6

GEARS

Gears are toothed cylindrical wheels used for transmitting mechanical power from one rotating shaft to another. Several types of gears are in common use. This chapter introduces various types of gears and details the design, specification and selection of spur gears in particular.

LEARNING OBJECTIVES

At the end of this section you should be:

- familiar with gear nomenclature;
- able to select a suitable gear type for different applications;
- able to determine gear train ratios;
- able to determine the bending stress for a spur gear using the Lewis formula;
- able to select appropriate gears for a compound gearbox using spur gears;
- able to select appropriate gears for an epicyclic gearbox using spur gears.

6.1 Introduction

Many power producing sources or prime movers such as internal combustion (IC) engines, gas turbine engines and electric motors produce power in the form of rotary motion. The operating characteristics of prime movers vary according to the type and size as discussed in Chapter 3, Section 3.1, and a change of speed is often necessary to [illegible] the output [illegible] characteristics of prime movers to useful torque conversions. When transmitting power from a source to the required point of application, a series of devices is available including gears, belts, pulleys,

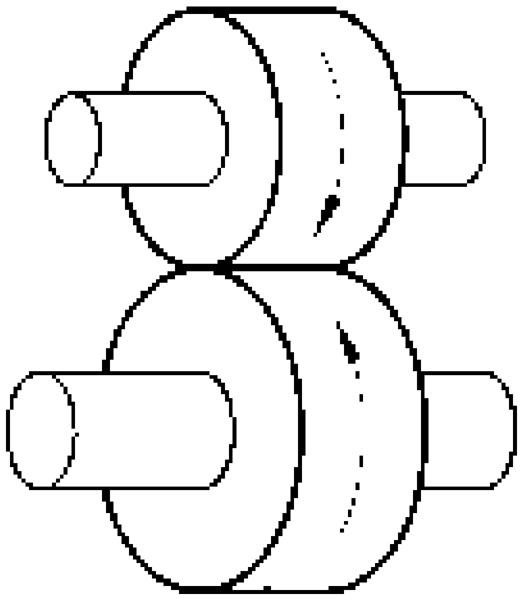


Figure 6.1 [illegible]

chains, hydraulic and electrical systems. Generally, if the distances of power transmission are large, gear drives are not ideal and chain and belt drives can be considered, which are introduced in Chapter 7. However, where compact, efficient or high-speed drive is required, gear trains offer a competitive [illegible]. Additional benefits of gears include [illegible] configurations of [illegible] any angle between input and output shafts, and suitability to operate in arduous conditions.

Simple velocity and speed changes can be achieved by running plain discs of different diameters together, Figure 6.1, or alternatively cones for turning corners as well, Figure 6.2. However, the torque capacity of such systems is limited by the frictional properties of the surfaces in contact.

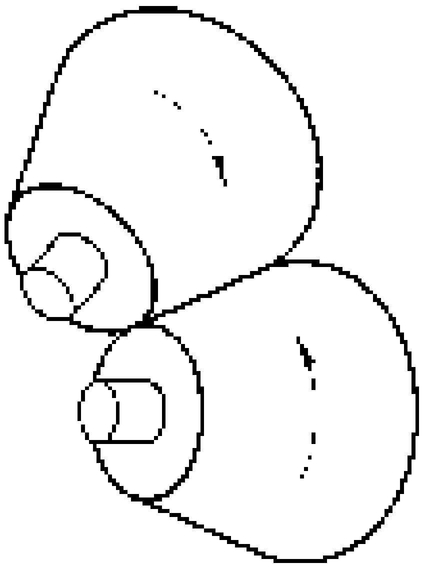


Figure 6.2 Cone drives.

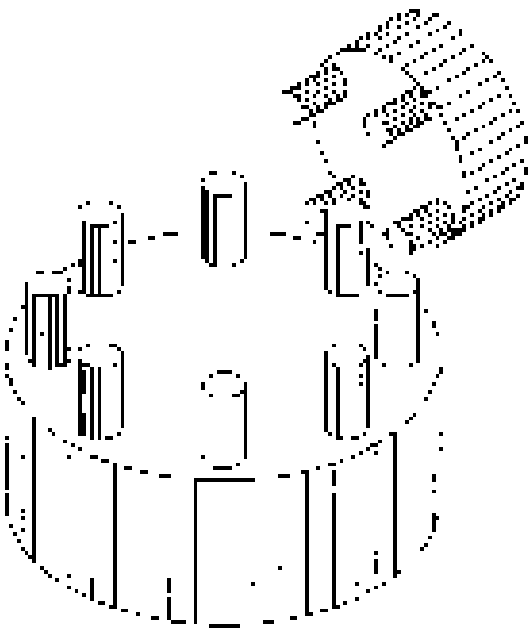


Figure 6.3 Primitive tooth gears.

of teeth to the surfaces of the discs or cones makes the drive positive, ensuring synchronization, and substantially increases the torque capability.

Primitive gear trains can consist of teeth or pegs located on discs as illustrated in Figures 6.3 and 6.4. The problem with gears of these simple teeth is that

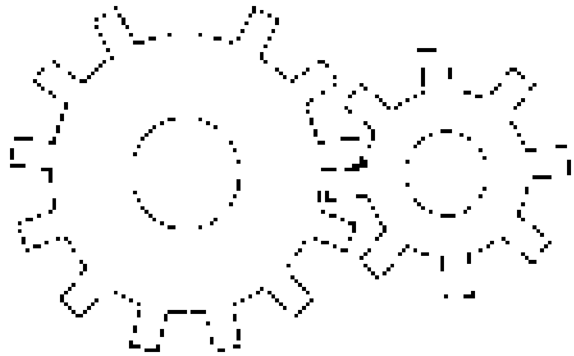


Figure 6.4 Primitive gears.

the velocity ratio is not constant and changes as the teeth go through the meshing cycle causing noise and vibration problems at elevated speeds. The solution to this problem can be achieved by using a profile on the gear teeth which gives a constant velocity ratio throughout the meshing cycle. Several different geometric forms can be used, but the 20° degree involute form is generally used in current professional engineering practice.

Gears can be divided into several broad classifications.

1. Parallel axis gears:
   (a) spur gears (see Figure 6.5),
   (b) helical gears (see Figures 6.6 and 6.7),
   (c) internal gears.
2. Non-parallel, coplanar gears (intersecting axes):
   (a) bevel gears (see Figure 6.8),
   (b) face gears,
   (c) conical involute gearing.
3. Non-parallel, noncoplanar gears (nonintersecting axes):
   (a) crossed axis helicals (see Figure 6.9),
   (b) cylindrical worm gearing (see Figure 6.10),
   (c) single enveloping worm gearing,
   (d) double enveloping worm gearing,
   (e) hypoid gears,
   (f) spiroid and helicon gearing,
   (g) face gears (off centre).
4. Special gear types:
   (a) square and rectangular gears,
   (b) elliptical gears.

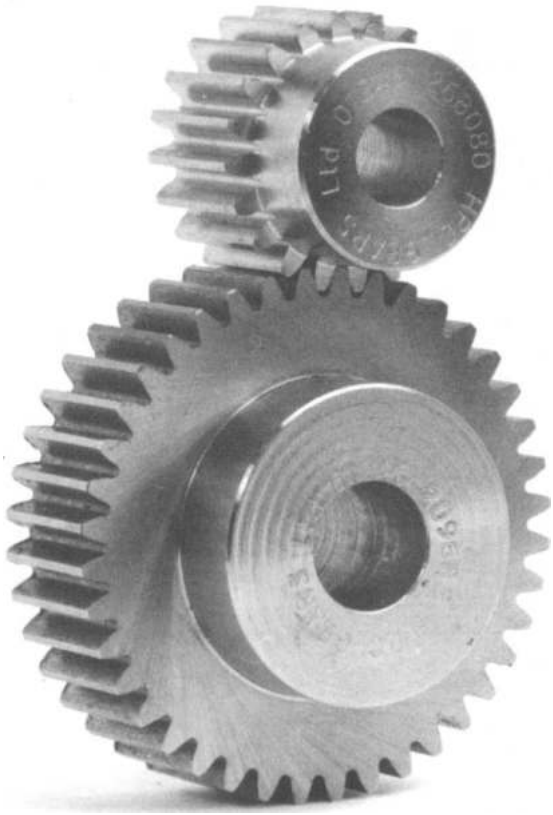


Figure 6.5 Spur gears. Photograph courtesy of [illegible] Hi-Precision Components Ltd

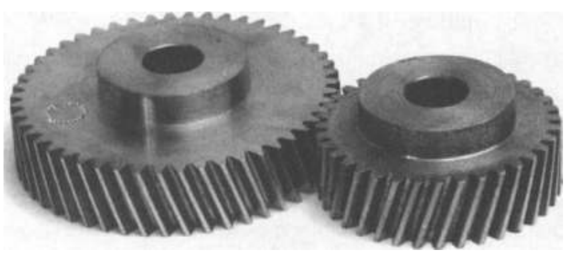


Figure 6.6 Helical gears. Photograph courtesy of [illegible] Hi-Precision Components Ltd

(5) worm gears,
(6) [illegible] gears.

Spur gears (Figure 6.5) are the cheapest of all types for parallel shaft applications. Their straight teeth allow running engagement or disengagement using sliding shaft and clutch mechanisms. Typical applications of spur gears include [illegible] motor vehicle gearboxes, machine tool drives, conveyor systems, electric motor gearboxes, rolling [illegible] The majority of power [illegible] and [illegible] steel. Other materials used

Figure 6.7 Double helical gears

Figure 6.8 Bevel gears

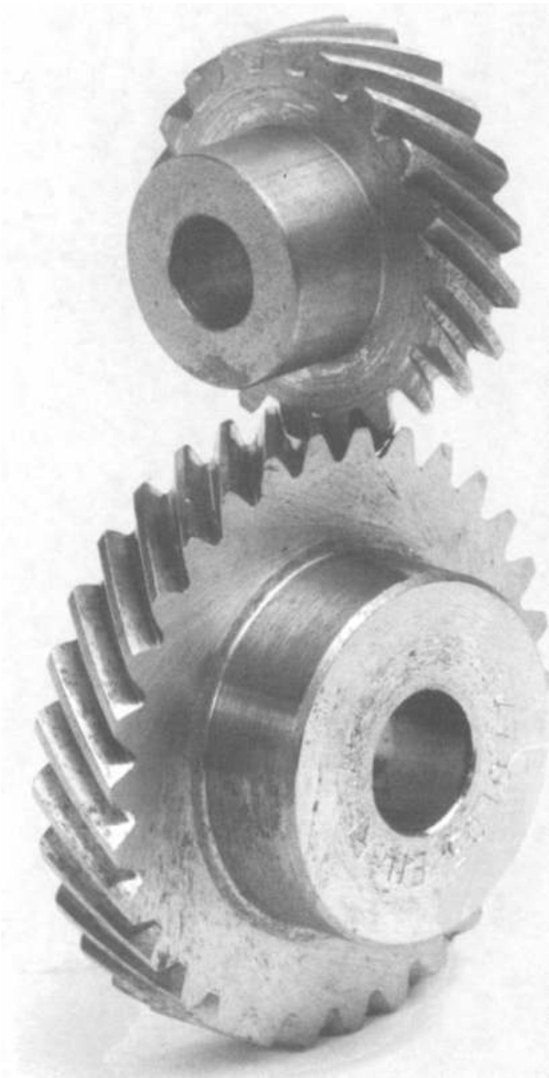


Figure 6.9 Crossed axis helical gears. Photograph courtesy of Hinchliffe Precision Components Ltd

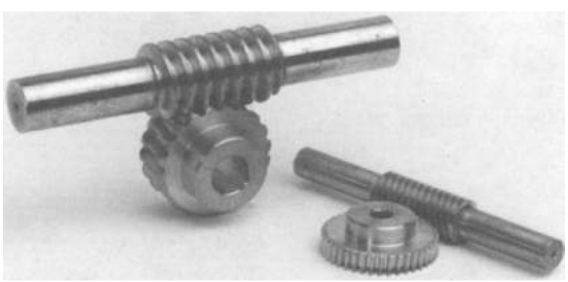


Figure 6.10 Worm gears. Photograph courtesy of Hinchliffe Precision Components Ltd

Table 6.1 Typical material matches for gears and pinions

Gear material	Pinion material
Cast iron	Cast iron
Cast iron	Cast steel
Carbon steel	Alloy steel
Alloy steel	Alloy steel
Alloy steel	Case-hardened steel

include iron, bronze, brass and polymers such as polyamide (e.g. nylon), polyacetal (e.g. Delrin) and PTFE (e.g. Teflon). Typical material matches are listed in Table 6.1. A harder material may sometimes be used for a pinion, than for the mating gear wheel, as the pinion teeth will experience more use than those on the gear wheel. Steel gears are often supplied in unhardened form to allow for guide bores, keyways or splines to be added before hardening and finishing.

A helical gear is a cylindrical gear whose teeth form are helixes, as shown in Figure 6.9. Common helix angles are 15° to 30°. Helical gears are typically used for heavy duty high speed (>25 m/s) power transmission, turbine drives, locomotive gearboxes and marine propulsion drives. Helical gears are generally more expensive than spur gears. Noise levels are lower than for spur gears because teeth in mesh make point contact rather than line contact. The forces arising from a meshing pair of helical gears can be resolved into three components: axial, radial and tangential. Axial loads are often called thrust loads, the bearings used to support the gear shafts must be able to withstand the axial or thrust force component. A solution to this problem is the use of double helical gears, as illustrated in Figure 6.7. These eliminate the need for thrust bearings because the axial force components cancel each other. While spur gears are generally cheaper, a comparable helical gear will be smaller.

Bevel gears have teeth cut on conical blanks (see Figure 6.8) and a gear pair can connect non-parallel intersecting shafts. Bevel gears are used in motor vehicle differential drives, valve control and mechanical instruments.

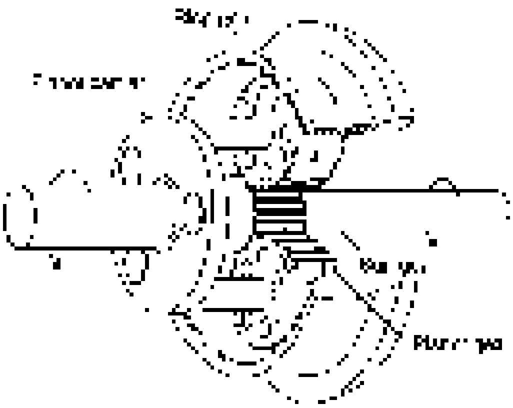


Figure 6.11 Epicyclic gears (reproduced from Townsend, 1992).

A worm gear is a cylindrical helical gear with one or more threads (see Figure 6.10). A worm wheel is a cylindrical gear with flanks cut in such a way as to ensure line contact with the flanks of the worm gear. Worm gears are used for steering gear, winch blocks, low speed gearboxes, rotating tables and remote valve control. Worm gear sets are capable of high speed reduction and high load applications where nonparallel, noninteracting shafts are used. The 90° configuration is most common. Frictional heat generation is high in worm gears, so continuous lubrication is required and provision for heat dissipation must be made.

Some gear axes can be allowed to rotate about others. In such cases, the gear trains are called planetary or epicyclic (see Figure 6.11). Planetary trains always include a sun gear, a planet carrier or arm, and one or more planet gears.

All gear mechanisms and gear trains demand continuous lubrication, which must be pressure fed for high speed gears to avoid a combination of high temperatures on the oil. Plastic gears, for example nylon, can be used in certain applications and have the advantage that there is no need for lubrication, but are only suitable for low speed applications. Plastic gears can [illegible] make the design [illegible].

Generally the pinion of a pair of gears should have the largest number of teeth consistent with adequate resistance to failure by bending stress in

Table 6.2 Useful range of gear ratios

Gear	Ratio range	Pitch line velocity (m/s)	Efficiency
Spur	1:1–5:1	25	98–99%
Helical	1:1–10:1	50	98–99%
Double helical	1:1–15:1	150	98–99%
Bevel	1:1–4:1	20	
Worm	5:1–75:1	30	20–98%
Crossed helical	1:1–6:1		

the teeth. In general, for a given diameter, the larger the number of teeth the finer the pitch and consequently the smoother and quieter the gear and the greater the ability to [illegible]. Table 6.2 lists the range of gear ratios and [illegible] typically achievable.

When the number of teeth selected for a gear wheel is equal to the number of the pinion multiplied by an integer, and the number of teeth on the pinion [illegible] a particular tooth on the pinion will come into contact with the same [illegible] teeth in each revolution of the gear wheel. If there is no common factor between the numbers of teeth on the gear wheel and pinion, then each tooth on the gear wheel will come into contact with every tooth on the pinion in turn, the number of revolutions of contact between a particular pair of teeth being the product of the numbers of teeth on the gear wheel. The avoidance of an integer ratio between the number of teeth on the gear wheel and the pinion can be achieved by the addition of an extra tooth to the gear wheel, provided that there is no overriding need for an exact velocity ratio. This extra tooth is called a hunting tooth, and can have advantages for wear distribution (see Dudley (1984) for a fuller discussion).

Various definitions used for describing gear geometry are illustrated in Figure 6.12 and listed below. The smaller of two meshing gears is called the 'pinion'; the larger is called the 'gear wheel' or simply the 'gear'.

- Pitch circle. This is a theoretical circle on which calculations are based. Its diameter is called the pitch diameter.

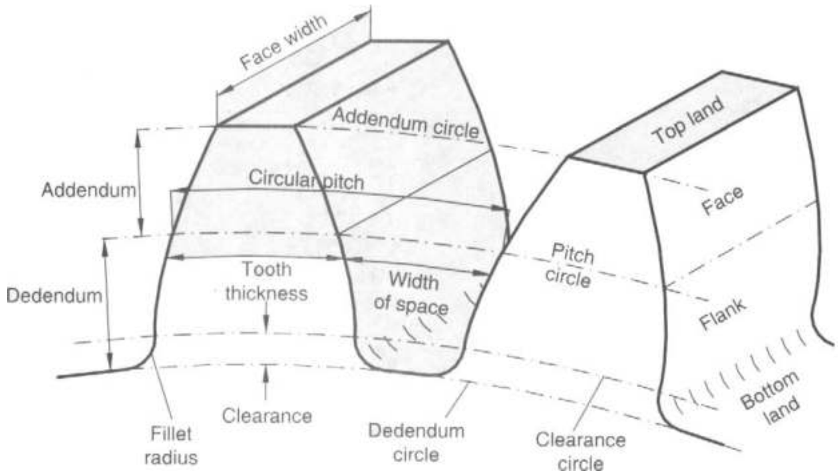


Figure 6.12 Spur gear schematic showing principle terminology.

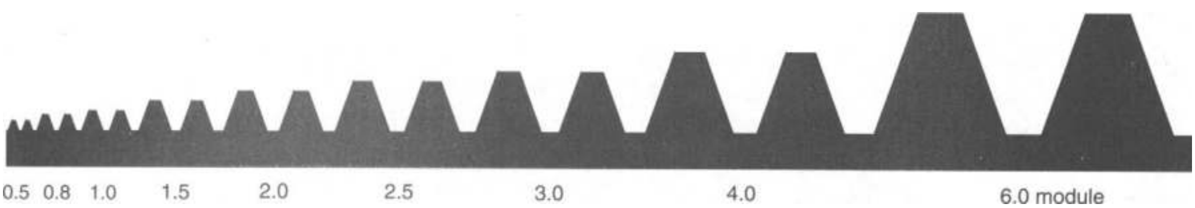


Figure 6.13 Gear tooth size as a function of the module.

where d is the pitch diameter (mm), [illegible] is the [illegible], N is the number of teeth. [illegible] from the [illegible] standard [illegible].

- **Circular pitch.** This is the distance [illegible] corresponding [illegible] measured along the pitch circle.

$$p = [illegible] \frac{\pi d}{N} \quad [illegible]$$

where p is the circular pitch [illegible] the [illegible] (mm), and N the number of teeth.

- **Module.** This is the ratio of the pitch diameter to the number of teeth. [illegible] The module is defined by the ratio of pitch diameter to number of teeth. Typically the height of a tooth is about 2.25 times the module. Various modules are illustrated in Figure 6.13.

$$m = \frac{d}{N} \quad [illegible]$$

- **Addendum** a. This is the radial distance from the pitch circle to the outside of the tooth.
- **Dedendum** b. This is the radial distance from the pitch circle to the bottom land.
- **Backlash.** The amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circle.

Prior to the use of the metric module, diametral pitch was commonly used. The diametral pitch is the ratio of the number of teeth in the gear to the pitch diameter, $P_d = N/d$ (usually in US English units, e.g. teeth per inch (tpi)). The relationship between diametral pitch P_d and the module (in mm) is $m = 25.4/P_d$.

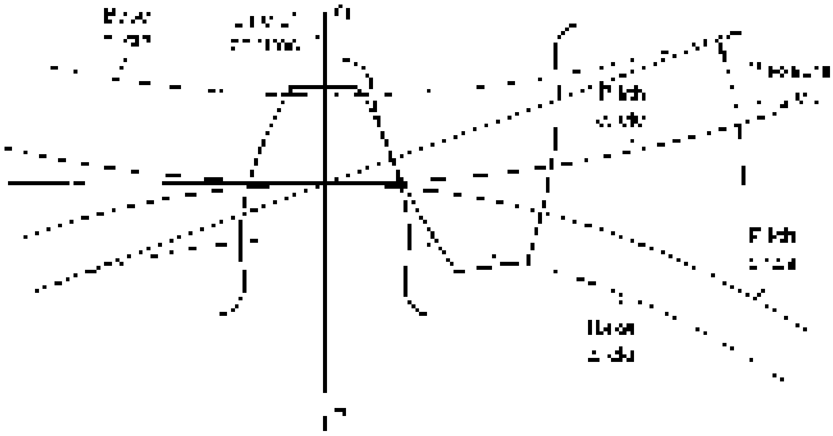


Figure 6.14 Schematic drawing of the line of action and pressure angle.

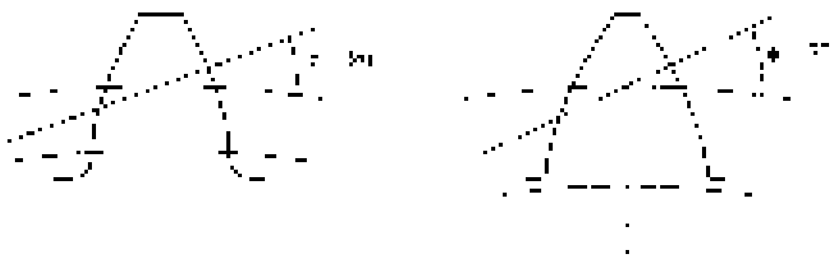


Figure 6.15 Full depth involute form for varying pressure angles.

Figure 6.14 shows two lines of centres $O_1 O_2$ connecting the centres of a pair of meshing gears. The angle ϕ is called the pressure angle. The pressure line (also called the generating line or line of action) is defined by the pressure angle. The resultant force vector between a pair of operating meshing gears acts along this line. The actual shape of the teeth of the gear teeth depends on the pressure angle chosen. Figure 6.15 shows two gear teeth of differing pressure angle for a gear with identical number of teeth, pitch and pitch diameter. Common standard pressure angles are 20° and 25° with the 20° form most widely used.

6.2 Construction of gear tooth profiles

The most widely used tooth form for spur gears is the full depth involute form as illustrated in Figure 6.16. An involute is one of a class of curves called conjugate curves. When two such gear teeth are in mesh, there is a constant velocity ratio between them. From the moment

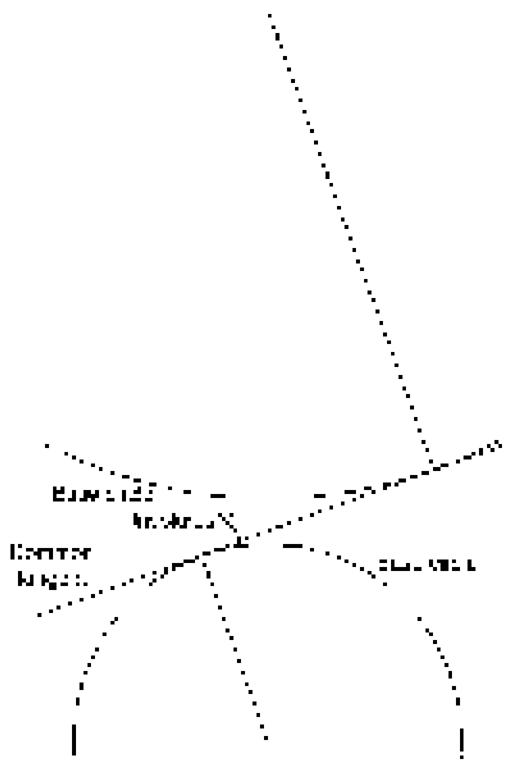


Figure 6.16 Schematic of the involute form.

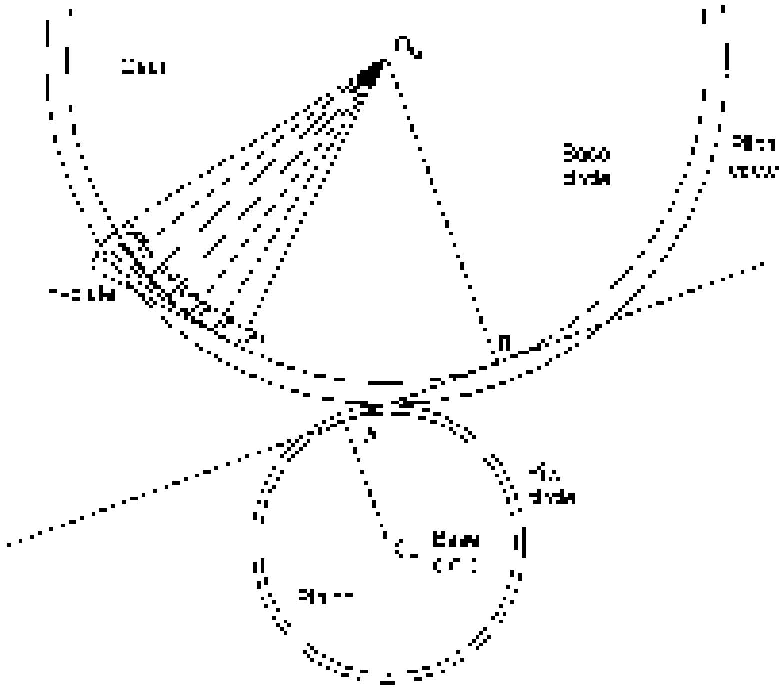


Figure 6.17 Construction of gear geometry

of teeth in contact, to the position of disengagement, the speed of the gear is in constant proportion to the speed of the pinion. The resulting motion of the gears is very smooth. If the velocity ratio were not constant, there would be accelerations and decelerations during the engagement and disengagement, causing vibration, noise and potentially damaging torsional oscillations.

The layout of gear geometry for a pair of meshing spur gears can be determined by the procedure set out below. This procedure remains relevant in computer aided design (CAD) drawing packages. It should be noted that gears are commonly available as standard items from specialist manufacturers and suppliers and need not necessarily be designed from scratch.

1. Calculate the pitch diameter and draw pitch circles tangential to each other (see Figure 6.17).

$$d = mN$$

$$d_p = mN_p$$

$$d_g = mN_g$$

In the example shown, the module has been selected as $m = 2.5$, the number of teeth on the pinion 20, and on the gear 50, so $d_p = 2.5 \times 20 = 50$ mm and $d_g = 2.5 \times 50 = 125$ mm.

2. Draw a line perpendicular to the line of centres through the pitch point. This is the point of tangency of the pitch circles. Draw the pressure line at an angle equal to the pressure angle from the perpendicular. It is called the pressure line because the resultant tooth force is along this line during meshing. Here the pressure angle is 20°.
3. Construct perpendiculars O_pA and O_gB to the pressure line through the centres of each gear. The radial distances of each of these lines are the radii of the base circles of the pinion and gear respectively. Draw the base circles.
4. Draw an involute curve on each base circle. This is illustrated on the gear. First divide the base circle in equal parts A_0, A_1, A_2, A_3, A_4, A_5, …, A_n. Construct radial lines O_gA_0, O_gA_1, O_gA_2, O_gA_3, …, O_gA_n. Construct perpendiculars to these radial lines. The involute begins at A_0. The second point is obtained by measuring

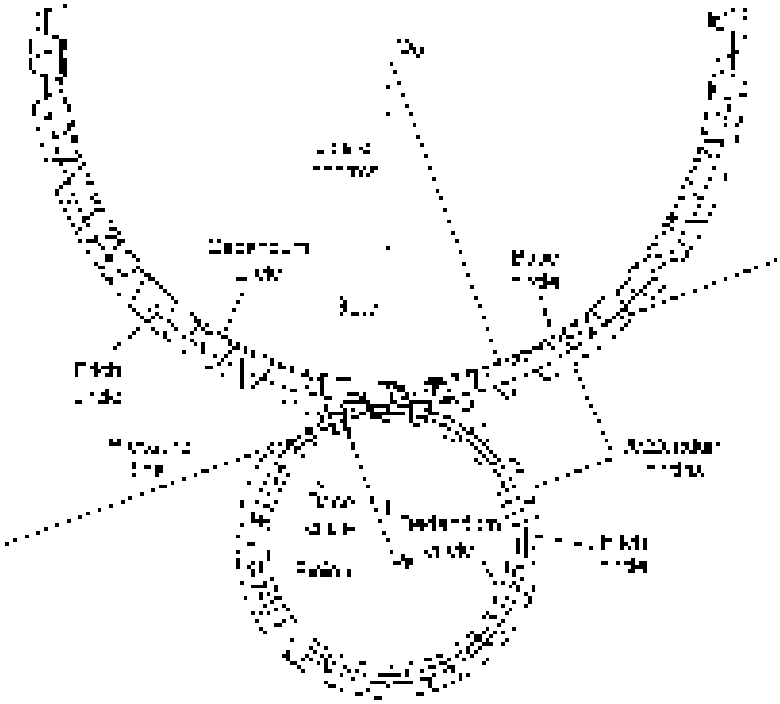


Figure 6.18 Layout of a pair of meshing spur gears.

off the distance A_1A_2 on the perpendicular through A_2. The next point is found by marking off twice the distance A_1A_2 on the perpendicular through A_3 and so on. The curve constructed through these points is the involute for the gear. The involute for the pinion is constructed in the same way on the base circle of the pinion.

5. Calculate the circular pitch:

$$p = \pi m$$

The width of the teeth and the width of the spaces are equal to half the circular pitch. Mark these distances off on the pitch circle (here $p = \pi \times 7.5$).

6. Draw the addendum and dedendum circles for the pinion and gear (see Figure 6.18). Here a [illegible] system (see Section 6.4 and Table 6.3) has been selected, so

$$a = m$$

$$b = 1.25m$$

Table 6.3 [illegible]

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	1.25m
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	2.25m
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]		0.25m
[illegible]		0.35m
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible]

7. Mirror the involute profile about a line constructed using a distance half the tooth width along the pitch circle and the gear centre. Using a polar array generate the teeth for the gear and pinion. Fillets as appropriate can be added to the tooth top and bottom lands.

Gears can be purchased as standard items from specialist manufacturers [illegible] for the detail needed for engineering drawings and general design purposes (unless you are designing gears or undertaking analysis). However, more comprehensive analysis is necessary for detailed stress, reliability and dynamic analysis.

6.3 Gear trains

A gear train is one or more pairs of gears operating together to transmit power. Figure 6.19 shows examples of a simple gear train, a reverted gear train and a compound gear train.

When two gears are in mesh, their pitch circles roll on each other without slippage. If r_1 is pitch radius of gear 1, r_2 pitch radius of gear 2, ω_1 is angular velocity of gear 1 and ω_2 angular velocity of gear 2, then the pitch line velocity is given by

$$V = |r_1\omega_1| = |r_2\omega_2| \qquad (6.4)$$

The velocity ratio is

$$\left|\frac{\omega_1}{\omega_2}\right| = \frac{r_2}{r_1} \qquad (6.5)$$

This can be defined in any of the following ways:

$$\frac{n_P}{n_G} = \frac{\omega_P}{\omega_G} = \frac{N_G}{N_P} = \frac{d_G}{d_P} \qquad (6.6)$$

where n_P and n_G are the angular velocities of the pinion and gear respectively (rpm), ω_P and ω_G are the rotational speeds of the pinion and gear respectively (rad/s), N_P and N_G are the numbers of teeth in the pinion and gear respectively, and d_P and d_G are the pitch diameters of the pinion and gear respectively (m).

Consider a pinion 1 driving a gear 2. The speed of the driven gear is

$$n_2 = \left|\frac{N_1}{N_2}\right| n_1 = \left|\frac{d_1}{d_2}\right| n_1 \qquad (6.7)$$

where n is the revolutions or rpm, N is the number of teeth and d is pitch diameter.

Equation 6.7 applies to any gear set (spur, helical, bevel or worm). For spur and parallel helical gears, the convention for direction is positive for anticlockwise rotation, i.e.

$$n_2 = -\frac{N_1}{N_2} n_1 \qquad (6.8)$$

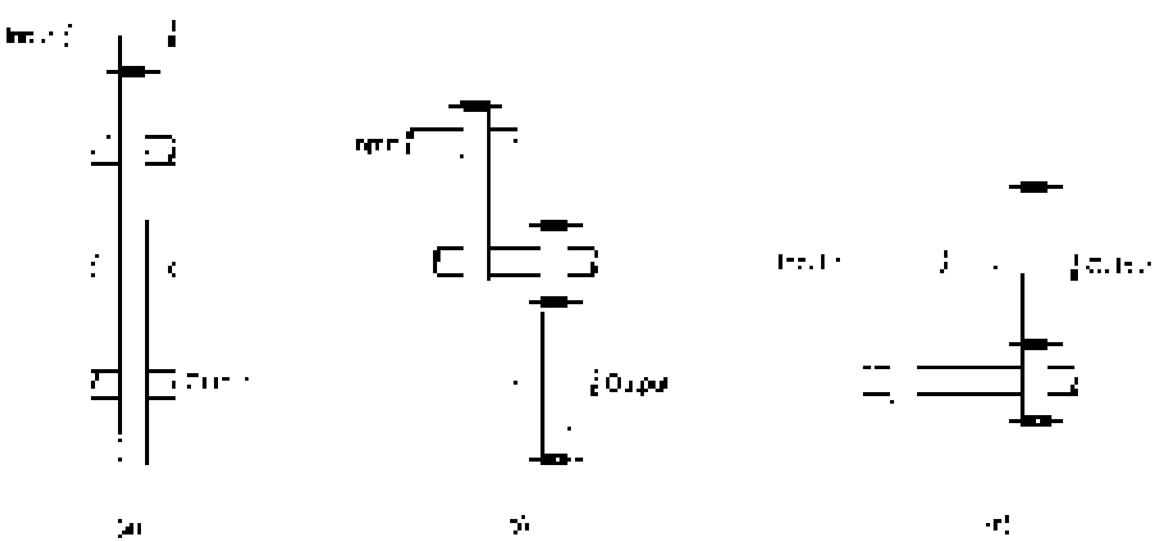


Figure 6.19 Gear trains. (a) Simple gear train. (b) Reverted compound gear train. (c) Non-reverted compound gear train.

Example 6.1

Consider the gear train shown in Figure 6.20. Calculate the speed of gear five.

Solution

$$n_2 = -\frac{N_1}{N_2} n_1$$

$$n_3 = -\frac{N_2}{N_3} n_2$$

$$n_4 = n_3 \text{ (on the same shaft)}$$

$$n_5 = -\frac{N_4}{N_5} n_4$$

$$n_5 = -\frac{N_4 N_2 N_1}{N_5 N_3 N_2} n_1$$

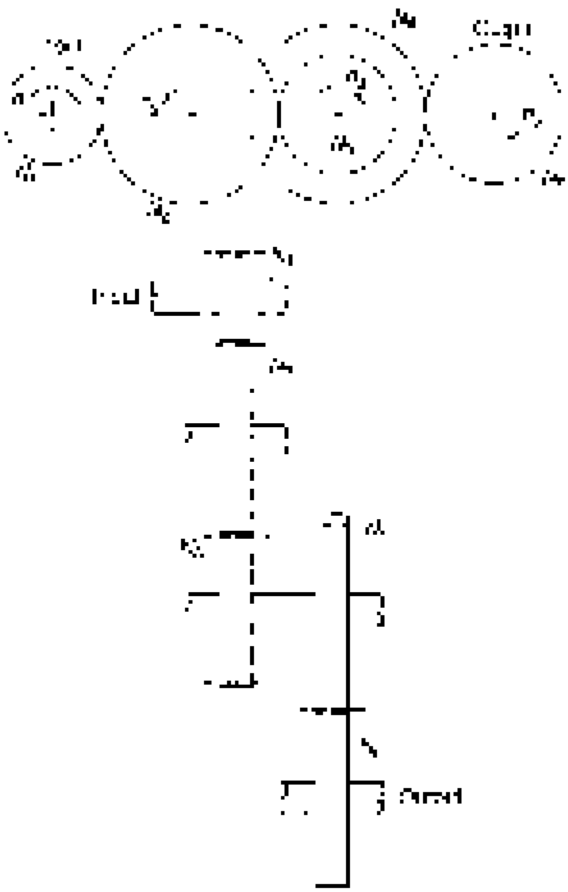


Figure 6.20 Example gear train

Example 6.2

For the double reduction gear train shown in Figure 6.21, if the input speed is 1750 rpm in a clockwise direction what is the output speed?

Solution

$$n_2 = -\frac{N_1}{N_2} n_1$$

$$n_3 = n_2 \text{ (on the same shaft)}$$

$$n_4 = -\frac{N_3}{N_4} n_3$$

$$n_4 = \frac{N_3}{N_4}\frac{N_1}{N_2} n_1 = \left(\frac{18}{54}\right)\left(\frac{20}{70}\right)(-1750)$$

$$= -166.7 \text{ rpm}$$

Example 6.3

For the double reduction gear train with an idler shown in Figure 6.22, if the input speed is 1750 rpm in a clockwise direction what is the output speed?

Solution

$$n_5 = -\frac{N_4}{N_5} n_4$$

$$n_4 = -\frac{N_3}{N_4} n_3$$

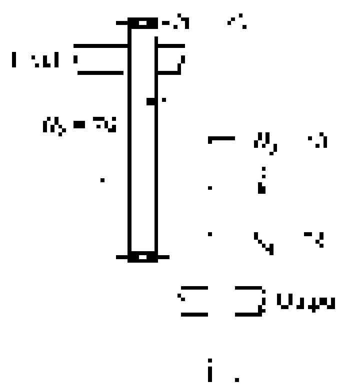


Figure 6.21 Double reduction gear train.

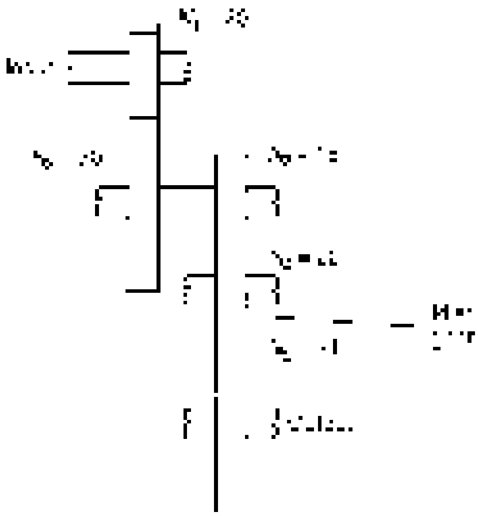


Figure 6.21 [illegible]

$$n_3 = n_2$$

$$n_2 = -\frac{N_1}{N_2} n_1$$

$$n_5 = -\frac{N_4}{N_5}\frac{N_3}{N_4}\frac{N_1}{N_2} n_1$$

$$= -\left(\frac{22}{54}\right)\left(\frac{18}{22}\right)\left(\frac{20}{70}\right)(-1750)$$

$$= 166.7 \text{ rpm}$$

Notice that the presence of the idler gear has caused the gear train output to reverse direction, but has not altered the gear ratio in comparison to the previous example.

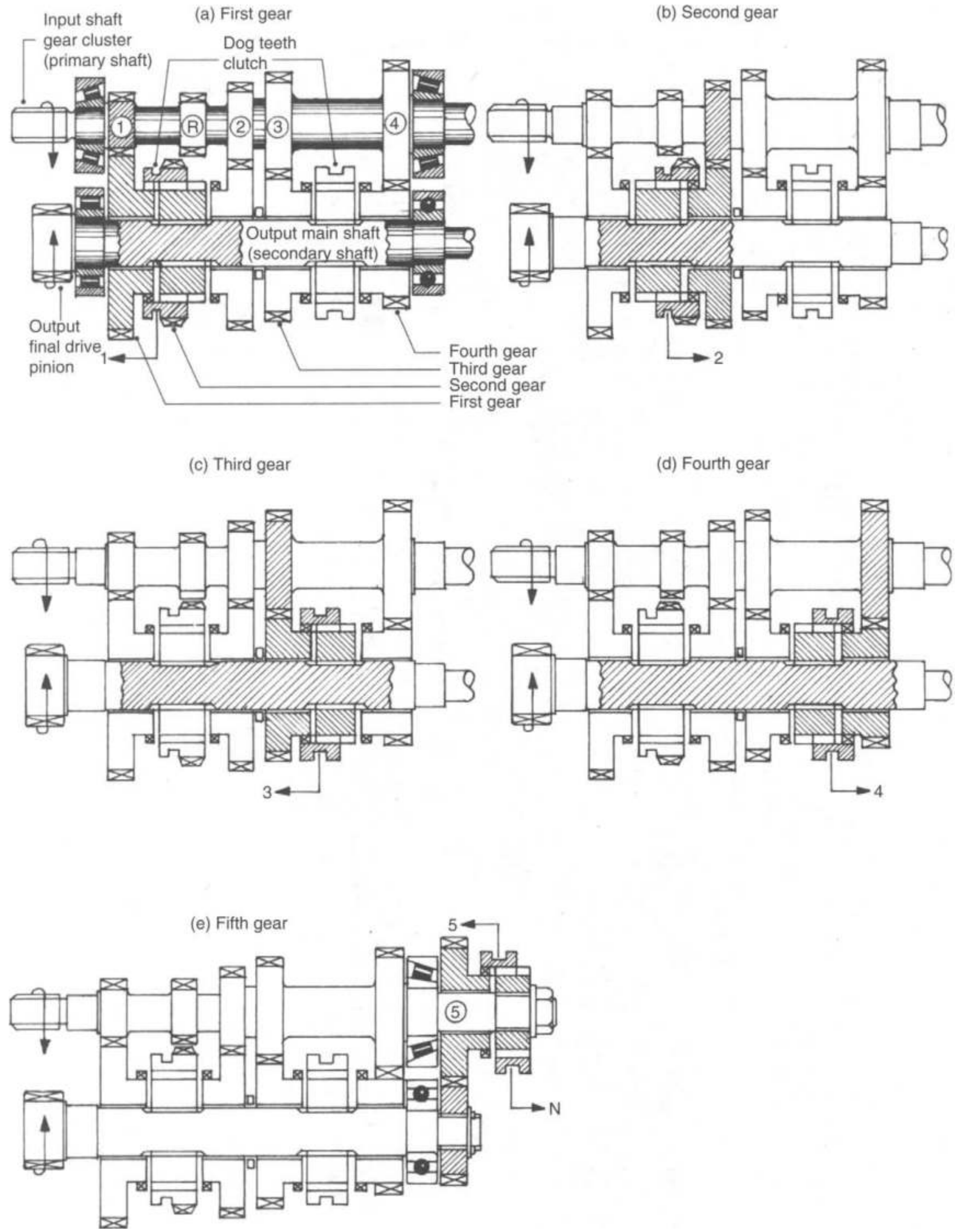
6.3.1 Manually shifted automotive transmissions

[illegible]

[illegible]

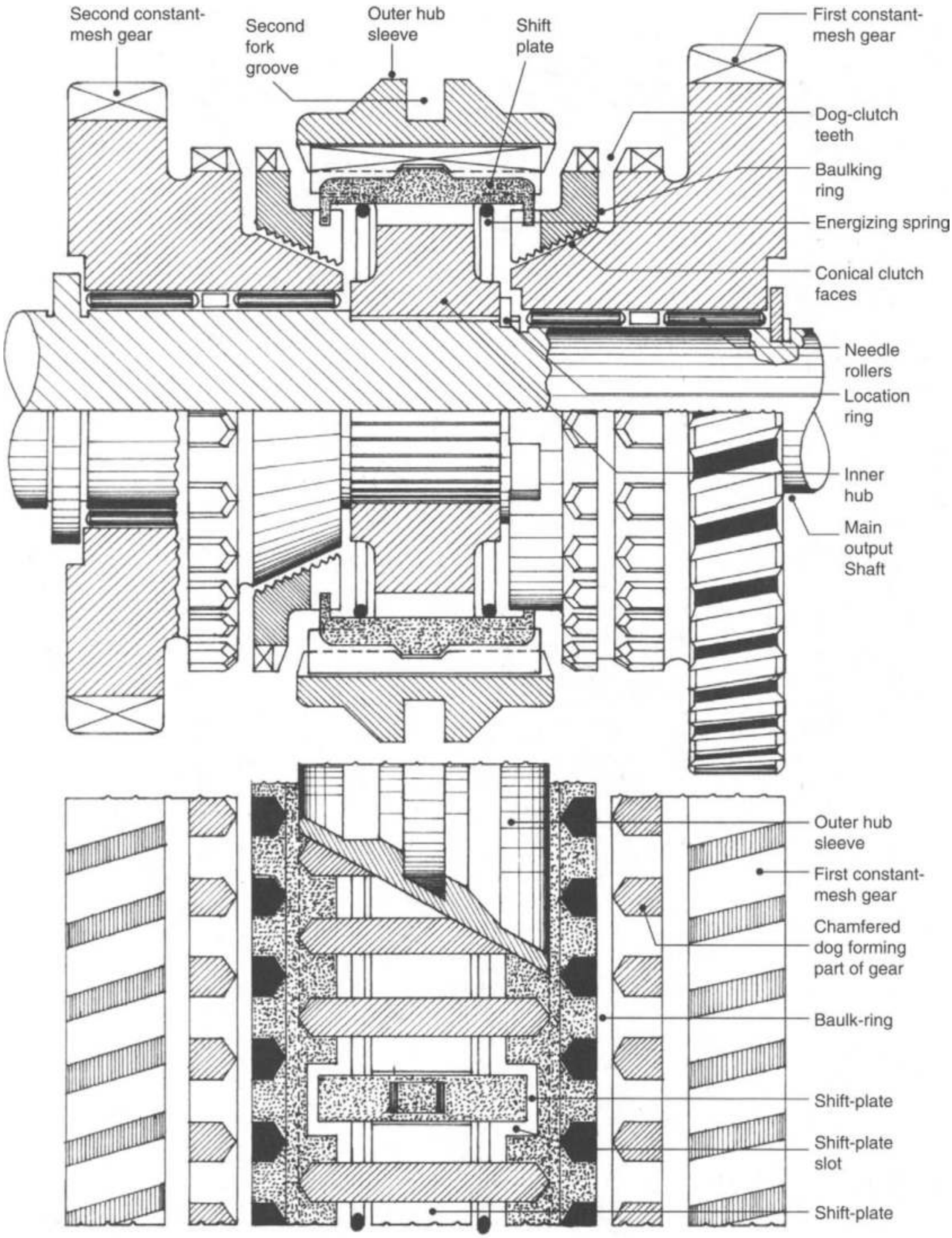
6.3.2 Epicyclic gear trains

[illegible]



(a) First gear
Input shaft gear cluster (primary shaft)
Dog teeth clutch
1
R
2
3
4
Output main shaft (secondary shaft)
Output final drive pinion
1
Fourth gear
Third gear
Second gear
First gear
(b) Second gear
2
(c) Third gear
3
(d) Fourth gear
4
(e) Fifth gear
5
5
N

Figure 6.23 [illegible]



(a) Neutral disengaged position

Figure 6.24 Positive baulking synchromesh [illegible]

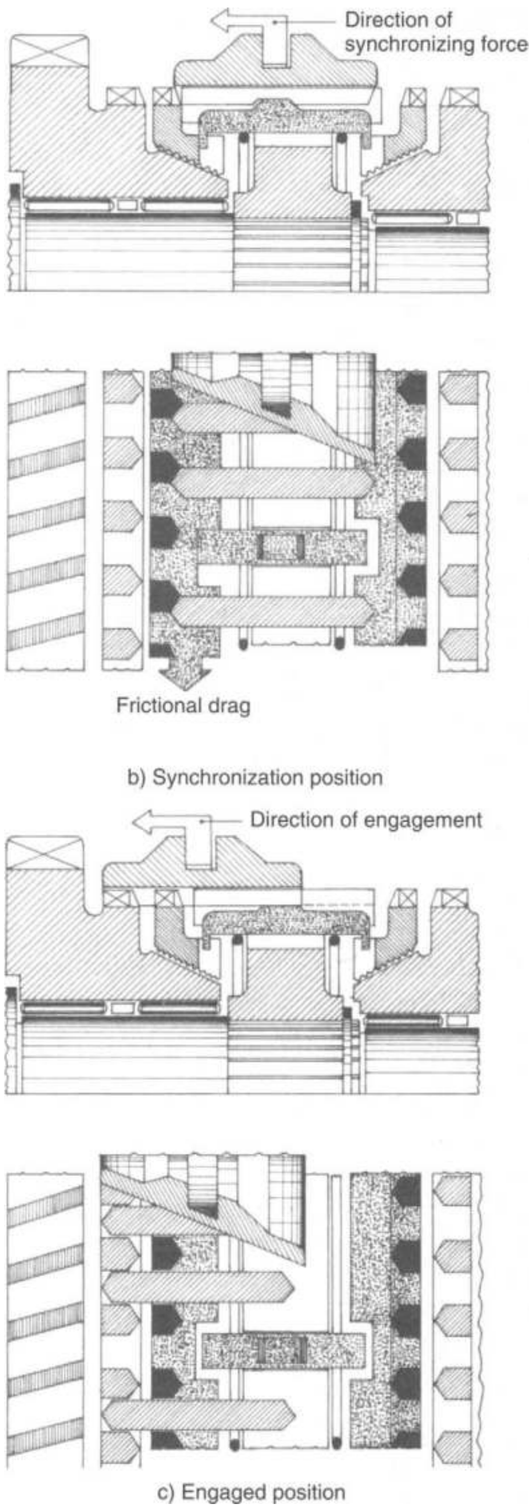


Figure 6.34 (Continued)

If [illegible] represents the speed of the first gear in the [illegible] and [illegible] represents the [illegible] gear [illegible] then

$$[illegible] = [illegible] - [illegible] \tag{6.10}$$

$$[illegible] = [illegible] - [illegible] \tag{6.11}$$

Dividing Eq. 6.11 by 6.10 gives

$$[illegible] = [illegible] = \frac{\text{Product of driving gear numbers}}{\text{Product of driven gear numbers}} = [illegible] \tag{6.12}$$

where [illegible] the [illegible] values [illegible] gears; the [illegible] value is [illegible] if the [illegible] gear [illegible] in the same [illegible] as the first.

Equation 6.12 can be rearranged to give the [illegible] defined as [illegible], which for an [illegible] with the [illegible] fixed [illegible] is given by

$$[illegible] = [illegible] + 1 \tag{6.13}$$

For calculation of [illegible] may also be necessary to determine the speed of the planetary gears; velocity difference equations can be used for this [illegible] gear train [illegible]

$$[illegible] = [illegible] - [illegible] \tag{6.14}$$

$$[illegible] = [illegible] - [illegible] \tag{6.15}$$

[illegible] knowledge of [illegible] and [illegible] these equations can be used to determine [illegible]

[illegible] spacing of the planetary gears is desired, the [illegible] given in Eq. 6.16 must be an integer. [illegible] spacing of [illegible] [illegible] planet [illegible]

$$\frac{[illegible] + [illegible]}{\text{Number of planets}} = \text{integer} \tag{6.16}$$

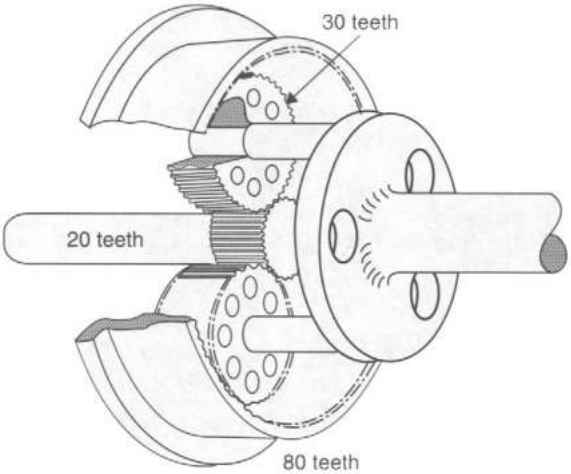


Figure 6.25 Epicyclic gear train

Example 6.4

For the epicyclic gearbox illustrated in Figure 6.25, determine the speed and direction of the final drive and also the speed and direction of the planetary gears. The teeth numbers of the sun, planets and ring gear are 20, 30 and 80, respectively. The speed and direction of the sun gear is 1000 rpm clockwise and the ring gear is held stationary.

Solution

From Eq. 6.13,

$$n_{arm} = \frac{n_{sun}}{(80/20) + 1} = \frac{-1000}{5} = -200 \text{ rpm}$$

The speed of the final drive is 200 rpm clockwise. The reduction ratio for the gearbox is given by $n_{sun}/n_{arm} = 1000/200 = 5$. To determine the speed of the planets use Eqs 6.14 and 6.15. The planets and sun are in mesh, so

$$\frac{n_{planet}/n_{arm}}{n_{sun}/n_{arm}} = -\frac{N_S}{N_P}$$

$$\frac{n_{planet} - n_{arm}}{n_{sun} - n_{arm}} = -\frac{N_S}{N_P}$$

$$\frac{n_{planet} - (-200)}{-1000 - (-200)} = -\frac{20}{30}$$

$$n_{planet} = -\frac{20}{30} \times (-800) - 200 = 333 \text{ rpm}$$

The speed of rotation of the planetary gears is 333 rpm counter-clockwise.

Table 6.4 Preferred values for the module m [illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table 6.5 Preferred standard gear teeth numbers [illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

6.4 Tooth systems

[illegible] Table [illegible] [illegible] full depth [illegible] with pressure angles of 20° and 25°.

[illegible] Table 6.5 lists the preferred [illegible]

The [illegible] surfaces. Section [illegible] spur gear [illegible] and Section [illegible]

6.5 Force analysis

Figure [illegible] gears in mesh. The force acting [illegible] φ can be resolved into two components. The

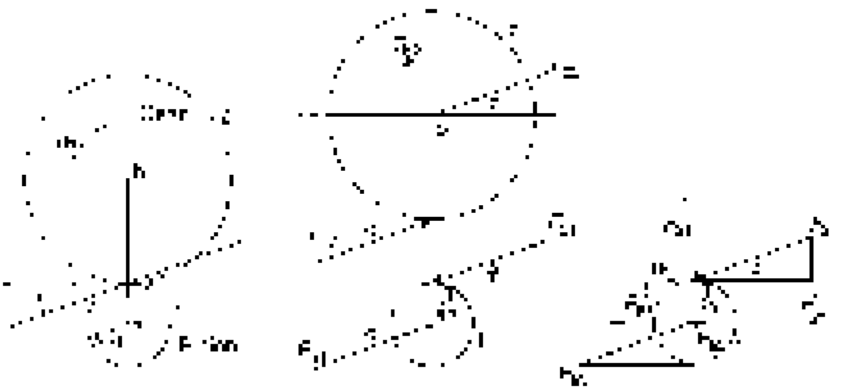


Figure 6.26 Forces on spur gears in mesh

tangential component F_t and the radial component F_r. The radial component serves no useful purpose. The tangential component F_t transfers the load from one gear to the other. If W_t is defined as the transmitted load, $W_t = F_t$. The transmitted load is related to the power transmitted through the gears by the equation:

$$W_t = \frac{P}{V} \qquad (6.27)$$

where W_t is the transmitted load (N), P is the power (W) and V is the pitch line velocity (m/s).

Alternatively the transmitted load can be defined by

$$W_t = \frac{60\times 10^3 H}{\pi d n} \qquad (6.28)$$

where W_t is transmitted load (kN), H is power (kW), d is pitch diameter (mm) and n is speed (rpm).

6.5.1 Introduction to gear stresses

Gears are prone to two principal types of stress: bending stress at the root of the teeth due to the transmitted load and contact stresses on the flank of the teeth due to repeated impact, or sliding contact, of one tooth surface against another. A simple method of calculating bending stresses is presented in Section 6.5.2 and this is useful within a gear selection procedure given in Section 6.6. However, it cannot be used by itself to determine a gear's geometry in the conceptual phase of evolution. For designing a gear train for highly loaded applications or where the life of the components is important, for the determination of the critical parts, national and international standards for gears are available such as the AGMA, DIN and ISO standards for gears. These are introduced in Chapter 7 in which AGMA standards for spur and helical gears are introduced.

6.5.2 Bending stresses

The calculation of bending stress in gear teeth can be based on the Lewis formula:

$$\sigma = \frac{W_t}{F m Y} \qquad (6.29)$$

where W_t is the transmitted load (N), F is the face width (m), m is the module (m) and Y is the Lewis form factor (see Table 6.x).

When teeth mesh, the load is delivered to the teeth with some degree of impact. The velocity factor is used to account for this and is given in the case of cut or milled profile gears by the Barth equation (6.30):

$$K_v = \frac{6.1}{6.1 + V} \qquad (6.30)$$

Table 6.6 [illegible]

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
17	[illegible]	[illegible]
18	[illegible]	[illegible]
19	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
100	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
Rack	[illegible]	[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Example 6.5

A 20° full depth spur pinion is to transmit 1.25 kW at 850 rpm. The pinion has 18 teeth. Determine the Lewis bending stress if the module is 2 and the face width is 25 mm.

Solution

Calculating the pinion pitch diameter

$$d_p = mN_p = 2 \times 18 = 36\ \text{mm}$$

Calculating the pitch line velocity

$$V = \frac{d_p}{2} \times 10^{-3} \times n\frac{2\pi}{60}$$

$$= \frac{0.036}{2} \times 850 \times 0.1047 = 1.602\ \text{m/s}$$

Calculating the velocity factor

$$K_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 1.602} = 0.7920$$

Calculating the transmitted load

$$W_t = \frac{\text{Power}}{V} = \frac{1250}{1.602} = 780.2\ \text{N}$$

From Table 6.6, for $N_P = 18$, the Lewis form factor $Y = 0.29327$.

The Lewis Equation for bending stress gives:

$$\sigma = \frac{W_t}{K_v F m Y}$$

$$= \frac{780.2}{0.792 \times 0.025 \times 0.002 \times 0.29327}$$

$$= 67.18 \times 10^6\ \text{N/m}^2 = 67.18\ \text{MPa}.$$

6.6 Simple gear selection procedure

[illegible]

1. [illegible]

Table 6.7 Spur gears: 1.0 module stock data [illegible]

Part number	Teeth	PCD (mm)	Outside diameter (mm)	Boss diameter (mm)	Bore diameter (mm)	Part number	Teeth	PCD (mm)	Outside diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible] maximum gear ratios. Note [illegible] the minimum [illegible] permissible when using a pressure angle of 20° [illegible]. Then either use standard teeth numbers as listed in Table 6.5 or as [illegible] stock gear catalogue.

2. Select a suitable [illegible] listed in the stock gear catalogues.
3. Select a module [illegible] from Table [illegible] stock gear catalogue (see Tables [illegible]) which give examples [illegible] stock gears [illegible].
4. Calculate the pitch diameter, $d = mN$.
5. Calculate the pitch line velocity, $V = (d/2) \times$ [illegible] does not exceed the guidelines given in Table [illegible].
6. Calculate the dynamic factor, K_v = [illegible].
7. Calculate the [illegible] load, W_t = [illegible].
8. Calculate the [illegible] permissible [illegible] using the Lewis formula in the form,

$$F = \frac{W_t}{K_v m Y \sigma_p} \quad (6.27)$$

Table 6.5 Spur gears 1.5 module, heavy duty steel 817M40, 655M13, face width 20 mm

Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)	Part number	Teeth	PCD (mm)	Outer diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
SC15-9	9	[illegible]	[illegible]	12	6	SC15-47	47	[illegible]	[illegible]	[illegible]	15
SC15-10	10	[illegible]	[illegible]	[illegible]	6	SC15-48	48	[illegible]	[illegible]	[illegible]	15
SC15-11	11	[illegible]	[illegible]	[illegible]	6	SC15-49	49	[illegible]	[illegible]	[illegible]	15
SC15-12	12	[illegible]	[illegible]	[illegible]	8	SC15-50	50	[illegible]	[illegible]	[illegible]	15
SC15-13	13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-14	14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-15	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-16	16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-17	17	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-18	18	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-19	19	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-20	20	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-21	21	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-22	22	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-23	23	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-24	24	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-25	25	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-26	26	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-27	27	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-28	28	[illegible]	[illegible]	30	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-29	29	[illegible]	[illegible]	30	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
SC15-30	30	[illegible]	[illegible]	30	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
SC15-31	31	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
SC15-32	32	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-33	33	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-34	34	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-35	35	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
SC15-36	36	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
SC15-37	37	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
SC15-38	38	57.00	60.00	[illegible]	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
SC15-39	39	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
SC15-40	40	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
SC15-41	41	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
SC15-42	42	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
SC15-43	43	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
SC15-44	44	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
SC15-45	45	[illegible]	[illegible]	[illegible]	15	SC15-120	120	[illegible]	[illegible]	[illegible]	15
SC15-46	46	[illegible]	72.00	[illegible]	15						

The Lewis form factor, Y, can be obtained from Table [illegible].

The permissible bending stress, σ_p, can be taken as σ_y/factor of safety where the factor of safety is set by experience, but may range from [illegible] to [illegible] remaining the value of σ_y related to the appropriate material in the stock gear catalogue. Certain plastics are suitable for use in gears especially in applications where low weight, low friction, high corrosion resistance, low wear and quiet operation are beneficial. The strength of plastics is usually significantly lower than that of metals. Plastics are often formed using fillers to improve strength, wear, impact resistance, temperature performance, as well as other properties and it is therefore difficult to specify or standardize properties for plastics and these need to be obtained directly from the manufacturer or [illegible]

Table 6.10 [illegible]

Part number	Teeth	PCD (mm)	Outside diameter (mm)	Boss diameter (mm)	Bore diameter (mm)	Part number	Teeth	PCD (mm)	Outside diameter (mm)	Boss diameter (mm)	Bore diameter (mm)
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						

[illegible]

Example 6.6

A gearbox is required to transmit 18 kW from a shaft rotating at 2650 rpm. The desired output speed is approximately 12 000 rpm. For space limitation and standardization reasons a double step-up gearbox is requested with equal ratios. Using the limited selection of gears presented in Tables 6.7 to 6.10, select suitable gears for the gear wheels and pinions.

Solution

Overall ratio = 12 000/2650 = 4.528.

First stage ratio = $\sqrt{4.528}$ = 2.128.

This could be achieved using a gear with 38 teeth and pinion with 18 teeth (ratio = 38/18 = 2.11).

The gear materials listed in Tables 6.7 to 6.10 are 817M40 and 655M13 steels. From

Table 6.11 Permissible bending stresses for various common spur gear materials

Material	Treatment	UTS (MPa)	Permissible bending stress σ_p (MPa)
[illegible]		[illegible]	[illegible]
[illegible]		[illegible]	[illegible]
[illegible]		[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

Table 6.11, the 655M13 is the stronger steel and this is selected for this example prior to a more detailed consideration. For 655M13 case hardened steel gears, the permissible stress σ_p = 345 MPa.

Calculations for gear 1: $Y_{38} = 0.37727$, $n = 2650$ rpm.

m	1.5	2.0
$d = mN$ (mm)	57	76
$V = \frac{d}{2} \times 10^{-3} \times n\frac{2\pi}{60}$ (m/s)	7.9	10.5
$W_t = \frac{\text{Power}}{V}$ (N)	2276	1707
$K_v = \frac{6.1}{6.1 + V}$	0.4357	0.3675
$F = \frac{W_t}{K_v m Y \sigma_p}$ (m)	0.027	0.018

$m = 1.5$ gives a face width value greater than the catalogue value of 20 mm, so try $m = 2$. $m = 2$ gives a face width value less than the catalogue value of 25 mm, so design is OK.

Calculations for pinion 1: $Y_{18} = 0.29327$, $n = 5594$ rpm. (No need to do calculations for $m = 1.5$, because it has now been rejected.)

m	2.0
d (mm)	36
V (m/s)	10.5
W_t (N)	1707
K_v	0.3676
F (m)	0.023

$m = 1.5$ gives a face width value greater than the catalogue value of 20 mm, so try $m = 2$. $m = 2$ gives a face width value less than the catalogue value of 25 mm, so design is OK.

Calculations for gear 2: $Y_{38} = 0.37727$, $n =$ 5594 rpm.

m	2.0
d (mm)	76
V (m/s)	22.26
W_t (N)	808.6
K_v	0.215
F (m)	0.0144

$m = 2$ gives a face width value lower than catalogue specification, so the design is OK.

Calculations for pinion 2: $Y_{18} = 0.29327$, $n = 11\,810$ rpm.

m	2.0
d (mm)	36
V (m/s)	22.26
W_t (N)	808.6
K_v	0.215
F (m)	0.0186

$m = 2$ gives a face width value lower than catalogue specification, so the design is OK. A general arrangement for the solution is shown in Figure 6.27.

Example 6.7

A gearbox for a cordless hand tool, Figure 6.28, is required to transmit 5 W from an electric motor running at 3000 rpm to an output drive running at approximately 75 rpm. The specification for the gearbox is that it should be reliable, efficient, maintenance free for the life of the device, light and fit within an enclosure of 50 mm diameter. The length is not critical.

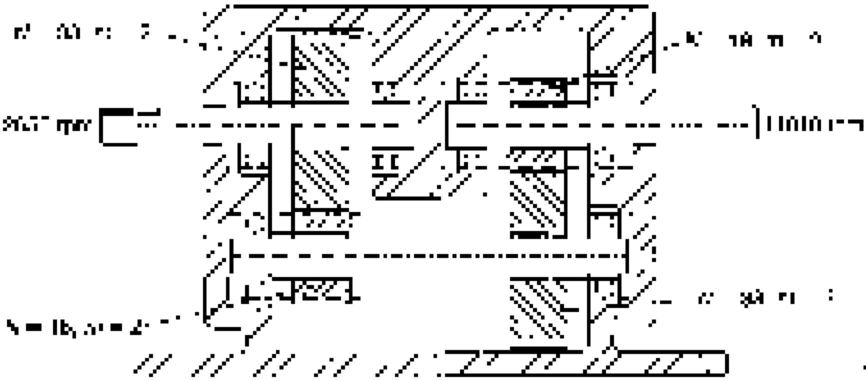

Figure 6.27 [illegible]

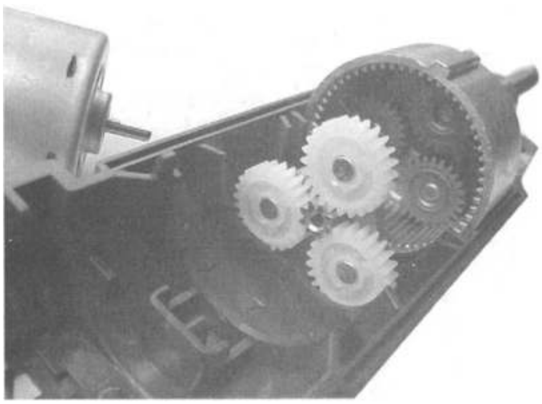

Figure 6.30 [illegible]

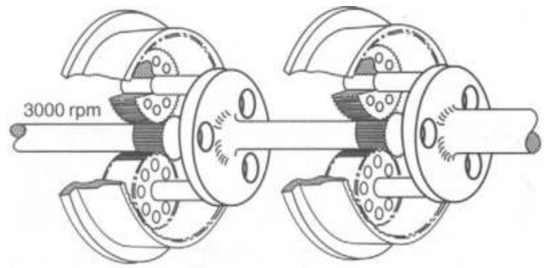

Figure 6.29 Two stage reduction epicyclic gear train (after Townsend [illegible])

The tolerance on the output speed is acceptable at ±10 per cent. Specify an appropriate gear train including values for the module, teeth numbers and gear materials.

Solution

The reduction ratio 3000/75 is quite large and could be achieved by a compound gear train, a worm and wheel or a two stage reduction epicyclic. The restriction on the maximum diameter precludes the use of compound gear trains, especially if the drive input is axial as would be expected from an electric motor. The high efficiency demand rules out the use of a worm and wheel.

We want an overall ratio of 40 (=3000/75). This could be achieved in a two-stage reduction epicyclic gearbox as illustrated in Figure 6.29. For identical stages the gear ratio of each stage would be $\sqrt{40} = 6.3$ ($6.3 \times 6.3 \approx 39.7$).

Try $N_S = 18$, $N_R = 72$. This gives a reduction ratio per stage of $N_R/N_S + 1 = 72/18 + 1 = 5$. Overall reduction ratio of the two stage gear train $= 5 \times 5 = 25$. No good.

Try $N_S = 12$, $N_R = 72$. This gives a reduction ratio of 7. Overall reduction ratio $7 \times 7 = 49$. No good.

Try $N_S = 12$, $N_R = 63$. This gives a reduction ratio of 6.25. Overall reduction ratio $6.25 \times 6.25 = 39.1$. OK.

Can this fit in the space?

Propose a module. Say $m = 0.5$.

With $N_S = 12$, $d_{sun} = mN = 0.5 \times 12 = 6$ mm.

With $N_R = 63$, $d_{ring} = mN = 0.5 \times 63 = 31.5$ mm.

Room available for planet gear is $31.5/2 - 6/2 = 12.75$ mm. With a module of 0.5 there are no gears with this diameter. Impossible solution.

Try $N_S = 12$, $N_R = 66$. This gives a reduction ratio of 6.5. Reduction ratio $6.5 \times 6.5 = 42.3$.

This would give an output speed of 71 rpm. This is within the tolerance allowable on the output speed. OK.

Can this fit in the space?

Propose a module. Say $m = 0.5$

With $N_S = 12$, $d_{sun} = mN = 0.5 \times 12 = 6$ mm.

With $N_R = 66$, $d_{ring} = mN = 0.5 \times 66 = 33$ mm.

Room available for planet gear is $33/2 - 6/2 = 13.5$ mm.

With a module of 0.5, the planetary gears would have $13.5/0.5 = 27$ teeth. $N_p = 27$.

Proposed design: $N_S = 12$, $N_P = 27$, $N_R = 66$, module 0.5.

What face width should the gears have? For a module of 0.5 HPC Gears Ltd offers standard 0.5 module gears with a face width of 5 mm available in Tufnol, delrin, brass and 045M10 steel.

The Lewis formula can be used as an approximate check to determine whether the gears with a face width of 5 mm manufactured from a particular material are strong enough.

Analysing the input sun gear

$$V_S = \frac{6 \times 10^{-3}}{2} \times 3000 \times \frac{2\pi}{60} = 0.942 \text{ m/s}$$

$$K_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 0.942} = 0.866$$

$$W_t = \frac{\text{power}}{V} = \frac{5}{0.942} = 5.3 \text{ N}$$

From Table 6.6

$$Y_{12} = 0.2296$$

Try Tufnol

$$\sigma_p = 31 \text{ MPa}$$

$$F = \frac{W_t}{K_v m Y \sigma_p}$$

$$= \frac{5.3}{0.866 \times 0.0005 \times 0.2296 \times 31 \times 10^6}$$

$$= 1.72 \times 10^{-3} \text{ m}$$

This face width of 1.7 mm is less than the 5 mm available, indicating that the gear will be more than strong enough. If this figure had been greater than 5 mm, then another material such as steel or brass could be considered.

Analysing the planets

$$n_{arm} = 3000/6.5 = 461.5 \text{ rpm}$$

To determine the speed of the planets use Eqs 6.14 and 6.15

$$\frac{n_P - n_{arm}}{n_S - n_{arm}} = -\frac{N_S}{N_P}$$

$$\frac{n_P - (-461.5)}{-3000 - (-461.5)} = -\frac{12}{27}$$

$$n_P = -\frac{12}{27} \times (-2538) - 461.5 = 666.7 \text{ rpm}$$

$$V_p = \frac{13.5 \times 10^{-3}}{2} \times 666.7 \times \frac{2\pi}{60} = 0.47 \text{ m/s}$$

$$K_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 0.47} = 0.928$$

The power is divided between three planets, so:

$$W_t = \frac{\text{power}}{V} = \frac{5/3}{0.47} = 3.55 \text{ N}$$

From Table 6.6, $Y_{27} = 0.34385$. Try Tufnol

$$\sigma_p = 31\text{ MPa}$$

$$F = \frac{W_t}{K_v m Y \sigma_p}$$

$$= \frac{3.55}{0.928 \times 0.0005 \times 0.34385 \times 31 \times 10^6}$$

$$= 0.72\text{m}$$

This face width of 0.72 mm is less than the 5 mm available, indicating that the gear will be more than strong enough.

Analysing the ring gear, the stress on the ring gear will be less than on the planets so does not necessarily need to be analysed at this stage.

Analysing the stage 2 sun gear

$(n_{sun} = n_{planet\ carrier})$

$$V_S = \frac{6 \times 10^{-3}}{2} \times 461.5 \times \frac{2\pi}{60} = 0.145\text{ m/s}$$

$$K_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 0.145} = 0.98$$

$$W_t = \frac{\text{power}}{V} = \frac{5}{0.145} = 34.5\text{ N}$$

From Table 6.6, $Y_{12} = 0.2296$

Try Tufnol

$$\sigma_p = 31\text{ MPa}$$

$$F = \frac{W_t}{K_v m Y \sigma_p}$$

$$= \frac{34.5}{0.98 \times 0.0005 \times 0.2296 \times 31 \times 10^6}$$

$$= 0.0099\text{m}$$

This is larger than the available standard size (5 mm), so try a stronger material.

Using 045M10 steel, $\sigma_p = 117$ MPa

$$F = \frac{W_t}{K_v m Y \sigma_p}$$

$$= \frac{34.5}{0.98 \times 0.0005 \times 0.2296 \times 117 \times 10^6}$$

$$= 2.62 \times 10^{-3}\text{ m}$$

This face width of 2.62 mm is less than the 5 mm available, indicating that the gear will be more than strong enough. So the sun gear must be manufactured from 045M10 steel. Manufacturing expediency might dictate that the planet carrier should also be made from the same steel.

Analysing the planets

$$\frac{n_P - (-71)}{-461.5 - (-71)} = -\frac{12}{27}$$

$$n_P = -\frac{12}{27} \times (-390.5) - 71 = 102.6\text{ rpm}$$

$$V_p = \frac{13.5 \times 10^{-3}}{2} \times 102.6 \times \frac{2\pi}{60}$$

$$= 0.0725\text{ m/s}$$

$$K_v = \frac{6.1}{6.1 + V} = \frac{6.1}{6.1 + 0.0725} = 0.99$$

The power is divided between three planets, so

$$W_t = \frac{\text{power}}{V} = \frac{5/3}{0.0725} = 23.0\text{ N}$$

$$Y_{27} = 0.34385$$

Try Tufnol

$$\sigma_p = 31\text{ MPa}$$

$$F = \frac{W_t}{K_v m Y \sigma_p}$$

$$= \frac{23.0}{0.99 \times 0.0005 \times 0.34285 \times 31 \times 10^6}$$

$$= 4.4 \times 10^{-3}\text{ m}$$

This face width of 4.4 mm is less than the 5 mm available, indicating that the gear will be more than strong enough.

Design summary:

Stage 1: module 0.5, F = 5 mm, $N_S = 12$ (Tufnol), $N_P = 27$ (three planets, Tufnol), $N_R = 66$ (Tufnol).

Stage 2: module 0.5, $F = 5$ mm, $N_S = 12$ (steel), $N_P = 27$ (three planets, Tufnol), $N_R = 66$ (Tufnol).

6.7 Conclusions

[illegible] Chapter 7.

References and sources of information

Books and papers

[illegible]

[illegible]

[illegible]

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Standards

[illegible]

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Websites

[illegible]

Nomenclature

Generally, preferred SI units have been stated.

a	addendum (mm)
b	dedendum (mm)
c	clearance (mm)
d	pitch diameter (mm)
d_G	pitch diameter of the gear (mm)
d_P	pitch diameter of the pinion (mm)
d_{in}	pitch diameter of the driving gear (mm)
d_{out}	pitch diameter of the driven gear (mm)
e	efficiency
F	face width (mm)
F_r	radial force component (N)
F_t	tangential force component (N)
h_k	working depth (mm)
h_t	whole depth (mm)
H	power (W)
K_v	velocity factor
m	module (mm)
n	rotational speed (rpm)
n_P	rotational speed of the pinion (rpm)
n_G	rotational speed of the gear (rpm)
N	number of teeth
N_G	number of teeth in the gear
N_P	number of teeth in the pinion or driven gear
N_{in}	number of teeth in the driving gear
N_t	number of teeth in the train
OD	outside diameter (mm)
p	circular pitch (mm)
PCD	pitch circle diameter (mm)
P_d	diametral pitch (teeth per inch)
t	thickness (mm)
r	pitch radius of the rack (mm)
t	tooth thickness (mm)
w	width of tooth land (mm)
V	velocity (m/s)
W_t	transmitted load (N)
Y	Lewis form factor
ϕ	pressure angle (°)
σ	bending stress (N/m²)
σ_b	bending stress (MPa)
σ_{all}	allowable bending stress (MPa)
ω	angular velocity (rad/s)
ω_G	angular velocity of the gear (rad/s)
ω_P	angular velocity of the pinion (rad/s)

Worksheet

6.1 For the gear train illustrated in Figure 6.38, determine the output speed and direction of rotation if the input shaft rotates at 1490 rpm clockwise. Gears A to D have a module of 1.5 and gears E to H a module of 2.

6.2 What type of gear would be selected for the following applications:

(a) To transmit power between two shafts with intersecting axes at 90° to each other.

(b) To transmit power between two shafts with non-intersecting axes at 90° to each other.

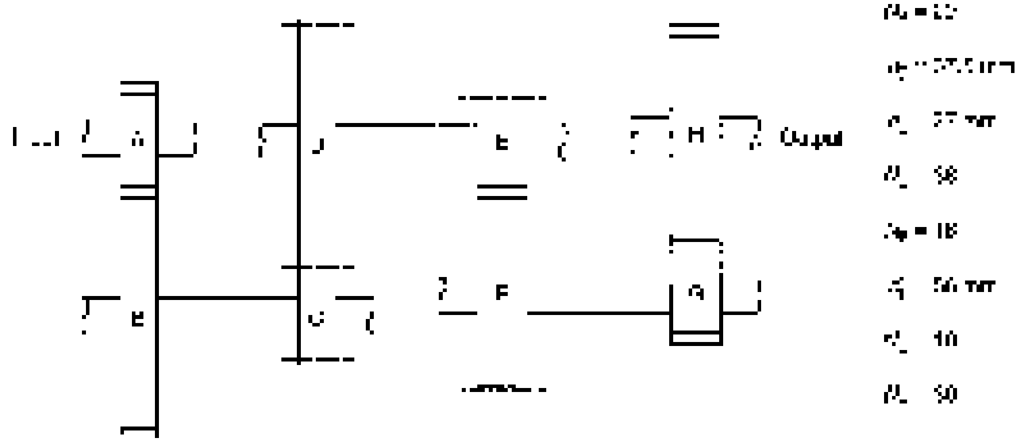

Figure 6.38 Gear train.

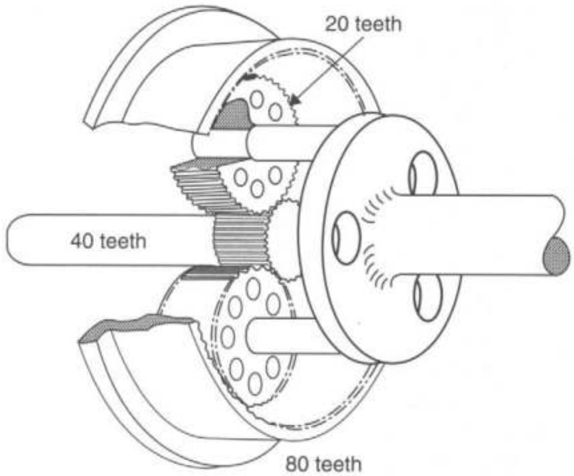

Figure 6.31 Epicyclic gears (reproduced from Townsend 1992).

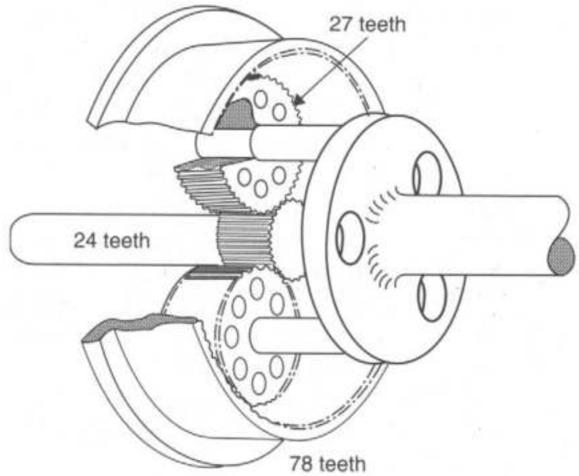

Figure 6.32 Epicyclic gears (reproduced from Townsend 1992).

(a) The pinion ... 2.0 kW ... [illegible] ... which material might be specified for the pinion?
(b) If ... [illegible] ... epicyclic gearbox. What material might be specified for [illegible]?

6.3 Determine the [illegible] ratio ... and the speed and direction of rotation of the output carrier for the epicyclic gearbox illustrated in Figure 6.31. The sun, planets and annulus gears have 40, 20 and 80 teeth, respectively. The sun gear is the input and rotates at 2000 rpm clockwise. The planet carrier is the output. The annulus gear is stationary.

6.4 Determine the train [illegible] ... and the speed and direction of the output carrier for the epicyclic gearbox illustrated in Figure 6.32. The sun, planet and annulus gears have 24, 27 and 78 teeth, respectively. The sun gear is rotating at 5000 rpm clockwise. The sun gear is the input and the planet carrier the output. The annulus gear is [illegible] ... [illegible].

6.5 A 20° full depth spur pinion is to transmit 1.75 kW at 2000 rpm. If the pinion has 17 teeth with a module of 2, determine a suitable value for the face width based on the Lewis formula if the bending stress should not exceed 50 MPa.

6.6 A 20° full depth spur pinion is required to transmit 1.5 kW at a speed of 1000 rpm. If the pinion has 18 teeth and is manufactured from heavy duty 817M40 steel, select a suitable gear from the limited choice given in Tables 6.7–6.10, specifying the module and face width based on the Lewis formula.

6.7 A gearbox is required to transmit 4 kW from a shaft rotating at 2400 rpm. The desired output speed is approximately 800 rpm. Using the Lewis formula to determine the gear strength based upon bending stresses, select suitable gears from the limited selection presented in Tables 6.7–6.10.

6.8 A 20° full depth spur pinion is required to transmit 1 kW at a speed of 1200 rpm. If the pinion has 21 teeth and is manufactured from heavy duty 817M40 steel, select a suitable gear from the limited choice given in Tables 6.7–6.10, specifying the module and face width based on the Lewis formula.

6.9 A gearbox is required to transmit [illegible] kW from a shaft rotating at 2000 rpm. The desired output speed is approximately 500 rpm. [illegible] ... the same ... Select suitable gears.

6.10 A [illegible] reduction gearbox is required to transmit 15 kW from a shaft rotating at [illegible] rpm. The desired output speed is approximately [illegible] rpm. Using the Lewis formula to determine gear strength [illegible] bending stress, select suitable gears from the [illegible] selection presented in Table 6.[illegible].

6.11 A gearbox is required to transmit [illegible] kW from a shaft rotating at 1470 rpm. The desired output speed is approximately [illegible] rpm. The output must rotate in the same direction as the input and along the same axis. Select and specify appropriate gears for this gearbox.

6.12 An epicyclic gearbox is required to transmit [illegible] kW from a shaft rotating at [illegible] rpm. The desired output speed is approximately [illegible] rpm. Using [illegible], determine the number of teeth in each gear for a single stage reduction gearbox.

Answers

6.1 [illegible] rpm, clockwise.
6.2 (a) Bevel, (b) crossed axis helical gears, (c) helical gears or herringbone teeth, (d) spur gears, [illegible]
6.3 [illegible] rpm, anticlockwise.
6.4 [illegible] rpm, [illegible]
6.5 [illegible]
6.6 $m = 2$, $F = 25$ [illegible] would be acceptable.
6.7 No unique solution. $m = 3$, $F = 25$ mm, $N_1 = 18$, $N_2 = 48$, $N_3 = 18$, $N_4 =$ [illegible], case hardened 655M13 [illegible]
6.8 No unique solution. $m = 2$, $N = 24$, $F =$ [illegible] mm [illegible]
6.9 No unique solution.
6.10 No unique solution.
6.11 No unique solution.
6.12 No unique solution. $N_S = 18$, $N_P =$ [illegible], $N_A = 60$ would be acceptable.

Learning objectives achievement

[illegible]	[illegible]	✔	✗ [illegible]
[illegible]	[illegible]	✔	✗ [illegible]
[illegible]		✔	✗ [illegible]
[illegible]	[illegible]	✔	✗ [illegible]
[illegible]	[illegible]	✔	✗ [illegible]
[illegible]	[illegible]	✔	✗ [illegible]