

Karnaugh Maps (2)

Lecture 11

COE211-Digital Logic Design

الفصل الدراسي الأول 1442-2020 Fall

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Simplification Using K-Maps (1)

- Based on the **Unifying Theorem:**

$$A + A' = 1$$

In a K-map, **each cell containing a '1' corresponds to a minterm** of a given function F

- **Each group of adjacent cells containing '1'**
 - A group must have size *in powers of twos*: 1, 2, 4, 8, ... then **corresponds to a simpler product term** of F
 - ✓ Grouping 2 adjacent squares eliminates 1 variable, grouping 4 squares eliminates 2 variables, grouping 8 squares eliminates 3 variables, and so on.
 - In general, **grouping 2^n squares eliminates n variables.**

Simplification Using K-Maps (2)

- Group as many squares as possible:
 - **The larger the group is, the fewer the number of literals** in the resulting product term.
- Select as few groups as possible to cover all the squares (minterms) of the function
 - **The fewer the groups, the fewer the number of product terms** in the minimized function.

Example 1 (1)

- Example:

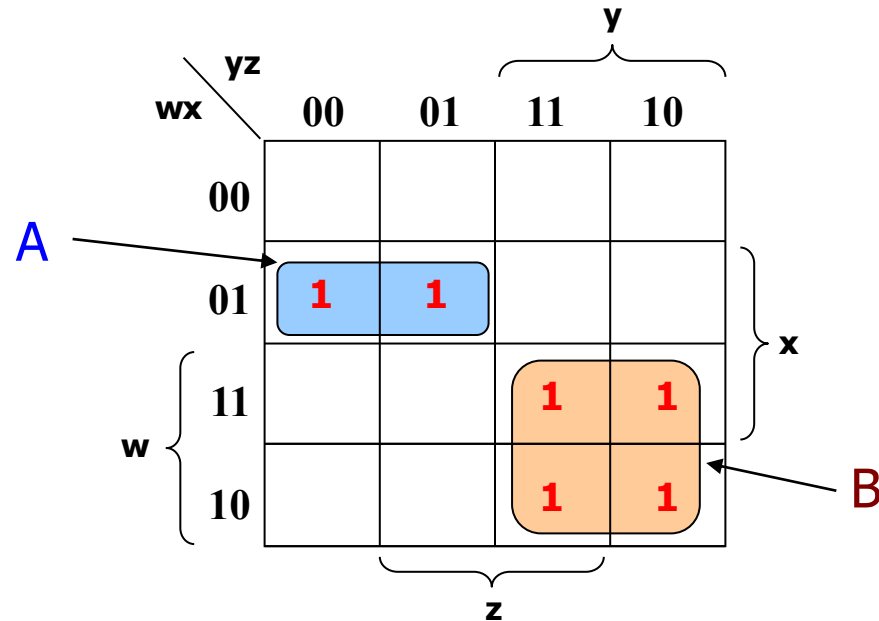
$$F(w,x,y,z) = w'xy'z' + w'xy'z + wx'yz' + wx'yz + wxyz' + wxyz$$
$$= \Sigma m(4, 5, 10, 11, 14, 15)$$

		y			
		00	01	11	10
w	wx				
	00				
	01	1	1		
	11			1	1
10			1	1	

(cells with '0' are not shown for clarity)

Example 1 (2)

- Each group of adjacent minterms (group size in powers of twos) corresponds to a possible **product term** of the given function.

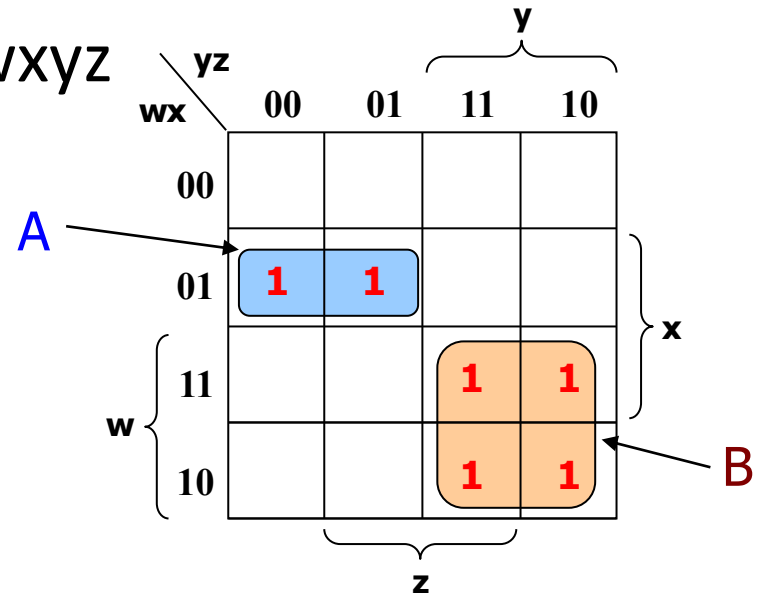


Example 1 (3)

- There are 2 groups of minterms: A and B, where:

$$\begin{aligned} A &= w'xy'z' + w'xy'z \\ &= w'xy'(z' + z) \\ &= w'xy' \end{aligned}$$

$$\begin{aligned} B &= wx'yz' + wx'yz + wxyz' + wxyz \\ &= wx'y(z' + z) + wxy(z' + z) \\ &= wx'y + wxy \\ &= w(x'+x)y \\ &= wy \end{aligned}$$



Example 1 (4)

- Each product term of a group, $w'xy'$ and wy , represents the *sum of minterms* in that group
- Boolean function is therefore the sum of product terms (SOP) which represent all groups of the minterms of the function:

$$F(w,x,y,z) = A + B = w'xy' + wy$$

Notes

- Larger groups correspond to product terms of fewer literals
- In the case of a 4-variable K-map:

1 cell = 4 literals, e.g.: $wxyz$, $w'xy'z$

2 cells = 3 literals, e.g.: wxy , $wy'z'$

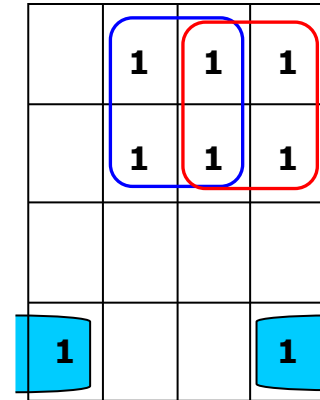
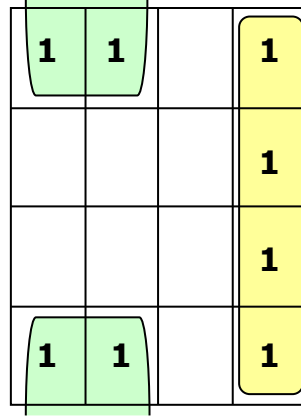
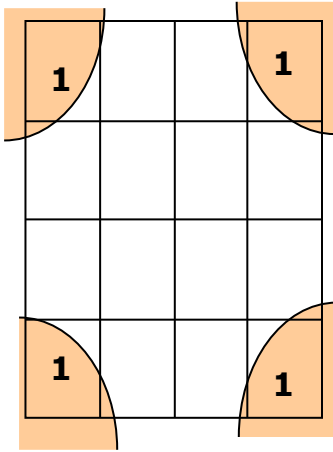
4 cells = 2 literals, e.g.: wx , $x'y$

8 cells = 1 literal, e.g.: w , y' , z

16 cells = no literal, e.g.: 1

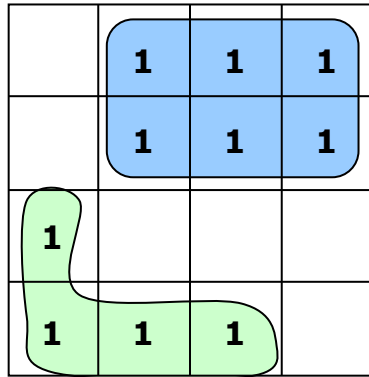
Other Grouping Forms

- Other possible valid groupings of a 4-variable K-map include:

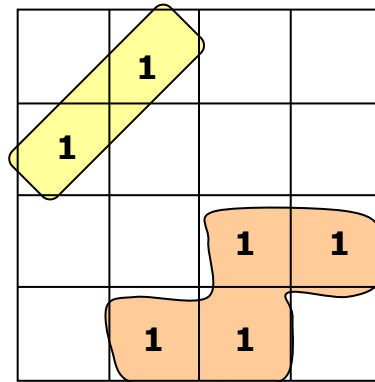


Grouping Rules

- Groups of minterms must be:
 - 1) rectangular, and
 - 2) have size in powers of 2's
- Otherwise they are *invalid* groups.
- Some examples of *invalid groups*:



x



x

Converting to Minterms Forms

- The K-map of a function is easily drawn when the function is given in canonical sum-of-products, or sum-of-minterms form
- What if the function is not in sum-of-minterms?
 - Convert it to sum-of-products (SOP) form.
 - Expand the SOP expression into sum-of-minterms expression, or fill in the K-map directly based on the SOP expression.

Example 2

- Example:

$$\begin{aligned}
 f(A,B,C,D) &= A(C+D)'(B'+D') + C(B+C'+A'D) \\
 &= A(C'D')(B'+D') + BC + CC' + A'CD \\
 &= AB'C'D' + AC'D' + BC + A'CD
 \end{aligned}$$

$$\begin{aligned}
 &AB'C'D' + AC'D' + BC + A'CD \\
 = &AB'C'D' + AC'D'(B+B') + BC + A'CD \\
 = &AB'C'D' + ABC'D' + AB'C'D' + BC(A+A') + A'CD \\
 = &AB'C'D' + ABC'D' + ABC + A'BC + A'CD \\
 = &AB'C'D' + ABC'D' + ABC(D+D') + A'BC(D+D') + A'CD(B+B') \\
 = &AB'C'D' + ABC'D' + ABCD + ABCD' + A'BCD + A'BCD' + A'B'CD
 \end{aligned}$$

