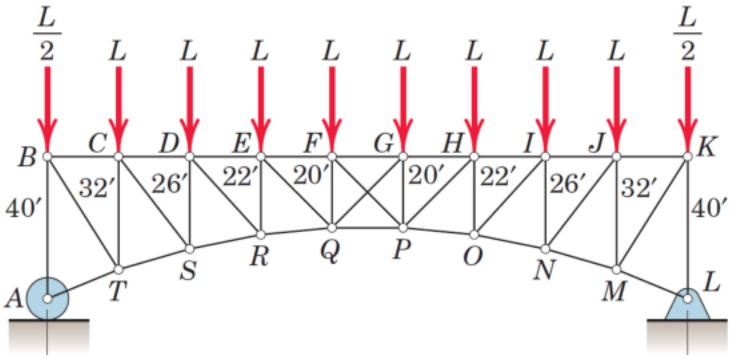
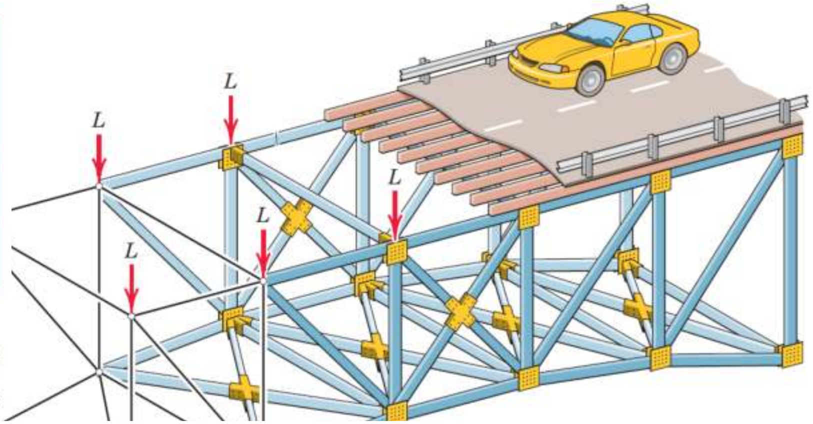
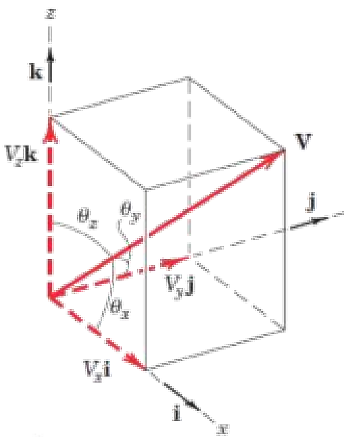


١٤٢٧
2006
جامعة تبوك
University of Tabuk

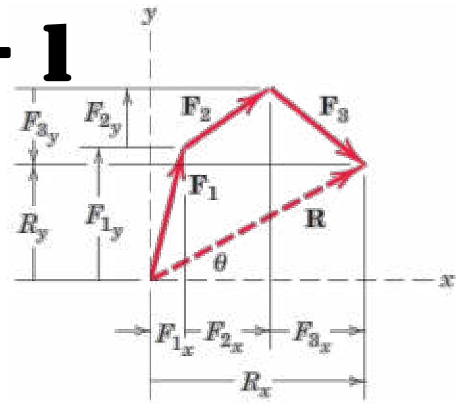


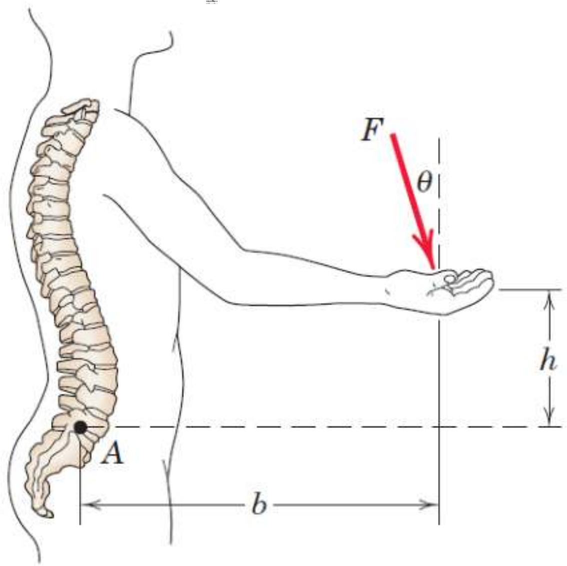


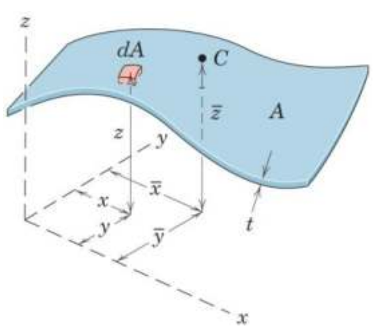
ENGINEERING MECHANICS – 1
ENG 203

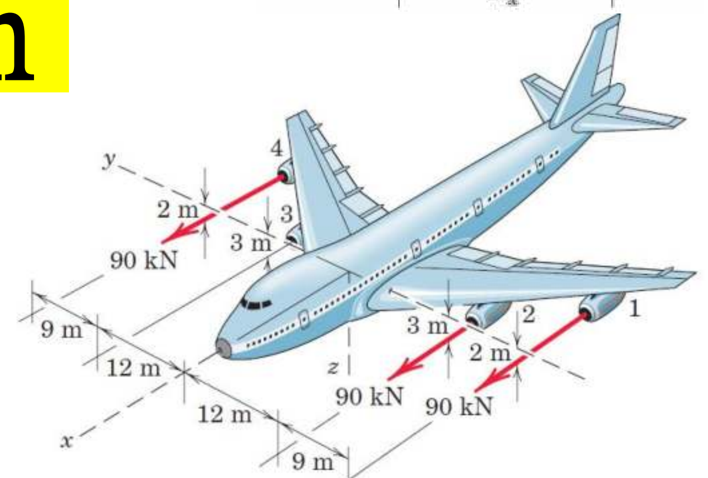
CHAPTER - 3

Equilibrium









CHAPTER OUTLINE

- ✓ ***Equilibrium in Two Dimensions***
- ✓ ***System Isolation and the Free-Body Diagram***
- ✓ ***Equilibrium Conditions***

Introduction

When a body is in equilibrium, the **resultant** of *all forces acting on it is* ***zero***. → *Thus, the resultant force* ***R*** *and the resultant couple* ***M*** *are both* ***zero,*** and we have the ***equilibrium equations:***

$$\mathbf{R} = \Sigma \mathbf{F} = \mathbf{0} \qquad \mathbf{M} = \Sigma \mathbf{M} = \mathbf{0}$$

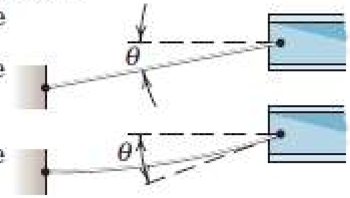
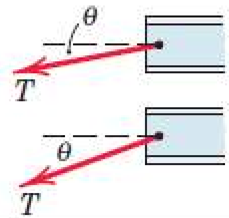
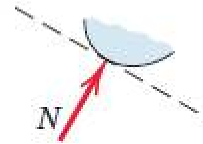
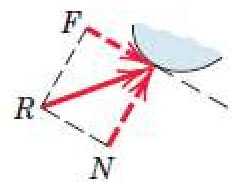
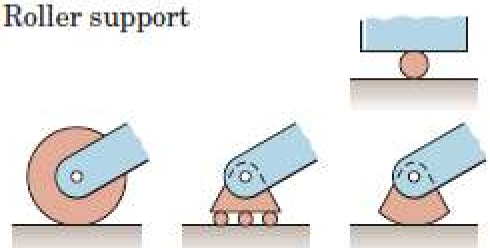
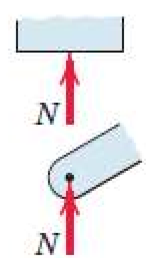
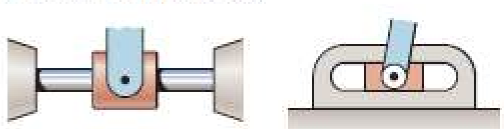
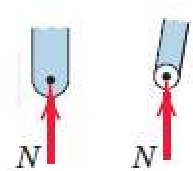
System Isolation and the Free-Body Diagram

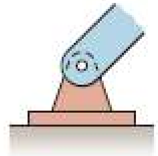
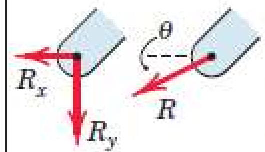
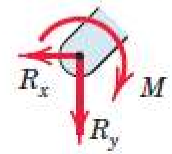
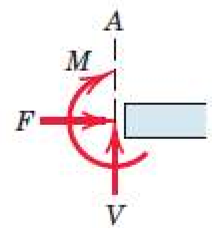
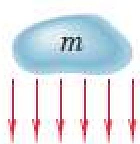
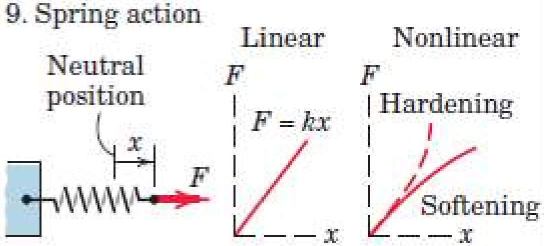
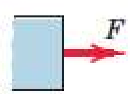
- ✓ **The free-body diagram is the most important single step in the solution of problems in mechanics.**

- ✓ The diagram shows ***all forces applied*** to the system by mechanical contact with other bodies, which are ***imagined to be removed***.

- ✓ The **following Table** shows the common types of force application on mechanical systems for analysis in two dimensions.

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS

Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible  	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential component *F* (frictional force) as well as a normal component *N* of the resultant contact force *R*.
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.

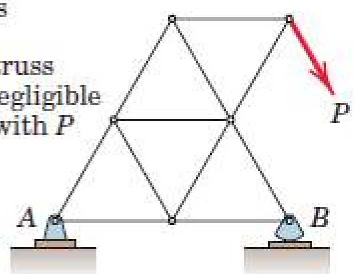
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)		
Type of Contact and Force Origin	**Action on Body to Be Isolated**	
6. Pin connection 	Pin free to turn  Pin not free to turn 	A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 		A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 		The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 		Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

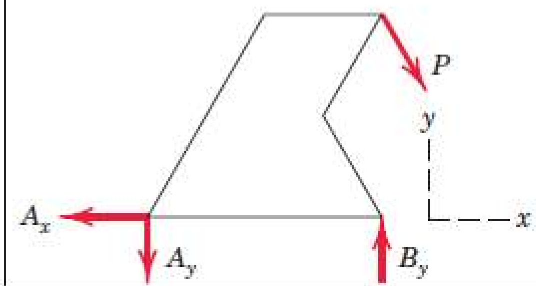
SAMPLE FREE-BODY DIAGRAMS

Mechanical System | **Free-Body Diagram of Isolated Body**

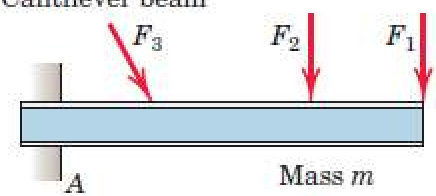
1. Plane truss

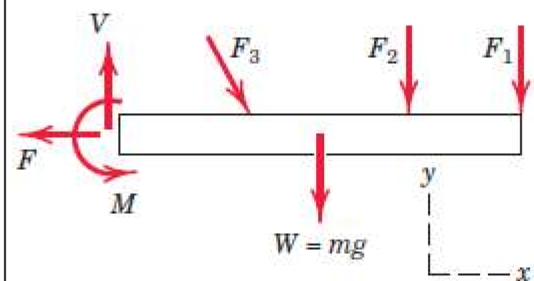
Weight of truss assumed negligible compared with P





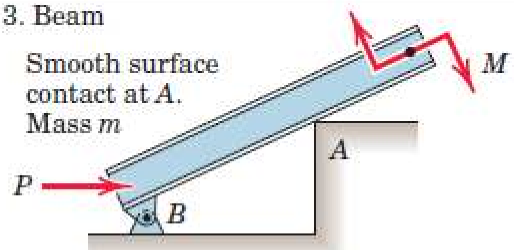
2. Cantilever beam

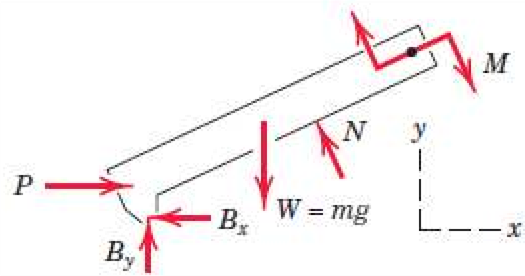




3. Beam

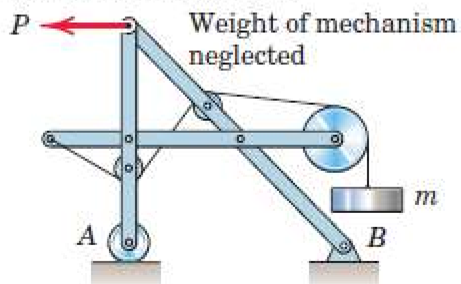
Smooth surface contact at A. Mass m

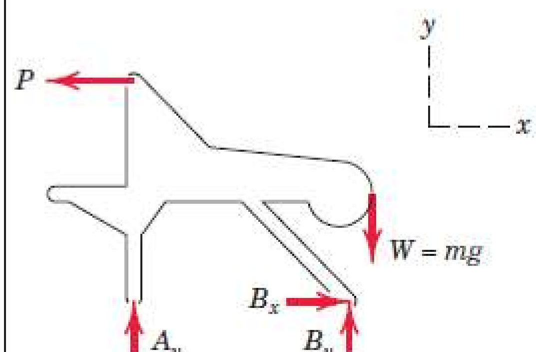




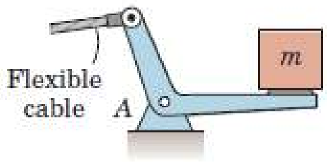
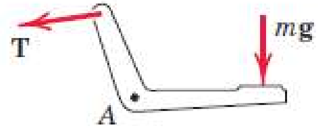
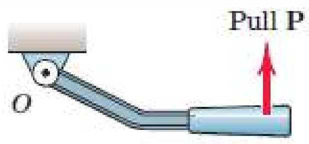
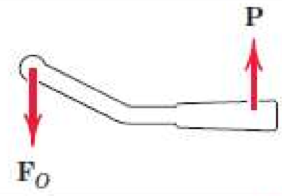
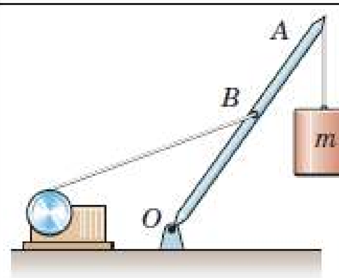
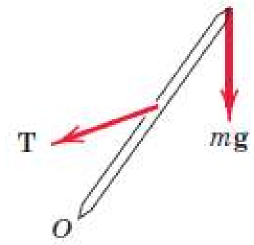
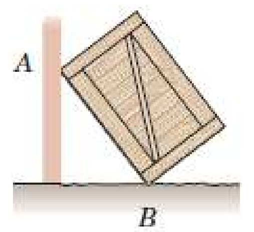
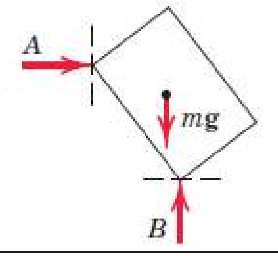
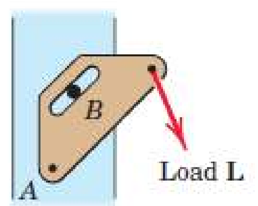
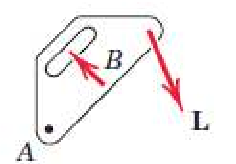
4. Rigid system of interconnected bodies analyzed as a single unit

Weight of mechanism neglected

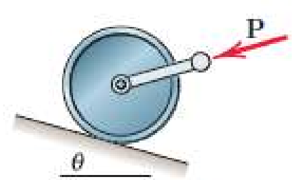
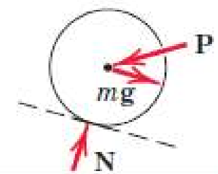
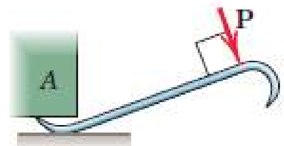
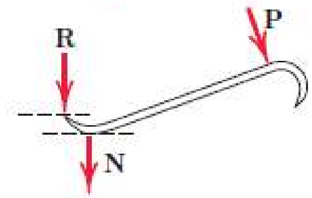
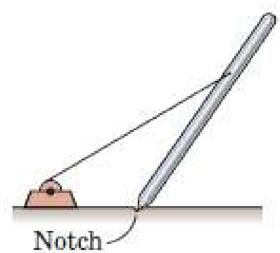
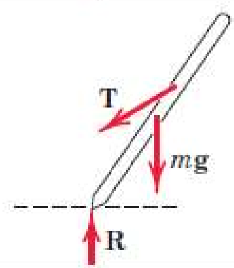
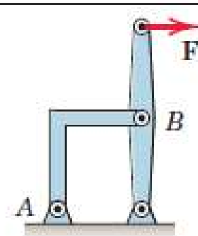
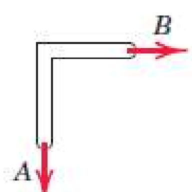
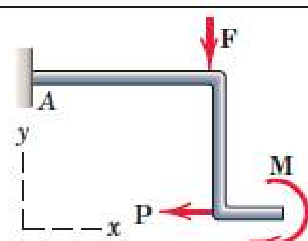
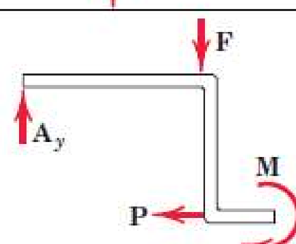




Free-Body Diagram Exercises

<table>
<tr><th></th><th>Body</th><th>Incomplete FBD</th></tr>
<tr><td>1. Bell crank supporting mass m with pin support at A.</td><td>
</td><td>
</td></tr>
<tr><td>2. Control lever applying torque to shaft at O.</td><td>
</td><td>
</td></tr>
<tr><td>3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.</td><td>
</td><td>
</td></tr>
<tr><td>4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.</td><td>
</td><td>
</td></tr>
<tr><td>5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.</td><td>
</td><td>
</td></tr>
</table>

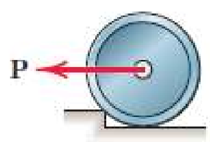
Free-Body Diagram Exercises

	Body	Wrong or Incomplete FBD
1. Lawn roller of mass m being pushed up incline θ.		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		
4. Supporting angle bracket for frame; pin joints.		
5. Bent rod welded to support at A and subjected to two forces and couple.		

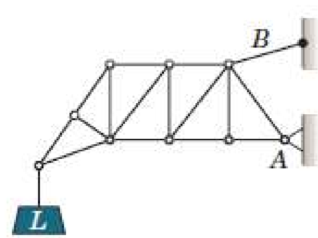
Free-Body Diagram Exercises

1. Uniform horizontal bar of mass m suspended by vertical cable at A and supported by rough inclined surface at B.

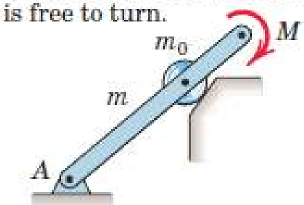
2. Wheel of mass m on verge of being rolled over curb by pull **P**.



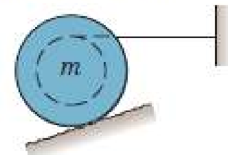
3. Loaded truss supported by pin joint at A and by cable at B.



4. Uniform bar of mass m and roller of mass m_0 taken together. Subjected to couple M and supported as shown. Roller is free to turn.

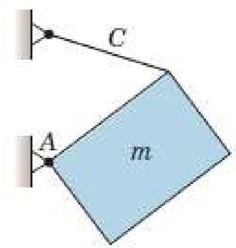


5. Uniform grooved wheel of mass m supported by a rough surface and by action of horizontal cable.

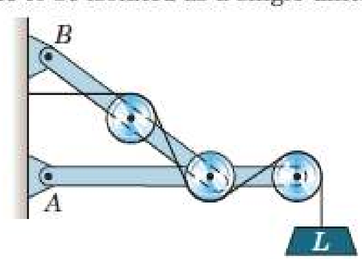


6. Bar, initially horizontal but deflected under load **L**. Pinned to rigid support at each end.

7. Uniform heavy plate of mass m supported in vertical plane by cable C and hinge A.



8. Entire frame, pulleys, and contacting cable to be isolated as a single unit.

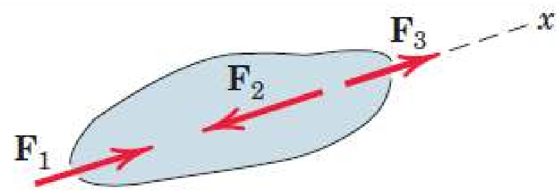
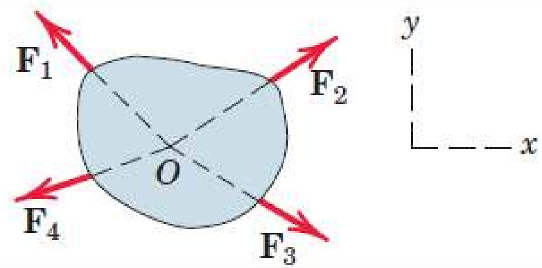
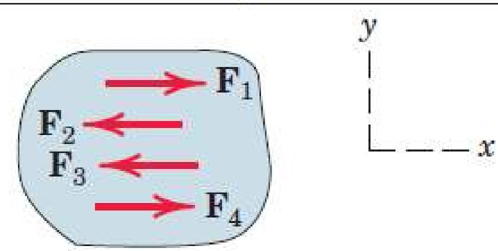
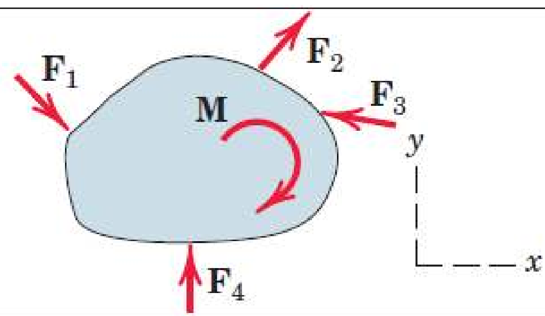


Equilibrium Conditions

- ✓ We defined equilibrium as the condition in which the resultant of all forces and moments acting on a body is zero.
- ✓ Stated in another way, a body is in equilibrium if all forces and moments applied to it are in balance.
- ✓ These requirements are contained in the vector equations of equilibrium, which in two dimensions may be written in scalar form as:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0$$

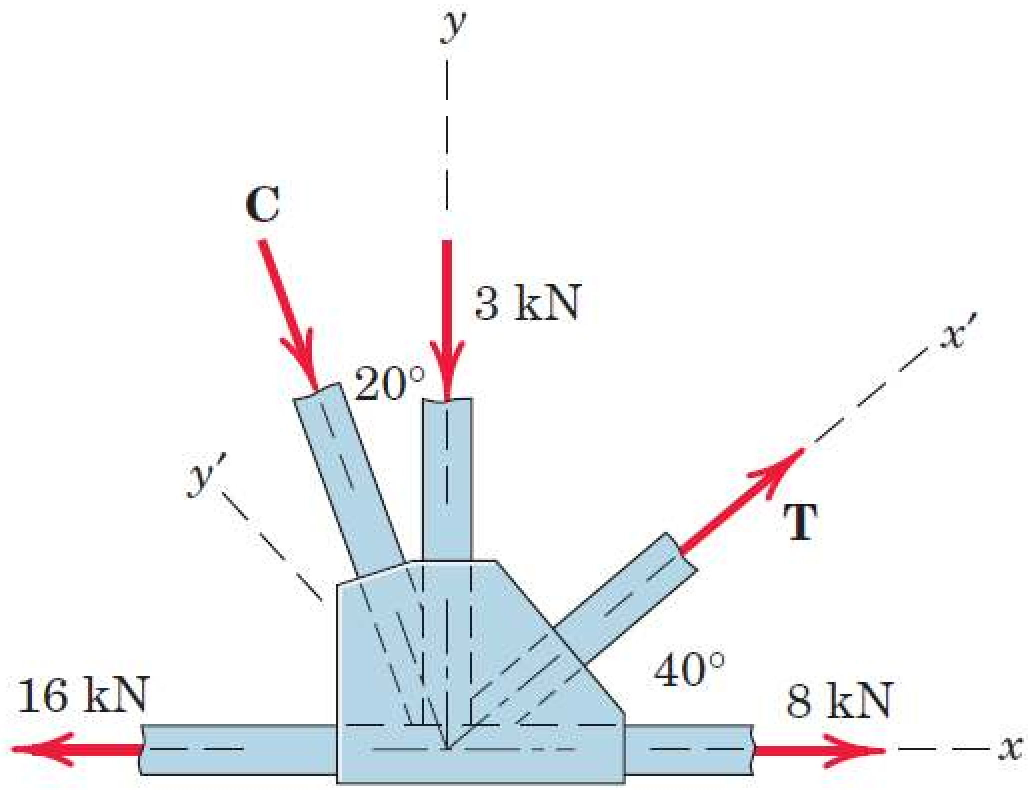
Equilibrium Conditions

<table>
<tr><th colspan="3">CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS</th></tr>
<tr><th>Force System</th><th>Free-Body Diagram</th><th>Independent Equations</th></tr>
<tr><td>1. Collinear</td><td></td><td>$\Sigma F_x = 0$</td></tr>
<tr><td>2. Concurrent at a point</td><td></td><td>$\Sigma F_x = 0$
$\Sigma F_y = 0$</td></tr>
<tr><td>3. Parallel</td><td></td><td>$\Sigma F_x = 0 \quad \Sigma M_z = 0$</td></tr>
<tr><td>4. General</td><td></td><td>$\Sigma F_x = 0 \quad \Sigma M_z = 0$
$\Sigma F_y = 0$</td></tr>
</table>

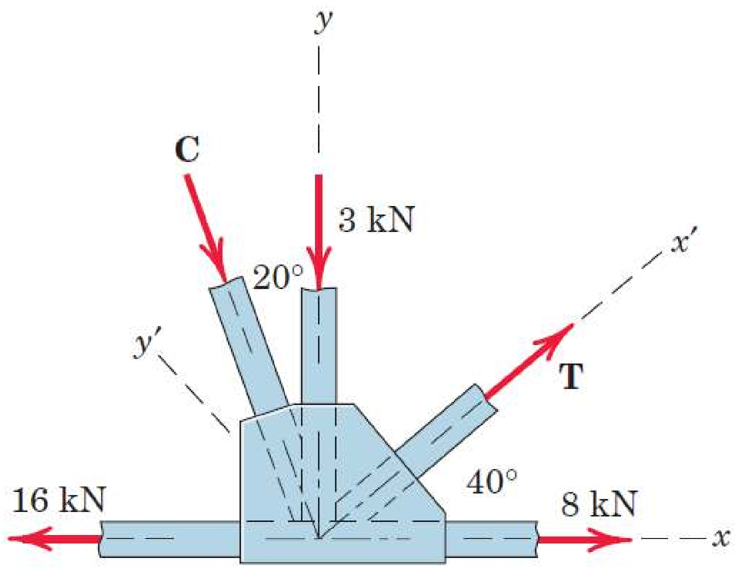
Exercises

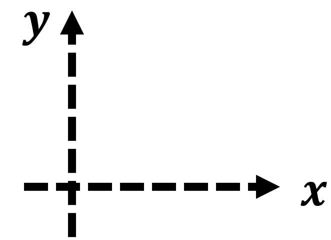
Exercise – 1

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.



Solution - Exercise – 1





Solution 1 (scalar algebra). For the x-y axes as shown we have

$[\Sigma F_x = 0]$ $$8 + T\cos 40° + C\sin 20° - 16 = 0$$

$$0.766T + 0.342C = 8 \qquad (a)$$

$[\Sigma F_y = 0]$ $$T\sin 40° - C\cos 20° - 3 = 0$$

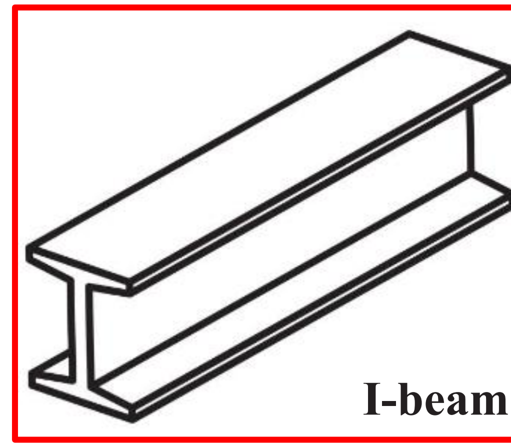
$$0.643T - 0.940C = 3 \qquad (b)$$

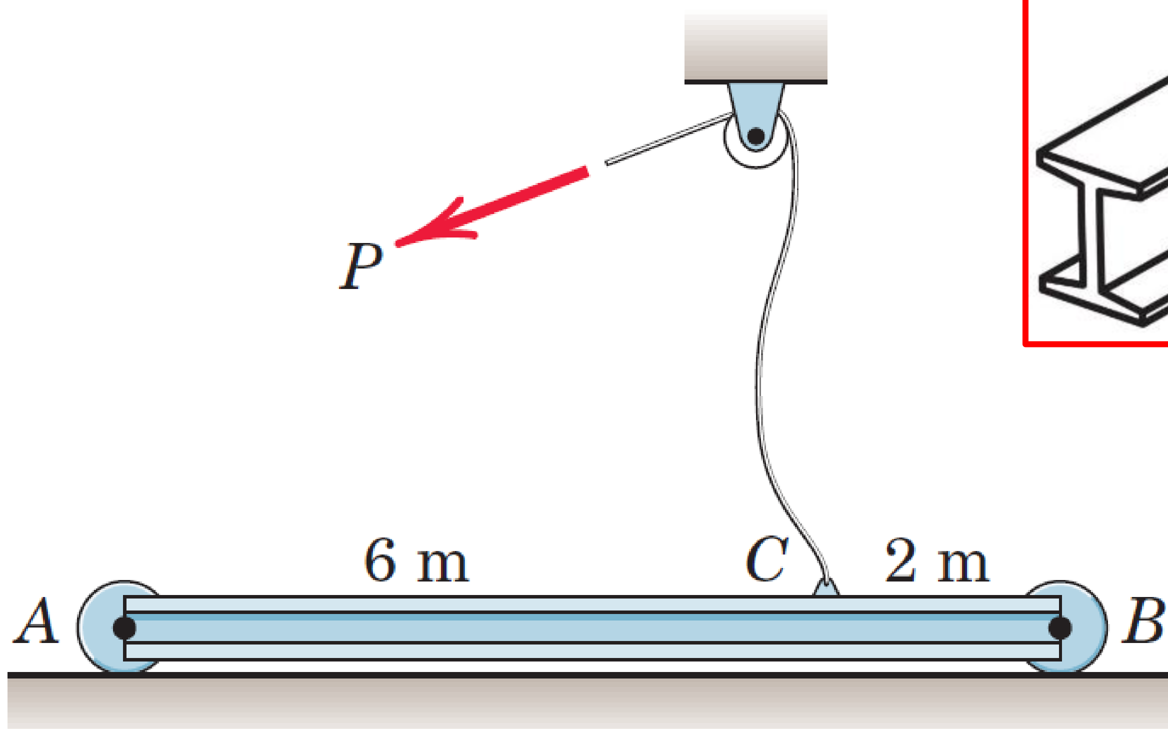
Simultaneous solution of Eqs. (a) and (b) produces

$$T = 9.09 \text{ kN} \qquad C = 3.03 \text{ kN} \qquad \textit{Ans.}$$

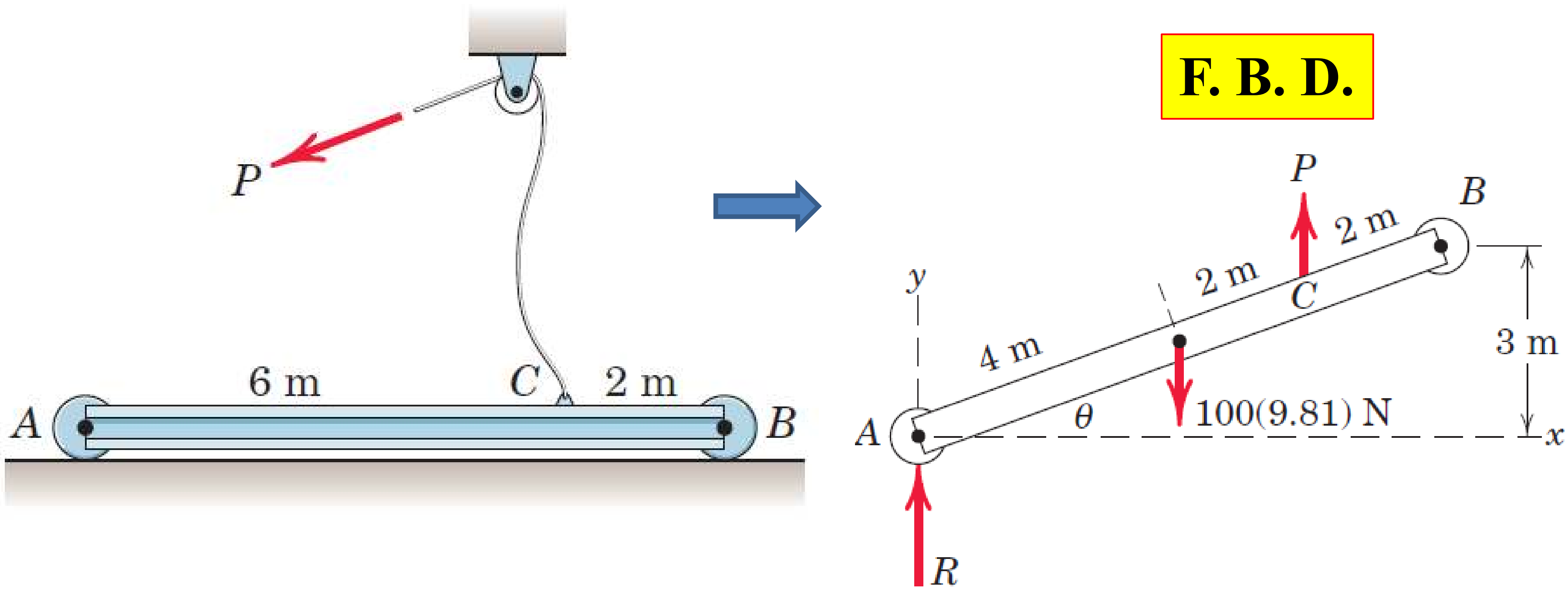
Exercise – 2

The uniform **100-kg** "***I-beam***" is supported initially by its end rollers on the horizontal surface at ***A*** and ***B***. By means of the cable at ***C*** it is desired to elevate end ***B*** to a position **3 m** above end ***A***. Determine the required tension ***P***, the reaction at ***A***, and the **angle** made by the beam with the horizontal in the elevated position.





Solution - Exercise – 2



$$[\Sigma M_A = 0] \qquad P(6\cos\theta) - 981(4\cos\theta) = 0 \qquad P = 654 \text{ N}$$

Equilibrium of vertical forces requires

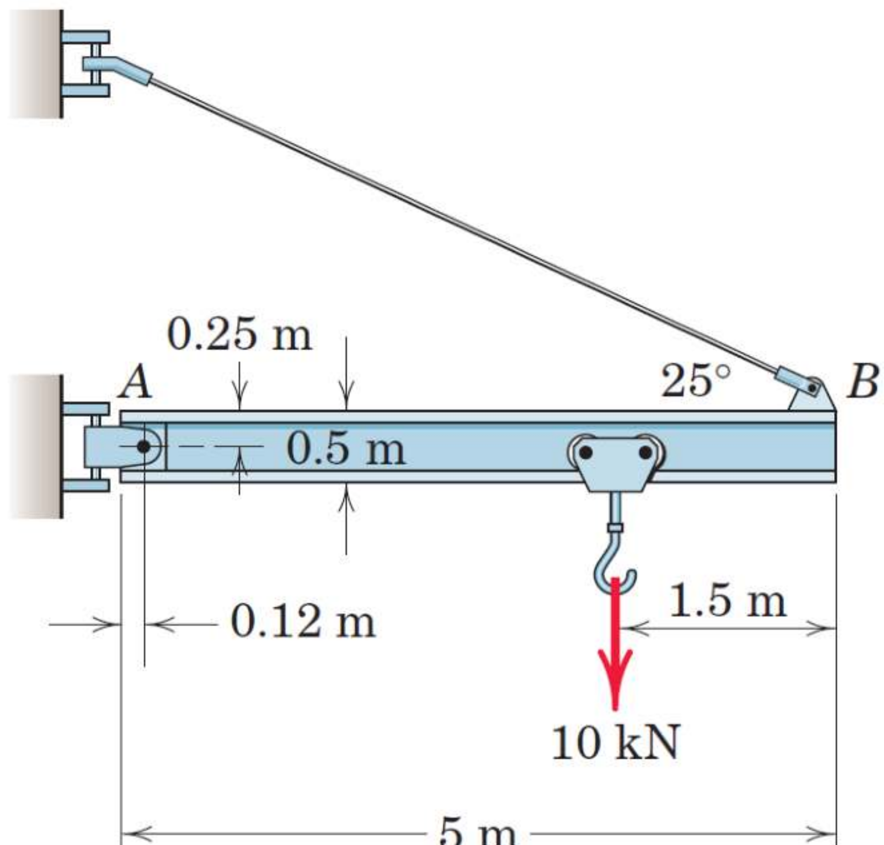
$$[\Sigma F_y = 0] \qquad 654 + R - 981 = 0 \qquad R = 327 \text{ N}$$

The angle θ depends only on the specified geometry and is

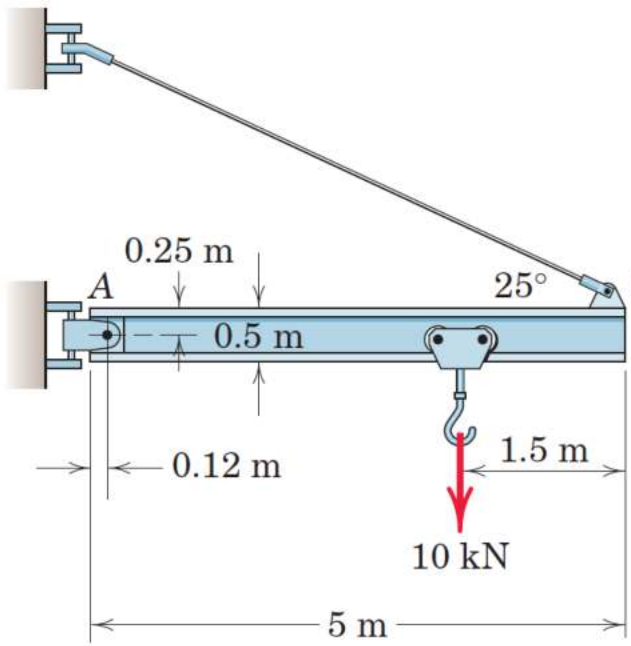
$$\sin\theta = 3/8 \qquad \theta = 22.0^\circ$$

Exercise – 3

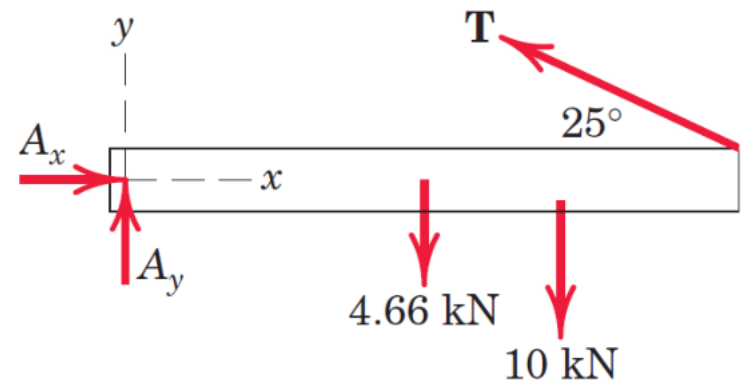
Determine the magnitude ***T*** of the tension in the supporting cable and the magnitude of the force on the pin at ***A*** for the jib crane shown. The beam ***AB*** is a standard **0.5-m** "***I-beam***" with a mass of **95 kg** ***per meter*** of length.



Solution - Exercise – 3



F. B. D.



The weight of the beam is $95(10^{-3})(5)9.81 = 4.66$ kN

$[\Sigma M_A = 0]$ $\quad (T \cos 25°)0.25 + (T \sin 25°)(5 - 0.12)$
$$- 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$$

from which $\quad T = 19.61$ kN $\quad$ *Ans.*

Equating the sums of forces in the x- and y-directions to zero gives

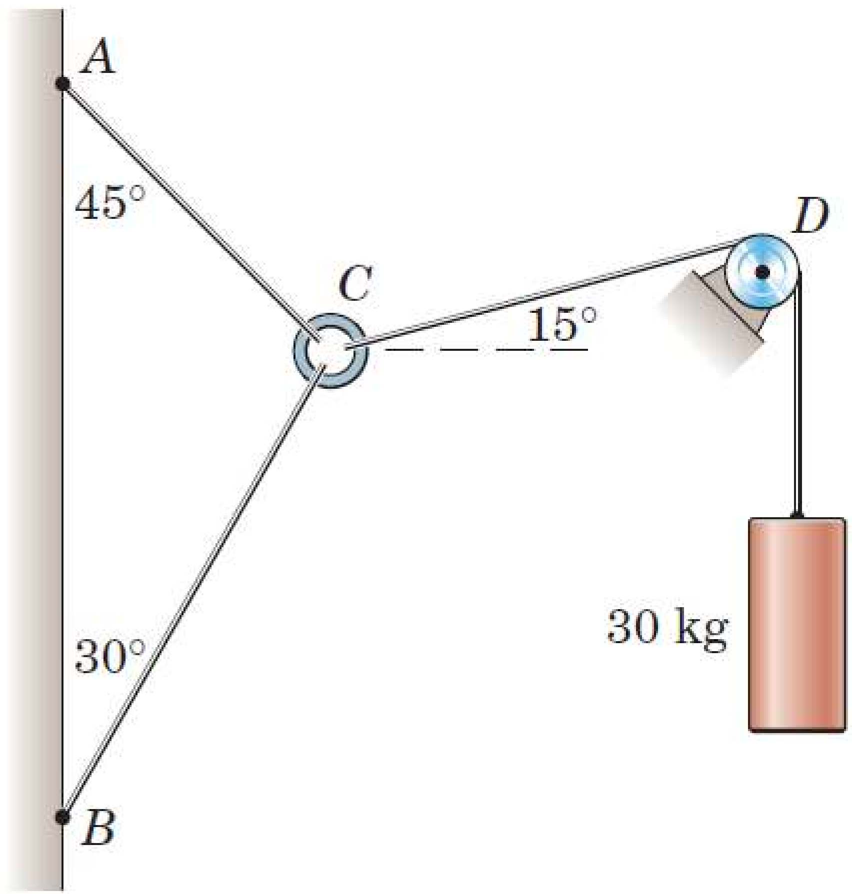
$[\Sigma F_x = 0]$ $\quad A_x - 19.61 \cos 25° = 0 \quad A_x = 17.77$ kN

$[\Sigma F_y = 0]$ $\quad A_y + 19.61 \sin 25° - 4.66 - 10 = 0 \quad A_y = 6.37$ kN

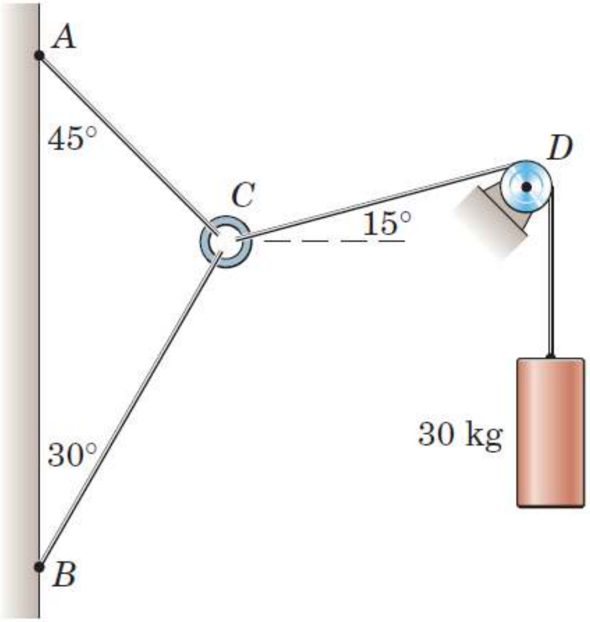
$[A = \sqrt{A_x^2 + A_y^2}]$ $\quad A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88$ kN $\quad$ *Ans.*

Exercise – 4

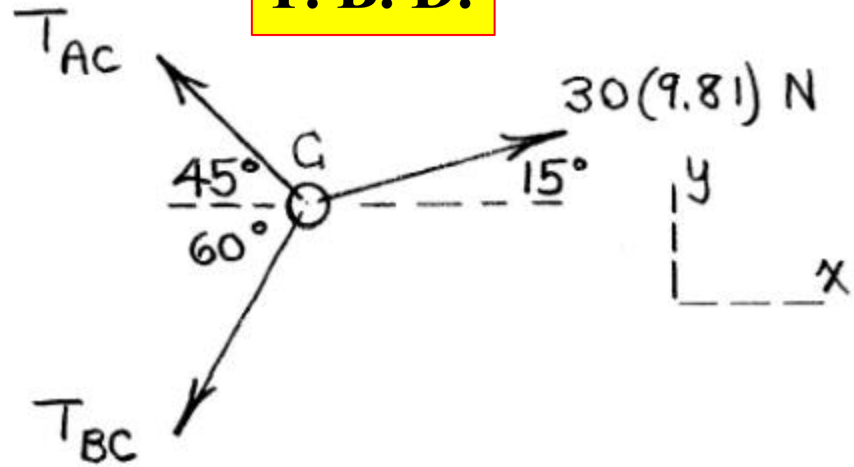
Three cables are joined at the junction ring *C*. Determine the tensions in cables ***AC*** and ***BC*** caused by the weight of the **30-kg** cylinder.



Solution - Exercise – 4



F. B. D.



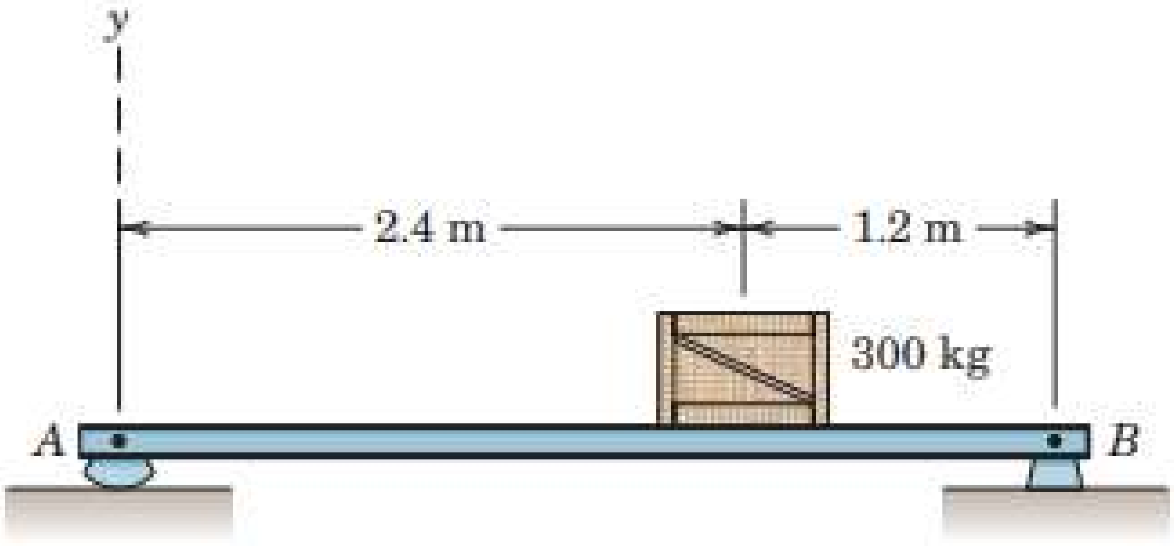
$$\begin{cases} \sum F_x = 0: -T_{AC}\cos 45° - T_{BC}\cos 60° + 30(9.81)\cos 15° = 0 \\ \sum F_y = 0: T_{AC}\sin 45° - T_{BC}\sin 60° + 30(9.81)\sin 15° = 0 \end{cases}$$

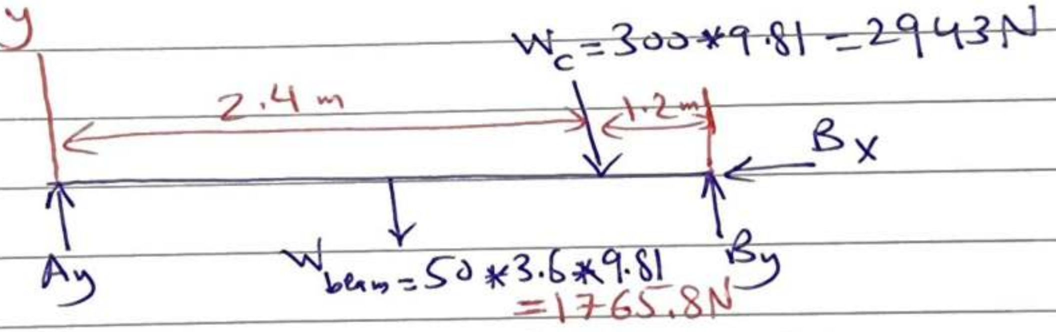
Solve simultaneously to obtain

$$\begin{cases} T_{AC} = 215 \text{ N} \\ T_{BC} = 264 \text{ N} \end{cases}$$

Exercise – 5

3/4 The uniform beam has a mass of 50 kg per meter of length. Determine the reactions at the supports.





$m_{beam} = 50\ kg/m * 3.6\ m = 180\ kg$

$\Rightarrow W_{beam} = 180 * 9.81 = 1765.8\ N$

$\Sigma F_x = 0 \Rightarrow \boxed{B_x = 0}$

$\circlearrowleft \Sigma M_B = 0 \Rightarrow -Ay * (2.4 + 1.2) + 1765.8 * \left(\frac{2.4 + 1.2}{2}\right)$

$+ 2943 (1.2) = 0$

$\Rightarrow Ay = \frac{6710.04}{3.6} \Rightarrow \boxed{Ay = 1863.9N}$

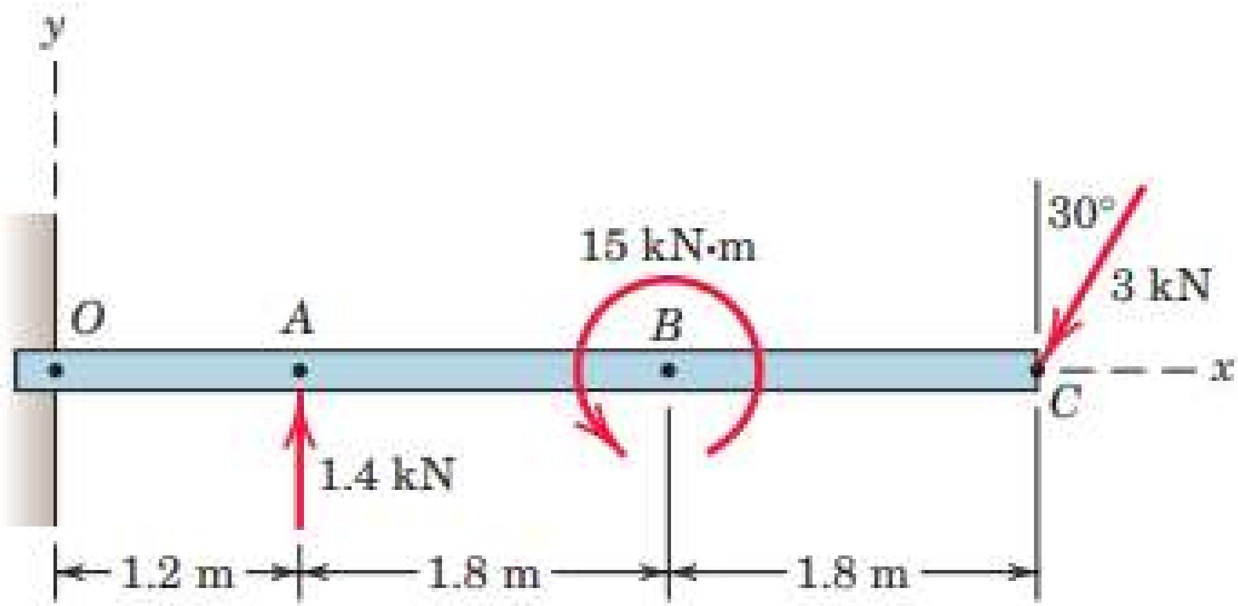
$\Sigma F_y = 0 \Rightarrow Ay - W_{beam} - W_c + By = 0$

$\Rightarrow 1863.9 - 1765.8 - 2943 + By = 0$

$\Rightarrow \therefore \boxed{By = 2844.9N}$

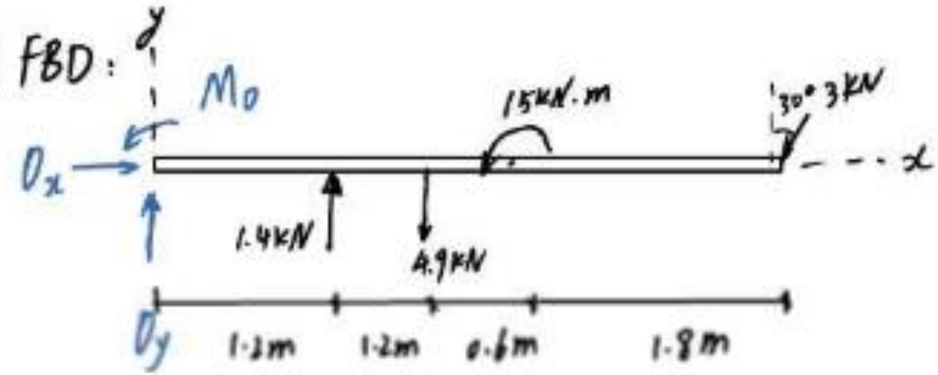
Exercise – 6

3/5 The 500-kg uniform beam is subjected to the three external loads shown. Compute the reactions at the support point *O*. The *x-y* plane is vertical.



Solution:

1) FBD:



2) EOE.

$\xrightarrow{+} \Sigma F_x = 0 \qquad O_x - 3(\sin 30°) = 0$

$O_x = 1.5\ kN\ (\rightarrow)$

$+\uparrow \Sigma F_y = 0 \qquad O_y + 1.4 - 4.9 - 3(\cos 30°) = 0$

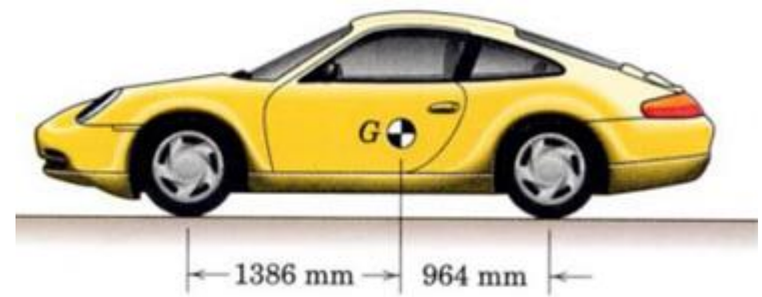
$O_y = 6.1\ kN\ (\uparrow)$

$\circlearrowleft + \Sigma M_O = 0 \qquad M_O + 1.4(1.2) - 4.9(2.4) + 15 - 3(\cos 30°)(4.8) = 0$

$M_O = 7.6\ kN.m\ (\circlearrowleft)$

Exercise – 7

The mass center G of the 1400 kg rear engine car is located as shown in the figure. Determine the normal force under each tire when the car is in equilibrium state any assumptions.



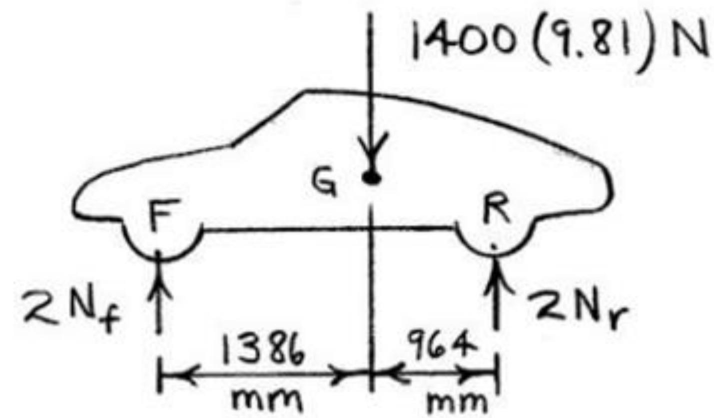
Solution - Exercise – 7

$+\uparrow \Sigma F = 0 : \quad 2N_f + 2N_r - 1400(9.81) = 0$

$\circlearrowleft + \Sigma M_F = 0 : \quad -1400(9.81)(1386) + 2N_r(1386 + 964) = 0$

Solution : $\begin{cases} N_f = 2820 \text{ N} \\ N_r = 4050 \text{ N} \end{cases}$

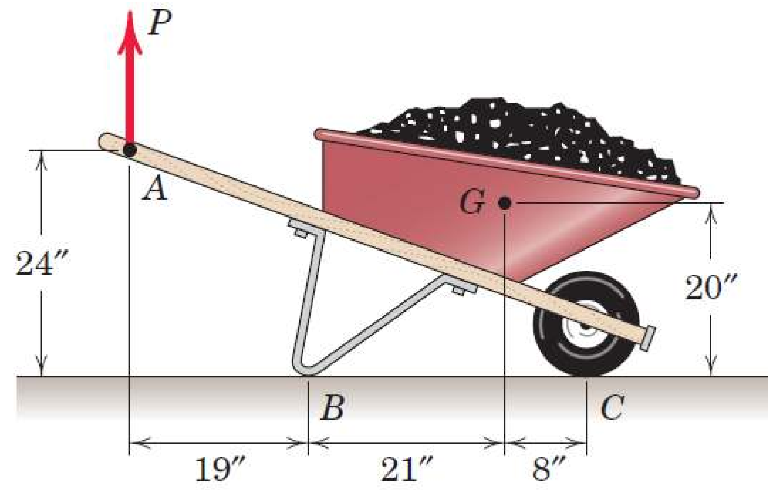
Assumes G midway between left and right wheels.



F. B. D.

Exercise – 8

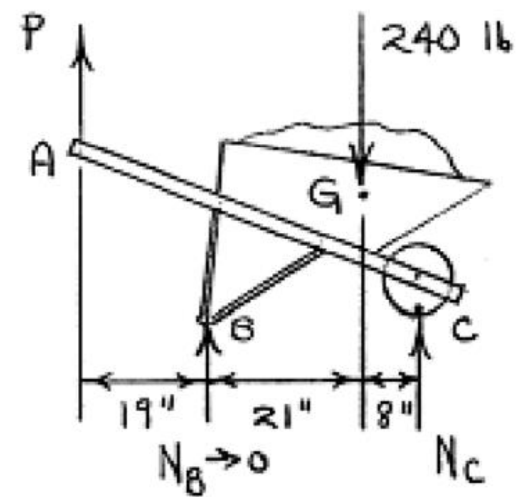
Determine the magnitude P of the vertical force required to lift the wheelbarrow free of the ground at point B. The combined weight of the wheelbarrow and its load is 240 lb with center of gravity at G.



Solution - Exercise – 8

$$\circlearrowleft + \sum M_C = 0: \quad P(48) - 240(8) = 0$$

$$\underline{P = 40 \text{ lb}}$$

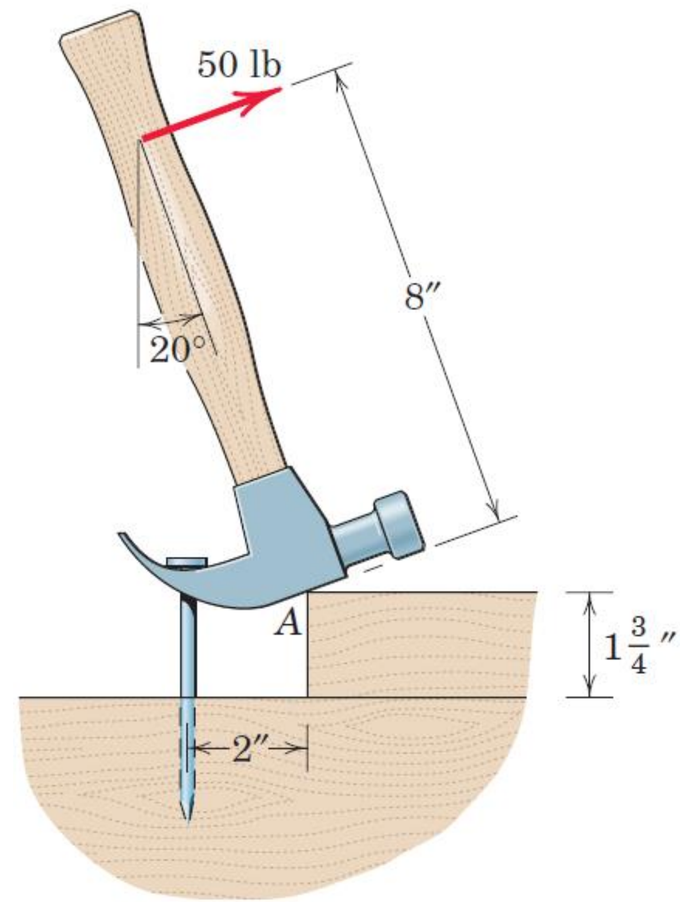


F. B. D.

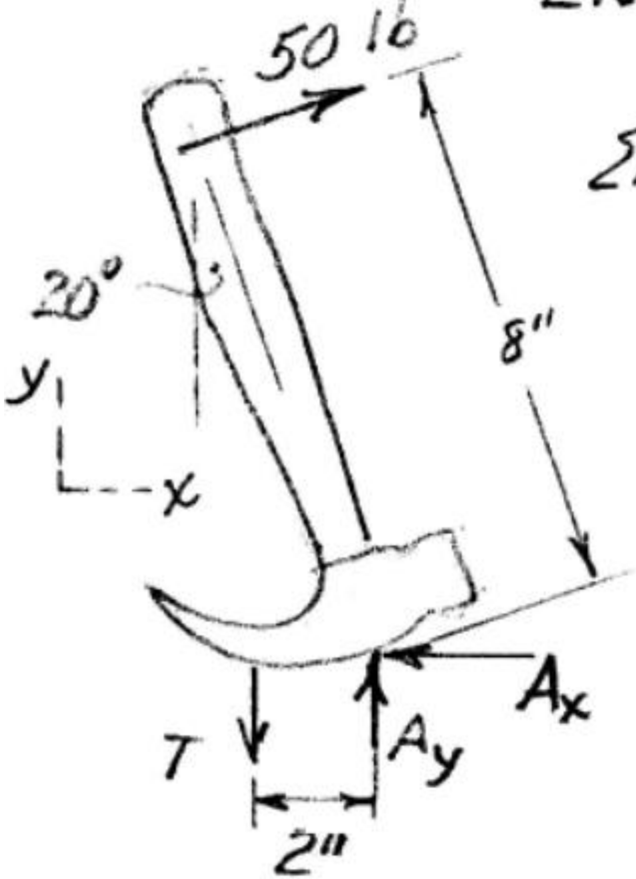
Exercise – 9

A block placed under the head of the claw hammer as shown greatly facilitates the extraction of the nail.

If a 50-lb pull on the handle is required to pull the nail, calculate the tension *T* in the nail and the magnitude *A* of the force exerted by the hammer head on the block. The contacting surfaces at *A* are sufficiently rough to prevent slipping.



Solution - Exercise – 9



$\Sigma M_A = 0;\ 50(8) - 2T = 0,\ \underline{T = 200\ lb}$

$\Sigma F_x = 0;\ 50\cos 20° - A_x = 0$

$A_x = 46.98\ lb$

$\Sigma F_y = 0;\ A_y + 50\sin 20° - 200 = 0$

$A_y = 182.9\ lb$

$A = \sqrt{(46.98)^2 + (182.9)^2} = \underline{188.8\ lb}$

F. B. D.