

Logic Gates (2)

Lecture 09

COE211-Digital Logic Design

الفصل الدراسي الأول 1442-2020 Fall

جامعة طيبة فرع ينبع - كلية علوم وهندسة الحاسبات - شطر الطالبات



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Midterm

- The midterm is scheduled to be on Wednesday 21/10/2020 from 9:30AM to 10:30AM
- The schedule can be changed in two cases:
 - A student has more than 2 midterms on the same day
 - A student has 2 exams at the same time

Universal Gates: NAND and NOR

- AND/OR/NOT gates are sufficient for building any Boolean functions
- However, other gates are also used because:
 - (i) usefulness
 - (ii) economical on transistors
 - (iii) self-sufficient

NAND/NOR: economical, self-sufficient

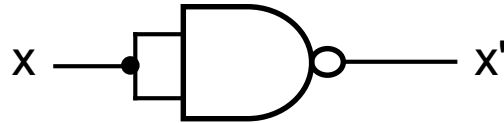
XOR: useful (e.g. parity bit generation)

NAND Gate

- NAND gate is **self-sufficient** (you can build any logic circuit with it)
- Can be used to implement AND/OR/NOT

Implementations using NAND Gate (1)

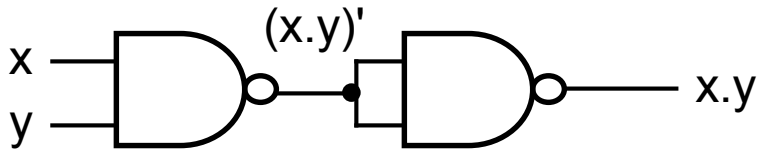
- Implementing an inverter using NAND gate:



$$(x.x)' = x' \quad (\text{T1: idempotency})$$

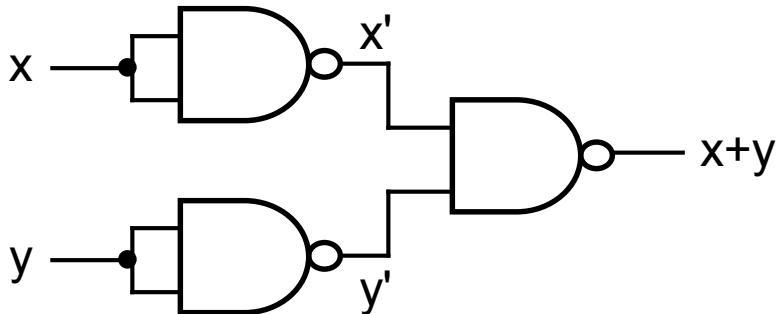
Implementations using NAND Gate (2)

- Implementing AND using NAND gates:



$$\begin{aligned} ((xy)'(xy)')' &= ((xy)')' && \text{idempotency} \\ &= (xy) && \text{involution} \end{aligned}$$

- Implementing OR using NAND gates:



$$\begin{aligned} ((xx)'(yy)')' &= (x'y')' && \text{idempotency} \\ &= x''+y'' && \text{DeMorgan} \\ &= x+y && \text{involution} \end{aligned}$$

Implementations using NAND Gate (3)

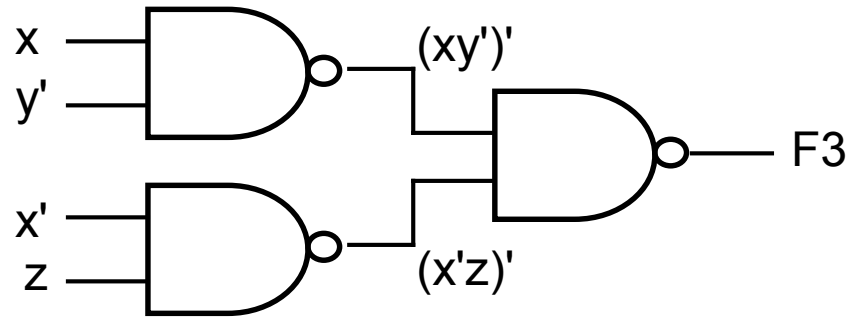
- Possible to implement any Boolean expression using NAND gates
- *Procedure:*
 1. Obtain sum-of-products Boolean expression:
(e.g.,) $F3 = xy' + x'z$
 2. Use DeMorgan theorem to obtain expression using 2-level NAND gates:

$$(e.g.,) F3 = xy' + x'z$$

$$= (xy' + x'z)'' \quad \text{involution}$$

$$= ((xy')' \cdot (x'z)')' \quad \text{DeMorgan}$$

Implementations using NAND Gate (4)



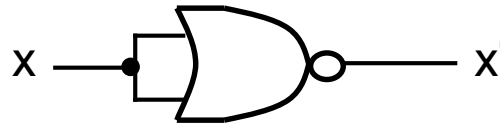
$$F3 = ((xy')' \cdot (x'z)')' = xy' + x'z$$

NOR Gate

- NOR gate is also self-sufficient
- Can be used to implement AND/OR/NOT

Implementations using NOR Gate (1)

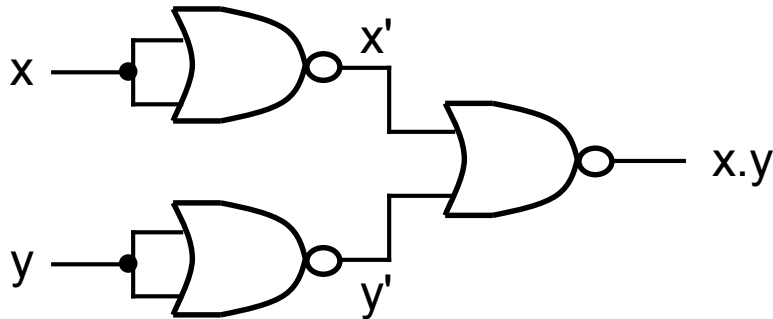
- Implementing an inverter using NOR gate:



$$(x+x)' = x' \quad (\text{T1: idempotency})$$

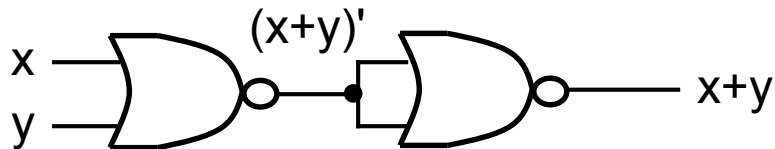
Implementations using NOR Gate (2)

- Implementing AND using NOR gates:



$$\begin{aligned} ((x+x)' + (y+y)')' &= (x'+y')' && \text{idempotency} \\ &= x'' \cdot y'' && \text{DeMorgan} \\ &= x \cdot y && \text{involution} \end{aligned}$$

- Implementing OR using NOR gates:



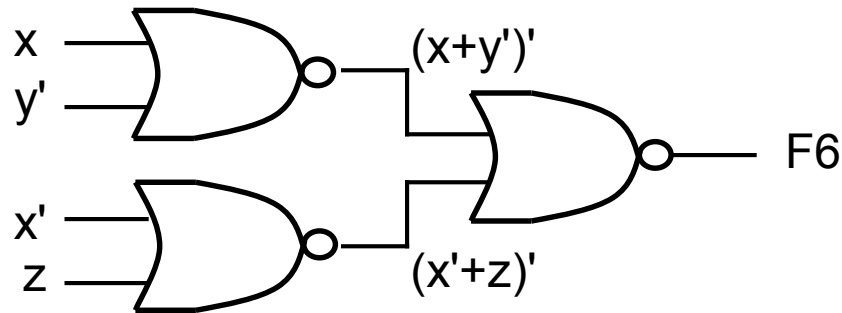
$$\begin{aligned} ((x+y)' + (x+y)')' &= ((x+y)')' && \text{idempotency} \\ &= (x+y) && \text{involution} \end{aligned}$$

Implementations using NOR Gate (3)

- Possible to implement boolean expression using NOR gates
- *Procedure:*
 1. Obtain product-of-sums Boolean expression:
(e.g.,) $F6 = (x+y')(x'+z)$
 2. Use DeMorgan theorem to obtain expression using 2-level NOR gates:

$$\begin{aligned} \text{(e.g.,) } F6 &= (x+y')(x'+z) \\ &= ((x+y')(x'+z))' \quad \text{involution} \\ &= ((x+y')'+(x'+z)')' \quad \text{DeMorgan} \end{aligned}$$

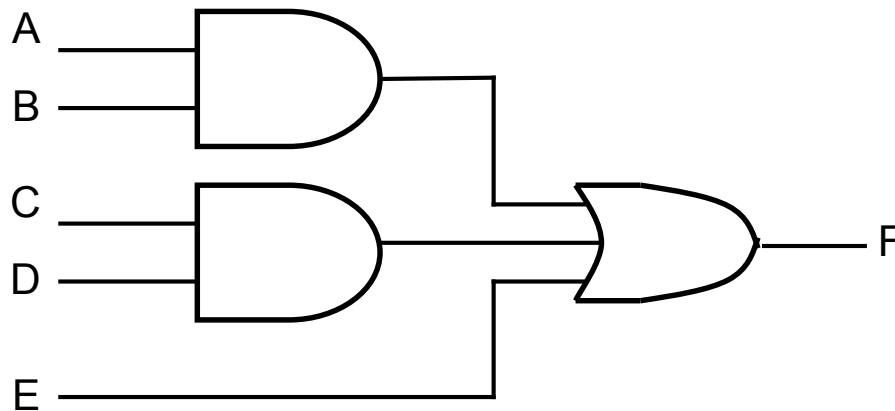
Implementations using NOR Gate (4)



$$F6 = ((x+y')'+(x'+z)')' = (x+y').(x'+z)$$

Implementation of SOP Expressions (1)

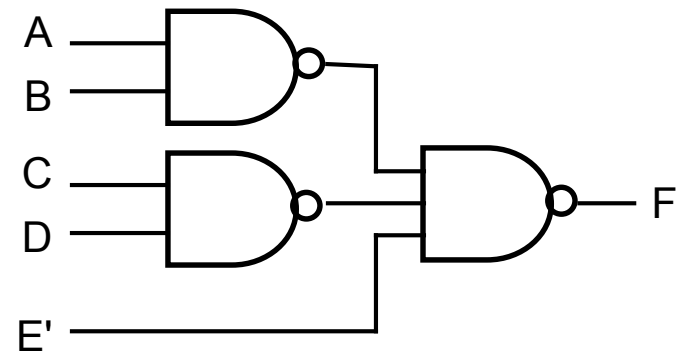
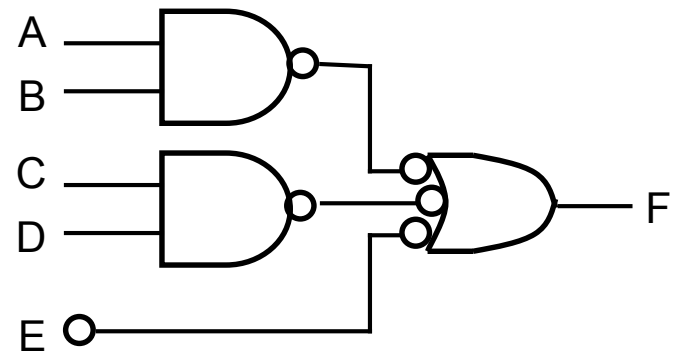
- Sum-of-Products (SOP) expressions can be implemented using:
 - 2-level AND-OR logic circuits
 - 2-level NAND logic circuits
- AND-OR logic circuit



$$F = AB + CD + E$$

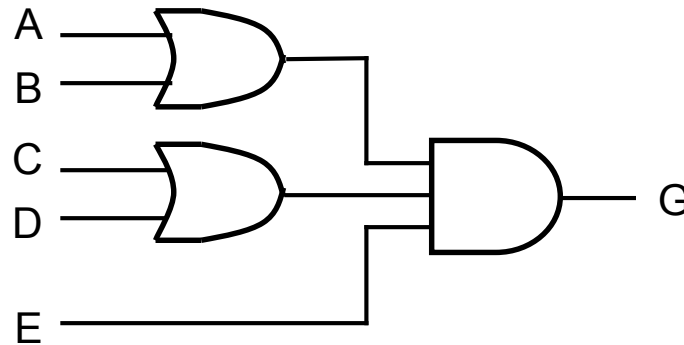
Implementation of SOP Expressions (2)

- NAND-NAND circuit (by circuit transformation):
 - a) add double bubbles
 - b) Change OR-with-inverted-inputs to NAND & bubbles at inputs to their complements



Implementation of POS Expressions (1)

- Product-of-Sums expressions can be implemented using:
 - 2-level OR-AND logic circuits
 - 2-level NOR logic circuits
- OR-AND logic circuit



$$G = (A+B).(C+D).E$$

Implementation of POS Expressions (2)

- NOR-NOR circuit (by circuit transformation):

- a) add double bubbles
- b) changed AND-with-inverted-inputs to NOR & bubbles at inputs to their complements

