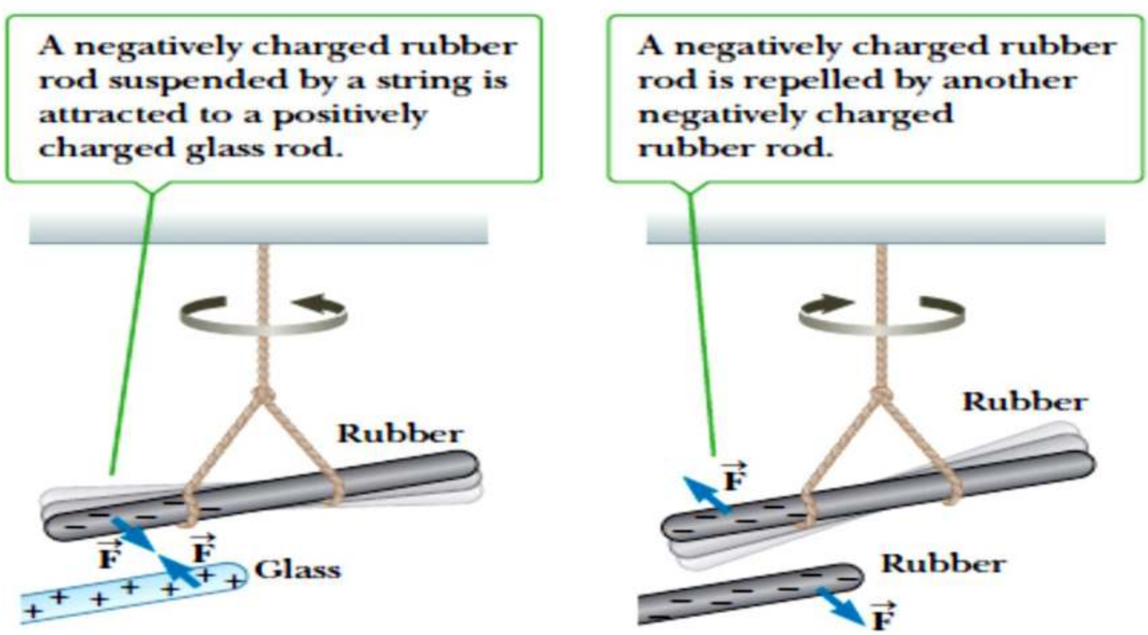
Chapter 3

Electricity and Magnetism

Electric Charges

After running a plastic comb through your hair, you will find that the comb attracts bits of paper. The attractive force is often strong enough to suspend the paper from the comb, defying the gravitational pull of the entire Earth. The same effect occurs with other rubbed materials, such as glass and hard rubber. These materials have become electrically charged.

Experiments also demonstrate that there are two kinds of electric charge, which Benjamin Franklin (1706–1790) named positive and negative. The following Figure illustrates the interaction of the two charges. A hard **rubber (or plastic) rod** that has been rubbed with **fur** is suspended by a piece of string.



When a **glass rod** that has been rubbed with **silk** is brought near the rubber rod, the rubber rod is attracted toward the glass rod. If two charged rubber rods (or two charged glass rods) are brought near each other, the force between them is repulsive. These observations may be explained by assuming the rubber and glass rods have acquired different kinds of excess charge.

We use the convention suggested by Franklin, where the excess **electric charge on the glass rod is called positive and that on the rubber rod is called negative.** On the basis of such observations, we conclude that **"like charges repel one another and unlike charges attract one another".** Objects usually contain equal amounts of positive and negative charge; electrical forces between objects arise when those objects have net negative or positive charges.

Coulomb's Law

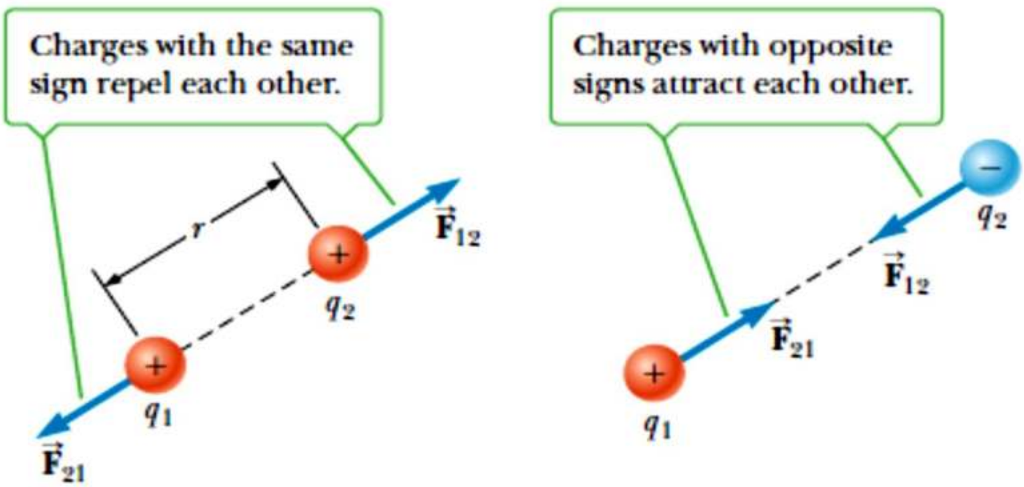
In 1785 Charles Coulomb (1736–1806) experimentally established the fundamental law of electric force between two stationary charged particles.

An **electric force** has the following properties:

1- It is directed along a line joining the two particles and is inversely proportional to the square of the separation distance r, between them.

2. It is proportional to the product of the magnitudes of the charges, $|q_1|$ and $|q_2|$, of the two particles.

3. It is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.



From these observations, Coulomb proposed the following mathematical form for the electric force between two charges:

The magnitude of the electric force $\vec{F}$ between charges q_1 and q_2 separated by a distance r is given by:

$$\vec{F} = \frac{kq_1q_2}{r^2}$$

where **k** is a constant called the **Coulomb constant** and equal **9×10^9 N.m^2/C^2**.

This equation, known as **Coulomb's law**, applies exactly only to point charges and to spherical distributions of charges, in which case r is the distance between the two centers of charge. Electric forces between unmoving charges are called **electrostatic forces**.

The Coulomb force is similar to the gravitational force. Both act at a distance without direct contact. Both are inversely proportional to the distance squared, with the force directed along a line connecting the two bodies. The mathematical form is the same, with the masses m_1 and m_2 in Newton's law replaced by q_1 and q_2 in Coulomb's law and with Newton's constant G $(6.67 \times 10^{-11}\, N.\frac{m^2}{kg^2})$ replaced by Coulomb's constant k. There are two important differences: **(1) electric forces can be either attractive or repulsive, but gravitational forces are always attractive, and (2) the electric force between charged elementary particles is far stronger than the gravitational force between the same particles.**

Example 22:

The electron and proton of a hydrogen atom are separated (on the average) by a distance of about 0.5×10^{-10} m .(a) Find the magnitudes of the electric force and the gravitational force that each particle exerts on the other, and the ratio of the electric force F_e to the gravitational force F_g. (where, e $=1.9\times10^{-19}$ C, $m_p= 1.67\times10^{-27}$ kg, $m_e= 9.11\times10^{-31}$ kg)

Solution

To find the electric force

$$\vec{F}_e = \frac{kq_1q_2}{r^2} = \frac{9\times10^9\, N.\frac{m^2}{C^2}.\ (1.6\times10^{-19}\, C)^2}{(0.5\times10^{-10}\, m)^2} = 9.22\times10^{-8}\ N$$

To find the gravitational force

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\, N.\frac{m^2}{kg^2}.(9.1\times10^{-31}kg)(\,1.67\times10^{-27}kg)}{(0.5\times10^{-10}\, m)^2}$$

$$= 4.05 \times 10^{-47} \; N$$

So,

$$\frac{F_e}{F_g} = \frac{9.22 \times 10^{-8}}{4.05 \times 10^{-47}} = 2.27 \times 10^{39}$$

Example 23:

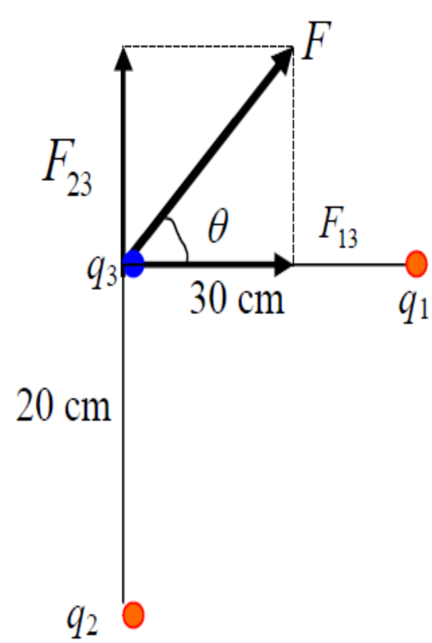
Calculate the electrical force between two charges, $q_1 = 5$ μC, $q_2 = -3$ μC. where the distance between them is equal to 15 cm, and Coulomb's constant equal to 9×10^9 N.m^2 / C^2.

Solution

Example 24:

Consider three point charges at the corners of a triangle, as shown in the Figure illustrated, where q_1= 6 μC, q_2= -10 μC and q_3= -8 μC.

(a) Find the components of the force $\vec{F}_{23}$ exerted by q_2 on q_3.



(b) Find the components of the force $\vec{F}_{13}$ exerted by q_1 on q_3. (c) Find the resultant force on q_3, in terms of components and also in terms of magnitude and direction.

Solution

The Electric Field

The gravitational force and the electrostatic force are both capable of acting through space, producing an effect even when there isn't any physical contact between the objects involved. **An electric field is said to exist in the region of space around a charged object**. The electric field exerts an electric force on any other charged object within the field.

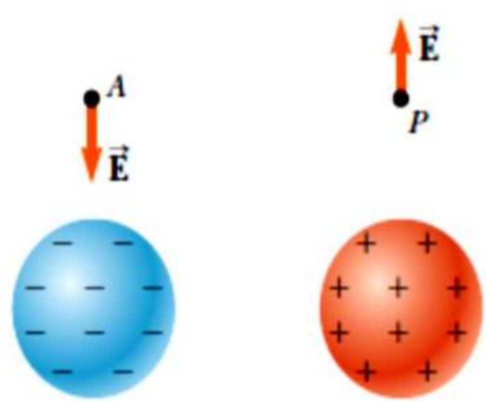
The illustrated figure shows an object with a small positive charge q_0 placed near a second object with a much larger positive charge Q. The electric field $\vec{E}$ produced by a charge Q at the location of a small "test" charge q_0 is defined as the electric force $\vec{F}$ exerted by Q on q_0 divided by the test charge q_0:

$$\vec{E} = \frac{\vec{F}}{q}$$

SI unit: newton per coulomb (N/C)

So, **<u>the electric field $\vec{E}$ at any point</u>** can be defined as: **the electric force acting on the positive electrical charge unit at this point.**

The electric field always has the same direction as the electric force on the test charge. As shown in the opposite figure, when a positive test charge is used, the electric field at point A is vertical and downward when the charged sphere is negative.



While when the charged sphere is positive, the electric field at point A is vertical and upward.

Once the electric field due to a given arrangement of charges is known at some point, the force on any particle with charge q placed at that point can be calculated from:

$$\vec{F} = \vec{E} \,.\, q_0$$

Consider a point charge Q located a distance r from a test charge q_0. According to Coulomb's law, the magnitude of the electric force of the charge Q on the test charge is:

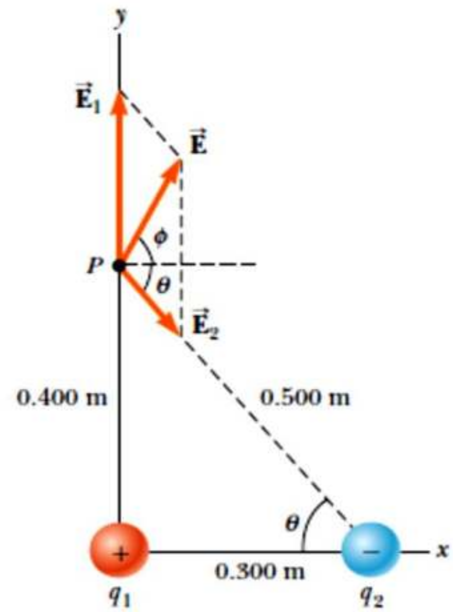
$$F = \frac{k\,Q\,q_0}{r^2}$$

Because the magnitude of the electric field at the position of the test charge is defined as $E = F/q_0$, we see that the magnitude of the electric field due to the charge Q at the position of q_0 is

$$E = \frac{F}{q_0} = \frac{k\,Q}{r^2}$$

Example 25:

Charge q_1 = 7μC is at the origin, and charge q_2 = 25 μC is on the x-axis, 0.3 m from the origin. (a) Find the magnitude and direction of the electric field at point P, which has coordinates (0, 0.4) m. (b) Find the force on a charge of 2 × 10^{-8} C placed at P.

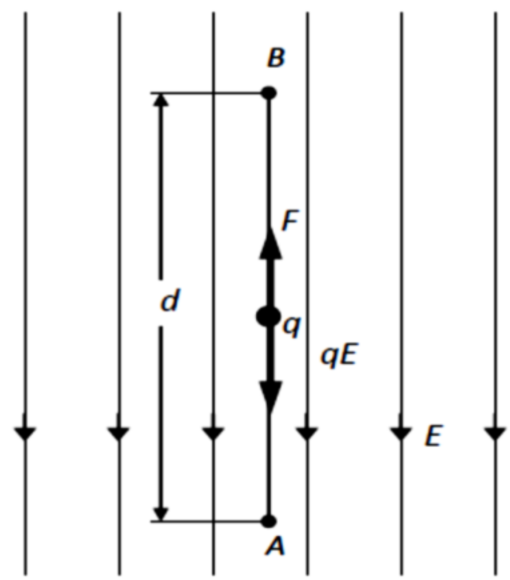


Solution

Potential Difference and Electric Potential

In the previous sections, we learned that when you put an electric charge in the electric field, it influenced by an electrical force with the magnitude of + qE and its direction is in the same direction of the electric field. In order to remain this charge in equilibrium, (i.e. remain in its place) it must be affected by an equivalent force, but in the opposite direction -qE. This means that, it has to be doing a work by an external factor against the electrical force to keep the charge movement always in equilibrium.

This work done against the electrical force to transfer a positive test unit charge q_0 from point A to point B is known as, the potential difference between the two points. i.e.



$$\Delta V = V_B - V_A = \frac{W_{AB}}{q_o} \qquad \frac{Joule}{C} = Volt$$

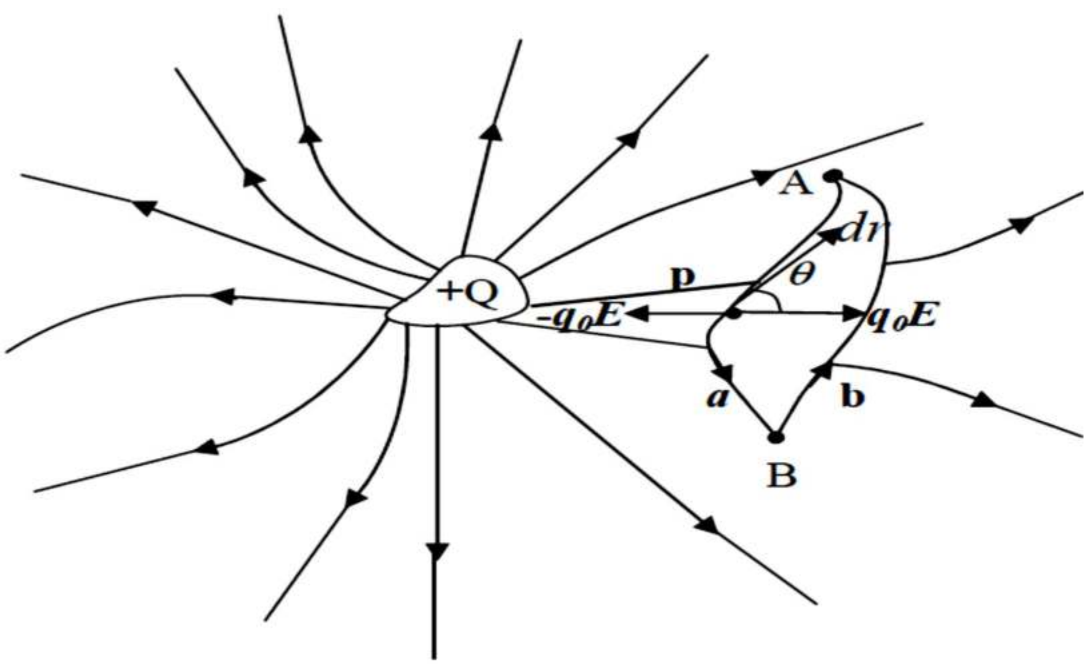
To calculate the potential at any point, it was agreed that the potential of the very distant points from the charges is equal to zero. In our case, if we chose point A at infinity, the potential V_A becomes equal to zero. And by compensating in the previous equation we get the potential at point B where: $V_B = \frac{W}{q_o}$

So, **the electric potential at any point** can be defined as: **"The work done for the unity of charge to transfer a small positive test charge from infinity to that point".**

Calculating the Potential from the Field

We can calculate the potential difference between any two points i and f in an electric field if we know the electric field $\vec{E}$ vector all along any path connecting those points. To make the calculation, we find the work done on a positive test charge by the field as the charge moves from i to f, and then use the previous equation.

Suppose charge q_0 move along the path (a) without accelerating and consider first the work done to move the charge a differential displacement dr and then make an integration process along the path from A to B.



If the amount of the electric field strength at the element (dr) is E and makes an angle θ with it, the test charge influenced by force + qE and be in the same direction of the field, therefore the force which done by the external factor to move the charge without acceleration is -qE and be contrary to the direction of the field, and the work done becomes:

$$W_{AB} = \int_A^B F\ \cos\theta\ dr\ = -q_o \int_A^B E\ \cos\theta\ dr$$

And the electrical potential difference is:

$$V_B - V_A = \frac{W_{AB}}{q_o} = -\int_A^B E\ \cos\theta\ dr$$

In special cases, where the field is uniform and parallel to the path of the charge, the charge movement in the opposite direction of the field strength makes angle θ equal to 180 ° and thus the electrical potential difference is:

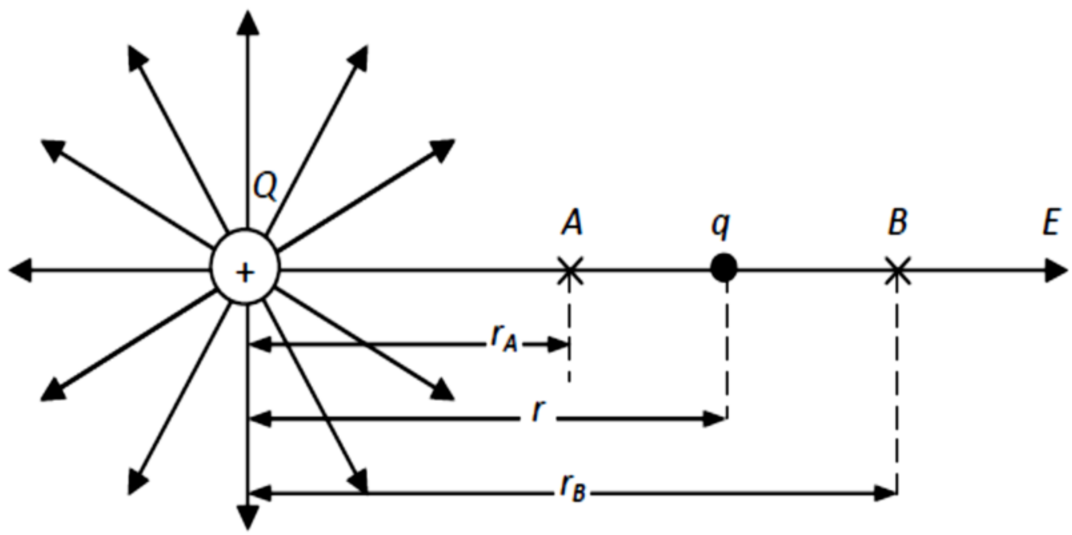
$$V_B - V_A = -\int_A^B E\ \cos 180\ dr\ = E\int_A^B dr\ = E\ d$$

Where d represents the path length between points A and B.

Therefore, it is clear that it can express the electric field strength of unity **volts / meter**.

Electric potential for a point charge

To find the potential difference between two points, such as A and B, lies in the electric field E for a point charge Q on the extension of the line passing along the center of the charge as the figure:



The electric field E for a point charge Q at any point at distance r from the relation:

$$E = \frac{k\,Q}{r^2}$$

The potential difference between two points, A and B, lies at distances r_A, r_B from the point charge Q where, θ = 0, is given from:

$$V_B - V_A = -\int_A^B E\ \cos 0\ dr = -\int_{r_A}^{r_B} E\ dr$$

$$= -\int_{r_A}^{r_B} \frac{k\,Q}{r^2}\,dr = -k\,Q\int_{r_A}^{r_B} \frac{dr}{r^2}$$

$$V_B - V_A = \mathrm{k}\,\mathrm{Q}\,\frac{1}{r}\bigg|_{r_A}^{r_B} = \mathrm{k}\,\mathrm{Q}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

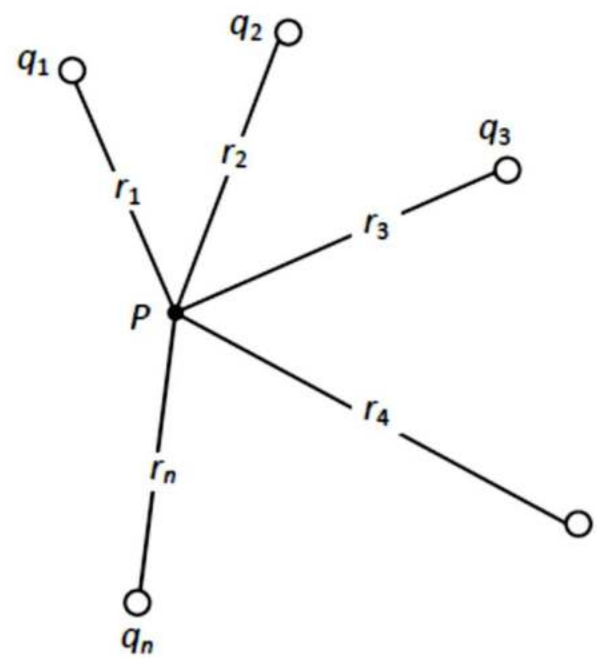
So,

$$V_B = \mathrm{k}\,\mathrm{Q}\,\frac{1}{r_B} \qquad , \qquad V_A = \mathrm{k}\,\mathrm{Q}\,\frac{1}{r_A}$$

And that represents the potential at any point.

Electric potential for a group of charges

To calculate the electric potential for n number of point charges at a point such as p, we calculate the potential for each individual charge, ignoring the existence of other charges, and then collect algebraically the values of these potentials.



$$\mathrm{V_p} = \mathrm{V_1} + \mathrm{V_2} + \mathrm{V_3} + \mathrm{V_4} + \cdots$$

$$\mathrm{V_p} = \mathrm{k}\left[\frac{\mathrm{q_1}}{\mathrm{r_1}} + \frac{\mathrm{q_2}}{\mathrm{r_2}} + \frac{\mathrm{q_3}}{\mathrm{r_3}} + \frac{\mathrm{q_4}}{\mathrm{r_4}} + \cdots\right]$$

So,

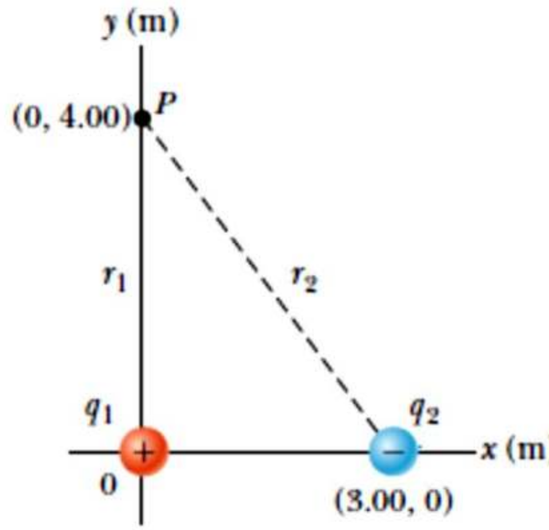
$$\mathrm{V_p} = \mathrm{k}\sum_{i}^{n}\frac{\mathrm{q_i}}{\mathrm{r_i}}$$

Example 26:

From the opposite figure, calculate:

1- The electric potential at point P due to the two charges ($q_1 = 5\mu C$, $q_2 = -2\mu C$).

2- The work required to bring a third point charge of $4\mu C$ from infinity to P.



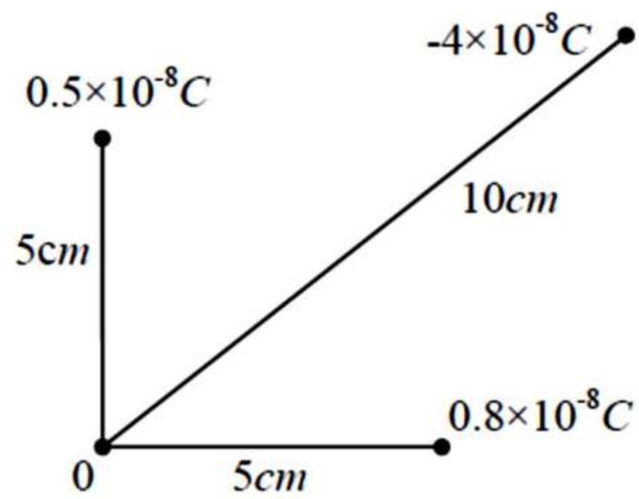
Solution

Example 27:

From the opposite figure, calculate:

1- The electric potential at point 0 due to the three charges.

2- The work required to bring an electron (q= 1.6×10^{-19} C) from infinity to 0.



Solution

<u>**Electric Current (I):**</u>

In this section, we study the flow of electric charges through a piece of material. The amount of flow depends on both the material through which the charges are passing and the potential difference across the material. Whenever there is a net flow of charge through some region, an electric current is said to exist. So, the electric current is a flow of electric charge.

To define current more precisely, suppose charges are moving perpendicular to a surface of area A (This area could be the cross-sectional area of a wire, for example.) The current is defined as the rate at which charge flows through this surface. If **ΔQ** is the amount of charge that passes through this surface in a time interval **Δt**, the current **I** is equal to the charge that passes through A per unit time:

$$I = \frac{\Delta Q}{\Delta t}$$

The SI unit of current is the **ampere** [A], with **1 A = 1 coulomb/sec**.

So <u>**Electrical current**</u> is generally defined as "**the rate of flow of the electric charges" or "the amount of electric charge passing through the conductor per second.**

And the <u>**Ampere**</u> can be defined as "**The electrical current passing through the conductor when the amount of electric charge passing in one second is one coulomb".**

The Directions of Currents

Because flow has a direction, we have implicitly introduced a convention that **the direction of current corresponds to the direction in which positive charges are flowing**. The flowing charges inside wires are negatively charged electrons that move in the opposite direction of the current. Electric currents flow in conductors: solids (metals, semiconductors), liquids (electrolytes, ionized) and gases (ionized), but the flow is impeded in non-conductors or insulators.

Current Density (J):

The current density (J) in the conductor is defined as "**the current intensity per unit area".**

$$J = \frac{I}{A} \qquad Amp./m^2$$

Example 28:

Calculate the number of charges passing through a conductor with cross-section area of $5cm^2$ if the electric current intensity passed in it is 3A in 6sec. As well as calculate the current density.

Solution

Conductivity (σ) and ohm's law:

When a potential difference is applied between the terminals of conductor, an electric field and current density arises into the conductor. If the potential difference is constant, proportionality relationship between the current density and electric field exist where:

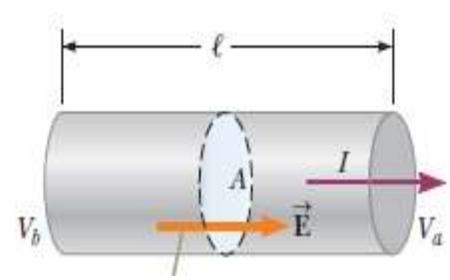
$$J \propto E \qquad \therefore J = \sigma E$$

where σ is called the conductivity of the material, and its unit is **Amp/volt.m** .

The above equation is known as **Ohm's law**, where **Ohm's law** is defined as **"at constant temperature, the current density passing through conductor is directly proportion with the electric field intensity arises on it".**

Resistance (R):

To obtain a more useful form of Ohm's law for practical applications, consider a segment of straight wire of length ℓ and cross-sectional area A, as shown in the Figure.



Suppose a potential difference $\Delta V = V_b - V_a$ is applied between the ends of the wire, creating an electric field $\vec{E}$ and a current I. Assuming $\vec{E}$ to be uniform, we then have $\Delta V = E\,\ell$

The magnitude of the current density can then be written as

$$J = \sigma E = \sigma \frac{V}{\ell}$$

With J = I / A , the potential difference becomes

$$V = \frac{\ell J}{\sigma} = \frac{\ell}{\sigma}\frac{I}{A} = \left(\frac{\ell}{\sigma A}\right) I$$

The term $\frac{\ell}{\sigma A}$ known as the **resistance of the conductor** R and can be defined as **"the impedance which the electric current found it when pass through conductor" or "the ratio between the potential difference between the terminals of the conductor and electrical current passing through it".**

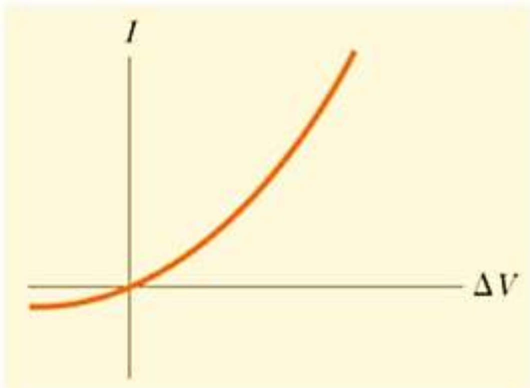
$$R = \frac{\ell}{\sigma A} = \frac{V}{I}$$

This equation is considered as another form of **ohm's law** which can be defined according to this equation as **"at constant temperature, the electric potential between the terminals of a conductor is directly proportion with the current intensity passing through it".**

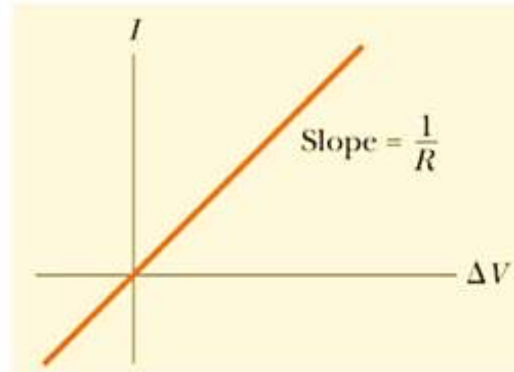
The SI unit of Resistance is the **ohm Ω** where 1Ω = volt/Ampere

Ohm defined as **"the resistance of conductor, where one amp of electric current intensity passes through it when the potential difference between its terminals is one volt".**

Material that obeys this relation is said to be ohmic as seen in figure a; otherwise, the material is non-ohmic as seen in figure b.



(b)



(a)

Resistivity (ρ):

The resistivity ρ of a material is defined as "the reciprocal of conductivity", so:

$$\rho = \frac{1}{\sigma} = \frac{R\ A}{\ell}$$

Therefore, the unit of the resistivity is **Ω**.m. and we can write the resistance as:

$$R = \rho \frac{\ell}{A}$$

Example 29:

Calculate the resistance of copper wire 10 cm long, and the cross-section area of 3×10^{-4} cm^2 and resistivity 1.7×10^{-8} Ω.m.

Solution

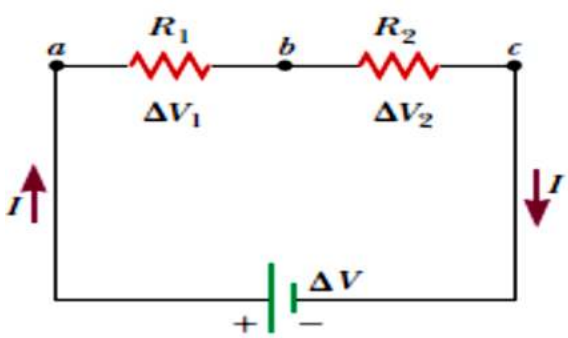
Example 30:

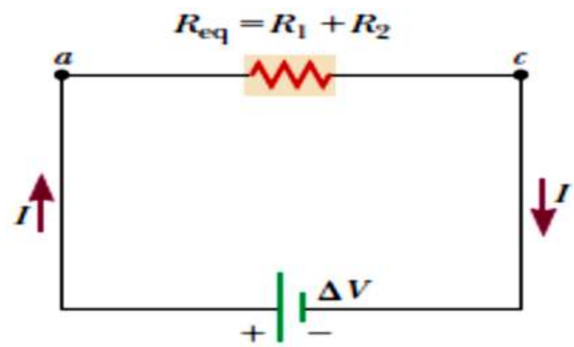
Calculate the electrical current intensity passing in an aluminum wire of 20 cm long, cross-section area of 5×10^{-3} cm^2 and resistivity 2.82×10^{-8} Ω.m, if the potential difference between its ends is 10V.

Solution

Resistances in Series

When two or more resistors are connected end to end, they are said to be in series. The resistors could be simple devices, such as light bulbs or heating elements.





This connection is characterized by:

1- The current is the same in all resistors $I = I_1 = I_2$

2- The potential difference is divided on the resistors

$$\Delta V = \Delta V_1 + \Delta V_2$$

So, $\Delta V = I R_{eq}$, $V_1 = I R_1$, $V_2 = I R_2$

$$\Delta V = I R_1 + I R_2 = I (R_1 + R_2)$$

$$I R_{eq} = I (R_1 + R_2)$$

$$R_{eq} = R_1 + R_2$$

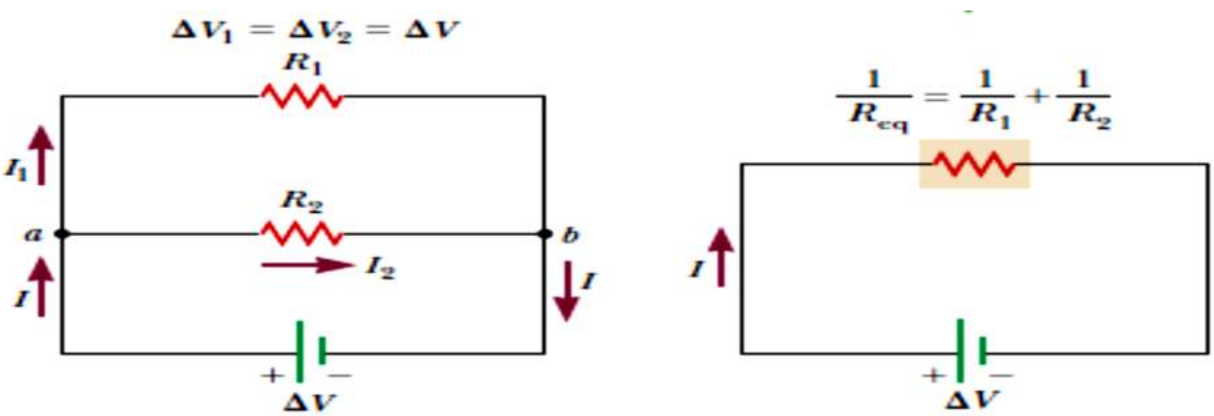
the equivalent resistance of three or more resistors connected in series is:

$$R_{eq} = R_1 + R_2 + R_3 + \ldots\ldots\ldots$$

Note that, the equivalent resistance is greater than any of the individual resistances.

Resistances in Parallel

When two or more resistors are connected where the ends of each one is connected to the ends of the others in the same side, they are said to be in parallel.



This connection is characterized by:

1- The potential differences across the resistors are the same

$$\Delta V = \Delta V_1 = \Delta V_2$$

2- The electric current is divided on the resistors. $I = I_1 + I_2$

So,

$$I = \frac{\Delta V}{R_{eq}}, \qquad I_1 = \frac{\Delta V}{R_1}, \qquad I_2 = \frac{\Delta V}{R_2}$$

$$I = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

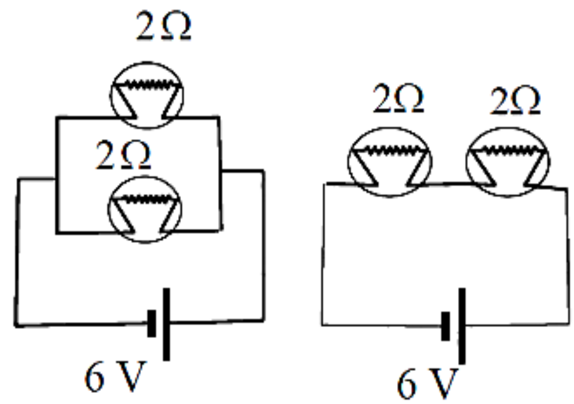
$$\frac{\Delta V}{R_{eq}} = \Delta V \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

The equivalent resistance of three or more resistors connected in series is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Note that, the equivalent resistance is smaller than any of the individual resistances.



Example 31:

In the opposite figure, Find:

1- The equivalent resistance of each circuit.

2- The current and the potential difference on each lamp in each circuit.

Solution

Example 32:

Three resistors ($R_1 = 6\Omega$, $R_2 = 4\Omega$, $R_3 = 2\Omega$) are connected with battery of 12V, calculate:

1- The equivalent resistance when connected in series and when connected in parallel.

2- The current intensity passes in each resistance in each case (series and parallel)

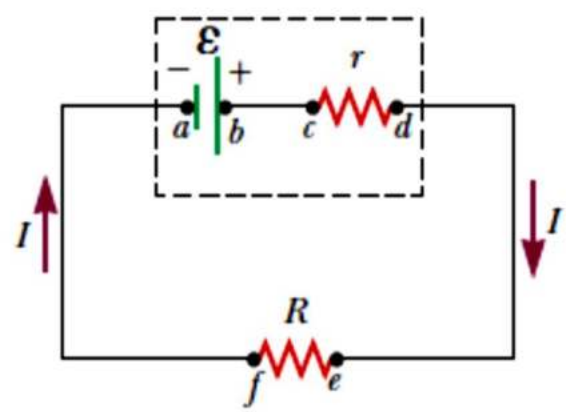
3- The potential difference on each resistance in each case.

Solution

The electromotive force (emf.):

A current is maintained in a closed circuit by a source of emf among such sources are any devices (for example, batteries and generators) that increase the potential energy of the circulating charges. A source of emf can be thought of as a "charge pump" that forces electrons to move in a direction opposite the electrostatic field inside the source. The emf of a source is the work done per unit charge; hence the SI unit of emf is the volt.

Consider the circuit consisting of a battery connected to a resistor. We assume the connecting wires have no resistance. If we neglect the internal resistance of the battery, the potential drop across the battery (the terminal voltage) equals the emf of the battery.



Because a real battery always has some internal resistance r, however, the terminal voltage is not equal to the emf. That may be understood through the definition of both the electromotive force and potential difference as follows:

The electromotive force (ε):

It is the work done to transfer the charge unity (Coulomb) in all the electric circuit outside and inside the source.

And from the definition of the potential difference:

It is the work done to transfer the charge unity between two points in the circuit outside the source.

Thus, the electromotive force can be divided into two parts:
1- external potential: to transfer charges through the external resistance R.
2- Internal potential, to transfer charges through the internal resistance of the source r.

$$\therefore \varepsilon = IR + Ir = I(R + r)$$

Due to V = IR, So:

$$\varepsilon = V + Ir \qquad or \qquad V = \varepsilon - Ir$$

Example 33:

Battery with emf 12V and internal resistance 0.05Ω connected to 3Ω resistance, Calculate: the current and the terminal voltage of the battery.

Solution

Kirchhoff's rules

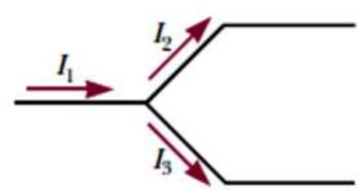
As demonstrated in the preceding section, we can analyze simple circuits using Ohm's law and the rules for series and parallel combinations of resistors. There are, however, many ways in which resistors can be connected so that the circuits formed can't be reduced to a single equivalent resistor. The procedure for analyzing more complex circuits can be facilitated by the use of two simple rules called Kirchhoff's rules:

Kirchhoff's first rule:

"The algebraic sum of all currents branching from any point in f a closed circuit is equal to zero". Or, **"The sum of the currents entering any junction must equal the sum of the currents leaving that junction".** (This rule is often referred to as the junction rule).

$$\sum I = 0 \qquad or \qquad \sum I_{in} = \sum I_{out}$$

If we apply this rule to the junction in opposite figure:



$$I_1 - I_2 - I_3 = 0 \qquad or \qquad I_1 = I_2 + I_3$$

Kirchhoff's second rule:

"The sum of the potential differences across all the elements around any closed circuit loop must be zero". (This rule is usually called the loop rule.)

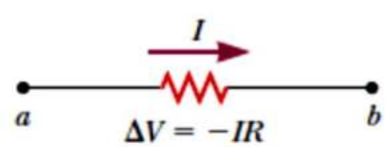
$$\sum_{\text{closed loop}} \Delta V = 0$$

When applying Kirchhoff's rules, you must make two decisions at the beginning of the problem:

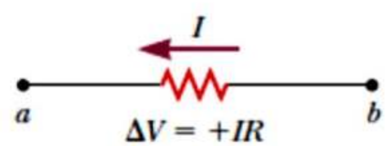
1- Assign symbols and directions to the currents in all branches of the circuit. Don't worry about guessing the direction of a current incorrectly; the resulting answer will be negative, but its magnitude will be correct. (Because the equations are linear in the currents, all currents are to the first power.)

2- When applying the loop rule, you must choose a direction for traversing the loop and be consistent in going either clockwise or counterclockwise. As you traverse the loop, record voltage drops and rises according to the following rules, where it is assumed that

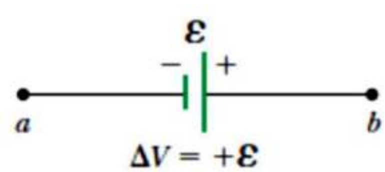
(a) If a resistor is traversed in the direction of the current, the change in electric potential across the resistor is –IR.



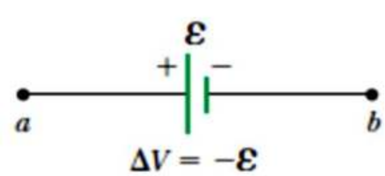
(b) If a resistor is traversed in the direction opposite the current, the change in electric potential across the resistor is +IR.



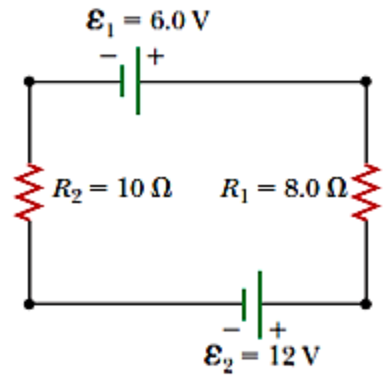
(c) If a source of emf is traversed in the direction of the emf (from - to + on the terminals), the change in electric potential is +ε.



(d) If a source of emf is traversed in the direction opposite the emf (from + to - on the terminals), the change in electric potential is –ε.



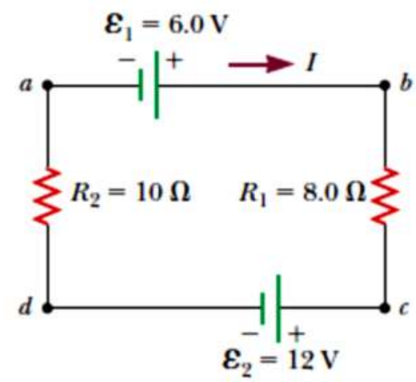
Example 34:



Find the currents in the circuit shown in opposite figure by using Kirchhoff's rules.

Solution

We do not need Kirchhoff's rules to analyze this simple circuit, but let's use them anyway simply to see how they are applied. There are no junctions in this single-loop circuit; therefore, the current is the same in all elements.



Let's assume the current is clockwise as shown in the figure. Traversing the circuit in the clockwise direction, starting at a, we see that a → b represents a potential difference of $+\varepsilon_1$, b → c represents a potential difference of $-IR_1$, c → d represents a potential difference of $-\varepsilon_2$, and d → a represents a potential difference of $-IR_2$.

Apply Kirchhoff's loop rule to the single loop in the circuit:

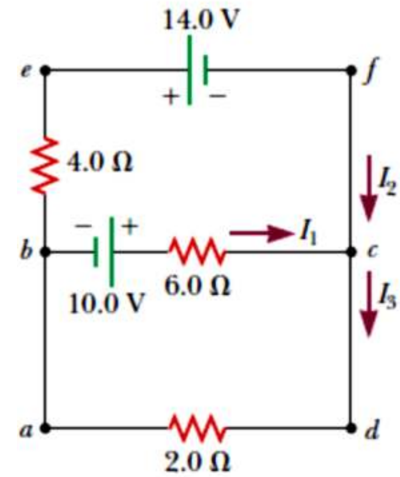
$$\sum_{\text{closed loop}} \Delta V = 0$$

$$\varepsilon_1 - IR_1 - \varepsilon_2 - IR_2 = 0$$

$$I = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{6 - 12}{8 + 10} = -0.33\ A$$

The negative sign for I indicate that the direction of the current is opposite the assumed direction.

Example 35:

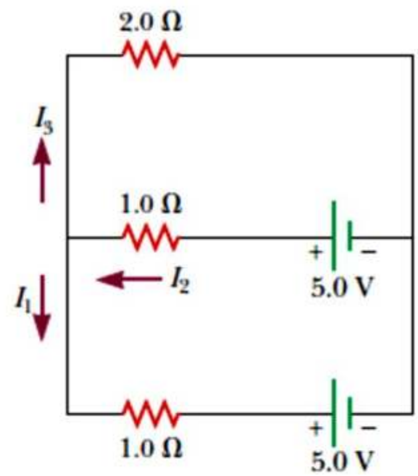


Find the currents I_1, I_2, and I_3 in the circuit shown in the figure:

Solution

Example 36:

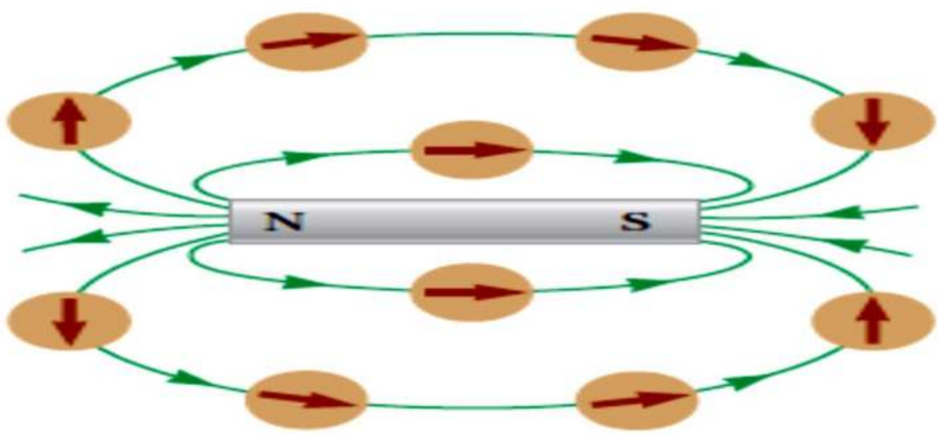
Find the currents I_1, I_2, and I_3 in the circuit shown in the figure:



Solution

Magnetic Fields and Forces

In our study of electricity, we described the interactions between charged objects in terms of electric fields. Recall that an electric field surrounds any electric charge. In addition to containing an electric field, the region of space surrounding any moving electric charge also contains a magnetic field. A magnetic field also surrounds a magnetic substance making up a permanent magnet.



The symbol $\vec{B}$ has been used to represent a magnetic field; the direction of the magnetic field $\vec{B}$ at any location is the direction in which a compass needle points at that location. As with the electric field, we can represent the magnetic field by means of drawings with magnetic field lines.

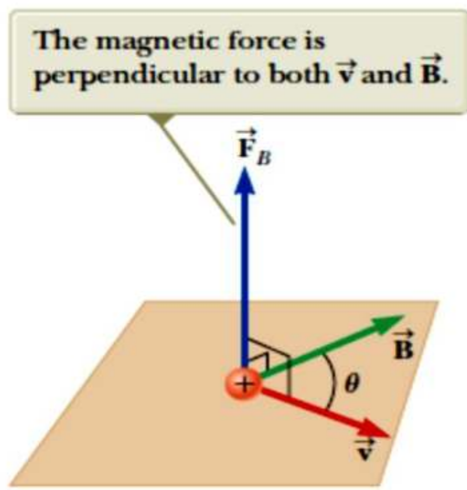
We can define a magnetic field $\vec{B}$ at some point in space in terms of the magnetic force $\vec{F}_B$ the field exerts on a charged particle moving with a velocity $\vec{v}$, which we call the test object.

Let's assume no electric or gravitational fields are present at the location of the test object. Experiments on various charged particles moving in a magnetic field give the following results:

• The magnitude $\vec{F}_B$ of the magnetic force exerted on the particle is proportional to the charge q and to the speed $\vec{v}$ of the particle.

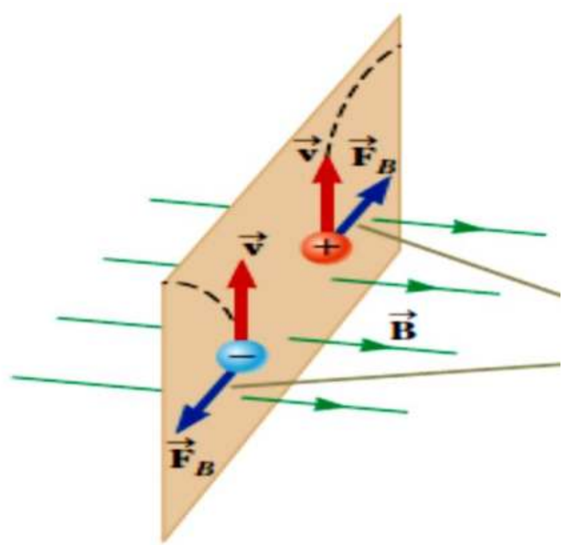
• When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.

• When the particle's velocity vector makes any angle θ ≠ 0 with the magnetic field, the magnetic force acts in a direction perpendicular to both $\vec{v}$ and $\vec{B}$; that is, $\vec{F}_B$ is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$.



• The magnitude of the magnetic force exerted on the moving particle is proportional to sin θ, where θ is the angle the particle's velocity vector makes with the direction of $\vec{B}$.

• The magnetic force exerted on a positive charge is in the direction opposite to the direction of the magnetic force exerted on a negative charge moving in the same direction.



We can summarize these observations by writing the magnetic force in the form

$$\vec{F} = q\,(\vec{v} \times \vec{B})$$

or

$$F_B = B\,q\,v\sin\theta \qquad \Rightarrow\ B = \frac{F_B}{q\,\vec{v}\sin\theta}$$

where θ is the angle the particle's velocity vector makes with the direction of $\vec{B}$.

The SI unit of magnetic field is the **N .sec / C .m,** or **N/Amp .m**, which is called the **tesla (T)**

A non-SI magnetic-field unit in common use, called **the gauss (G),** is related to the tesla through the conversion **1 T = 10^4 G**.

Electric and magnetic forces have some important differences:

• The electric force vector is along the direction of the electric field, whereas the magnetic force vector is perpendicular to the magnetic field.

• The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.

Example 37:

An electron in an old-style television picture tube moves toward the front of the tube with a speed of 8×10^6 m/s along the x axis. Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude 0.025 T, directed at an angle of 60° to the x axis and lying in the xy plane. Calculate the magnetic force on the electron, and the acceleration of it. ($e = 1.6\times10^{-19}$ C).

Solution:

Motion of a Charged Particle in a Uniform Magnetic Field

Consider the special case of a positively charged particle moving in a uniform magnetic field with the initial velocity vector of the particle perpendicular to the field. Let's assume the direction of the magnetic field is into the page. As the particle changes the direction of its velocity in response to the magnetic force, the magnetic force remains perpendicular to the velocity, so, the path of the particle is a circle. Due to q was a positive charge, the rotation is counterclockwise in a magnetic field directed into the page. If q were negative, the rotation would be clockwise. We use the particle under a net force model to write Newton's second law for the particle:

$$\sum F = F_B = ma$$

Because the particle moves in a circle, we also model it as a particle in uniform circular motion and we replace the acceleration with centripetal acceleration:

$$F_B = qvB = \frac{m\,v^2}{r} \qquad \Rightarrow \quad r = \frac{m\,v}{q\,B}$$

That is, the radius of the path is proportional to the linear momentum m*v* of the particle and inversely proportional to the magnitude of the charge on the particle and to the magnitude of the magnetic field. The angular speed of the particle is:

$$\omega = \frac{v}{r} = \frac{q\,B}{m}$$

The period of the motion (the time interval the particle requires to complete one revolution) is:

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

These results show that, the angular speed of the particle and the period of the circular motion do not depend on the speed of the particle or on the radius of the orbit. The angular speed *v* is often referred to as the cyclotron frequency because charged particles circulate at this angular frequency in the type of accelerator called a cyclotron.

Example 38:

A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35T magnetic field perpendicular to the velocity of the proton. Find the speed of the proton, the angular frequency, the frequency and the period of the motion. ($m_p = 1.67\times10^{-27}$kg, $q_p = 1.6\times10^{-19}$C)

Solution: