Chapter # 3

PROPERTIES OF PURE SUBSTANCES

Objectives

- Demonstrate understanding of key concepts including phase and pure substance, state principle for simple compressible systems, *p-v-T* surface, saturation temperature and saturation pressure, two-phase liquid-vapor mixture, quality, enthalpy, and specific heats.
- Sketch *T-v*, *p-v*, and phase diagrams, and locate states on these diagrams.
- Retrieve property data from Tables A-1 through A-23.
- Learn and apply the ideal and non-ideal gas model for thermodynamic analysis.
- Examine the moving boundary work or *P.dV* work commonly encountered in reciprocating devices such as automotive engines and compressors.

MOVING BOUNDARY WORK

Work done due to moving boundary

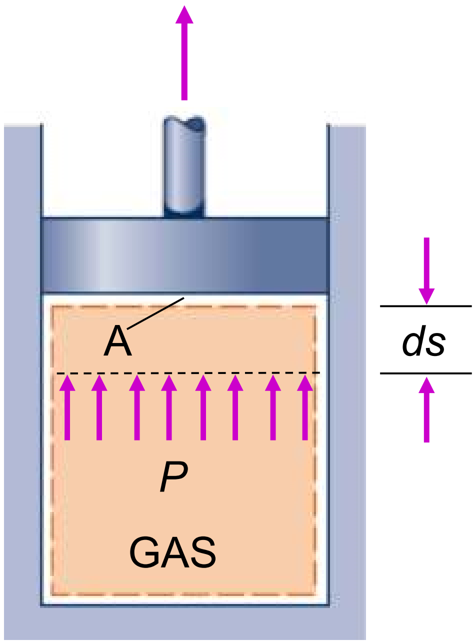
Work = Force × Displacement

Force F due to acting pressure $F = PA$

For a differential amount of displacement ds

Work done

$$\delta W = Fds = PAds \quad \text{Or} \quad \delta W = PdV$$



MOVING BOUNDARY WORK

Total Work Done

$$W_b = W_{12} = \int_1^2 P.dV$$

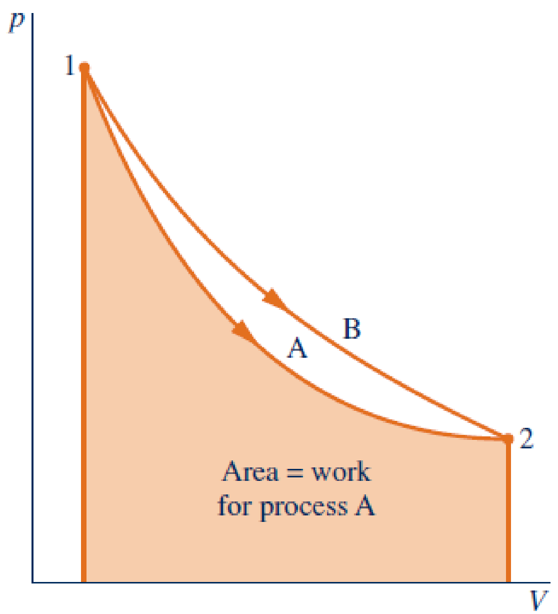
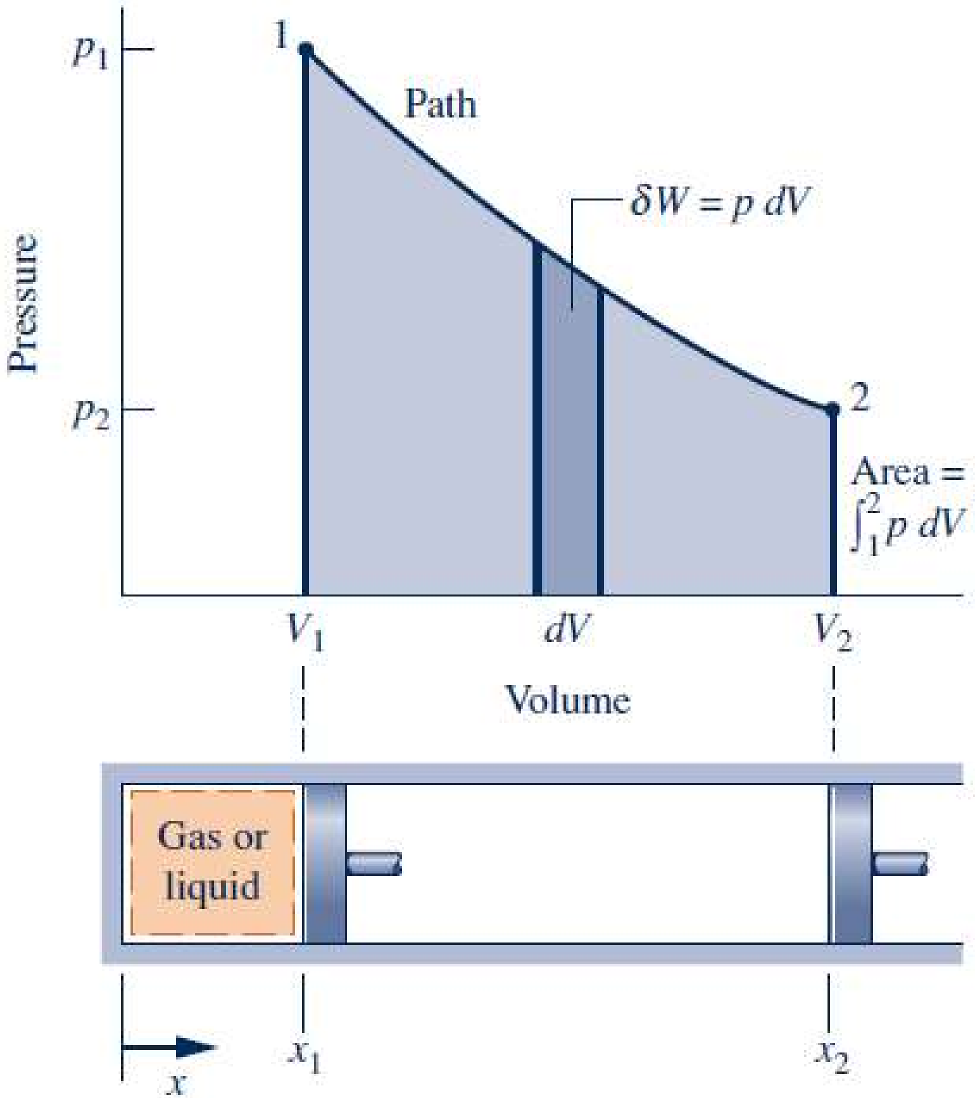


Fig. 2.8 Illustration that work depends on the process.

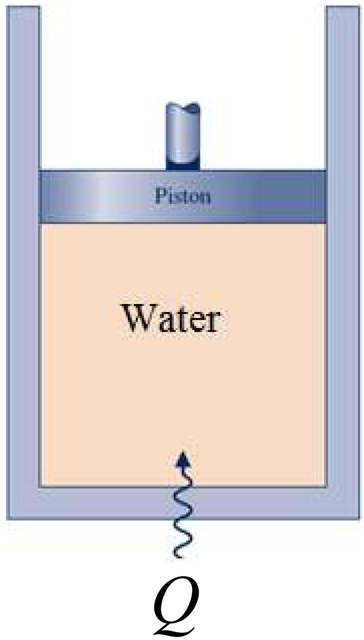


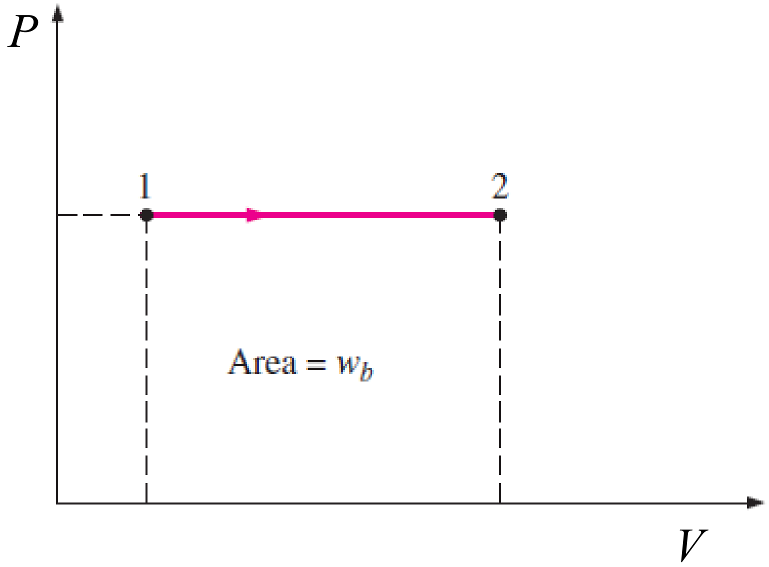
W_b is positive $\rightarrow$ for expansion

W_b is negative $\rightarrow$ for compression

WORK DONE FOR VARIOUS PROCESSES

Isobaric Process (Constant Pressure Process)



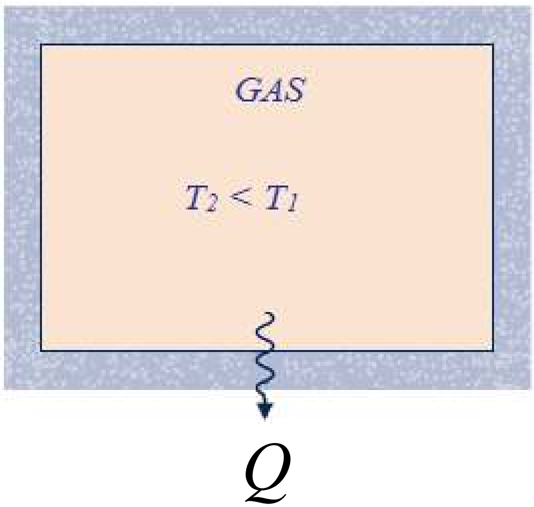


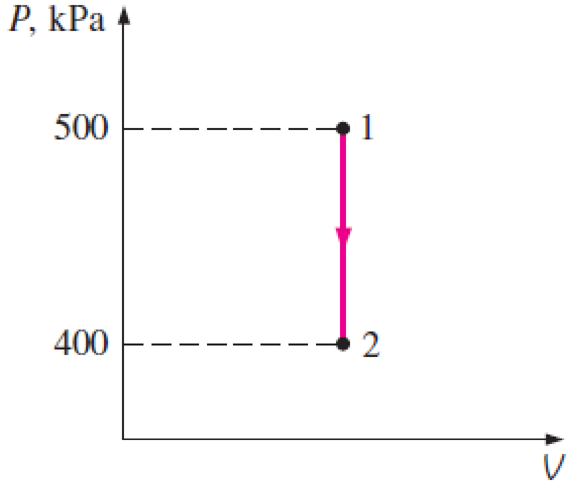
$$W_{12} = \int_1^2 P.dV$$ Area under the curve 1- 2 on the *PV* diagram

$$W_{12} = P(V_2 - V_1)$$

WORK DONE FOR VARIOUS PROCESSES

Isochoric Process (Constant Volume Process)



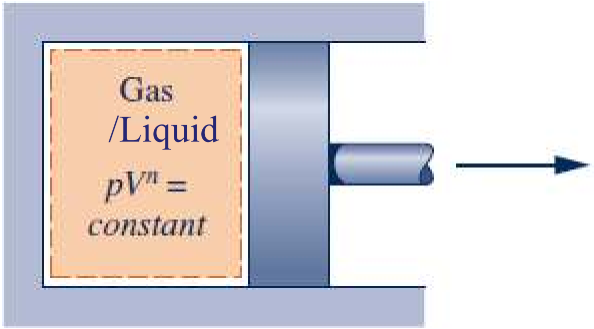


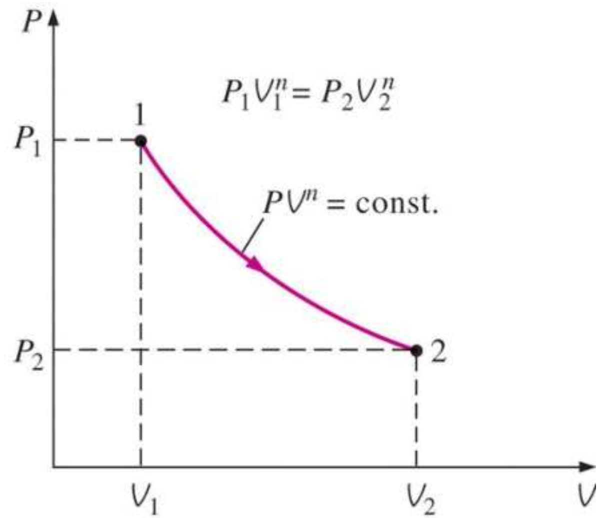
$$W_{12} = \int_1^2 P.dV$$ Area under the curve 1- 2 on the PV diagram

$$W_{12} = 0$$

WORK DONE FOR VARIOUS PROCESSES

Polytropic Process



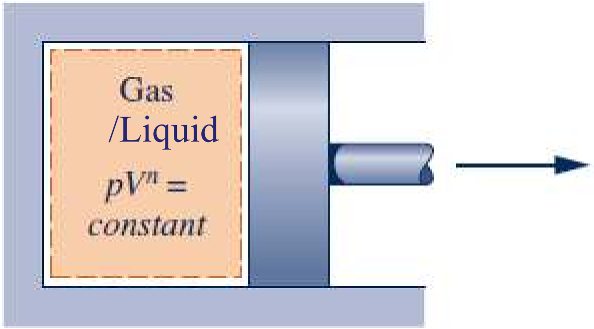


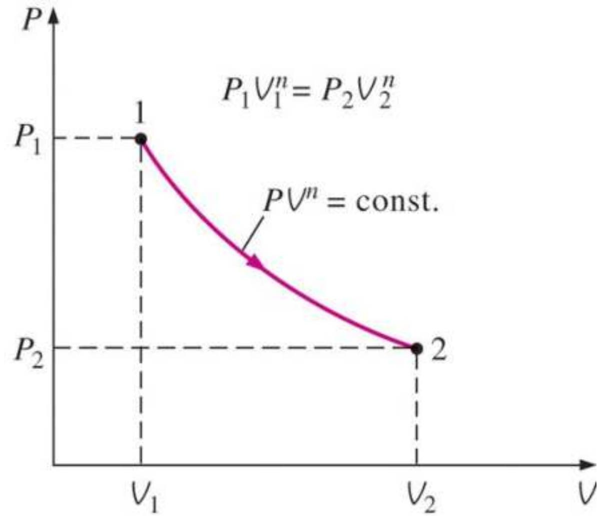
►The exponent, *n*, may take on any value from –∞ to +∞ depending on the particular process.

- ►For any gas (or liquid), when ***n* = 0**, the process is a constant-pressure (isobaric) process.
- ►For any gas (or liquid), when ***n* = ±∞**, the process is a constant-volume (isometric) process.
- ►For a gas modeled as an ideal gas, when *n* = 1, the process is a constant-temperature (isothermal) process.

WORK DONE FOR VARIOUS PROCESSES

Polytropic Process (n≠1)





$$W_{12} = \int_1^2 P.dV$$ Area under the curve 1- 2 on the PV diagram

$$PV^n = C \Rightarrow P = \frac{C}{V^n}$$

$$W_{12} = \int_1^2 C\frac{dV}{V^n} = C\int_1^2 V^{-n}dV$$

$$= C\left|\frac{V^{-n+1}}{-n+1}\right|_1^2 = C\left(\frac{V_2^{1-n} - V_1^{1-n}}{1-n}\right)$$

Now, $PV^n = C \Rightarrow C = P_1V_1^n = P_2V_2^n$

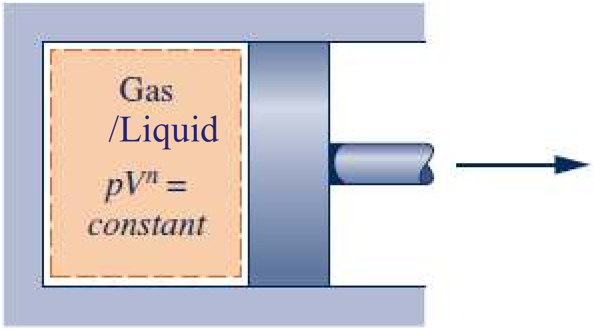
$$W_{12} = \frac{\left(P_2V_2^n\right)V_2^{-n+1} - \left(P_1V_1^n\right)V_1^{-n+1}}{1-n}$$

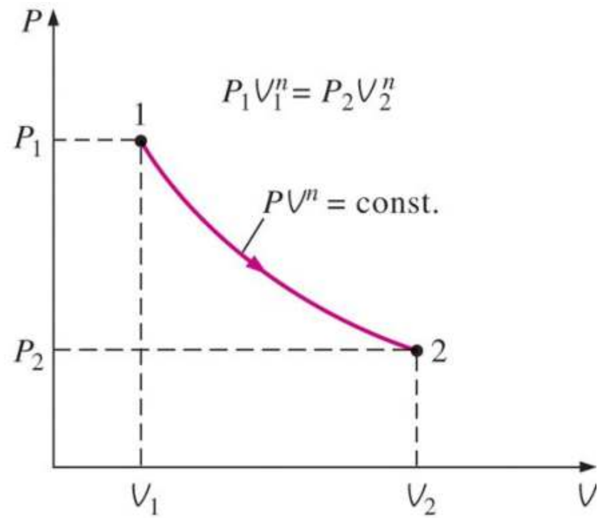
$$W_{12} = \frac{P_2V_2 - P_1V_1}{1-n}$$

$n \neq 1$

WORK DONE FOR VARIOUS PROCESSES

Polytropic Process (n=1)





$$W_{12} = \int_1^2 P.dV$$ Area under the curve 1- 2 on the PV diagram

$$PV = \text{Constant}(C)$$

$$PV = C \Rightarrow P = \frac{C}{V}$$

$$W_{12} = \int_1^2 C\frac{dV}{V} = C\ln\left(\frac{V_2}{V_1}\right)$$

$$P_1V_1 = P_2V_2 = C$$

$$W_{12} = C\ln\left(\frac{V_2}{V_1}\right)$$

$$W_{12} = C\ln\left(\frac{P_1}{P_2}\right)$$

WORK DONE FOR VARIOUS PROCESSES

Polytropic Process (Ideal gas case)

$$W_{12} = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{mR\left(T_2 - T_1\right)}{1-n}$$

$n \neq 1$

$$W_{12} = C\ln\left(\frac{V_2}{V_1}\right) = C\ln\left(\frac{P_1}{P_2}\right)$$

where $C = P_1V_1 = P_2V_2 = mRT_0$ $\quad n = 1$

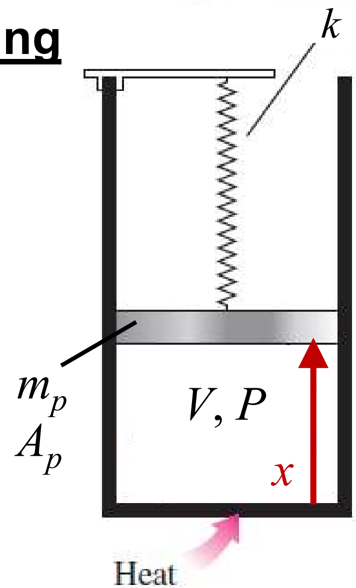
Combining $\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^n$ with the ideal gas equation, gives the following expressions:

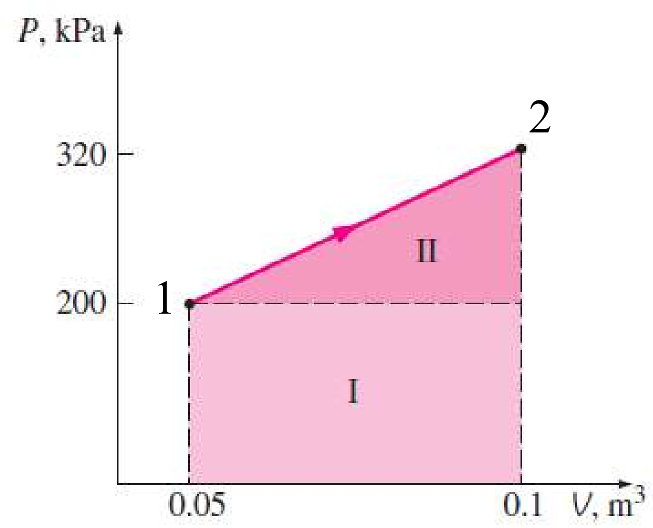
$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(n-1)/n} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

WORK DONE FOR VARIOUS PROCESSES

Work Done in a Piston-Cylinder-Spring

$$W_{12} = \int_1^2 P.dV$$





$$+\uparrow \sum F_y = 0$$

$$PA_p = P_{atm}A_p + m_p g + k(x - x_0)$$

x_0: piston position for a relaxed spring

$$P = P_{atm} + \frac{m_p g}{A_p} + \frac{k}{A_p}(x - x_0) \quad \text{or} \quad P = P_{atm} + \frac{m_p g}{A_p} + \frac{k}{A_p^2}(V - V_0) = C_1 + C_2 V$$

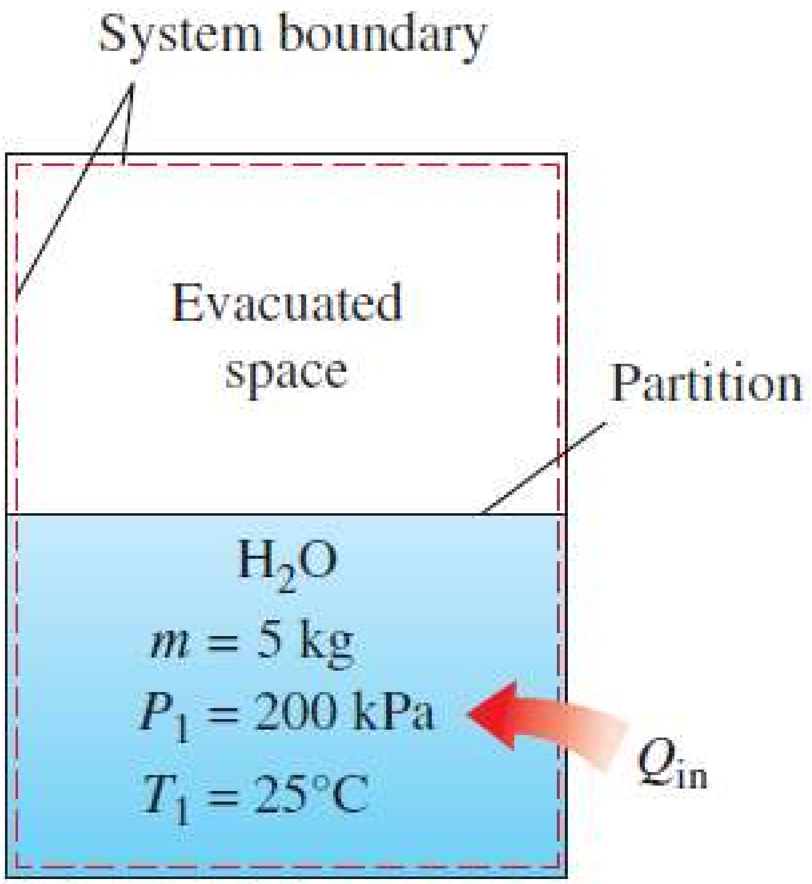
$$\Rightarrow C_2 = Slope = \frac{\Delta P}{\Delta V} = \frac{k}{A_p^2}$$

Area (of trapezoid) under the curve 1-2 on the PV diagram

$$W_{12} = \frac{(P_1 + P_2)}{2}(V_2 - V_1)$$

UNRESTRAINED EXPANSION

$$W_{12} = \int_{1}^{2} P.dV$$

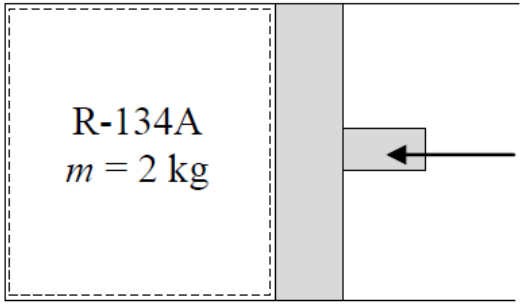


<u>During unrestrained expansion, there is no work exchanged with the surroundings</u>

Problem

2 kg of **Refrigerant 134A** undergoes a polytropic process in a piston–cylinder assembly from an initial state of saturated vapor at 200 kPa to a final state of 1200 kPa, 80 °C. *Determine the work for the process, in kJ.*

System: Closed, R-134a, W_{12}=?



State 1
P_1 = 200 kPa
m_1 = 2 kg
x_1 = 1

polytropic →

State 2
P_2 = 1200 kPa
$m_1 = m_2$
T_2 = 80 °C

$$W_{12} = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{m(P_2v_2 - P_1v_1)}{1-n}$$

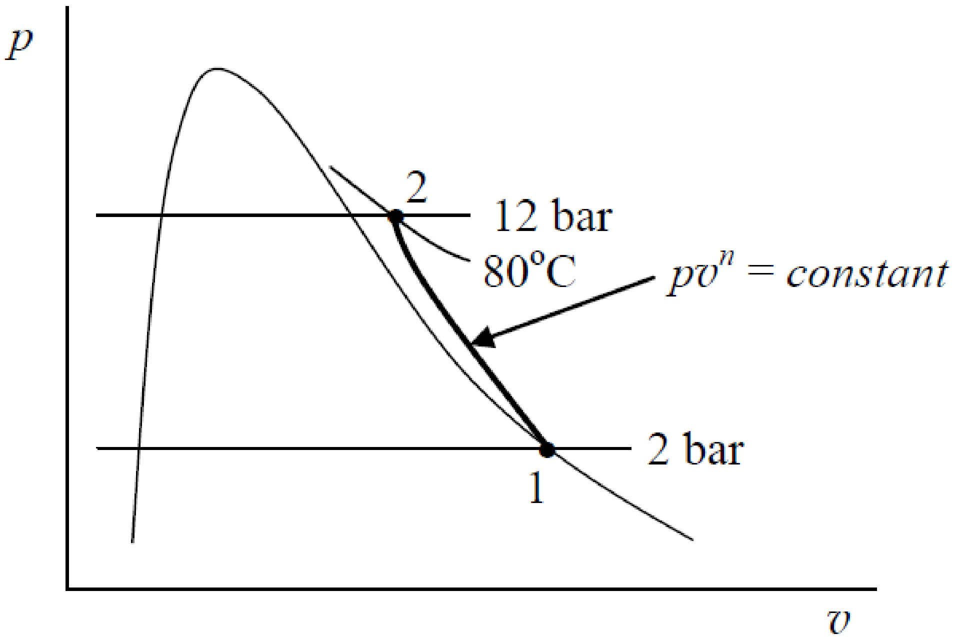
$$v_1 = v_g = 0.0993\, m^3/\text{kg} \quad ; \qquad v_2 = 0.02051\, m^3/\text{kg}$$

$$P_1v_1^n = P_2v_2^n \quad \Rightarrow \frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^n \quad \Rightarrow n = \frac{\ln\left(P_1/P_2\right)}{\ln\left(v_2/v_1\right)}$$

$$\Rightarrow n = \frac{\ln\left(2/12\right)}{\ln\left(0.02051/0.0993\right)} = 1.136$$

Problem

$$W_{12} = \frac{2(1200(0.02051) - 200(0.0993))}{1-n} = -69.88\ kJ$$



Problem

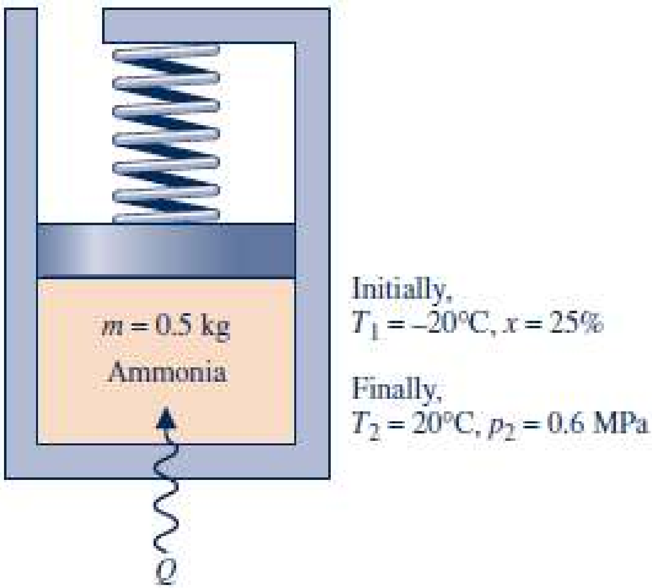
As shown in Fig., 0.5 kg of **Ammonia** is contained in a piston–cylinder assembly, initially at -20 °C and a quality of 25%. As the refrigerant is slowly heated to a final state of 120 °C, 0.6 MPa, its pressure varies linearly with specific volume. *For the refrigerant, (a) show the process on a sketch of the P–v diagram and (b) evaluate the work transfer, in kJ.*

System: Closed, Ammonia, W_{12}=?

State 1
T_1 = -20 °C
m_1 = 0.5 kg
x_1 = 0.25

Linear spring →

State 2
P_2 = 600 kPa
$m_1 = m_2$
T_2 = 120 °C



$$W_{12} = \frac{(P_1 + P_2)}{2}(V_2 - V_1) = m\frac{(P_1 + P_2)}{2}(v_2 - v_1)$$

$$v_1 = v_f + x_1(v_g - v_f) = 0.0015038 + 0.25(0.6233 - 0.0015038) = 0.157\ m^3/\text{kg}$$

$$v_2 = 0.22155\ m^3/\text{kg} \quad ; \qquad P_1 = P_{sat} = 190.19\ kPa$$

$$W_{12} = 0.5\frac{(190.19 + 600)}{2}(0.22155 - 0.157) = \boxed{12.75\ kJ}$$

Problem

Air is contained in a piston–cylinder assembly, initially at 275.8 kPa and 333 K. The air expands in a polytropic process with *n* = 1.4 until the volume is doubled. Modeling the air as an ideal gas, *determine (a) the final temperature, in Kelvin, and pressure, in kPa, and (b) the work transfer, in kJ/kg of air.*

System: Closed, Air, w_{12}=?, T_2 =? , P_2 =?

State 1		State 2
$T_1 = 333$ K	Polytropic(n=1.4)	$V_2 = 2V_1$
$P_1 = 275.8$ kPa	Ideal gas	$m_1 = m_2$

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^n \Rightarrow P_2 = 275.8\left(\frac{1}{2}\right)^{1.4} = \boxed{104.5\ kPa}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} \Rightarrow T_2 = 333\left(\frac{1}{2}\right)^{1.4-1} = \boxed{252.4\ K}$$

Problem

$$W_{12} = \frac{P_2V_2 - P_1V_1}{1-n} \quad \Rightarrow w_{12} = \frac{(P_2v_2 - P_1v_1)}{1-n} = \frac{(2P_2v_1 - P_1v_1)}{1-n} = \frac{v_1(2P_2 - P_1)}{1-n}$$

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287(333)}{275.8} = 0.3465\ m^3/kg$$

$$\Rightarrow w_{12} = \frac{(0.3465)(2(104.5) - 275.8)}{1-1.4}$$

$$\Rightarrow w_{12} = \boxed{57.86\ kJ/kg}$$

SPECIFIC HEAT

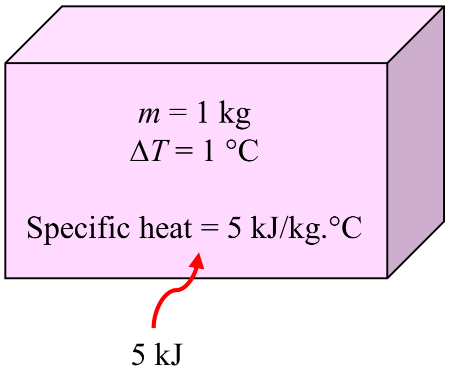
Energy required to raise the temperature of 1 kg of a substance by one degree.

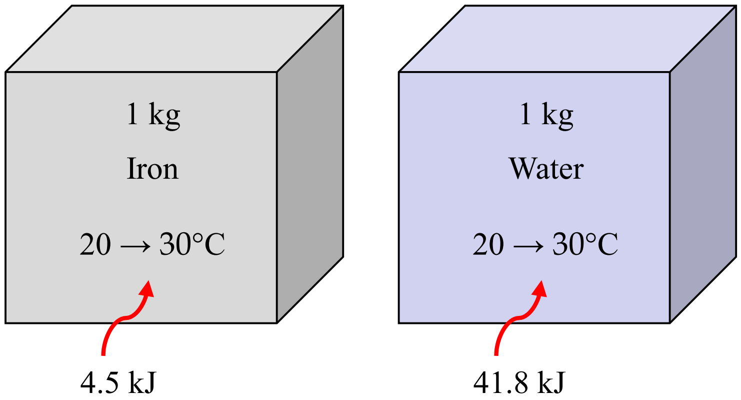
Represents the energy storage capability of a substance.

Units: J/kg/°C or J/kg/K

It is defined when there is no phase change i.e. only solid, only liquid or only vapor

It is a property.





It takes different amounts of energy to raise the temperature of different substances by the same amount.

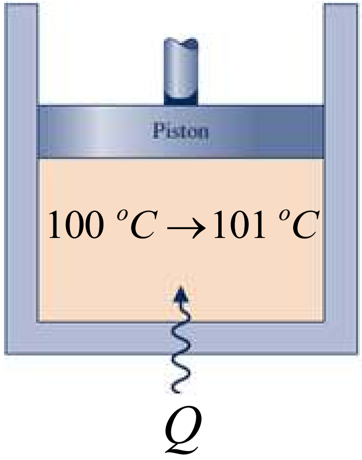
SPECIFIC HEAT OF A GAS

Specific Heat at Constant Volume (c_v)

Energy required to raise the temperature of 1kg of a (pure) substance by one degree at CONSTANT VOLUME.

At Constant Volume → Only internal energy increases, gas does not expand.

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

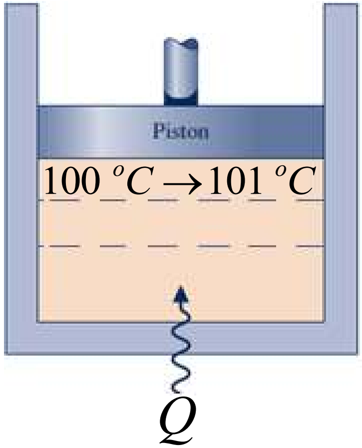


Specific Heat at Constant Pressure (c_p)

Energy required to raise the temperature of 1kg of a (pure) substance by one degree at CONSTANT PRESSURE.

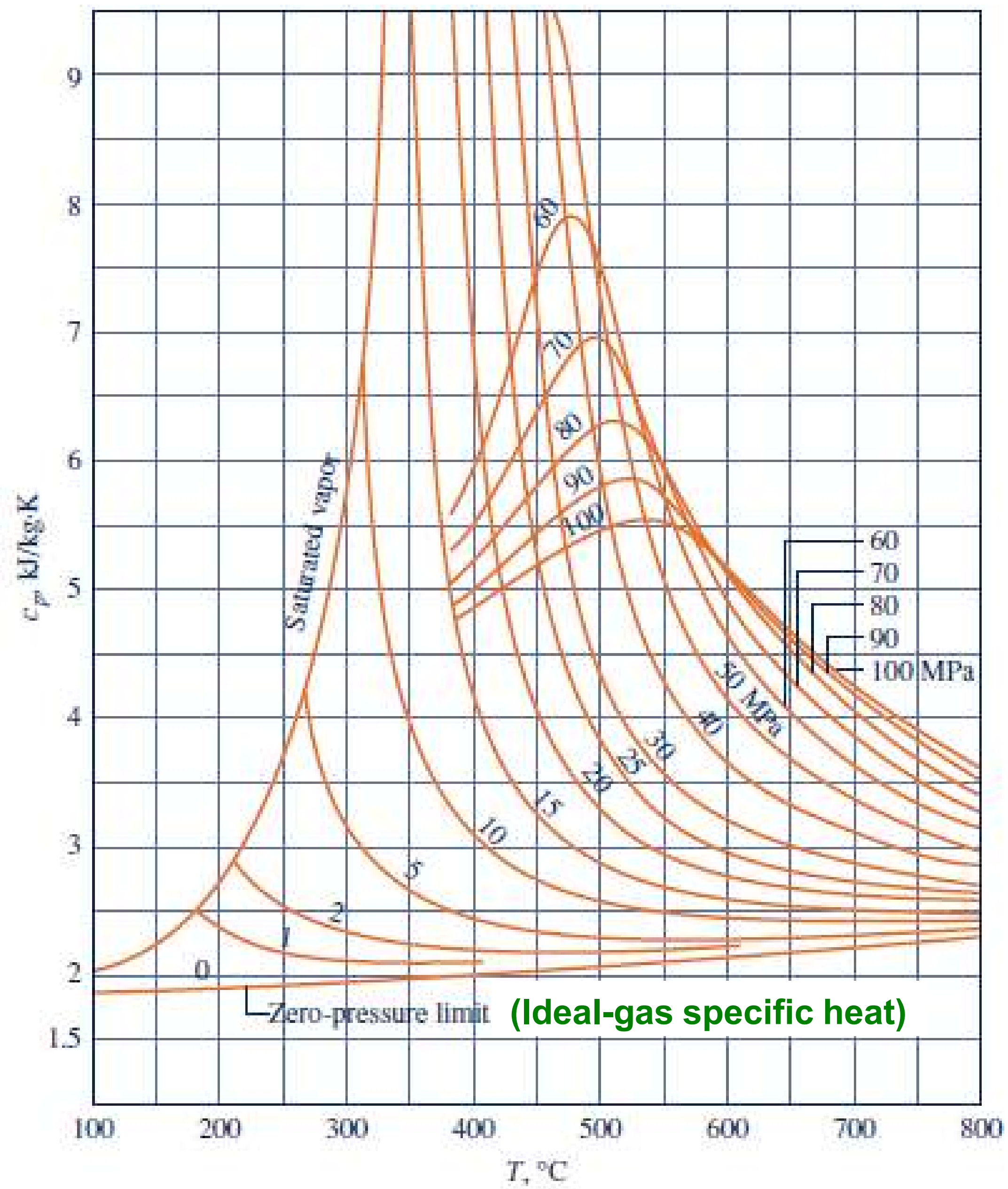
At Constant pressure → Internal energy increases, as well as the gas expands and does work.

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p$$



SPECIFIC HEAT OF A GAS

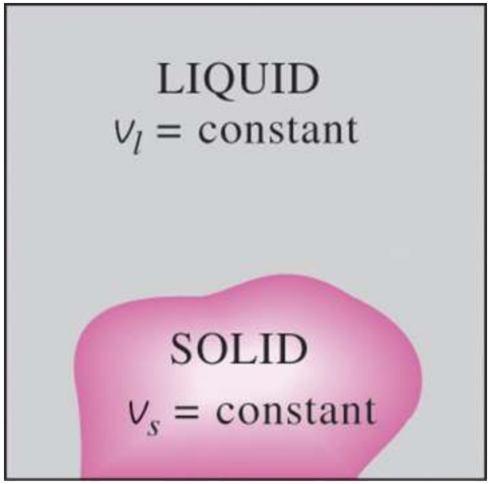
- c_p is always greater than c_v
- c_v and c_p are properties. Therefore, the equations are valid for *any* substance undergoing *any* process.
- c_v is related to the changes in *internal energy* and c_p to the changes in *enthalpy*.
- Specific heat ratio, $k = \dfrac{c_p}{c_v}$
  - Varies mildly with temperature.
  - For monatomic gases (helium, argon, etc.), its value is essentially constant at 1.667.
  - Many diatomic gases, including air, have a specific heat ratio of about 1.4 *at room temperature*.



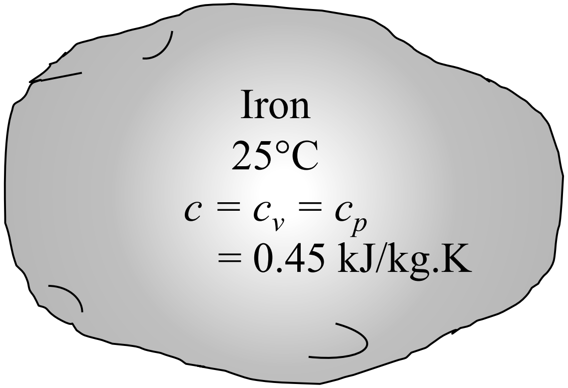
Internal Energy, Enthalpy, and Specific Heats of Solids and Liquids

Incompressible substance:
A substance whose specific volume (or density) is constant.
Solids and liquids are considered as ***incompressible substances***.
Internal energy is assumed to vary only with temperature.

The specific volumes of incompressible substances remain constant during a process.



The c_v and c_p values of incompressible substances are identical and are denoted by c.



$$h(T, p) = u(T) + pv$$

Differentiating with respect to temperature while holding pressure fixed to obtain

$$\left(\frac{\partial h}{\partial T}\right)_p = \frac{du}{dT}$$

$$c_p = c_v$$

Table A-19 contains specific heat values for some liquids and solids

Internal Energy, Enthalpy, and Specific Heats of Solids and Liquids

TABLE A-19

Properties of Selected Solids and Liquids: c_p, ρ, and κ

Substance	Specific Heat, c_p (kJ/kg · K)	Density, ρ (kg/m^3)	Thermal Conductivity, κ (W/m · K)
Selected Solids, 300K			
Aluminum	0.903	2700	237
Coal, anthracite	1.260	1350	0.26
Copper	0.385	8930	401
Granite	0.775	2630	2.79
Iron	0.447	7870	80.2
Lead	0.129	11300	35.3
Sand	0.800	1520	0.27
Silver	0.235	10500	429
Soil	1.840	2050	0.52
Steel (AISI 302)	0.480	8060	15.1
Tin	0.227	7310	66.6
Saturated Liquids			
Ammonia, 300K	4.818	599.8	0.465
Mercury, 300K	0.139	13529	8.540
Refrigerant 22, 300K	1.267	1183.1	0.085
Refrigerant 134a, 300K	1.434	1199.7	0.081
Unused Engine Oil, 300K	1.909	884.1	0.145
Water, 275K	4.211	999.9	0.574
300K	4.179	996.5	0.613
325K	4.182	987.1	0.645
350K	4.195	973.5	0.668
375K	4.220	956.8	0.681
400K	4.256	937.4	0.688

Internal Energy, Enthalpy, and Specific Heats of Solids and Liquids

Internal Energy Changes

$$du = c_v\, dT = c(T)\, dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c(T)\, dT \qquad (\text{kJ/kg})$$

$$\Delta u \cong c_{avg}\left(T_2 - T_1\right) \qquad \left(kJ/kg\right)$$

$$c_v = \frac{\int_{T_1}^{T_2} c_v(T)\, dT}{T_2 - T_1},$$

Enthalpy Changes

$$h = u + Pv$$

$$dh = du + v\, dP + P\, \overset{\nearrow 0}{dv} = du + v\, dP$$

$$\Delta h = \Delta u + v\Delta P \cong c_{avg}\Delta T + v\Delta P \qquad \left(kJ/kg\right)$$

For *solids*, the term $v\,\Delta P$ is insignificant and thus $\Delta h = \Delta u \cong c_{\text{avg}}\Delta T$.

For *liquids*, two special cases are commonly encountered:

1. *Constant-pressure processes*, as in heaters ($\Delta P = 0$): $\Delta h = \Delta u \cong c_{\text{avg}}\Delta T$
2. *Constant-temperature processes*, as in pumps ($\Delta T = 0$): $\Delta h = v\,\Delta P$

The enthalpy of a compressed liquid

$$h_{@P,T} \cong h_{f\,@\,T} + v_{f\,@\,T}(P - P_{\text{sat}\,@\,T})$$

A more accurate relation than $h_{@\,P,T} \cong h_{f\,@\,T}$

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

Joule showed experimentally that $u = u(T)$

For ideal gases, u, h, c_v, and c_p vary with temperature only.

$$u = u(T)$$
$$h = h(T)$$
$$c_v = c_v(T)$$
$$c_p = c_p(T)$$

Internal energy and enthalpy change of an ideal gas:

▶ $\boldsymbol{u = u(T)}$

$$du = c_v(T)\, dT$$

▶ $\boldsymbol{h = h(T)}$

$$dh = c_p(T)\, dT$$

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

$$\left.\begin{array}{l} h = u + Pv \\ Pv = RT \end{array}\right\} \Rightarrow h = u + RT \quad \Rightarrow dh = du + R.dT$$

$$but \quad dh = c_p dT \quad ; \quad du = c_v dT$$

The relationship between c_p, c_v and R

$$c_p(T) = c_v(T) + R$$

On a molar basis

$$\bar{c}_p(T) = \bar{c}_v(T) + \bar{R}$$

Keeping definition of ***k*** in mind,

$$c_p(T) = \frac{kR}{k-1}$$

$$c_v(T) = \frac{R}{k-1}$$

$\bar{R} \equiv$ Universal gas constant

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

$$du = c_v(T)\, dT \qquad dh = c_p(T)\, dT$$

$$\Delta u = u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T)dT \qquad \Delta h = h(T_2) - h(T_1) = \int_{T_1}^{T_2} c_p(T)dT$$

$$\Delta u \cong c_{v,avg} \Delta T$$ → (**Tables A-20**)

$$\Delta u = \int_1^2 c_v(T)\, dT$$ → (**Tables A-21**)

$$\Delta u = u_2 - u_1$$ → (**Tables A-22** to **A-23**)

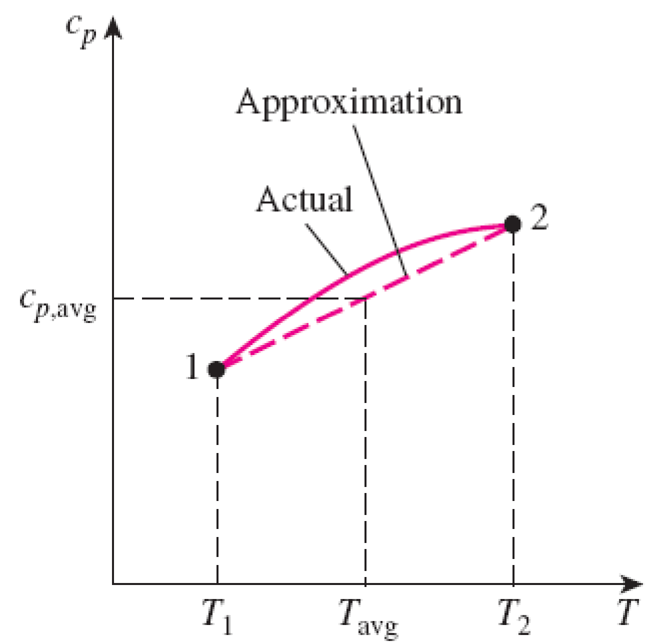
Three ways of calculating Δu. Similarly, for Δh.

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

Internal energy and enthalpy change when specific heat is taken constant at an average value

$$\Delta u = u_2 - u_1 \cong c_{v,avg}\left(T_2 - T_1\right) \qquad \left(kJ/kg\right)$$

$$\Delta h = h_2 - h_1 \cong c_{p,avg}\left(T_2 - T_1\right) \qquad \left(kJ/kg\right)$$



For small temperature intervals, the specific heats may be assumed to vary linearly with temperature.

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

TABLE A-20

Ideal Gas Specific Heats of Some Common Gases (kJ/kg · K)

Temp. K	Air c_p	Air c_v	Air k	Nitrogen, N_2 c_p	Nitrogen, N_2 c_v	Nitrogen, N_2 k	Oxygen, O_2 c_p	Oxygen, O_2 c_v	Oxygen, O_2 k	Temp. K
250	1.003	0.716	1.401	1.039	0.742	1.400	0.913	0.653	1.398	250
300	1.005	0.718	1.400	1.039	0.743	1.400	0.918	0.658	1.395	300
350	1.008	0.721	1.398	1.041	0.744	1.399	0.928	0.668	1.389	350
400	1.013	0.726	1.395	1.044	0.747	1.397	0.941	0.681	1.382	400
450	1.020	0.733	1.391	1.049	0.752	1.395	0.956	0.696	1.373	450
500	1.029	0.742	1.387	1.056	0.759	1.391	0.972	0.712	1.365	500
550	1.040	0.753	1.381	1.065	0.768	1.387	0.988	0.728	1.358	550
600	1.051	0.764	1.376	1.075	0.778	1.382	1.003	0.743	1.350	600
650	1.063	0.776	1.370	1.086	0.789	1.376	1.017	0.758	1.343	650
700	1.075	0.788	1.364	1.098	0.801	1.371	1.031	0.771	1.337	700
750	1.087	0.800	1.359	1.110	0.813	1.365	1.043	0.783	1.332	750
800	1.099	0.812	1.354	1.121	0.825	1.360	1.054	0.794	1.327	800
900	1.121	0.834	1.344	1.145	0.849	1.349	1.074	0.814	1.319	900
1000	1.142	0.855	1.336	1.167	0.870	1.341	1.090	0.830	1.313	1000

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

TABLE A-21

Variation of $\bar{c}_p$ with Temperature for Selected Ideal Gases

$$\frac{c_p}{R} = \frac{\bar{c}_p}{\bar{R}} = \alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4$$

T is in K, equations valid from 300 to 1000 K

Gas	α	$\beta \times 10^3$	$\gamma \times 10^6$	$\delta \times 10^9$	$\varepsilon \times 10^{12}$
CO	3.710	−1.619	3.692	−2.032	0.240
CO_2	2.401	8.735	−6.607	2.002	0
H_2	3.057	2.677	−5.810	5.521	−1.812
H_2O	4.070	−1.108	4.152	−2.964	0.807

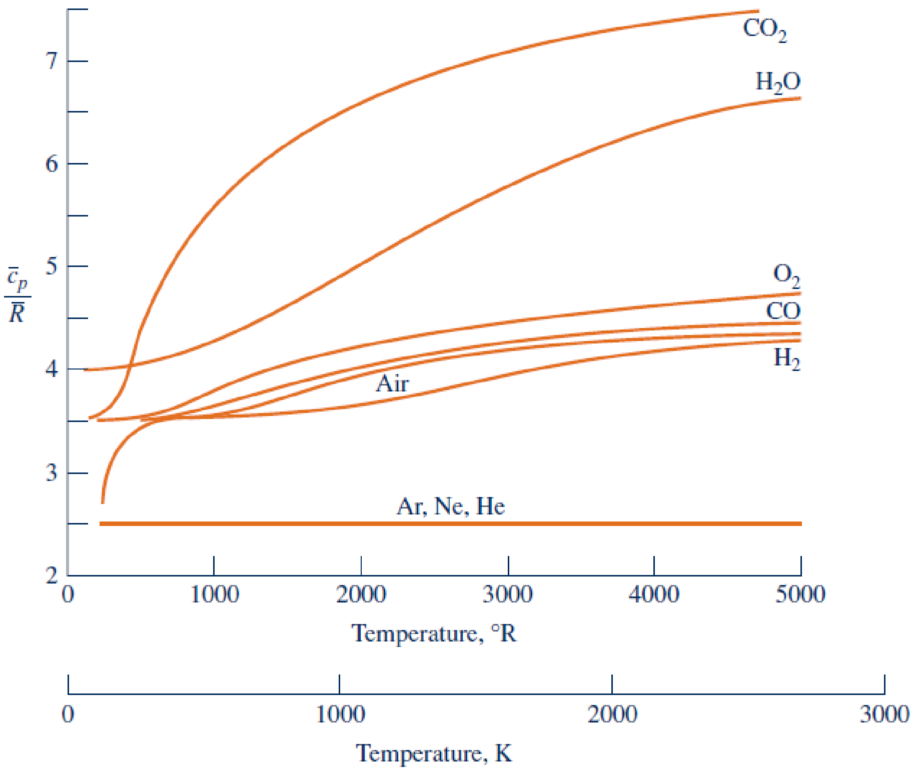
$$\Delta h = \int_{T_1}^{T_2} c_p(T)dT = R\int_{T_1}^{T_2}\left(\alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4\right)dT$$

Now, $c_v = c_p - R \quad \Rightarrow \Delta u = \int_{T_1}^{T_2}(c_p - R)dT$

$$\Delta u = R\int_{T_1}^{T_2}\left[\left(\alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4\right) - 1\right]dT$$

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

- At low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only.
- The specific heats of real gases at low pressures are called *ideal-gas specific heats*, or *zero-pressure specific heats*, and are often denoted c_{p0} and c_{v0}.



- *u* and *h* data for a number of gases have been tabulated (**Tables A-22** to **A-23**).
- These tables are obtained by choosing an arbitrary reference point and performing the integrations by treating state 1 as the reference state.

Air

T, K	u, kJ/kg	h, kJ/kg
0	0	0
.	.	.
.	.	.
300	214.07	300.19
310	221.25	310.24
.	.	.
.	.	.

In the preparation of ideal-gas tables, 0 K is chosen as the reference temperature.

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

TABLE A-22

Ideal Gas Properties of Air

T(K), h and u(kJ/

				when $\Delta s = 0$[1]	
T	h	u	s°	p_r	v_r
200	199.97	142.56	1.29559	0.3363	1707.
210	209.97	149.69	1.34444	0.3987	1512.
220	219.97	156.82	1.39105	0.4690	1346.
230	230.02	164.00	1.43557	0.5477	1205.
240	240.02	171.13	1.47824	0.6355	1084.
250	250.05	178.28	1.51917	0.7329	979.
260	260.09	185.45	1.55848	0.8405	887.8
270	270.11	192.60	1.59634	0.9590	808.0
280	280.13	199.75	1.63279	1.0889	738.0
285	285.14	203.33	1.65055	1.1584	706.1
290	290.16	206.91	1.66802	1.2311	676.1
295	295.17	210.49	1.68515	1.3068	647.9
300	300.19	214.07	1.70203	1.3860	621.2
305	305.22	217.67	1.71865	1.4686	596.0
310	310.24	221.25	1.73498	1.5546	572.3

Problem

Determine the enthalpy change, Δh, of nitrogen, in kJ/kg, as it is heated from 600 K to 1000 K, using *(a) the c_p value at the average temperature, (b) tabulated data of enthalpy, (c) the empirical specific heat equation as a function of temperature.*

Solution: *(a)* T_{avg} = 800 K $\Rightarrow c_{p@800K} = 1.121\, kJ/kg.K$

$$\Delta h = c_{p,avg}\left(T_2 - T_1\right) = 1.121(1000-600) = \boxed{448.4\ \text{kJ/kg}}$$

(**Tables A-20**)

(b) $\bar{h}_{600} = 17563\ \text{kJ/kmol}\ ;\ \bar{h}_{1000} = 30129\ \text{kJ/kmol} \quad \Rightarrow \Delta\bar{h} = 12566\ \text{kJ/kmol}$

(**Tables A-23**)

$$\Delta h = \frac{\Delta \bar{h}}{M_{N_2}} = \frac{12566}{28.01} = \boxed{448.6\, kJ/kg}$$

(**Tables A-1**)

(c) $R_{N_2} = 0.297\, kJ/kg.K$ (Can be determined from **Tables A-20**)

$$\Delta h = \int_{T_1}^{T_2} c_p(T)\,dT = R_{N_2}\int_{T_1}^{T_2}\left(\alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4\right)dT$$

Problem

$$\Delta h = R_{N_2}\left[\alpha T + \beta\frac{T^2}{2} + \gamma\frac{T^3}{3} + \delta\frac{T^4}{4} + \varepsilon\frac{T^5}{5}\right]_1^2$$

$$\Delta h = R_{N_2}\left[\alpha(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) + \frac{\gamma}{3}(T_2^3 - T_1^3) + \frac{\delta}{4}(T_2^4 - T_1^4) + \frac{\varepsilon}{5}(T_2^5 - T_1^5)\right]$$

(**Tables A-21**)

Gas	α	$\beta \times 10^3$	$\gamma \times 10^6$	$\delta \times 10^9$	$\varepsilon \times 10^{12}$
N_2	3.675	−1.208	2.324	−0.632	−0.226

$$\Delta h = \boxed{448.9\ kJ/kg}$$