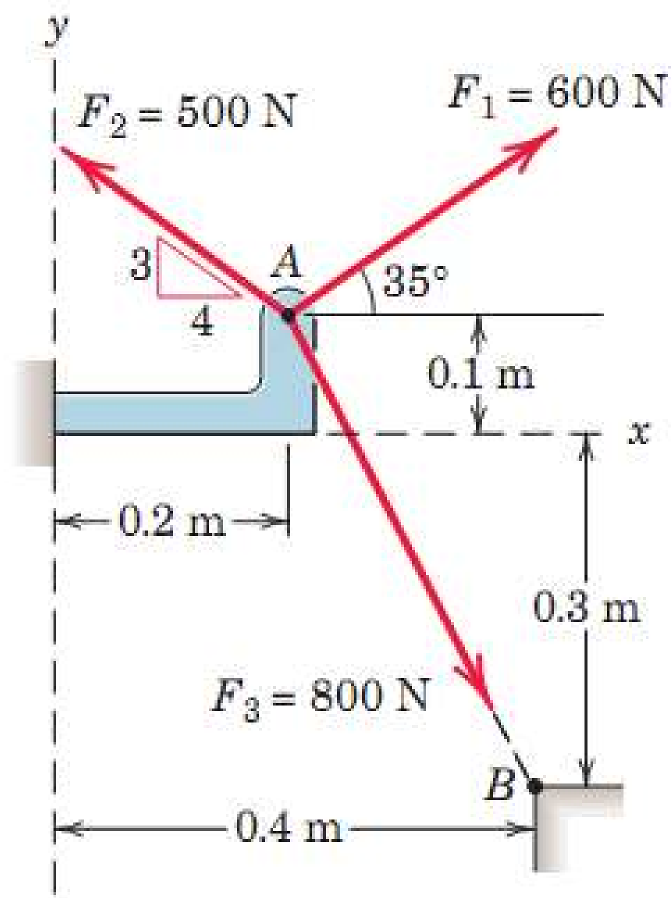
$$R_y = \Sigma F_y = 400 \sin 60^\circ = 346 \text{ N}$$

$$\Rightarrow \underline{R} = \underline{600\,\underline{i} + 346\,\underline{j} \text{ N}}$$

$$R = \sqrt{600^2 + 346^2} = \underline{693 \text{ N}}$$

Exercise – 3

The forces $\boldsymbol{F_1}$, $\boldsymbol{F_2}$, and $\boldsymbol{F_3}$, all of which act on point **A** of the bracket, are specified in three different ways. Determine the ***x*** **and** ***y*** **scalar components** of each of the three forces.



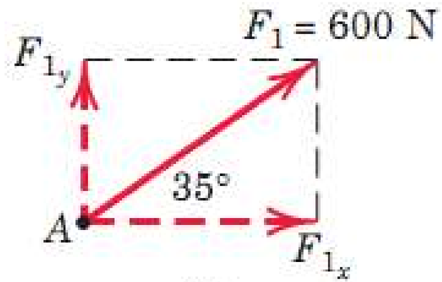
Solution - Exercise – 3

The scalar components of $\mathbf{F}_1$, from Fig. a, are

Fig. (a)

$$F_{1_x} = 600 \cos 35° = 491 \text{ N} \qquad \textit{Ans.}$$

$$F_{1_y} = 600 \sin 35° = 344 \text{ N} \qquad \textit{Ans.}$$



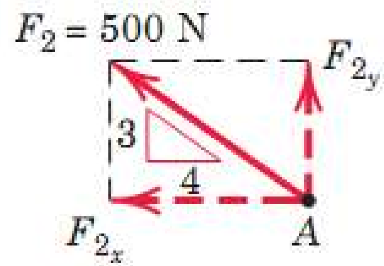
(*a*)

The scalar components of $\mathbf{F}_2$, from Fig. b, are

Fig. (b)

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N} \qquad \textit{Ans.}$$

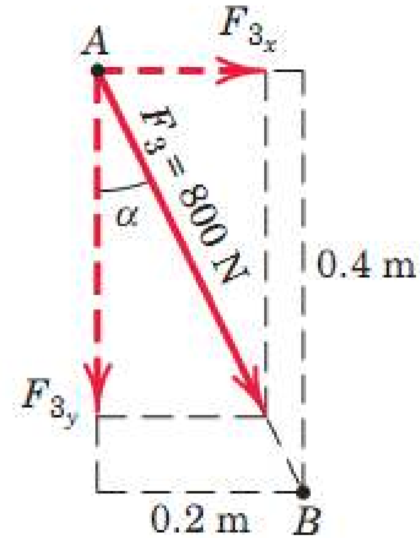
$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N} \qquad \textit{Ans.}$$



(*b*)

Fig. (c)

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6°$$



(*c*)

Then, $F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6° = 358 \text{ N}$ *Ans.*

$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6° = -716 \text{ N}$ *Ans.*

or:

$$\mathbf{F}_3 = F_3\mathbf{n}_{AB} = F_3 \frac{\overrightarrow{AB}}{AB} = 800\left[\frac{0.2\mathbf{i} - 0.4\mathbf{j}}{\sqrt{(0.2)^2 + (-0.4)^2}}\right]$$

$$= 800\,[0.447\mathbf{i} - 0.894\mathbf{j}]$$

$$= 358\mathbf{i} - 716\mathbf{j} \text{ N}$$

The required scalar components are then

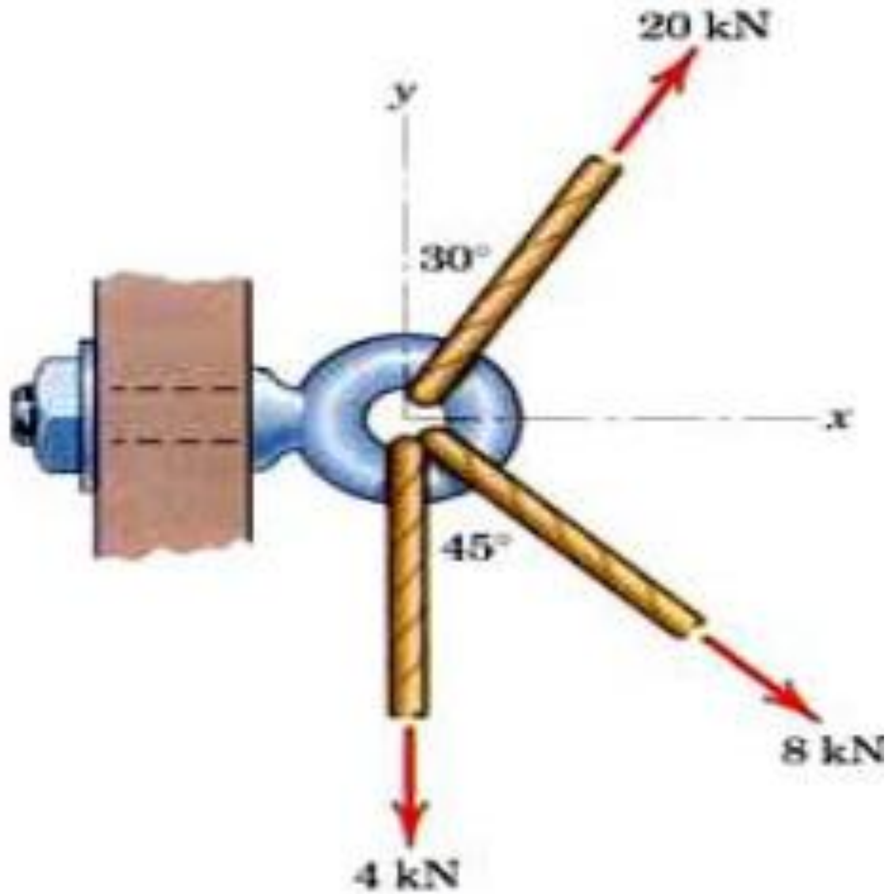
$$F_{3_x} = 358 \text{ N} \qquad \textit{Ans.}$$

$$F_{3_y} = -716 \text{ N} \qquad \textit{Ans.}$$

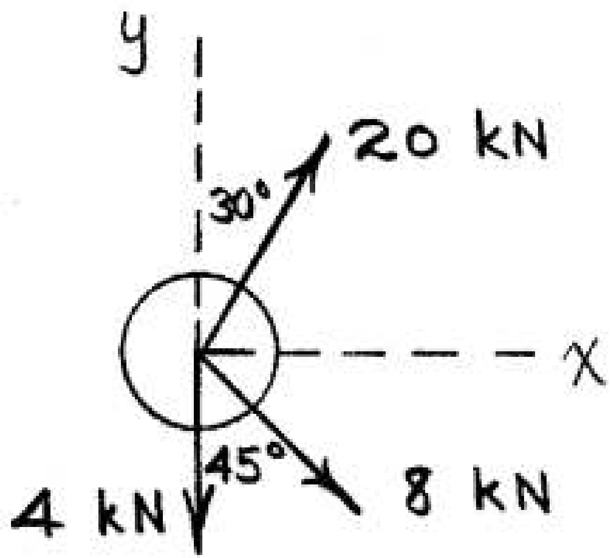
which agree with our previous results.

Exercise – 4

Determine the resultant R of the three tension forces acting on the eye bolt. Find the magnitude of R and the angle θ_x which R makes with the positive x- axis



Solution - Exercise – 4



$$R_x = \Sigma F_x = 20 \sin 30° + 8 \sin 45° = 15.66 \text{ kN}$$

$$R_y = \Sigma F_y = 20 \cos 30° - 8 \cos 45° - 4 = 7.66 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{17.43 \text{ kN}}$$

$$\theta_x = \tan^{-1}\left(R_y / R_x\right) = \underline{26.1°}$$

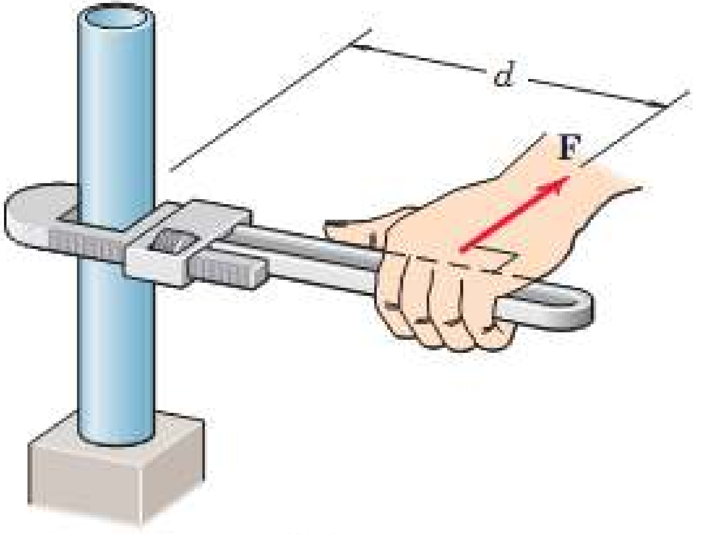
Moment

- Moment is also referred to as **torque**.

- The magnitude of this tendency <u>depends</u> on both:

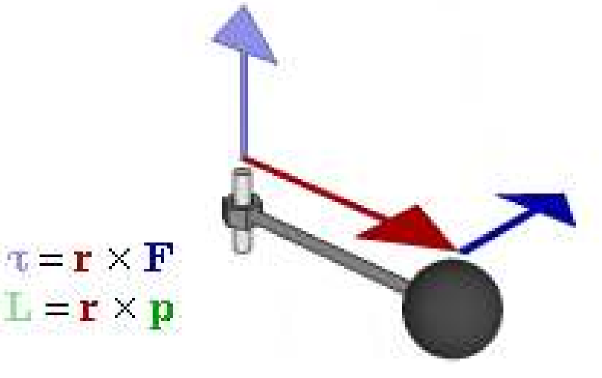
  ✓ **The <u>magnitude</u> *F*** of the force,

  ✓ and the effective **<u>length</u> *d*** of the wrench handle.



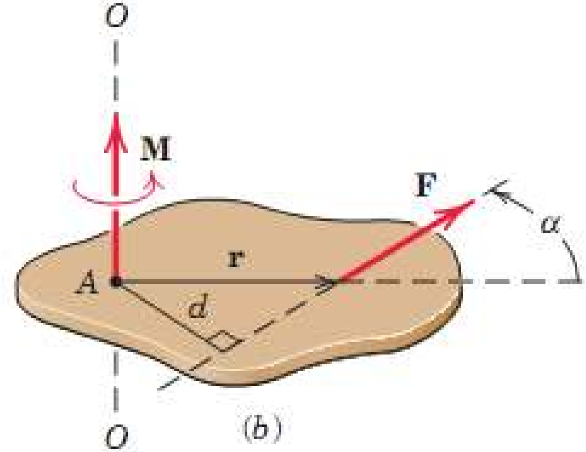
(a)

$$M = F.D$$

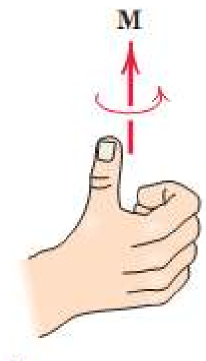


Moment about a Point

$$M = F.r.sin\alpha = F.d$$



(b)



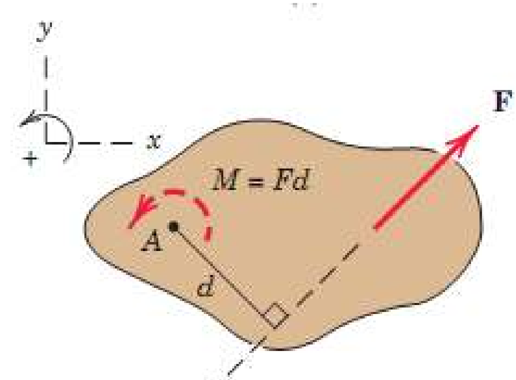
(c)

- ✓ ***The right-hand Rule..***

- ✓ *The* ***basic units*** *of moment in* ***SI*** *units are newton-meters* **(N. m)**, *and in the* ***U.S.*** *customary system are pound-feet* **(lb.ft.)**.

- ✓ **Moment directions:**

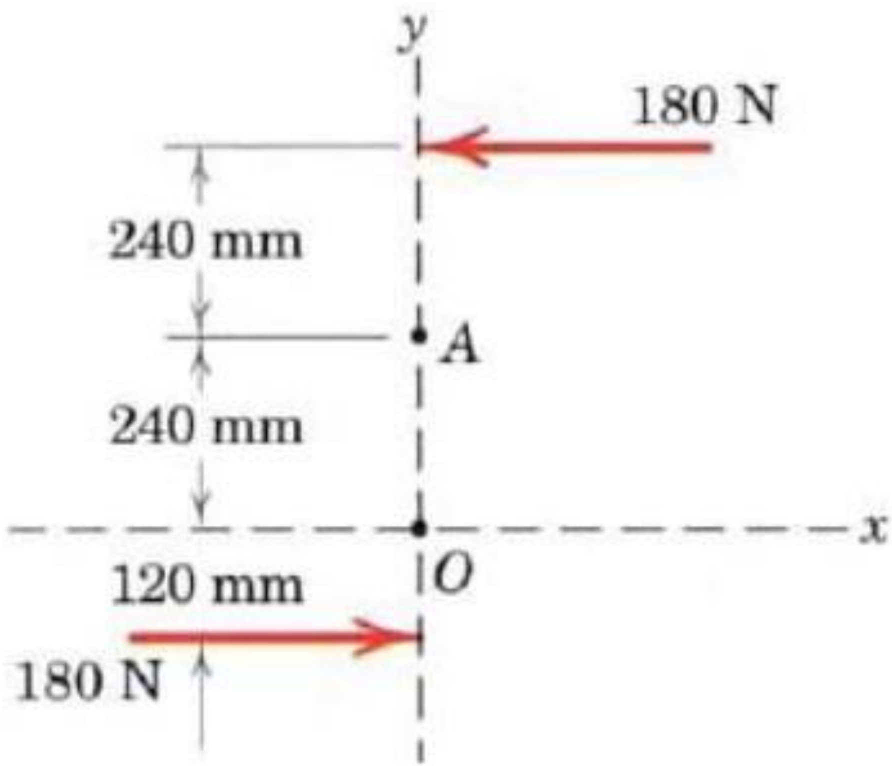
(+) for ***counter-clockwise*** moments, and
(-) for ***clockwise*** moments.



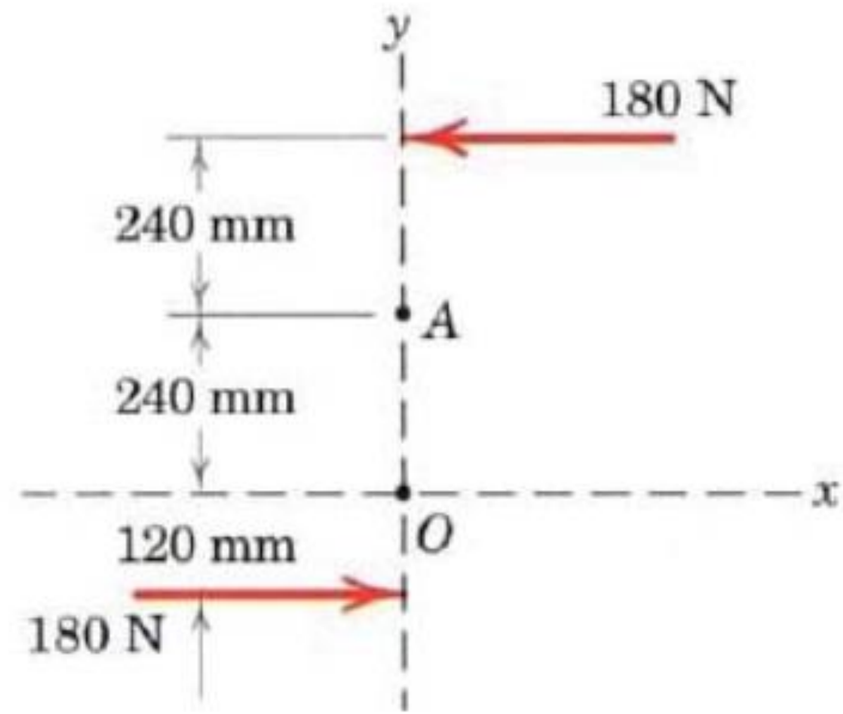
(d)

Exercise – 5

- Compute the combined moment of the two 180 N forces about (a) point O and (b) point A.



Solution - Exercise – 5



Ans. (a) M_o =108 N.m CCW

(b) M_A =108 N.m CCW

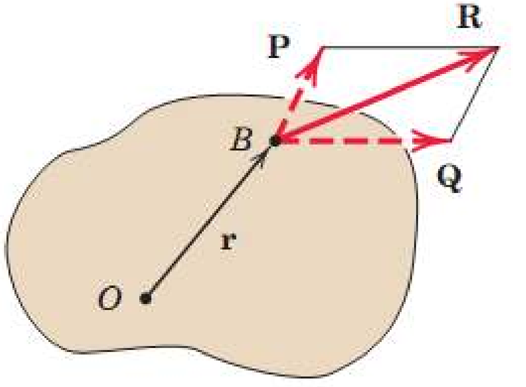
Varignon's Theorem

❖ The moment of a force about any point is **equal to the sum of the moments of the components** of the force about the same point.

Because **R =P+Q,** we may write:

$$\mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P} + \mathbf{Q})$$

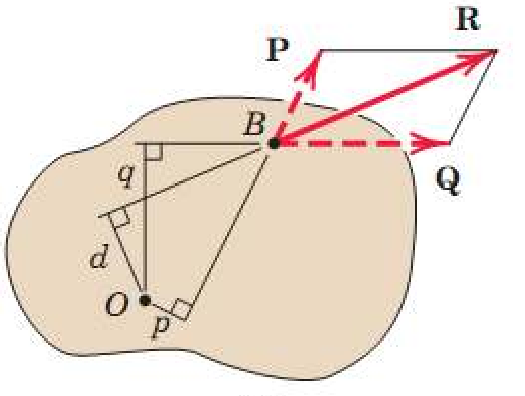
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}$$



(a)

we can resolve **R** into the components **P** and **Q**, and compute the moment as:

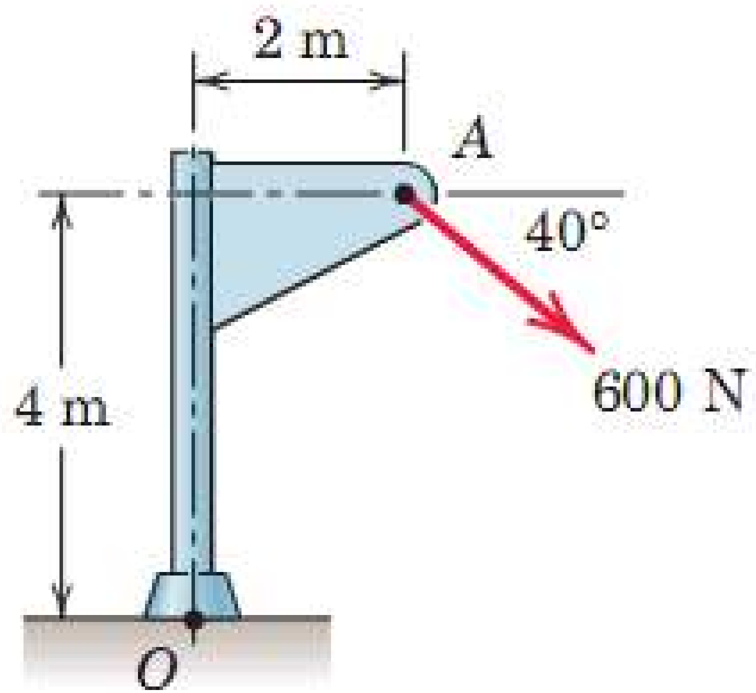
$$M_O = Rd = +pP - qQ$$



(b)

Exercise – 6

Calculate the magnitude of the moment about the base point ***O*** of the ***600-N*** force



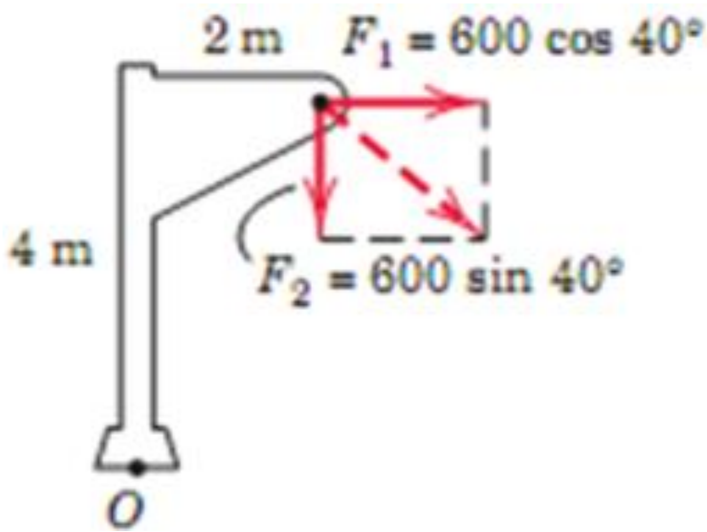
Solution - Exercise – 6

(II) Replace the force by its rectangular components at A,

$F_1 = 600 \cos 40° = 460$ N, $F_2 = 600 \sin 40° = 386$ N

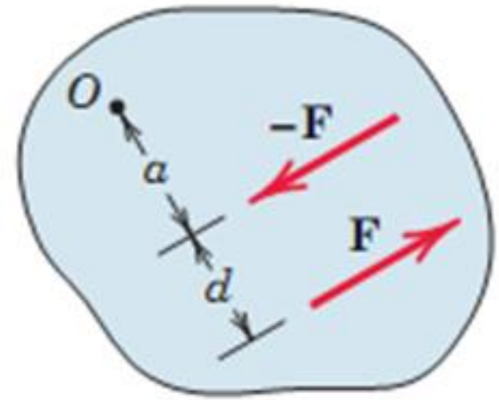
By Varignon's theorem, the moment becomes

$M_O = 460(4) + 386(2) = 2610$ N·m *Ans.*



Couple

✓ The moment produced by <u>two **equal, opposite**, and **non-collinear forces**</u> is called a **<u>COUPLE</u>.**



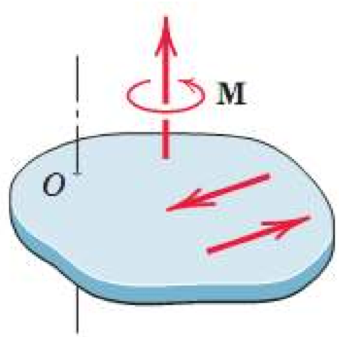
✓ The couple has a magnitude

$$M = F(a + d) - Fa$$

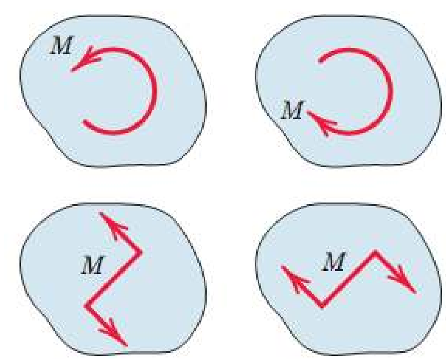
or: $$M = Fd$$

❖ The **<u>magnitude</u>** of the <u>couple</u> is **<u>independent</u>** of the **<u>distance</u>** between the forces → ***<u>moment of a couple</u>*** *has the* ***<u>same value</u>*** *for* ***<u>all moment centers</u>***.

❖ ***<u>Right-hand rule</u>***..



(c)



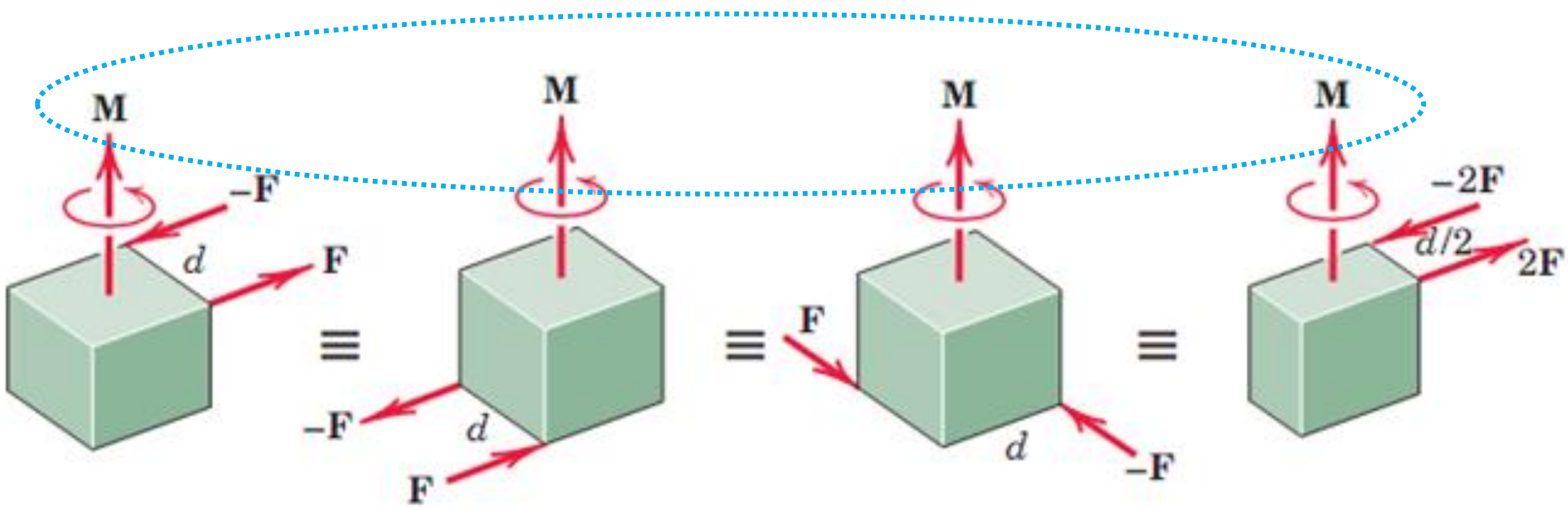
Counterclockwise couple

Clockwise couple

(d)

Equivalent Couples

Changing the values of ***F*** and ***d*** does not change a given couple as long as the product $\boldsymbol{M = F \,.\, d}$ remains the same.



Force - Couple Systems

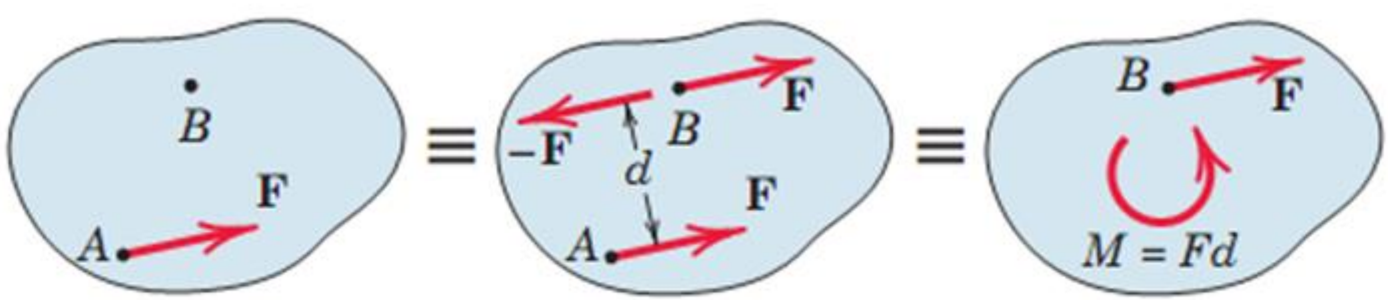
- ✓ The given force **F** acting at point **A** is replaced by an equal force **F** at some point **B** and the counter-clockwise couple

$$M = F \,.\, D$$

- ✓ *We now see that the original force at **A** and the equal and opposite one at **B** constitute the couple:*

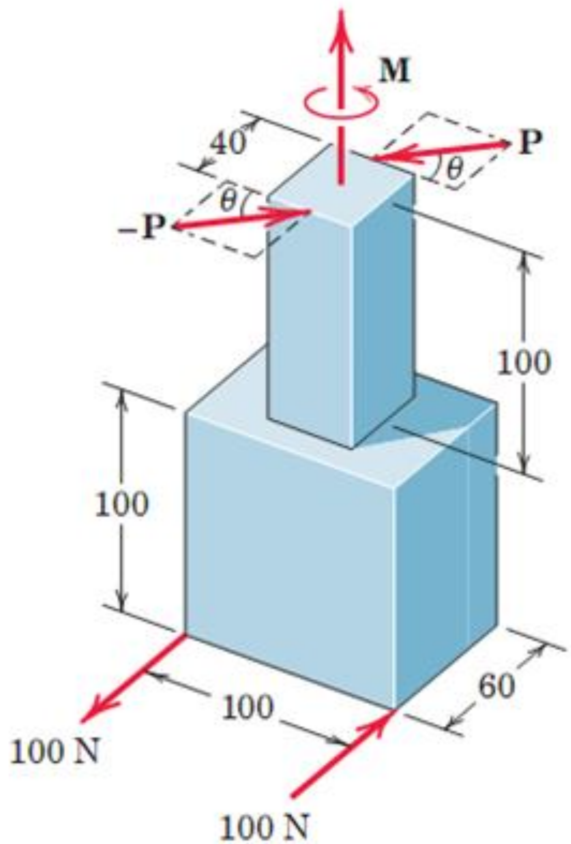
$$M = F \,.\, D$$

which is counter-clockwise.



Exercise – 7

The rigid structural member is subjected to a couple consisting of the two **100-N** forces. Replace this couple by an equivalent couple consisting of the two forces **P** and **-P,** each of which has a magnitude of **400 N**. Determine the proper angle .



Dimensions in millimeters

Equivalent Couples

$\boldsymbol{M_1 = M_2}$

Solution - Exercise – 7

The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is

$[M = Fd]$ $\qquad M = 100(0.1) = 10\ \text{N}\cdot\text{m}$

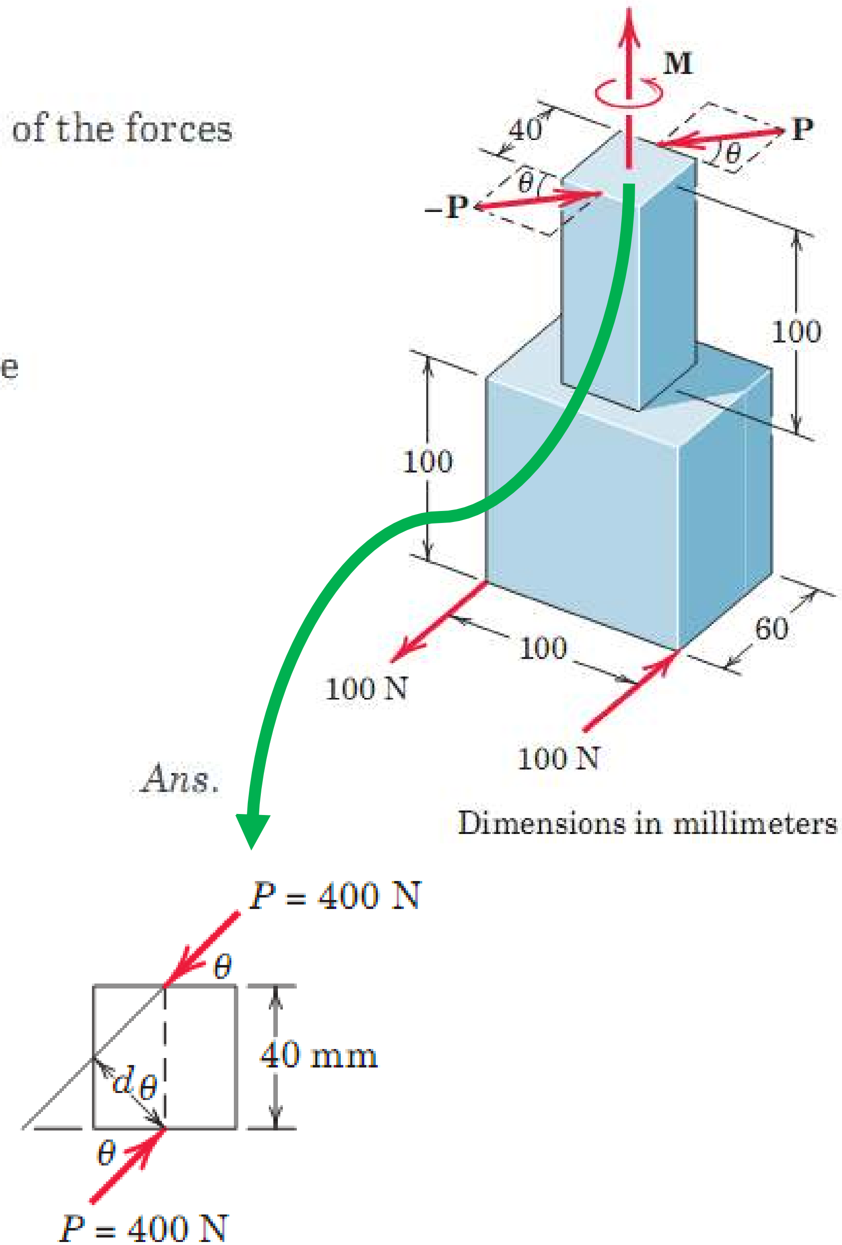
The forces **P** and −**P** produce a counterclockwise couple

$$M = 400(0.040)\cos\theta$$

Equating the two expressions gives

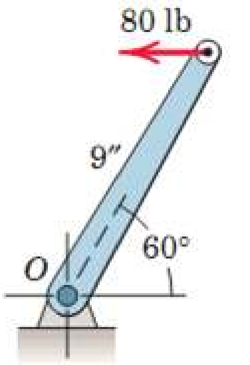
$$10 = (400)(0.040)\cos\theta$$

$$\theta = \cos^{-1}\frac{10}{16} = 51.3^\circ \qquad \textit{Ans.}$$



Exercise – 8

Replace the horizontal **80-lb** force acting on the lever by an equivalent system consisting of a ***force*** at ***O*** *and a* ***couple***.

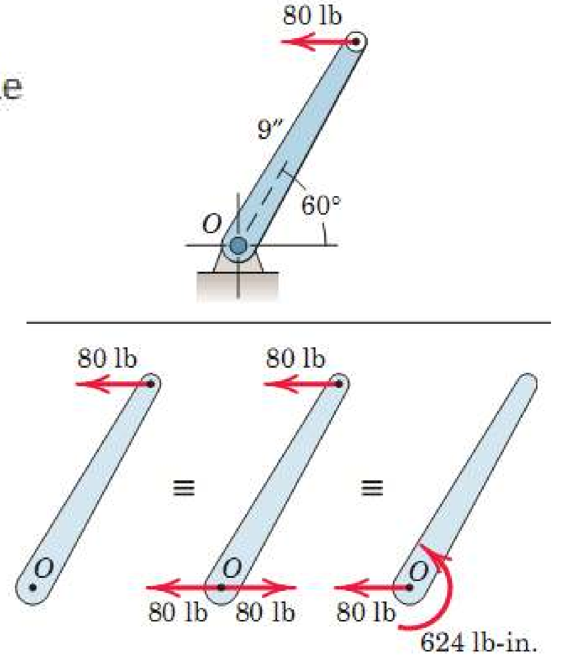


Solution - Exercise – 8

We apply two equal and opposite 80-lb forces at O and identify the counterclockwise couple

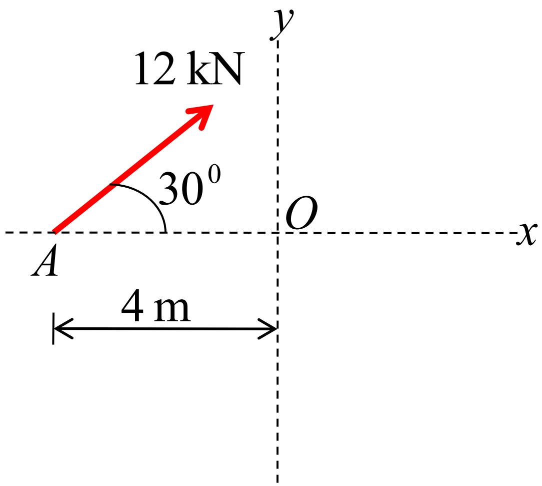
$$[M = Fd]$$

$$M = 80(9 \sin 60°) = 624 \text{ lb-in.} \qquad \textit{Ans.}$$

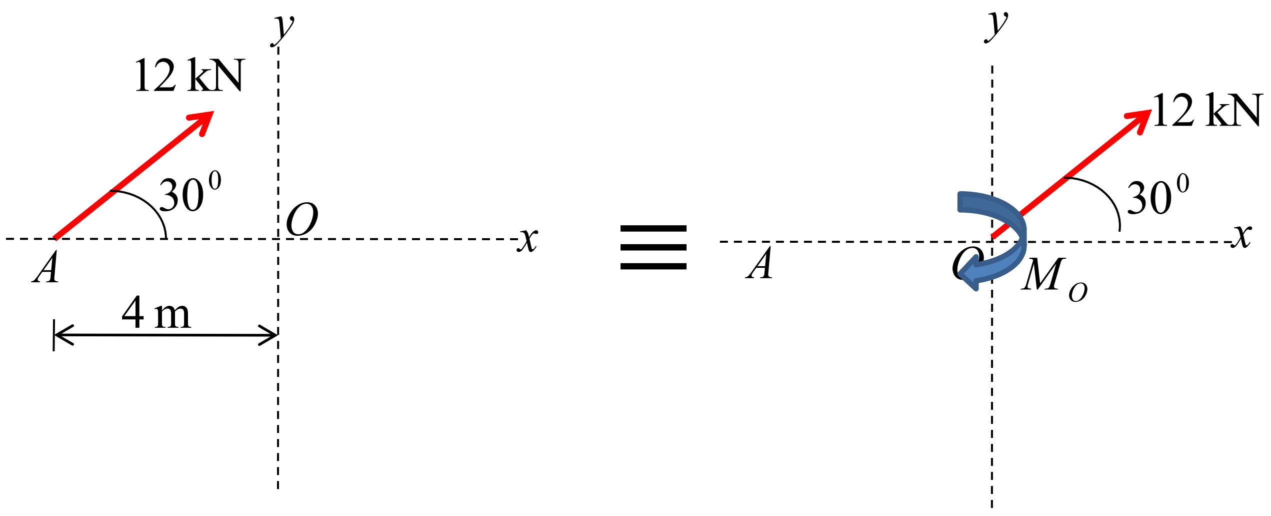


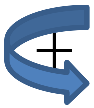
Exercise – 9

Replace the 12-kN force acting at point *A* by a force-couple system acting at point *O* .



Solution - Exercise – 9



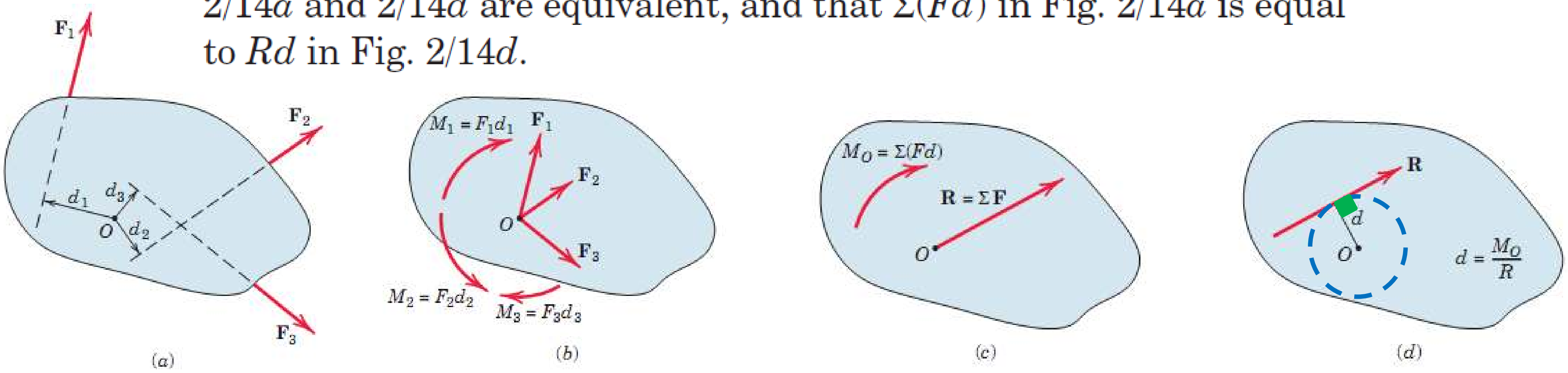

$M_O = -12 \times d = -12 \times 4 \sin 30^0 = -24 \text{ kNm}$

Resultant force & its **Line of Action**

Algebraic Method

We can use algebra to obtain the resultant force and its line of action as follows:

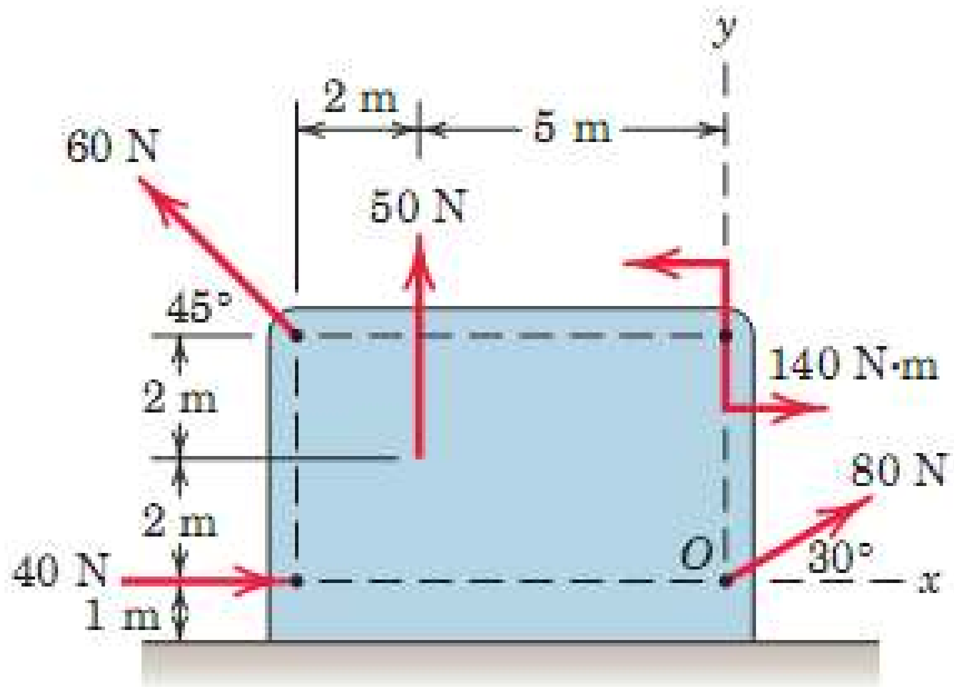
1. Choose a convenient reference point and move all forces to that point. This process is depicted for a three-force system in Figs. 2/14*a* and *b*, where M_1, M_2, and M_3 are the couples resulting from the transfer of forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ from their respective original lines of action to lines of action through point O.
2. Add all forces at O to form the resultant force $\mathbf{R}$, and add all couples to form the resultant couple M_O. We now have the single force–couple system, as shown in Fig. 2/14*c*.
3. In Fig. 2/14*d*, find the line of action of $\mathbf{R}$ by requiring $\mathbf{R}$ to have a moment of M_O about point O. Note that the force systems of Figs. 2/14*a* and 2/14*d* are equivalent, and that $\Sigma(Fd)$ in Fig. 2/14*a* is equal to Rd in Fig. 2/14*d*.



Extra Exercises

Exercise – 10

Determine the **resultant** of the **four forces** and **one couple** which act on the plate shown.



Solution - Exercise – 10

$[R_x = \Sigma F_x]$ $\quad R_x = 40 + 80 \cos 30^\circ - 60 \cos 45^\circ = 66.9 \text{ N}$

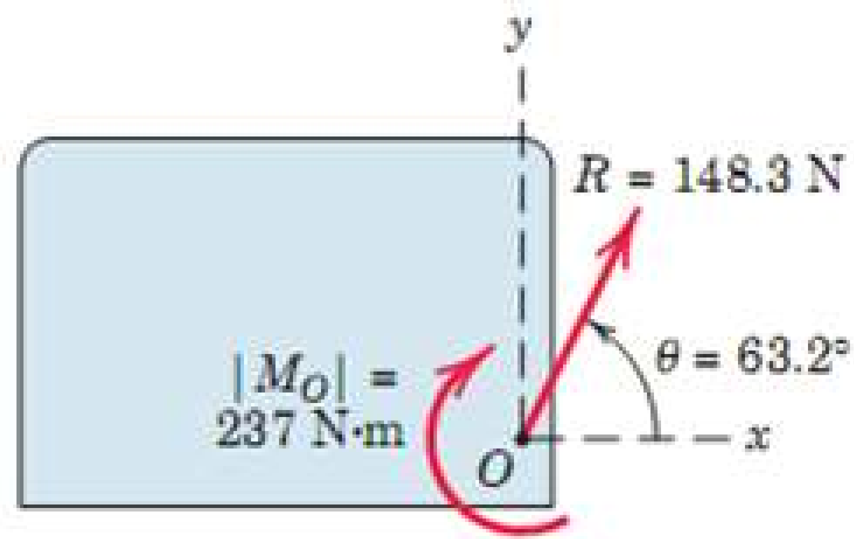
$[R_y = \Sigma F_y]$ $\quad R_y = 50 + 80 \sin 30^\circ + 60 \cos 45^\circ = 132.4 \text{ N}$

$[R = \sqrt{R_x^2 + R_y^2}]$ $\quad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3 \text{ N}$ *Ans.*

$\left[\theta = \tan^{-1} \dfrac{R_y}{R_x}\right]$ $\quad \theta = \tan^{-1} \dfrac{132.4}{66.9} = 63.2^\circ$ *Ans.*

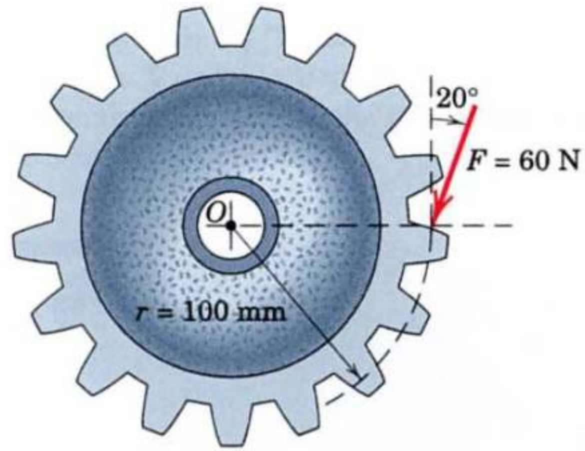
$[M_O = \Sigma(Fd)]$ $\quad M_O = 140 - 50(5) + 60 \cos 45^\circ(4) - 60 \sin 45^\circ(7)$

$= -237 \text{ N}\cdot\text{m}$

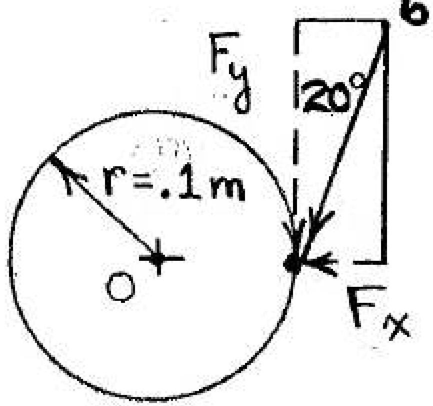


Exercise – 11

A force **F** of magnitude **60 N** is applied to the gear. Determine the moment of **F** about **O**.



Solution - Exercise – 11



60 N

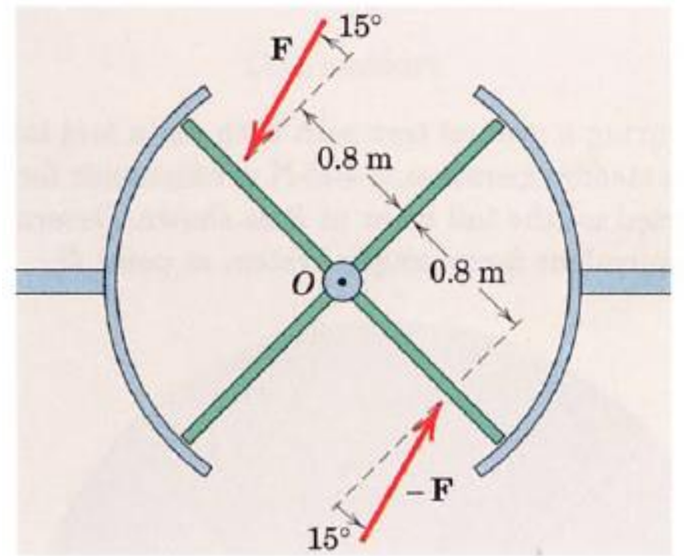
$$+\circlearrowleft M_o = r\,F_y$$

$$= (0.1)(60 \cos 20°)$$

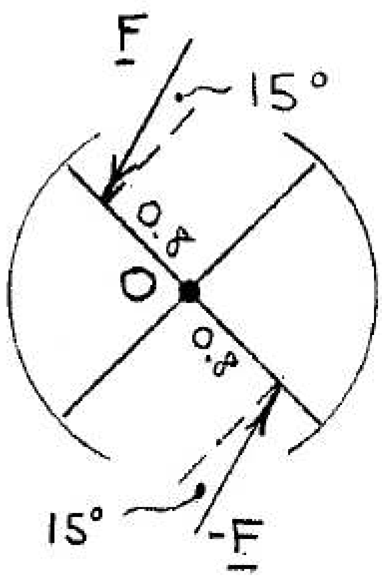
$$= \underline{5.64 \text{ N·m}}$$

Exercise – 12

The top view of a revolving entrance door is shown. Two persons imultaneously approach the door and exert forces of equal magnitudes as shown. If the resulting moment about the door pivot axis at **O** is **25 N.m**, determine the force magnitude **F**.



Solution - Exercise – 12



$$\circlearrowleft + \; M_o = \Sigma F d$$

$$25 = 2F(\cos 15°)(0.8)$$

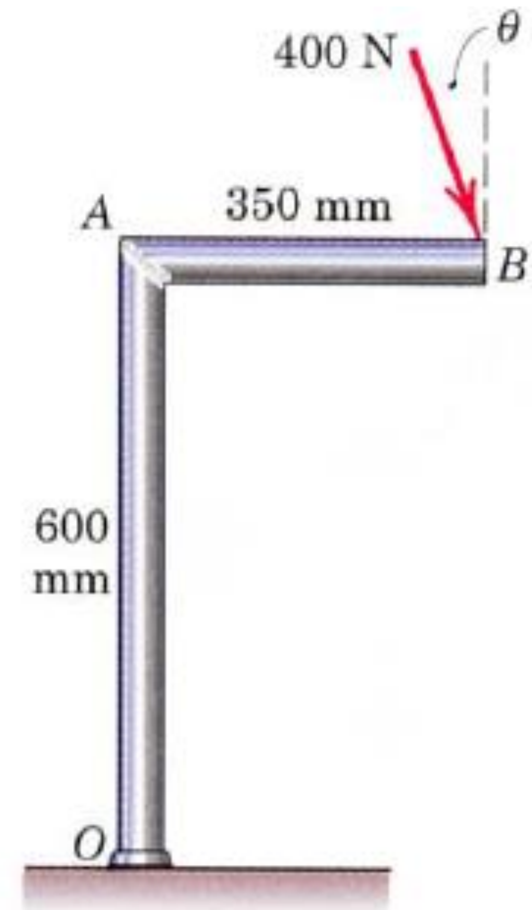
$$F = \underline{16.18 \text{ N}}$$

Exercise – 13

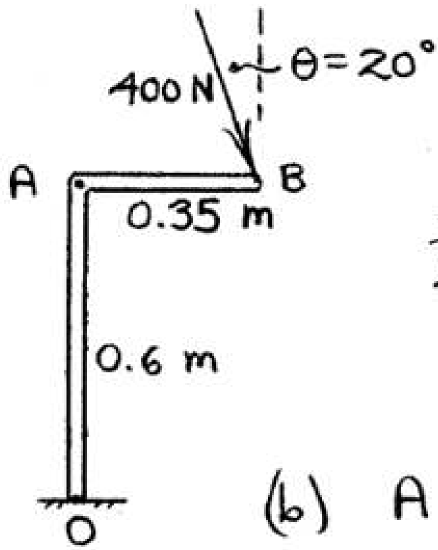
A **400 N** force is applied to the welded slender bar at an angle $\boldsymbol{\theta = 20^\circ}$. Determine the **equivalent force couple system** acting on the weld at:

a) point **A,** and
b) point **O**.

For which value of **θ** would the results of parts **(a)** and **(b)** be identical?



Solution - Exercise – 13



(a) At A:

$F = 400\ \text{N}$

$$\curvearrowright M_A = (400 \cos 20°)(0.35)$$

$$= \underline{131.6\ \text{N} \cdot \text{m CW}}$$

(b) At O:

$F = 400\ \text{N}$

$$\curvearrowright M_o = 400 \cos 20° (0.35) + 400 \sin 20° (0.6)$$

$$= \underline{214\ \text{N} \cdot \text{m CW}}$$

Part (a) and (b) results are the same if $\underline{\theta = 0 \text{ or } 180°.}$