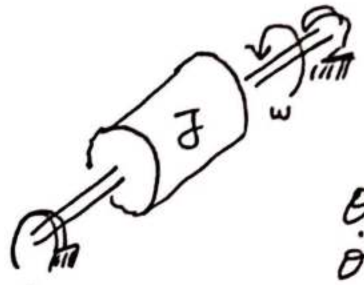
Correction of examples

Example 1 (page 7)



$\theta = \omega t$

$\dot{\theta} = \omega$

1) Newton's Second low

$$\sum T = J\ddot{\theta}$$

$$-T_b = J\ddot{\theta} \Rightarrow -b\omega = J\dot{\omega}$$

$$\Rightarrow J\dot{\omega} + b\omega = 0 \Rightarrow \boxed{\dot{\omega} + \frac{b}{J}\omega = 0}$$

2°/ using Laplac transform with $\omega(0) = \omega_0$

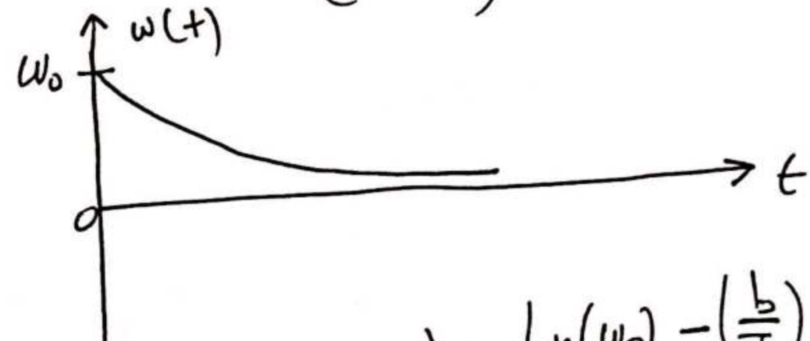
$$\dot{\omega} \longrightarrow s\,W(s) - \omega(0) = s\,W(s) - \omega_0$$

$$\Rightarrow \quad s\,W(s) + \frac{b}{J} W(s) - \omega_0 = 0$$

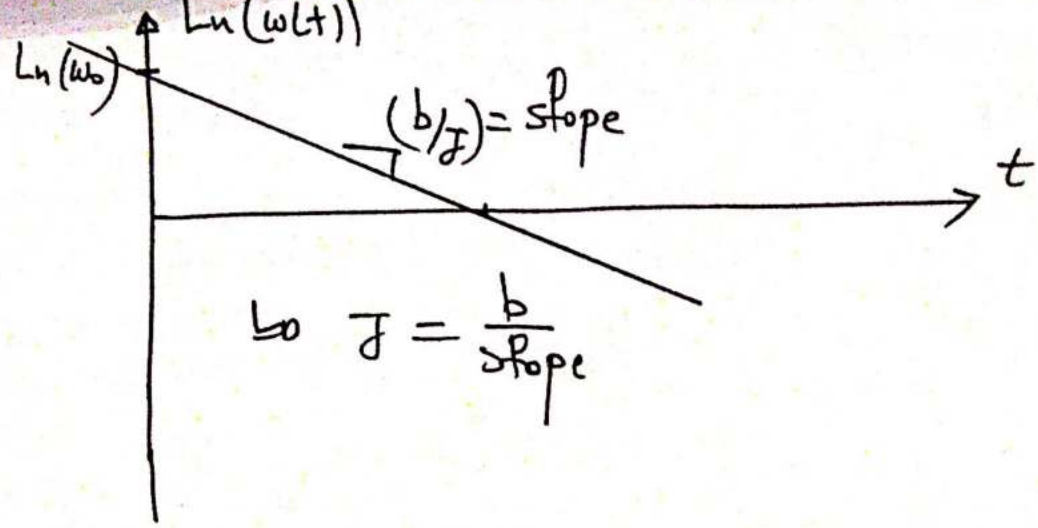
$$\Rightarrow \quad W(s)\left(s + b/J\right) = \omega_0$$

$$\Rightarrow \quad W(s) = \frac{\omega_0}{s + b/J}$$

$$\omega(t) = \mathcal{L}^{-1}\left(W(s)\right) = \omega_0 e^{-(b/J)t}$$

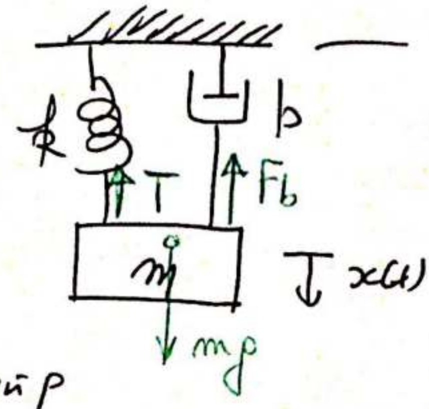


$$\ln(\omega(t)) = \ln(\omega_0) - \left(\frac{b}{J}\right)t$$



so $J = \frac{b}{slope}$

Example 2 (page 8)



1/ $\sum \vec{F}_{ext} = m\vec{a}$

$\vec{T} + \vec{F_b} + m\vec{g} = m\vec{a}$

proj $(\vec{x}) \Rightarrow$ and considering the static condition

$-kx - b\dot{x} = m\ddot{x}$

$\Rightarrow \boxed{m\ddot{x} + b\dot{x} + kx = 0}$

② the output of the system is the displacement x(t) using Laplace transform to solve the equation of motion (mathematical modelling)

for $m = 0.1$ / $b = 0.4$ / $k = 4$ / $x(0) = x_0$, $\dot{x}(0) = 0$

$m[S^2X(S) - Sx(0) - \dot{x}(0)] + b[SX(S) - x(0)] + kX(S) = 0$

$\Rightarrow m[S^2X(S) - Sx_0] + b(SX(S) - x_0) + kX(S) = 0$

$\Rightarrow X(S)[mS^2 + bS + k] = mSx_0 + bx_0$

$$\Rightarrow X(s) = \frac{x_0 (sm + b)}{ms^2 + bs + k}$$

Using the numerical values:

$$X(s) = \frac{x_0 (s+4)}{s^2 + 4s + 40}$$

$$x(t) \overset{?}{=} \mathcal{L}^{-1}(X(s))$$

$$s^2 + 4s + 40 = (s+2)^2 + 6^2$$

$$\Rightarrow X(s) = x_0 \left[\frac{s+4}{(s+2)^2 + 6^2} \right]$$

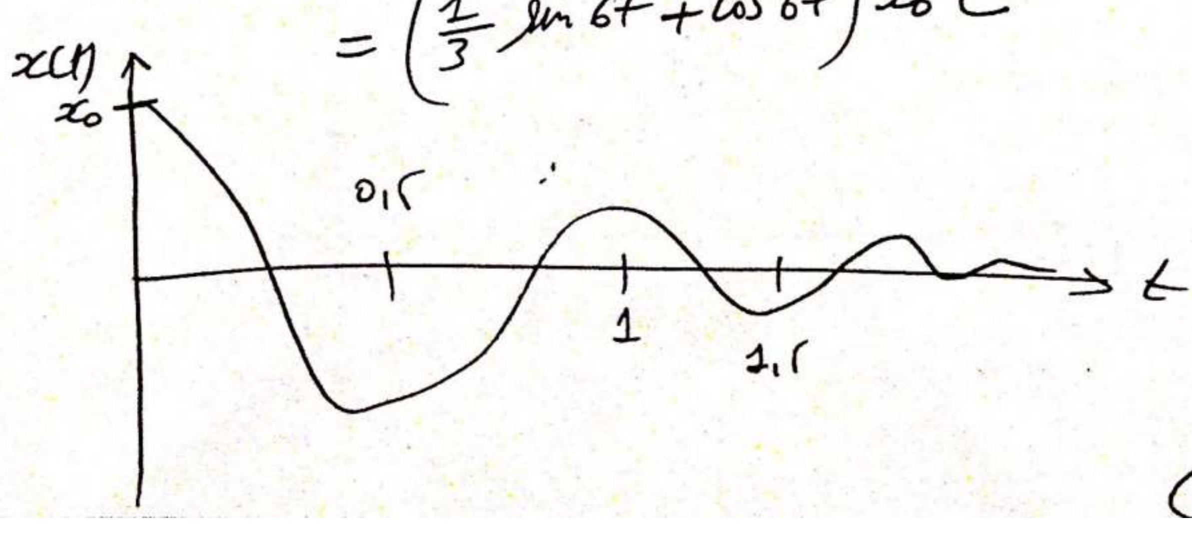
$$= x_0 \left[\frac{s+2}{(s+2)^2 + 6^2} + \frac{2}{(s+2)^2 + 6^2} \right]$$

$$= x_0 \left[\underbrace{\frac{s+2}{(s+2)^2 + 6^2}}_{\text{Line 20 } (e^{-2t}\cos 6t)} + \frac{1}{3} \underbrace{\frac{6}{(s+2)^2 + 6^2}}_{\text{Line 21 } (e^{-2t}\sin 6t)} \right]$$

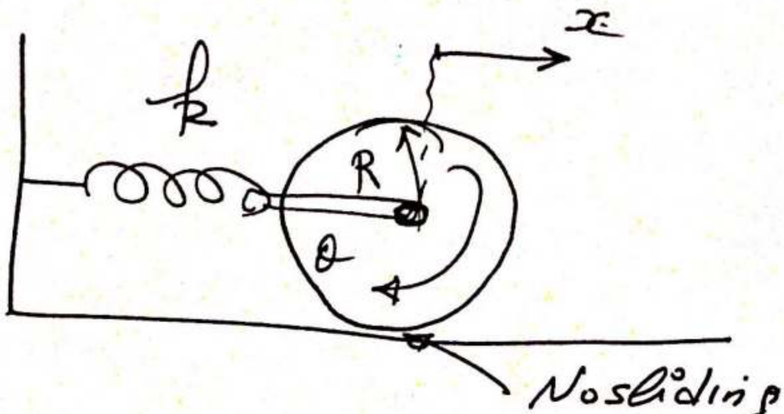
⇒ ~~x(t) =~~

$$\Rightarrow x(t) = \frac{1}{3} x_0 e^{-2t} \sin 6t + x_0 e^{-2t} \cos 6t$$

$$= \left(\frac{1}{3} \sin 6t + \cos 6t \right) x_0 e^{-2t}$$



Example 1 (page 18)



the Kinetic energy of the system is:

$$T = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} J\dot{\theta}^2$$

the potential energy of the system is:

$$U = \frac{1}{2} kx^2$$

the total mechanical energy is: then:

$$E = T + U$$

$$= \frac{1}{2} m\dot{x}^2 + \frac{1}{2} J\dot{\theta}^2 + \frac{1}{2} kx^2$$

Since the system is conservative $\Rightarrow$ No dissipation of energy $\Rightarrow E = cte \Rightarrow \frac{dE}{dt} = 0$

or $x \simeq R\theta$ and $J = \frac{1}{2} mR^2$

$$\Rightarrow E = \frac{1}{2} m\dot{x}^2 + \frac{1}{2}\left(\frac{1}{2}\frac{mR^2}{R^2}\right)\dot{x}^2 + \frac{1}{2} kx^2$$

$$\frac{dE}{dt} = m\dot{x}\ddot{x} + \frac{1}{2} m\dot{x}\ddot{x} + kx\dot{x} = 0$$

$$\Rightarrow \left[\left(m + \frac{1}{2}m\right)\ddot{x} + kx\right]\dot{x} = 0$$

$$\Rightarrow \frac{3}{2} m\ddot{x} + kx = 0 \quad \boxed{\ddot{x} + \left(\frac{2k}{3m}\right)x = 0}$$