Arithmetic
Operations on Digital and Binary Numbers (2)

Lecture 03
Book Chapter(s): 1

COE211-Digital Logic Design

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Binary Arithmetic Operations – Multiplication (1)

- To multiply two numbers, take each digit of the multiplier and multiply it with the multiplicand. This produces a number of partial products which are then added.

$(11001)_2$	$(214)_{10}$	Multiplicand
x $(10101)_2$	x $(152)_{10}$	Multiplier
$(11001)_2$	$(428)_{10}$	Partial products
$(11001)_2$	$(1070)_{10}$	
$+(11001)_2$	$+(214)_{10}$	
$(1000001101)_2$	$(32528)_{10}$	Result

Binary Arithmetic Operations – Multiplication (2)

- Digit multiplication table:

BINARY	DECIMAL
0 X 0 = 0	0 X 0= 0
0 X 1= 0	0 X 1= 0
1 X 0 = 0	…
1 X 1= 1	1 X 8 = 8
	1 X 9= 9
	…
	9 X 8 = 72
	9 X 9 = 81

Binary Arithmetic Operations – Division

- Can you figure out how this is done?
- Think of the division technique (shift & subtract) used for decimal numbers and apply it to binary numbers.

Signed Numbers Representation (1)

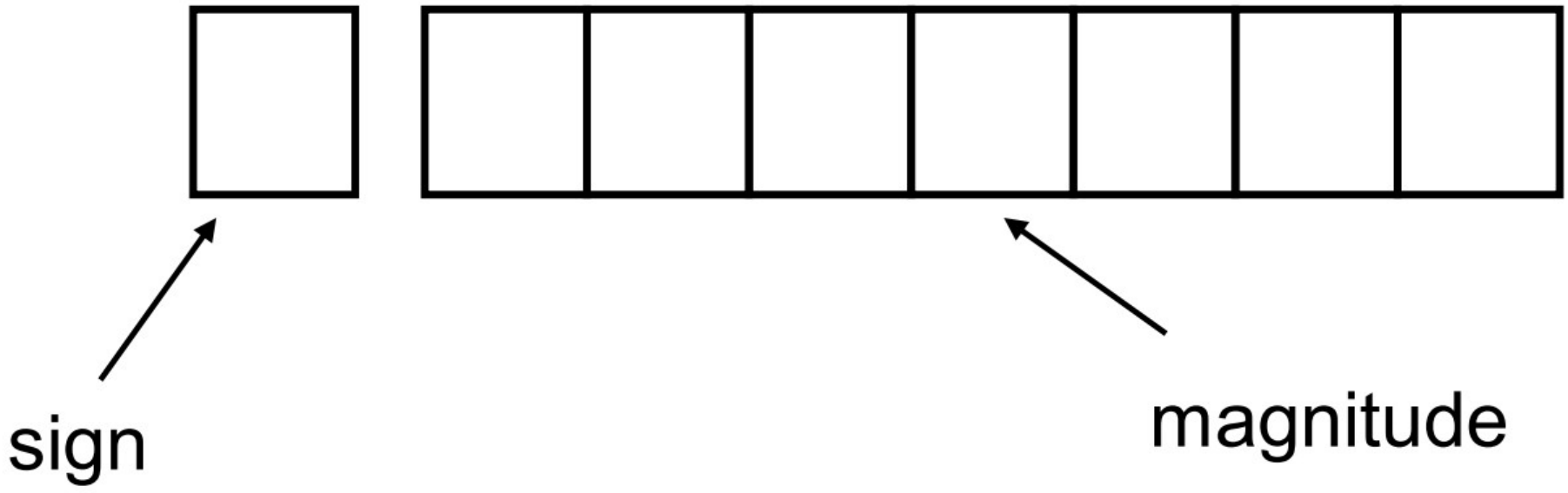
- Negative numbers are usually written by writing a minus sign in front.
  - Example:
    $-(12)_{10}$, $-(1100)_2$
- In computer memory of fixed width, this sign is usually represented by a bit:
  - 0 for +
  - 1 for -

Signed Numbers Representation (2)

- There are three common ways to represent signed numbers [positive (+ve) and negative numbers (-ve)] for binary numbers:
  1. **Sign-and-Magnitude**
  2. **1s Complement**
  3. **2s Complement**

Signed Numbers: Sign-and-Magnitude (1)

- Example: an 8-bit number can have 1-bit sign and 7-bits magnitude:



Signed Numbers: Sign-and-Magnitude (2)

- To negate a number, just invert the sign bit
- Examples:

$$-(0\ 0100001)_{sm} = (1\ 0100001)_{sm}$$
$$-(1\ 0000101)_{sm} = (0\ 0000101)_{sm}$$

Signed Numbers: 1s Complement (1)

- Complements are used in digital computers to simplify the subtraction operation and for logical manipulation.
- Given a number *x* which can be expressed as an *n*-bit binary number, its negative value can be obtained in 1s-complement representation using:

$$-x = 2^n - x - 1$$

Signed Numbers: 1s Complement (2)

- Example: With an 8-bit number 00001100, its negative value, expressed in 1s complement, is obtained as follows:

$$-(00001100)_2 = -(12)_{10}$$
$$= (2^8 - 12 - 1)_{10}$$
$$= (243)_{10}$$
$$= \mathbf{(11110011)_{1s}}$$

Signed Numbers: 1s Complement (3)

- Essential technique: invert all the bits
  - The 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's
- Examples:

  1s complement of $(00000001)_{1s} = (11111110)_{1s}$

  1s complement of $(01111111)_{1s} = (10000000)_{1s}$

- The most significant bit still represents the sign:
  0 = +ve; 1 = -ve.

Signed Numbers: 2s Complement (1)

- Given a number *x* which can be expressed as an *n*-bit binary number, its negative number can be obtained in 2s-complement representation using:

$$-x = 2^n - x$$

Signed Numbers: 2s Complement (2)

- Example: With an 8-bit number 00001100, its negative value in 2s complement is thus:

$$-(00001100)_2 = -(12)_{10}$$
$$= (2^8 - 12)_{10}$$
$$= (244)_{10}$$
$$= \mathbf{(11110100)_{2s}}$$

Signed Numbers: 2s Complement (3)

- Essential technique: invert all the bits and add 1
  - The complement of the complement restores the number to its original value
- Examples:

2s complement of

$(00000001)_{2s} = (11111110)_{1s}$ (invert)

$= (11111111)_{2s}$ (add 1)

2s complement of

$(01111110)_{2s} = (10000001)_{1s}$ (invert)

$= (10000010)_{2s}$ (add 1)

- The most significant bit still represents the sign:
  0 = +ve; 1 = -ve.

Signed Numbers: 2s Complement (4)

- The 2's complement can be formed by leaving all least significant 0's and the first 1 unchanged and replacing 1's with 0's and 0's with 1's in all other higher significant digits.
- Example:

The 2's complement of 1101100 is 0010100

The 2's complement of 0110111 is 1001001

Comparisons of Sign-and-Magnitude and Complements (1)

- Example: 4-bit signed number (*positive values*)

Value	Sign-and-Magnitude	1s Comp.	2s Comp.
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000

Comparisons of Sign-and-Magnitude and Complements (2)

- Example: 4-bit signed number (*negative values*)

Value	Sign-and-Magnitude	1s Comp.	2s Comp.
-0	1000	1111	-
-1	1001	1110	1111
-2	1010	1101	1110
-3	1011	1100	1101
-4	1100	1011	1100
-5	1101	1010	1011
-6	1110	1001	1010
-7	1111	1000	1001
-8	-	-	1000