Thurs

Thursday, November 11, 2021 10:26 AM

Evaluate $\iint_D (x + 2y)\, dA$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.

Solution:

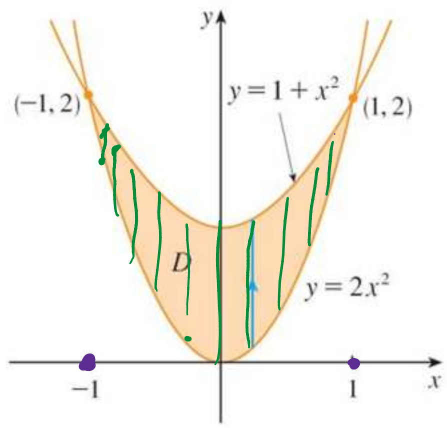
The parabolas intersect when $2x^2 = 1 + x^2$, that is, $x^2 = 1$, so $x = 1$.

$2x^2 = 1 + x^2 \Rightarrow x^2 = 1$

$x = \pm 1$

We note that the region D, sketched in Figure 8, is a type I region but not a type II region and we can write

$D = \{(x, y) \mid -1 \;\; x \;\; 1, 2x^2 \;\; y \;\; 1 + x^2\}$



$y_L = 2x^2, \quad y_u = 1 + x^2$

$$\int_{-1}^{1} \int_{2x^2}^{1+x^2} (x + 2y)\, dy\, dx$$

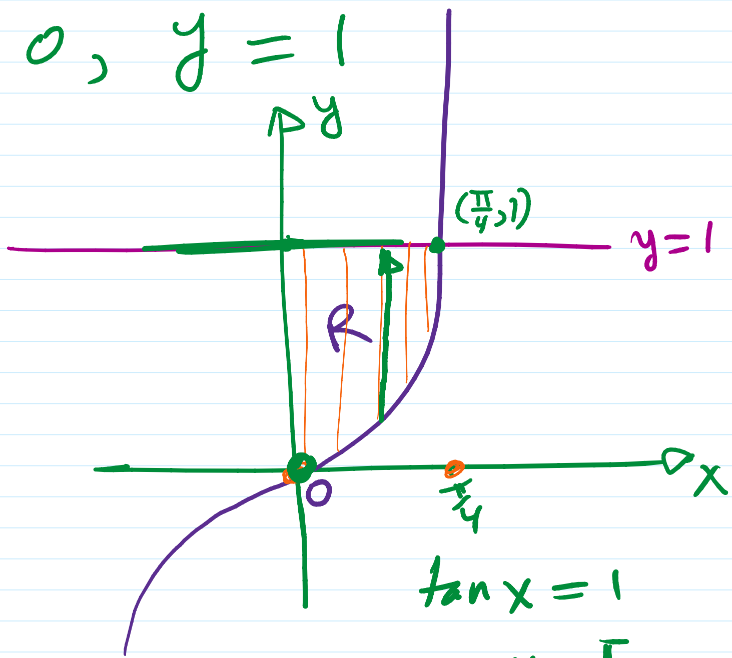
$x = \pm\sqrt{\frac{y}{2}}$

$x_L = -\sqrt{\frac{y}{2}}, \quad x_R = \sqrt{\frac{y}{2}}$

14/ $y = \tan x, \quad x = 0, \quad y = 1$

$$\int_0^1 \int_0^{\tan^{-1} y} f(x,y)\, dx\, dy$$

$$\int_0^{\frac{\pi}{4}} \int_{\tan x}^{1} f(x,y)\, dy\, dx$$



$\tan x = 1$

$x = \frac{\pi}{4}$

$\int_0 \int_{\tan x} \quad =$ $\quad \tan x = 1$

$x = \frac{\pi}{4}$

Finding Regions of Integration and Double Integrals

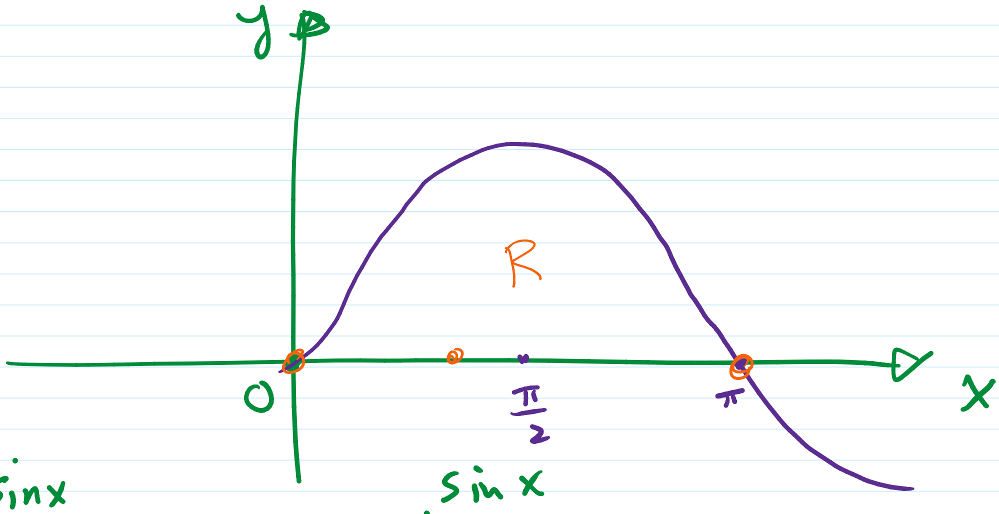
In Exercises 19–24, sketch the region of integration and evaluate t integral.

19. $\displaystyle\int_0^{\pi}\int_0^{x} x \sin y \, dy \, dx$ **20.** $\displaystyle\int_0^{\pi}\int_0^{\sin x} y \, dy \, dx$

21. $\displaystyle\int_1^{\ln 8}\int_0^{\ln y} e^{x+y} \, dx \, dy$ **22.** $\displaystyle\int_1^{2}\int_y^{y^2} dx \, dy$

23. $\displaystyle\int_0^{1}\int_0^{y^2} 3y^3 e^{xy} \, dx \, dy$ **24.** $\displaystyle\int_1^{4}\int_0^{\sqrt{x}} \frac{3}{2} e^{y/\sqrt{x}} \, dy \, dx$

$y = 0$ is the x-axis



$$\int_0^{\sin x} y \, dy = \frac{y^2}{2}\Big|_0^{\sin x} = \frac{1}{2}\sin^2 x$$

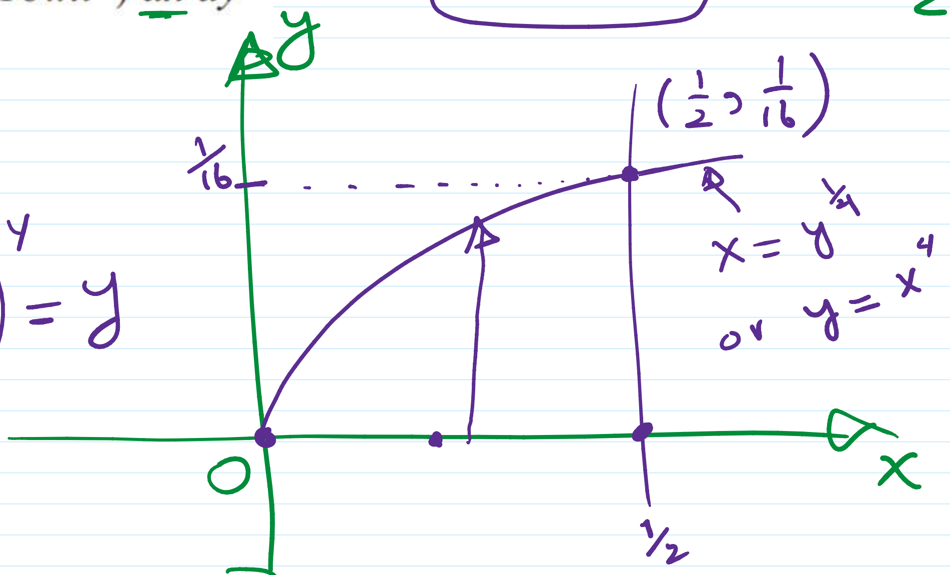
$$I = \frac{1}{2}\int_0^{\pi} \sin^2 x \, dx = \frac{1}{4}\int_0^{\pi}\left[1 - \cos 2x\right]dx$$

$$= \frac{1}{4}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi} = \frac{1}{4}\left[\pi - 0 - (0-0)\right]$$

$$= \frac{\pi}{4}$$

53. $\displaystyle\int_0^{1/16}\int_{y^{1/4}}^{1/2} \cos(16\pi x^5)\,dx\,dy$

$\boxed{x = y^{\frac{1}{4}}}$, $x = \frac{1}{2}$



$$\frac{1}{2} = y^{\frac{1}{4}} \Rightarrow \left(\frac{1}{2}\right)^4 = y$$

$$y = \frac{1}{16}$$

$$\int_0^{\frac{1}{2}}\left[\int_0^{x^4}\cos(16\pi x^5)\,dy\right]dx$$

$$I_{in} = \int_0^{x^4}\cos(16\pi x^5)\,dy = \cos(16\pi x^5)\,y\Big|_0^{x^4}$$

$$= x^4\cos(16\pi x^5)$$

$$I = \int_0^{\frac{1}{2}} x^4\cos(16\pi x^5)\,dx \quad , \quad u = 16\pi x^5$$

$$du = 80\pi x^4\,dx$$

$$\frac{du}{80\pi} = x^4\,dx$$

$$= \frac{1}{80\pi} \int_0^{} \cos u \, du$$

$$\frac{du}{80\pi} = x^4 dx$$

$$u\left(\frac{1}{2}\right) = 16\pi \left(\frac{1}{2}\right)^5$$