

Chapter 5

The Laws of Motion

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5.1 The Concept of Force

The word **force** refers to an interaction with an object by means of muscular activity and some change in the object's velocity.

When a coiled spring is pulled, as in Figure 5.1a, the spring stretches. When a stationary cart is pulled, as in Figure 5.1b, the cart moves. When a football is kicked, as in Figure 5.1c, it is both deformed and set in motion. These situations are all examples of a class of forces called **contact forces**. That is, they involve physical contact between two objects. Another class of forces, known as **field forces**, does not involve physical contact between two objects. These forces act through empty space. The gravitational force of attraction between two objects with mass, illustrated in Figure 5.1d, is an example of this class of force. Another common field force is the electric force that one electric charge exerts on another (Fig. 5.1e). A third example of a field force is the force a bar magnet exerts on a piece of iron (Fig. 5.1f).

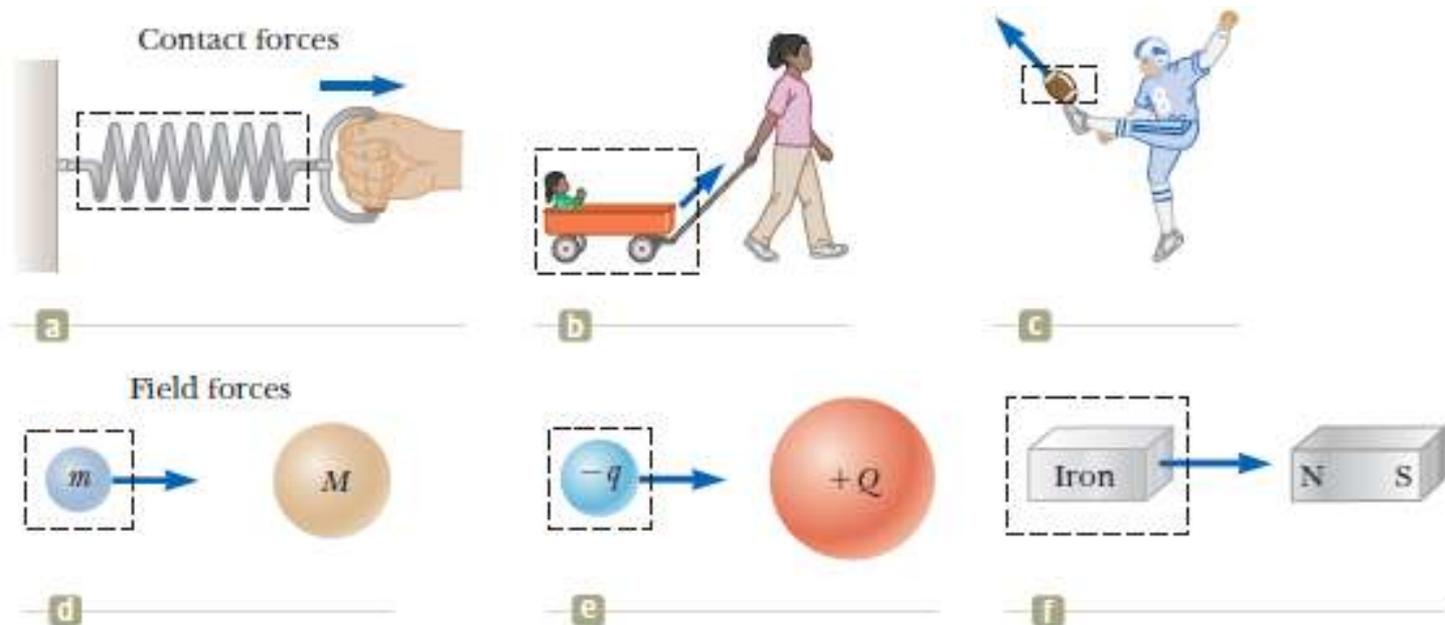


Figure 5.1

5.2 Newton's First Law and Inertial Frames

Newton's first law of motion, sometimes called the law of inertia, defines a special set of reference frames called inertial frames. This law can be stated as follows:

In the absence of external forces and when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line).

In other words, **when no force acts on an object, the acceleration of the object is zero.** From the first law, we conclude that any isolated object (one that does not interact with its environment) is either at rest or moving with constant velocity.

5.3 Mass

Mass is that property of an object that specifies how much resistance an object exhibits to changes in its velocity, the SI unit of mass is the kilogram. Experiments show that the greater the mass of an object, the less that object accelerates under the action of a given applied force.

Suppose a force acting on an object of mass m_1 produces a change in motion of the object that we can quantify with the object's acceleration \vec{a}_1 , and the same force acting on an object of mass m_2 produces an acceleration \vec{a}_2 . The ratio of the two masses is defined as the inverse ratio of the magnitudes of the accelerations produced by the force:

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1} \quad (5.1)$$

5.4 Newton's Second Law

Newton's first law explains what happens to an object when no forces act on it: it maintains its original motion; it either remains at rest or moves in a straight line with constant speed. Newton's second law answers the question of what happens to an object when one or more forces act on it.

When viewed from an inertial reference frame, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass:

$$\vec{\mathbf{a}} \propto \frac{\sum \vec{\mathbf{F}}}{m}$$

If we choose a proportionality constant of 1, we can relate mass, acceleration, and force through the following mathematical statement of Newton's second law:¹

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \quad (5.2)$$

Equation 5.2 is a vector expression and hence is equivalent to three component equations:

$$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_z = ma_z \quad (5.3)$$

The SI unit of force is the **newton (N)**. A force of 1 N is the force that, when acting on an object of mass 1 kg, produces an acceleration of 1 m/s². From this definition and Newton's second law, we see that the newton can be expressed in terms of the following fundamental units of mass, length, and time:

$$1 \text{ N} \equiv 1 \text{ kg} \cdot \text{m/s}^2 \quad (5.4)$$

Example 5.1 An Accelerating Hockey Puck

A hockey puck having a mass of 0.30 kg slides on the frictionless, horizontal surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck shown in Figure 5.4. The force \vec{F}_1 has a magnitude of 5.0 N, and is directed at $\theta = 20^\circ$ below the x axis. The force \vec{F}_2 has a magnitude of 8.0 N and its direction is $\phi = 60^\circ$ above the x axis. Determine both the magnitude and the direction of the puck's acceleration.

Solution

Find the component of the net force acting on the puck in the x direction:

Find the component of the net force acting on the puck in the y direction:

Use Newton's second law in component form (Eq.5.3) to find the x and y components of the puck's acceleration:

Substitute numerical values:

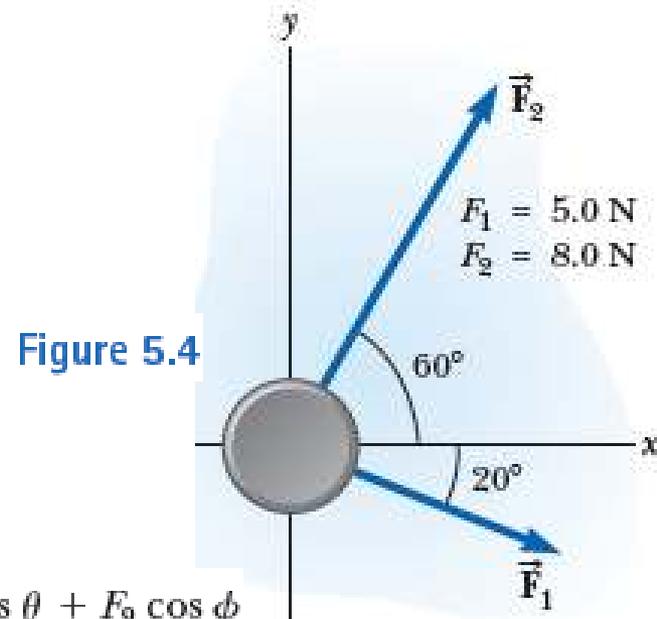


Figure 5.4

$$\sum F_x = F_{1x} + F_{2x} = F_1 \cos \theta + F_2 \cos \phi$$

$$\sum F_y = F_{1y} + F_{2y} = F_1 \sin \theta + F_2 \sin \phi$$

$$a_x = \frac{\sum F_x}{m} = \frac{F_1 \cos \theta + F_2 \cos \phi}{m}$$

$$a_y = \frac{\sum F_y}{m} = \frac{F_1 \sin \theta + F_2 \sin \phi}{m}$$

$$a_x = \frac{(5.0 \text{ N}) \cos(-20^\circ) + (8.0 \text{ N}) \cos(60^\circ)}{0.30 \text{ kg}} = 29 \text{ m/s}^2$$

$$a_y = \frac{(5.0 \text{ N}) \sin(-20^\circ) + (8.0 \text{ N}) \sin(60^\circ)}{0.30 \text{ kg}} = 17 \text{ m/s}^2$$

Find the magnitude of the acceleration:

$$a = \sqrt{(29 \text{ m/s}^2)^2 + (17 \text{ m/s}^2)^2} = 34 \text{ m/s}^2$$

Find the direction of the acceleration relative to the positive x axis:

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 31^\circ$$

5.5 The Gravitational Force and Weight

All objects are attracted to the Earth. The attractive force exerted by the Earth on an object is called the **gravitational force** \vec{F}_g .

$$\vec{F}_g = m\vec{g} \quad (5.5)$$

Therefore, the weight of an object, being defined as the magnitude of \vec{F}_g , is given by

$$F_g = mg \quad (5.6)$$

5.6 Newton's Third Law

If two objects interact, the force \vec{F}_{12} exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force \vec{F}_{21} exerted by object 2 on object 1:

$$\vec{F}_{12} = -\vec{F}_{21} \quad (5.7)$$

Figure 5.6 illustrates an extremely important step in solving problems involving forces. Figure 5.6a shows many of the forces in the situation: those acting on the monitor, one acting on the table, and one acting on the Earth. Figure 5.6b, by contrast, shows only the forces acting on one object, the monitor, and is called a force diagram or a diagram showing the forces on the object. The important pictorial representation in Figure 5.6c is called a **free-body diagram**.

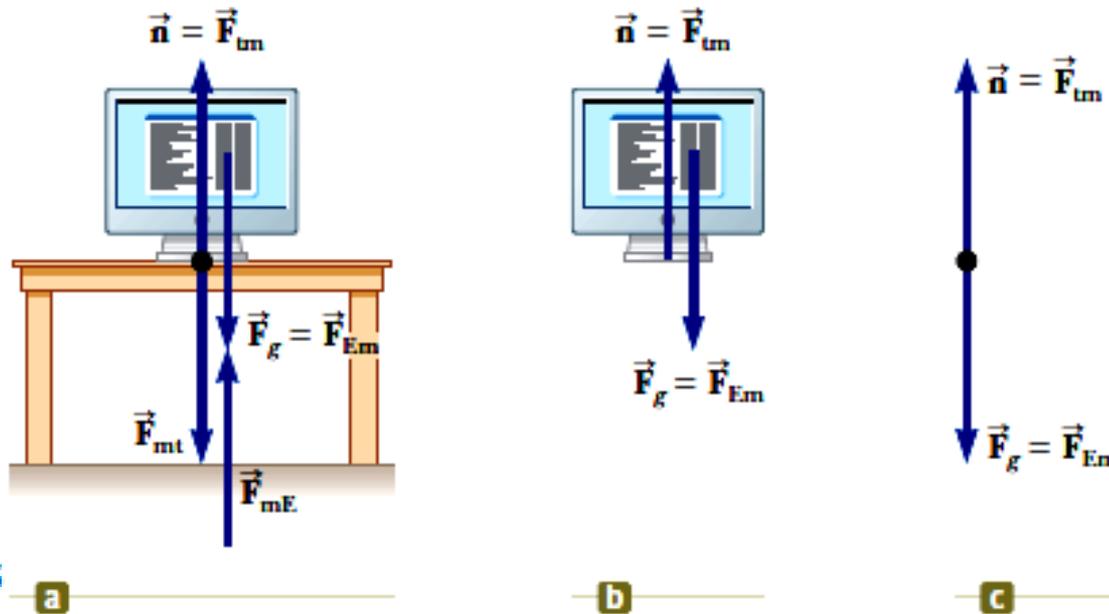


Figure 5.6

a

b

c

5.7 Analysis Models Using Newton's Second Law

Analysis Model: The Particle in Equilibrium

If the acceleration of an object modeled as a particle is zero, the object is treated with the **particle in equilibrium model**. In this model, the net force on the object is zero:

$$\sum \vec{F} = 0 \quad (5.8)$$

Consider a lamp suspended from a light chain fastened to the ceiling as in Figure 5.7a. The force diagram for the lamp (Fig. 5.7b) shows that the forces acting on the lamp are the downward gravitational force \vec{F}_g and the upward force \vec{T} exerted by the chain. Because there are no forces in the x direction, $\Sigma F_x = 0$ provides no helpful information. The condition $\Sigma F_y = 0$ gives

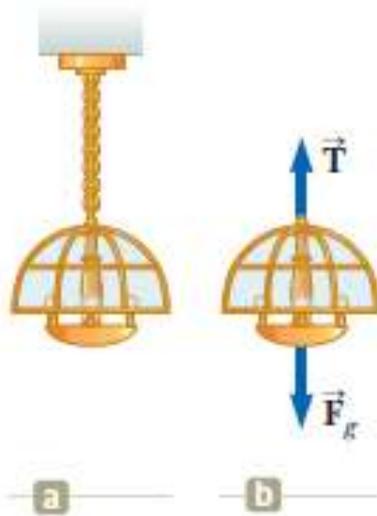
$$\Sigma F_y = T - F_g = 0 \text{ or } T = F_g$$


Figure 5.7

Analysis Model: The Particle Under a Net Force

If an object experiences an acceleration, its motion can be analyzed with the **particle under a net force** model. The appropriate equation for this model is Newton's second law, Equation 5.2:

$$\Sigma \vec{F} = m\vec{a} \quad (5.2)$$

Consider a crate being pulled to the right on a frictionless, horizontal floor as in Figure 5.8a. The forces acting on the crate are illustrated in the free-body diagram in Figure 5.8b.

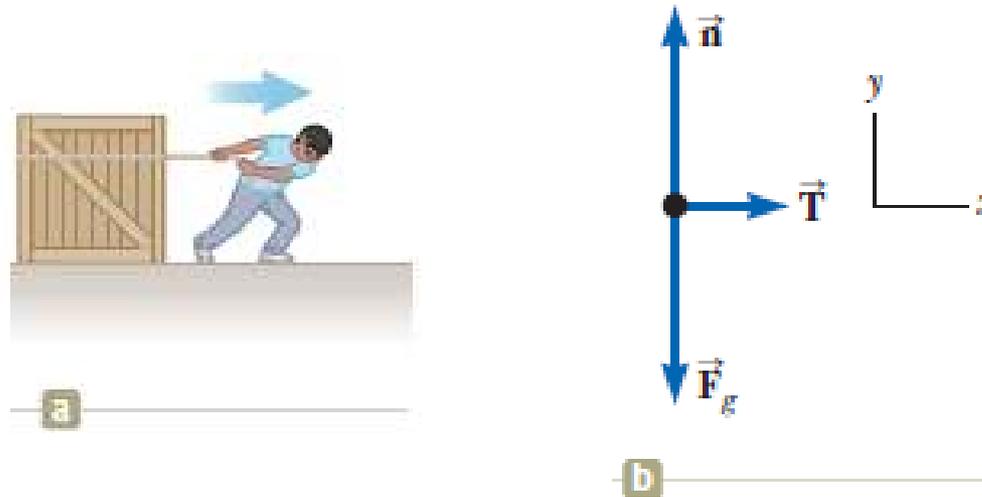


Figure 5.8

We can now apply Newton's second law in component form to the crate. The only force acting in the x direction is \vec{T} . Applying $\sum F_x = ma_x$ to the horizontal motion gives

$$\sum F_x = T = ma_x \quad \text{or} \quad a_x = \frac{T}{m}$$

No acceleration occurs in the y direction because the crate moves only horizontally. Therefore, we use the particle in equilibrium model in the y direction. Applying the y component of Equation 5.8 yields

$$\sum F_y = n - F_g = 0 \quad \text{or} \quad n = F_g$$

Example 5.4 A Traffic Light at Rest

A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support as in Figure 5.10a. The upper cables make angles of $\theta_1 = 37.0^\circ$ and $\theta_2 = 53.0^\circ$ with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 100 N. Does the traffic light remain hanging in this situation, or will one of the cables break?

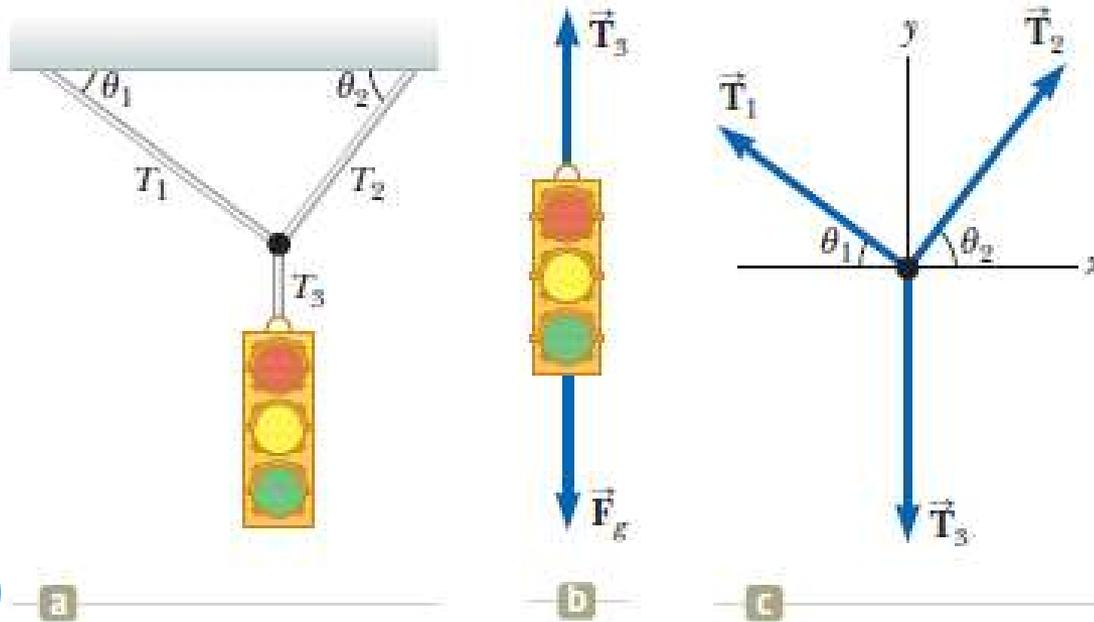


Figure 5.10

a

b

c

Solution

From the particle in equilibrium model, apply Equation 5.8 for the traffic light in the y direction:

$$\begin{aligned}\sum F_y = 0 &\rightarrow T_3 - F_g = 0 \\ T_3 &= F_g\end{aligned}$$

Force	x Component	y Component
\vec{T}_1	$-T_1 \cos \theta_1$	$T_1 \sin \theta_1$
\vec{T}_2	$T_2 \cos \theta_2$	$T_2 \sin \theta_2$
\vec{T}_3	0	$-F_g$

Choose the coordinate axes as shown in Figure 5.10c and resolve the forces acting on the knot into their components:

Apply the particle in equilibrium model to the knot:

$$(1) \quad \sum F_x = -T_1 \cos \theta_1 + T_2 \cos \theta_2 = 0$$

$$(2) \quad \sum F_y = T_1 \sin \theta_1 + T_2 \sin \theta_2 + (-F_g) = 0$$

Equation (1) shows that the horizontal components of \vec{T}_1 and \vec{T}_2 must be equal in magnitude, and Equation (2) shows that the sum of the vertical components of \vec{T}_1 and \vec{T}_2 must balance the downward force \vec{T}_3 , which is equal in magnitude to the weight of the light.

Solve Equation (1) for T2 in terms of T1:

$$(3) \quad T_2 = T_1 \left(\frac{\cos \theta_1}{\cos \theta_2} \right)$$

Substitute this value for T2 into Equation (2):

$$T_1 \sin \theta_1 + T_1 \left(\frac{\cos \theta_1}{\cos \theta_2} \right) (\sin \theta_2) - F_g = 0$$

Solve for T1:

$$T_1 = \frac{F_g}{\sin \theta_1 + \cos \theta_1 \tan \theta_2}$$

Substitute numerical values:

$$T_1 = \frac{122 \text{ N}}{\sin 37.0^\circ + \cos 37.0^\circ \tan 53.0^\circ} = 73.4 \text{ N}$$

Using Equation (3), solve for T2:

$$T_2 = (73.4 \text{ N}) \left(\frac{\cos 37.0^\circ}{\cos 53.0^\circ} \right) = 97.4 \text{ N}$$

Both values are less than 100 N (just barely for T2), so the cables will not break .

Example 5.6 The Runaway Car

A car of mass m is on an icy driveway inclined at an angle θ as in Figure 5.11a.

(A) Find the acceleration of the car, assuming the driveway is frictionless.

Solution

Apply these models to the car:

Solve Equation (1) for a_x :

$$(1) \quad \sum F_x = mg \sin \theta = ma_x$$

$$(2) \quad \sum F_y = n - mg \cos \theta = 0$$

$$(3) \quad a_x = g \sin \theta$$

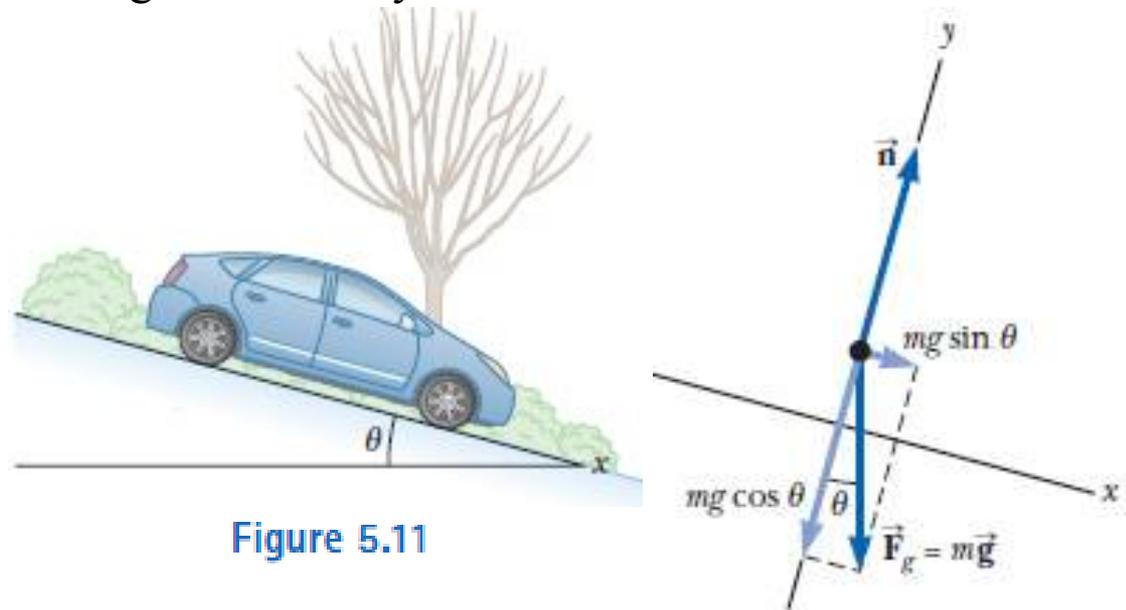


Figure 5.11

(B) Suppose the car is released from rest at the top of the incline and the distance from the car's front bumper to the bottom of the incline is d . How long does it take the front bumper to reach the bottom of the hill, and what is the car's speed as it arrives there?

Analyze Defining the initial position of the front bumper as $x_i = 0$ and its final position as $x_f = d$, and recognizing that $v_{xi} = 0$, choose Equation 2.16 from the particle under constant acceleration model, $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$:

Solve for t :

Use Equation 2.17, with $v_{xi} = 0$, to find the final velocity of the car:

$$d = \frac{1}{2}a_x t^2$$

$$(4) \quad t = \sqrt{\frac{2d}{a_x}} = \sqrt{\frac{2d}{g \sin \theta}}$$

$$v_{xf}^2 = 2a_x d$$

$$(5) \quad v_{xf} = \sqrt{2a_x d} = \sqrt{2gd \sin \theta}$$

Example 5.7 One Block Pushes Another

Two blocks of masses m_1 and m_2 , with $m_1 > m_2$, are placed in contact with each other on a frictionless, horizontal surface as in Figure 5.12a. A constant horizontal force \vec{F} is applied to m_1 as shown.

(A) Find the magnitude of the acceleration of the system.

Solution

First model the combination of two blocks as a single particle under a net force. Apply Newton's second law to the combination in the x direction to find the a

$$\sum F_x = F = (m_1 + m_2)a_x$$
$$(1) \quad a_x = \frac{F}{m_1 + m_2}$$

(B) Determine the magnitude of the contact force between the two blocks.

\vec{P}_{21} is the reaction force to \vec{P}_{12} , so $P_{21} = P_{12}$. Apply Newton's second law to m_2 :

$$(2) \quad \sum F_x = P_{12} = m_2 a_x$$

$$(3) \quad P_{12} = m_2 a_x = \left(\frac{m_2}{m_1 + m_2} \right) F$$

Apply Newton's second law to m_1 :

$$(4) \quad \sum F_x = F - P_{21} = F - P_{12} = m_1 a_x$$

$$P_{12} = F - m_1 a_x = F - m_1 \left(\frac{F}{m_1 + m_2} \right) = \left(\frac{m_2}{m_1 + m_2} \right) F$$

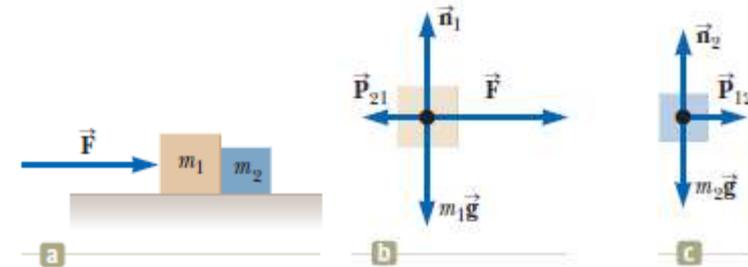


Figure 5.12

Example 5.8 Weighing a Fish in an Elevator

A person weighs a fish of mass m on a spring scale attached to the ceiling of an elevator as illustrated in Figure 5.13.

(A) Show that if the elevator accelerates either upward or downward, the spring scale gives a reading that is different from the weight of the fish.

Solution

Apply Newton's 2nd law to the fish:

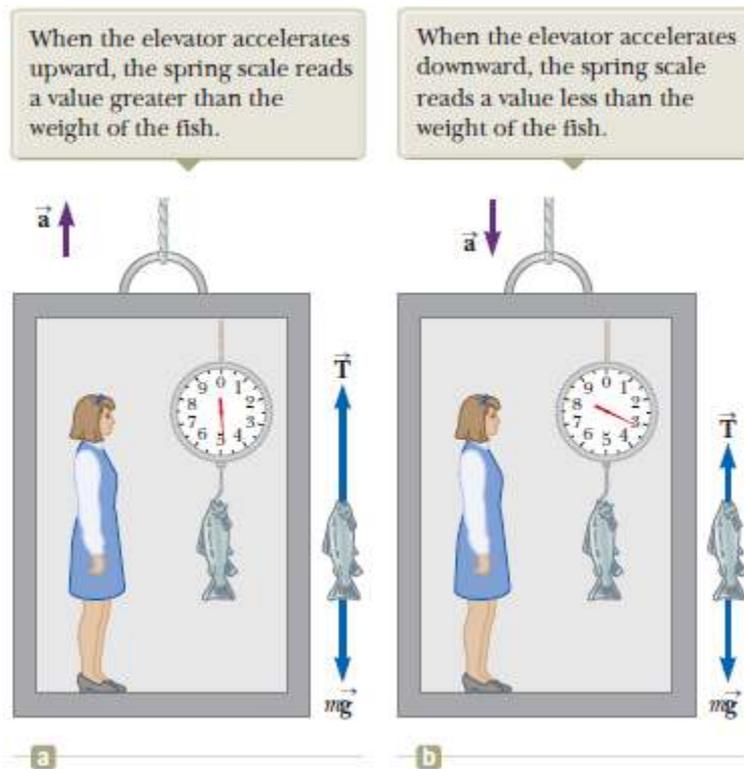
$$\sum F_y = T - mg = ma_y$$

Solve for T :

$$(1) \quad T = ma_y + mg = mg \left(\frac{a_y}{g} + 1 \right) = F_s \left(\frac{a_y}{g} + 1 \right)$$

where we have chosen upward as the positive y direction. We conclude from Equation (1) that the scale reading T is greater than the fish's weight mg if \vec{a} is upward, so a_y is positive (Fig. 5.13a), and that the reading is less than mg if \vec{a} is downward, so a_y is negative (Fig. 5.13b).

Figure 5.13



(B) Evaluate the scale readings for a 40.0-N fish if the elevator moves with an acceleration $a_y = \pm 2.00 \text{ m/s}^2$.

Evaluate the scale reading from Equation (1) if \vec{a} is upward:

$$T = (40.0 \text{ N}) \left(\frac{2.00 \text{ m/s}^2}{9.80 \text{ m/s}^2} + 1 \right) = 48.2 \text{ N}$$

Evaluate the scale reading from Equation (1) if \vec{a} is downward:

$$T = (40.0 \text{ N}) \left(\frac{-2.00 \text{ m/s}^2}{9.80 \text{ m/s}^2} + 1 \right) = 31.8 \text{ N}$$

Example 5.9 The Atwood Machine

When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass as in Figure 5.14a, the arrangement is called an Atwood machine. The device is sometimes used in the laboratory to determine the value of g . Determine the magnitude of the acceleration of the two objects and the tension in the lightweight string.

Solution

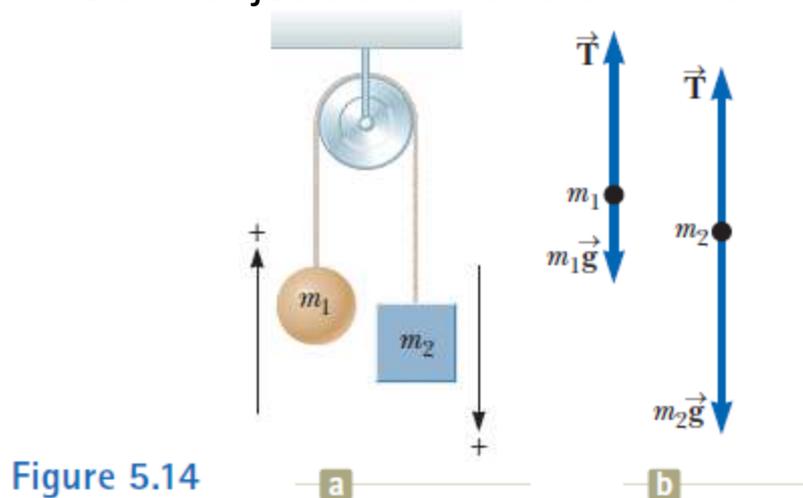


Figure 5.14

From the particle under a net force model, apply Newton's second law to object 1:

$$(1) \quad \sum F_y = T - m_1g = m_1a_y$$

Apply Newton's second law to object 2:

$$(2) \quad \sum F_y = m_2g - T = m_2a_y$$

Add Equation (2) to Equation (1), noticing that T cancels:

$$-m_1g + m_2g = m_1a_y + m_2a_y$$

Solve for the acceleration:

$$(3) \quad a_y = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g$$

Substitute Equation (3) into

Equation (1) to find T :

$$(4) \quad T = m_1(g + a_y) = \left(\frac{2m_1m_2}{m_1 + m_2} \right) g$$

Example 5.10 Acceleration of Two Objects Connected by a Cord

A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass as in Figure 5.15a. The block lies on a frictionless incline of angle θ . Find the magnitude of the acceleration of the two objects and the tension in the cord.

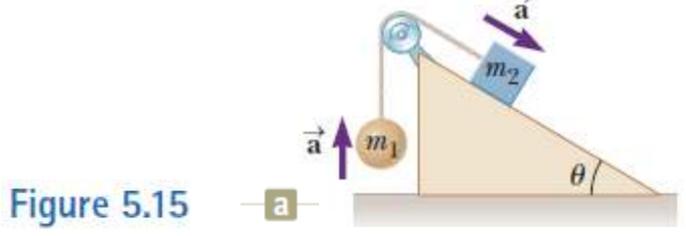
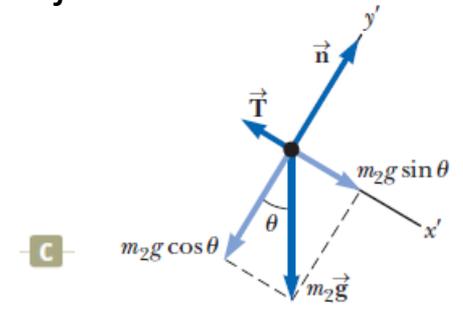
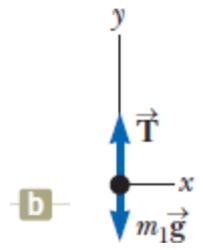


Figure 5.15



Apply Newton's second law in the y direction to the ball, choosing the upward direction as positive:

Apply the particle under a net force model to the Block in the x' direction and the particle in equilibrium model in the y' direction:

solve Equation (1) for T :

Substitute this expression for T into Equation (2):

Solve for a :

Substitute this expression for a into Equation (4) to find T :

$$(1) \quad \sum F_y = T - m_1g = m_1a_y = m_1a$$

$$(2) \quad \sum F_{x'} = m_2g \sin \theta - T = m_2a_{x'} = m_2a$$

$$(3) \quad \sum F_{y'} = n - m_2g \cos \theta = 0$$

$$(4) \quad T = m_1(g + a)$$

$$m_2g \sin \theta - m_1(g + a) = m_2a$$

$$(5) \quad a = \left(\frac{m_2 \sin \theta - m_1}{m_1 + m_2} \right) g$$

$$(6) \quad T = \left(\frac{m_1 m_2 (\sin \theta + 1)}{m_1 + m_2} \right) g$$

Forces of Friction

When an object is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. We call such resistance a **force of friction**. Forces of friction are very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.

Imagine that you are working in your garden and have filled a trash can with yard clippings. You then try to drag the trash can across the surface of your concrete patio as in Figure 5.16a. This surface is *real*, not an idealized, frictionless surface. If we apply an external horizontal force \vec{F} to the trash can, acting to the right, the trash can remains stationary when \vec{F} is small. The force on the trash can that counteracts \vec{F} and keeps it from moving acts toward the left and is called the **force of static friction** \vec{f}_s . As long as the trash can is not moving, $f_s = F$. Therefore, if \vec{F} is increased, \vec{f}_s also increases. Likewise, if \vec{F} decreases, \vec{f}_s also decreases.

When F exceeds $f_{s,max}$, the trash can moves and accelerates to the right. We call the friction force for an object in motion the **force of kinetic friction** \vec{f}_k .

The magnitude of the force of static friction between any two surfaces in contact can have the values

$$f_s \leq \mu_s n \tag{5.9}$$

where the dimensionless constant μ_s is called the **coefficient of static friction** and n is the magnitude of the normal force exerted by one surface on the other.

The magnitude of the force of kinetic friction acting between two surfaces is

$$f_k = \mu_k n \tag{5.10}$$

where μ_k is the **coefficient of kinetic friction**.

The values of μ_k and μ_s depend on the nature of the surfaces, but μ_k is generally less than μ_s . Typical values range from around 0.03 to 1.0.

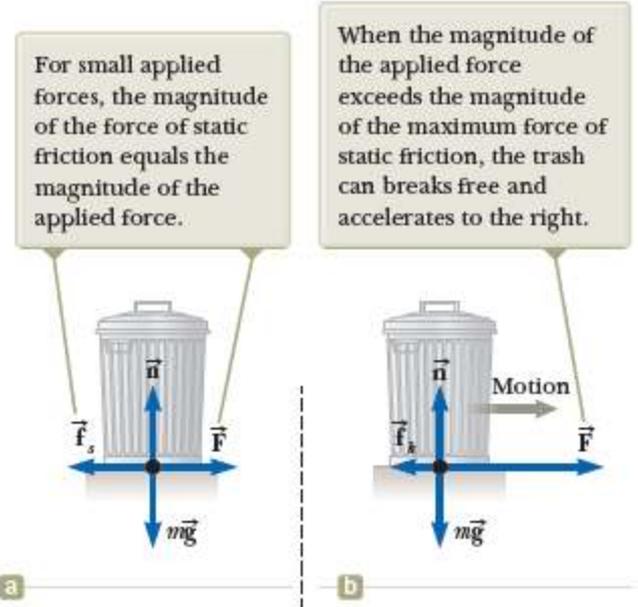


Table 5.1 Coefficients of Friction

	μ_s	μ_k
Rubber on concrete	1.0	0.8
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Glass on glass	0.94	0.4
Copper on steel	0.53	0.36
Wood on wood	0.25–0.5	0.2
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Teflon on Teflon	0.04	0.04
Ice on ice	0.1	0.03
Synovial joints in humans	0.01	0.003

Example 5.11 Experimental Determination of μ_s and μ_k

The following is a simple method of measuring coefficients of friction. Suppose a block is placed on a rough surface inclined relative to the horizontal as shown in Figure 5.18. The incline angle is increased until the block starts to move. Show that you can obtain μ_s by measuring the critical angle θ_c at which this slipping just occurs.

Solution

From the particle in equilibrium model, apply Equation 5.8 to the block in both the x and y directions:

$$(1) \sum F_x = mg \sin \theta - f_s = 0$$
$$(2) \sum F_y = n - mg \cos \theta = 0$$

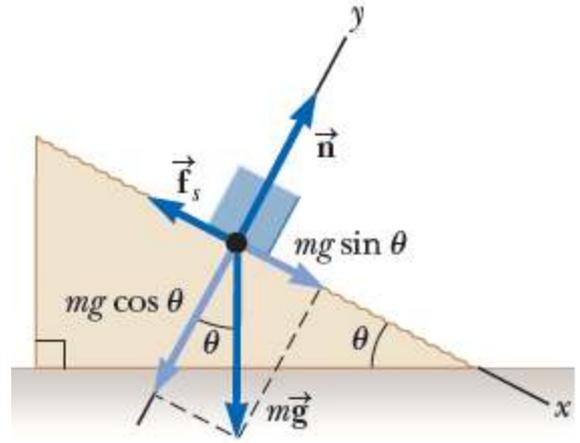
Substitute $mg = n/\cos \theta$ from Equation (2) into Equation (1):

$$(3) f_s = mg \sin \theta = \left(\frac{n}{\cos \theta} \right) \sin \theta = n \tan \theta$$

$$\mu_s n = n \tan \theta_c$$
$$\mu_s = \tan \theta_c$$

We have shown, as requested, that the coefficient of static friction is related only to the critical angle. For example, if the block just slips at $\theta_c = 20.0^\circ$, we find that $\mu_s = \tan 20.0^\circ = 0.364$.

Figure 5.18



Example 5.12 The Sliding Hockey Puck

A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.

Solution

Apply the particle under a net force model in the x direction to the puck:

$$(1) \sum F_x = -f_k = ma_x$$

Apply the particle in equilibrium model in the y direction to the puck:

$$(2) \sum F_y = n - mg = 0$$

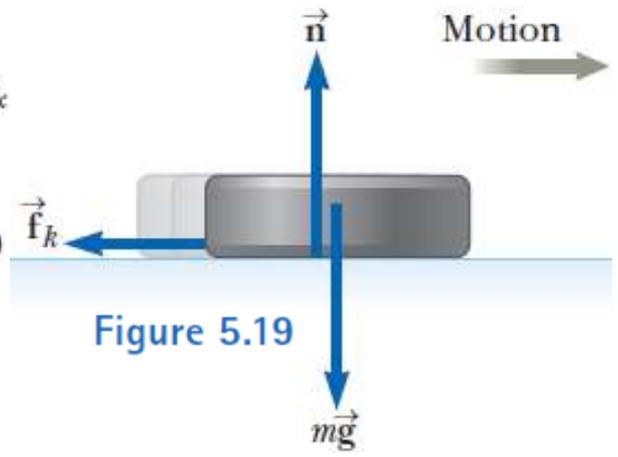


Figure 5.19

Substitute $n = mg$ from Equation (2) and $f_k = \mu_k n$ into Equation (1):

$$\begin{aligned}
 -\mu_k n &= -\mu_k mg = ma_x \\
 a_x &= -\mu_k g
 \end{aligned}$$

The negative sign means the acceleration is to the left in Figure 5.19.

Apply the particle under constant acceleration model to the puck, choosing Equation 2.17 from the model, $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$, with $x_i = 0$ and $v_{xf} = 0$:

$$\begin{aligned}
 0 &= v_{xi}^2 + 2a_x x_f = v_{xi}^2 - 2\mu_k g x_f \\
 \mu_k &= \frac{v_{xi}^2}{2g x_f}
 \end{aligned}$$

Solve for the coefficient of kinetic friction:

Substitute the numerical values:

$$\mu_k = \frac{(20.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(115 \text{ m})} = 0.177$$

Example 5.13 Acceleration of Two Connected Objects When Friction Is Present

A block of mass m_2 on a rough, horizontal surface is connected to a ball of mass m_1 by a lightweight cord over a lightweight, frictionless pulley as shown in Figure 5.20a. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown, and the block slides to the right. The coefficient of kinetic friction between the block and surface is μ_k . Determine the magnitude of the acceleration of the two objects.

Solution

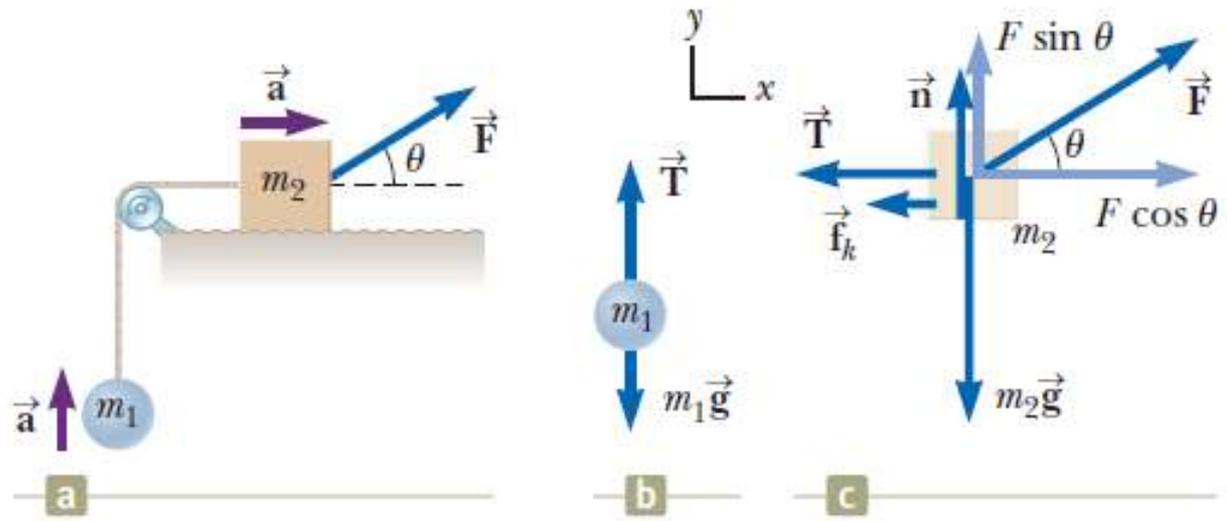


Figure 5.20

Apply the particle under a net force model to the block in the horizontal direction:

Because the block moves only horizontally, apply the particle in equilibrium model to the block in the vertical direction:

$$(1) \sum F_x = F \cos \theta - f_k - T = m_2 a_x = m_2 a$$

$$(2) \sum F_y = n + F \sin \theta - m_2 g = 0$$

Apply the particle under a net force model to the ball in the vertical direction:

$$(3) \quad \sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

Solve Equation (2) for n :

$$n = m_2 g - F \sin \theta$$

Substitute n into $f_k = \mu_k n$ from Equation 5.10:

$$(4) \quad f_k = \mu_k (m_2 g - F \sin \theta)$$

Substitute Equation (4) and the value of T from Equation (3) into Equation (1):

$$F \cos \theta - \mu_k (m_2 g - F \sin \theta) - m_1 (a + g) = m_2 a$$

Solve for a :

$$(5) \quad a = \frac{F(\cos \theta + \mu_k \sin \theta) - (m_1 + \mu_k m_2)g}{m_1 + m_2}$$