Differential Equations

MATH 222

L1

First Semester for Second Year

College of Science and Computer Engineering

Taibah University

Yanbu

Saudi Arabian

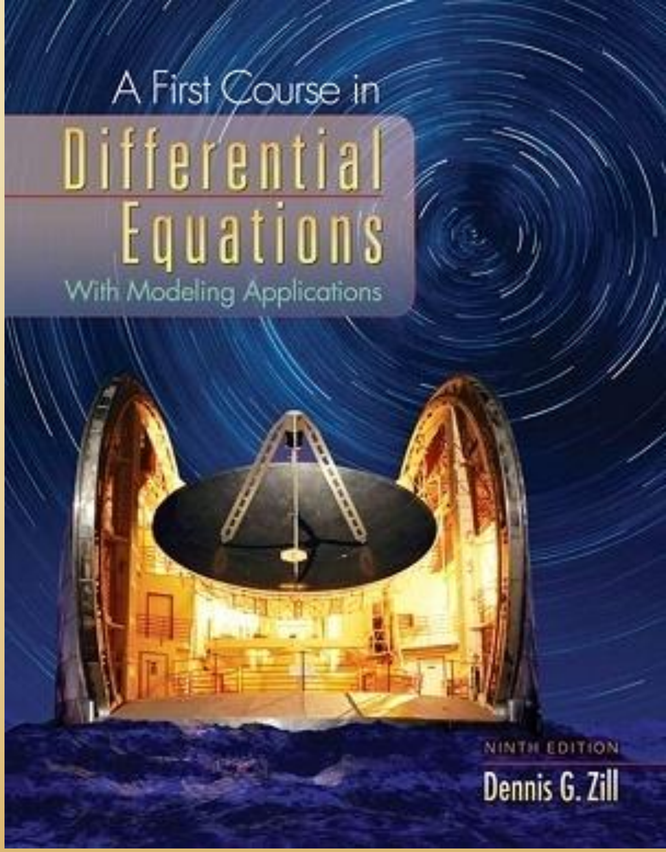
1442- 1443

Dr. Essam Othman Abdel-Rahman

Lecturer and Instructor

– Dr. Essam Othman Abdel-Rahman
- Room 209
- essamothman1@yahoo.com

Text



– **A First Course in Differential Equations with Modeling Applications, Ninth Edition . Dennis G. Zill**

Suggested Readings

– Elementary Differential Equations and Boundary Value Problems, 7th Edition.

– Fundamentals of Differential Equations, 8th Edition

COURSE OUTLINE FOR MATH 222
DIFFERENTIAL EQUATIONS

Chapter 1: Introduction to Differential Equations	Two weeks
Chapter 2: First-order Differential Equations	Two weeks
Chapter 4: Higher-order Differential Equations	Three weeks
Chapter 6: Series Solutions of Linear Equations	Three weeks
Chapter 7: The Laplace Transform	Three weeks
Testing and Review	One week
Total	Fourteen week

Assignments

– Coming of Class (10%)

. Minimum 50% or **don’t coming final exam**

– Homework assignments sheet and spoken (20%)

- Intended to support lecture material

– Midterm Exam (30% each)

– Final Exam (40% each)

CHAPTER 1
Introduction to **Differential Equations**

1.1 Definitions and Terminology

1.2 Initial-Value Problems

1.3 Differential Equation as Mathematical Models

1.1 Definitions and Terminology

DEFINITION: differential equation

An equation containing the derivative of one or more dependent variables, with respect to one or more independent variables is said to be a **differential equation** (DE).

(Zill, Definition 1.1, page 6).

1.1 Definitions and Terminology

Recall ***Calculus***

Definition of a Derivative

If $y = f(x)$, the derivative of y or $f(x)$

With respect to x is defined as

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The derivative is also denoted by y', $\frac{df}{dx}$ or $f'(x)$

1.1 Definitions and Terminology

Recall the Exponential function

$$y = f(x) = e^{2x}$$

➔dependent variable: y

➔independent variable: x

$$\frac{dy}{dx} = \frac{d(e^{2x})}{dx} = e^{2x}\left[\frac{d(2x)}{dx}\right] = 2e^{2x} = 2y$$

1.1 Definitions and Terminology

Differential Equation :

Equations that involve dependent variables and their derivatives with respect to the independent variables .

Differential Equations are classified by ***type***, ***order*** and ***linearity***.

1.1 Definitions and Terminology

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TYPE

There are two main *types* of differential equation: “ordinary” and “partial”.

1.1 Definitions and Terminology

Ordinary differential equation (ODE)

Differential equations that involve only **ONE** independent variable are called ordinary differential equations.

Examples:

$$\frac{dy}{dx}+5y=e^{x},\quad \frac{d^2y}{dx^2}-\frac{dy}{dx}+6y=0, \text{ and } \frac{dx}{dt}+\frac{dy}{dt}=2x+y$$

➔**only *ordinary* (or *total*) derivatives**

1.1 Definitions and Terminology

Partial differential equation (PDE)
Differential equations that involve two or more independent variables are called partial differential equations.

Examples:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} - 2\frac{\partial u}{\partial t} \quad \textbf{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

➔only *partial* derivatives

1.1 Definitions and Terminology

ORDER

The *order* of a differential equation is the order of the highest derivative found in the DE.

$$\frac{d^2y}{dx^2}+5\left(\frac{dy}{dx}\right)^3-4y=e^x$$

second order **first order**

1.1 Definitions and Terminology

$xy' - y^2 = e^x$ ➔first order $F(x, y, y') = 0$

Written in differential form: $M(x, y)dx + N(x, y)dy = 0$

$y'' = x^3$ ➔second order $F(x, y, y', y'') = 0$

1.1 Definitions and Terminology

LINEAR or NONLINEAR

An *n*-th order differential equation is said to be **linear** if the function $F(x, y, y', \ldots, y^{(n)}) = 0$ is linear in the variables $y, \; y', \ldots \; y^{(n-1)}$

$$\Rightarrow \quad a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \ldots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

➔there are no multiplications among **dependent variables** and **their derivatives**. All **coefficients** are functions of **independent variables**.

A **nonlinear** ODE is one that is not linear, i.e. does not have the above form.

1.1 Definitions and Terminology

LINEAR or NONLINEAR

$$(y-x)dx+4xdy=0 \quad \text{or} \quad 4x\frac{dy}{dx}+(y-x)=0$$

➔linear first-order ordinary differential equation

$$y''-2y'+y=0$$

➔linear second-order ordinary differential equation

$$\frac{d^3y}{dx^3}+3x\frac{dy}{dx}-5y=e^x$$

➔ linear third-order ordinary differential equation

1.1 Definitions and Terminology

LINEAR or NONLINEAR

$$(1-y)y' + 2y = e^{x}$$ coefficient depends on y

➔nonlinear first-order ordinary differential equation

$$\frac{d^2 y}{dx^2} + \sin(y) = 0$$

nonlinear function of y

➔nonlinear second-order ordinary differential equation

$$\frac{d^4 y}{dx^4} + y^2 = 0$$

power not 1

➔ nonlinear fourth-order ordinary differential equation

1.1 Definitions and Terminology

LINEAR or NONLINEAR

NOTE:

$$\sin(y) = y - \frac{y^3}{3!} + \frac{y^5}{5!} - \frac{y^7}{7!} + \dots$$

1.1 Definitions and Terminology

Solutions of ODEs

DEFINITION: solution of an ODE

Any function Φ , defined on an interval I and possessing at least n derivatives that are continuous

on I, which when substituted into an n-th order ODE reduces the equation to an identity, is said to be a **solution** of the equation on the interval.

(Zill, Definition 1.1, page 8).

1.1 Definitions and Terminology

Namely, a solution of an *n*-th order ODE is a function which possesses at least *n* derivatives and for which

$F(x, \ \phi(x), \ \phi^{'}(x), \ \phi^{(n)}(x)) = 0$ for all x in I

We say that *satisfies* the differential equation on I.

1.1 Definitions and Terminology

$$y'' - 2y' + y = 0 \quad ; y = xe^{x}$$

Verification of a solution by substitution

Example:

➔ $$y' = xe^{x} + e^{x}, \quad y'' = xe^{x} + 2e^{x}$$

➔ left hand side:

$$y'' - 2y' + y = (xe^{x} + 2e^{x}) - 2(xe^{x} + e^{x}) + xe^{x} = 0$$

right-hand side: 0

The DE possesses the constant y=0 ➔ trivial solution

1.1 Definitions and Terminology

EXERCISES 1.1

State the type and the order of the given differential equation? Determine whether the equation is linear or nonlinear?

1. $(1-x)y''-4xy'+5y=\cos x$

2. $x\frac{d^3y}{dx^3}-\left(\frac{dy}{dx}\right)^4+y=0$

3. $t^5y^{(4)}-t^3y''+6y=0$

4. $\frac{d^2u}{dr^2}+\frac{du}{dr}+u=\cos(r+u)$

5. $\frac{d^2y}{dx^2}=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

6. $\frac{d^2R}{dt^2}=-\frac{k}{R^2}$

7. $(\sin\theta)y'''-(\cos\theta)y'=2$

8. $\ddot{x}-\left(1-\frac{\dot{x}^2}{3}\right)\dot{x}+x=0$