

Karnaugh Maps (2)

Lecture 12

COE211-Digital Logic Design

الفصل الدراسي الأول 1442-2020 Fall

جامعة طيبة فرع ينبع - كلية علوم وهندسة الحاسبات - شطر الطالبات



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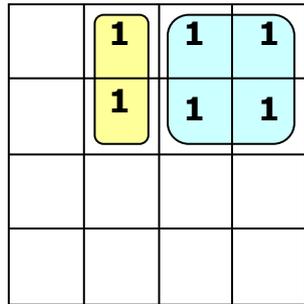
Simplest SOP Expressions

- To find the simplest possible *sum of products* (SOP) expression from a K-map, you need to obtain:
 - **minimum number of literals per product term**; and
 - **minimum number of product terms**
- This is achieved in K-map using:
 - ***bigger groupings of minterms*** (prime implicants) where possible; and
 - ***no redundant groupings*** (look for essential prime implicants)

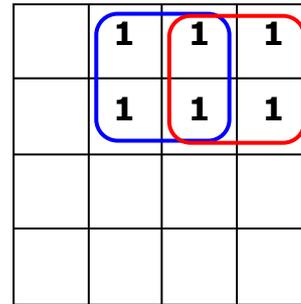
Implicant : a product term that could be used to cover minterms of the function.

Prime Implicant

- A **prime implicant** is a product term obtained by combining the **maximum possible number of minterms** from **adjacent** squares in the map.
- Use **bigger groupings** (prime implicants) where possible.



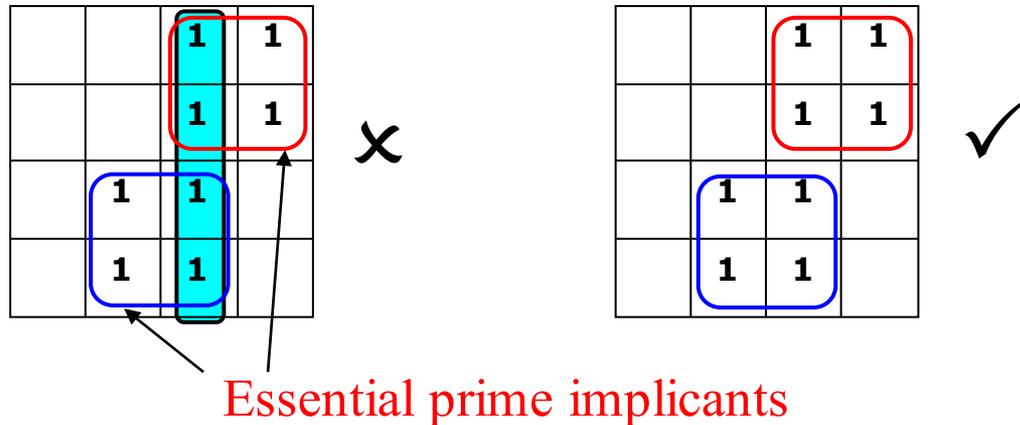
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Essential Prime Implicant

- No redundant groups:



- An **essential prime implicant** is a prime implicant that includes at least one minterm that is not covered by any other prime implicant.

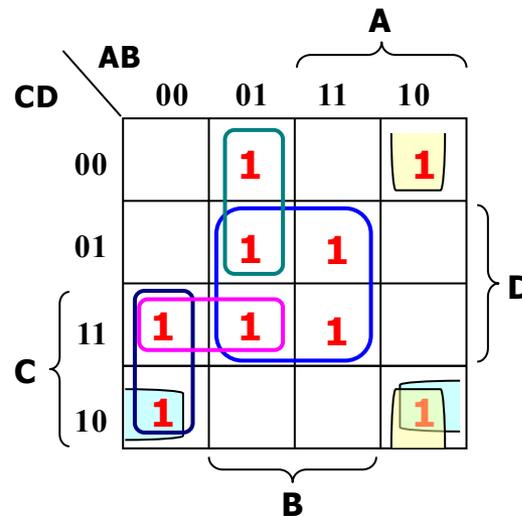
Algorithm

1. Circle all prime implicants on the K-map.
2. Identify and select all essential prime implicants for the cover.
3. Select a minimum subset of the remaining prime implicants to complete the cover, that is, to cover those minterms not covered by the essential prime implicants.

Example 1 (1)

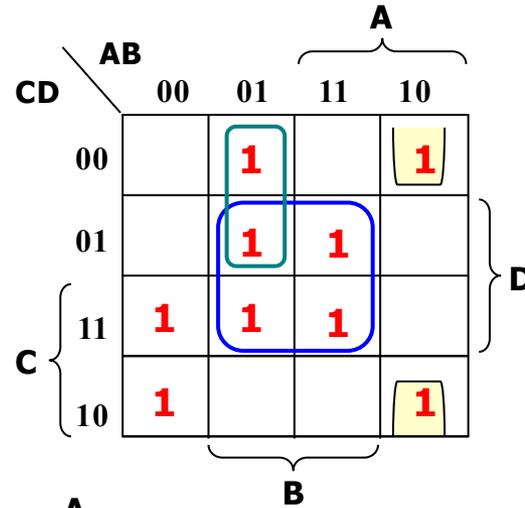
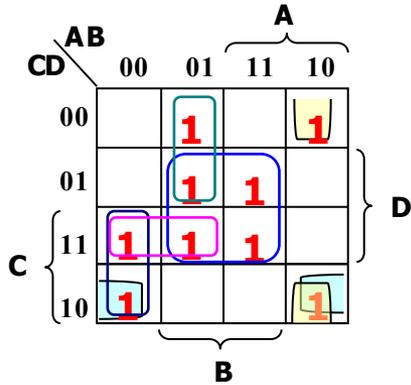
- Example:

$$f(A,B,C,D) = \sum m(2,3,4,5,7,8,10,13,15)$$

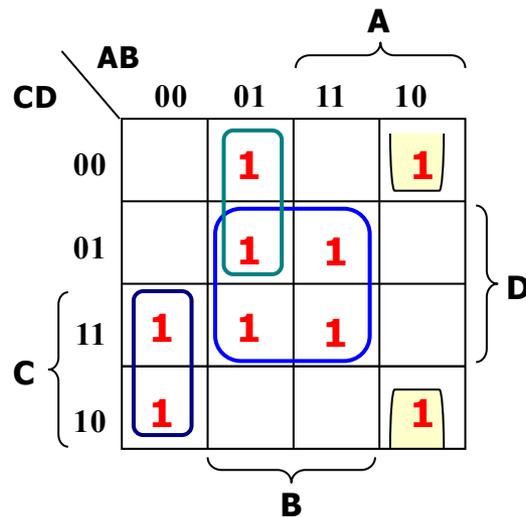


← All prime implicants

Example 1 (2)



← Essential prime implicants



← Minimum cover

Simplest POS Expressions

- **Simplified POS expression** can be obtained by grouping the maxterms (i.e., 0s) of given function.

Example 2 (1)

- Example:

Given $F = \sum m(0,1,2,3,5,7,8,9,10,11) \rightarrow$ we first draw the K-map, then group the maxterms together:

CD \ AB		A			
		00	01	11	10
C	00	1	0	0	1
	01	1	1	0	1
	11	1	1	0	1
	10	1	0	0	1

Example 2 (2)

CD		AB		A	
		00	01	11	10
C	00	1	0	0	1
	01	1	1	0	1
	11	1	1	0	1
	10	1	0	0	1

Groupings: A (columns 11, 10), B (columns 01, 10), D (rows 01, 11)

CD		AB		A	
		00	01	11	10
C	00	0	1	1	0
	01	0	0	1	0
	11	0	0	1	0
	10	0	1	1	0

Groupings: A (columns 11, 10), B (columns 01, 10), D (rows 01, 11)

- This gives the SOP of F' to be:

$$F' = BD' + AB$$

- To get POS of F , we have:

$$F = (BD' + AB)'$$

$$= (BD')'(AB)' \quad \text{DeMorgan}$$

$$= (B'+D)(A'+B') \quad \text{DeMorgan}$$