Differential Equations

MATH 222

L2

First Semester for Second Year

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1.2 Initial-Value Problem

DEFINITION: **initial value problem**

An **initial value problem** or IVP is a problem which consists of an *n-th* order ordinary differential equation along with n initial conditions defined at a point x_0 found in the interval of definition I

differential equation $\dfrac{d^n y}{dx^n} = f(x, y, y', \ldots, y^{(n-1)})$

initial conditions $y(x_0) = y_0, y'(x_0) = y_1, \ldots, y^{(n-1)}(x_0) = y_{n-1}$

where $y_0, y_1, \ldots, y_{n-1}$ are known constants.

1.2 Initial-Value Problem

First- and Second-Order IVPS

Solve: $$\frac{dy}{dx} = f(x, y)$$

Subject to: $$y(x_0) = y_0$$

Solve: $$\frac{d^2y}{dx^2} = f(x, y, y')$$

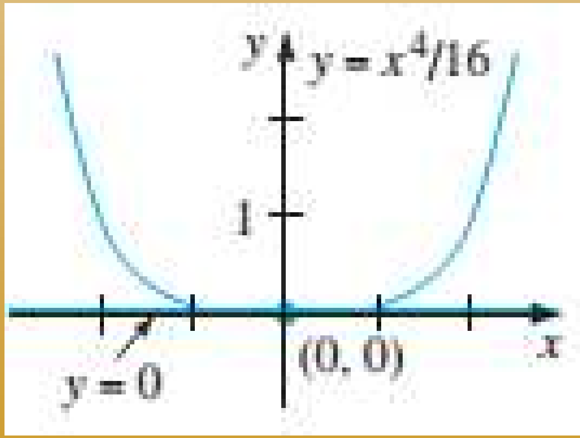
Subject to: $$y(x_0) = y_0, \quad y'(x_0) = y_1$$

1.2 Initial-Value Problem

Each of the functions $y = 0$ and $y = \frac{1}{16}x^4$ satisfies the differential equation $\frac{dy}{dx} = xy^{\frac{1}{2}}$ and the initial condition $y(0) = 0$, so the initial-value problem

$$\frac{dy}{dx} = xy^{\frac{1}{2}}, \quad y(0) = 0$$

has at least two solutions. As illustrated in Figure 1, the graphs of both functions pass through the same point (0, 0)



1.2 Initial-Value Problem

THEOREM: Existence of a Unique Solution

Let R be a rectangular region in the xy-plane defined by $a \le x \le b, \quad c \le y \le d$ that contains the point (x_0, y_0) in its interior. If $f(x, y)$ and $\partial f / \partial y$ are continuous on R, Then there exists some interval $I_0 : x_0 - h < x < x_0 + h, \; h > 0$ contained in $a \le x \le b$ and a unique function $y(x)$ defined on I_0 that is a solution of the initial value problem.

1.3 Differential Equation as Mathematical Models

POPULATION DYNAMICS

$$\frac{dP}{dt} \propto P \quad \text{or} \quad \frac{dP}{dt} = kP,$$

RADIOACTIVE DECAY

$$\frac{dA}{dt} \propto A \quad \text{or} \quad \frac{dA}{dt} = kA.$$

NEWTON'S LAW OF COOLING/WARMING

$$\frac{dT}{dt} \propto T - T_m \quad \text{or} \quad \frac{dT}{dt} = k(T - T_m),$$

SPREAD OF A DISEASE

$$\frac{dx}{dt} = kxy,$$

$$\frac{dx}{dt} = kx(n + 1 - x).$$

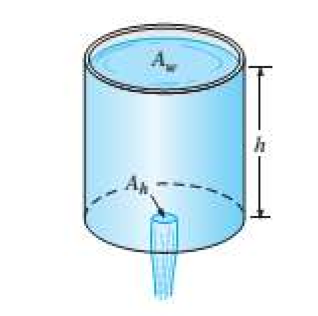
1.3 Differential Equation as Mathematical Models

CHEMICAL REACTIONS

$$CH_3Cl + NaOH \rightarrow CH_3OH + NaCl$$

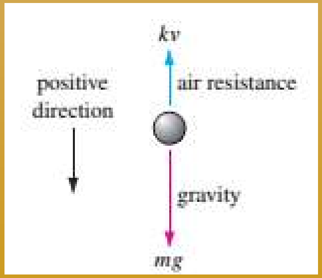
$$\frac{dX}{dt} = k(\alpha - X)(\beta - X),$$

DRAINING A TANK



$$\frac{dV}{dt} = -A_h\sqrt{2gh},$$

FALLING BODIES AND AIR RESISTANCE



$$m\frac{dv}{dt} = mg - kv.$$

EXERCISES

In Problems 1 and 2 , $y=\dfrac{1}{1+c_1 e^{-x}}$ **is a one–parameter family of solutions of the first-order DE** $y' = y - y^2$.

Find a solution of the first –order IVP consisting of this differential equation and the given initial condition.

1. $y(0) = -\dfrac{1}{3}$

2. $y(-1) = 2$