Chapter # 4

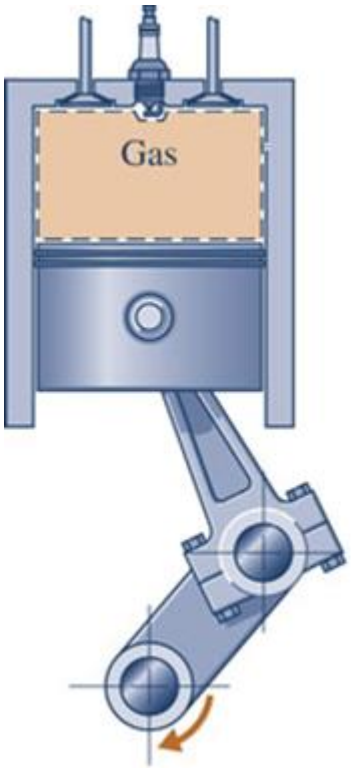
Control Volume Analysis Using Energy

OBJECTIVES

- Learn to distinguish between mass flow rate and volume flow rate, steady-state and transient analysis.
- Apply mass balances to control volumes.
- Develop appropriate engineering models for commonly encountered components in practice.
- Apply the first law of thermodynamics to control volume.

Closed & Open Systems

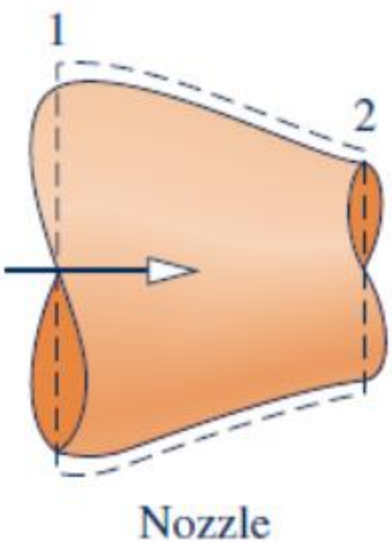
A Closed System (Control Mass)



Mass ✗

Energy ✓

An Open System (Control Volume)



Mass ✓

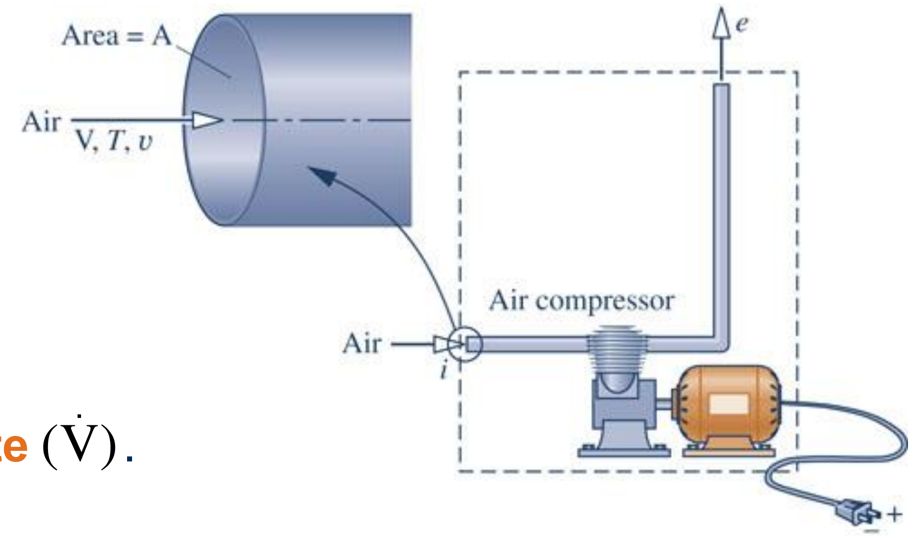
Energy ✓

Mass and Volume Flow Rates

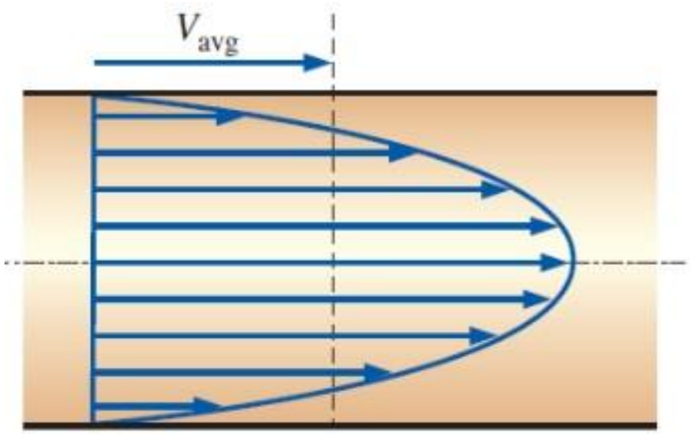
All intensive properties are ***uniform with position*** over each inlet or exit area ($\mathbf{A}$) through which matter flows

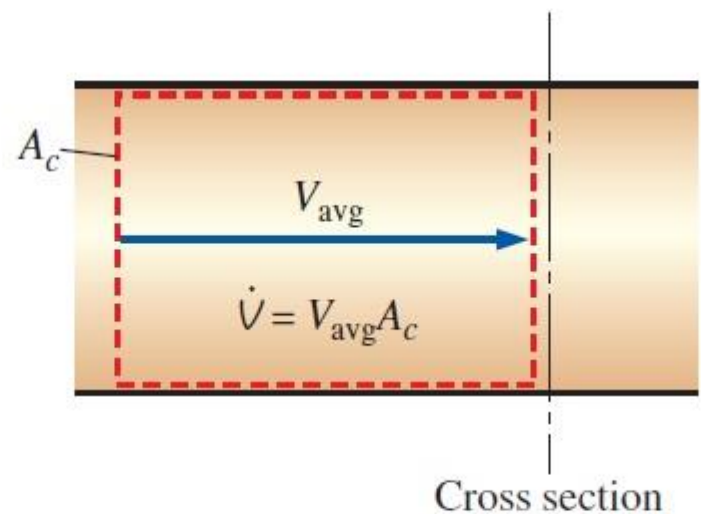
$$\dot{m} = \int_{A} \rho V_n \, dA$$

$$\dot{m} = \rho A V = \frac{AV}{v} \quad \text{where} \quad V = V_{avg}$$



The product ***AV*** is the **volumetric flow rate** $(\dot{V})$.
It is expressed in units of m^3/s.





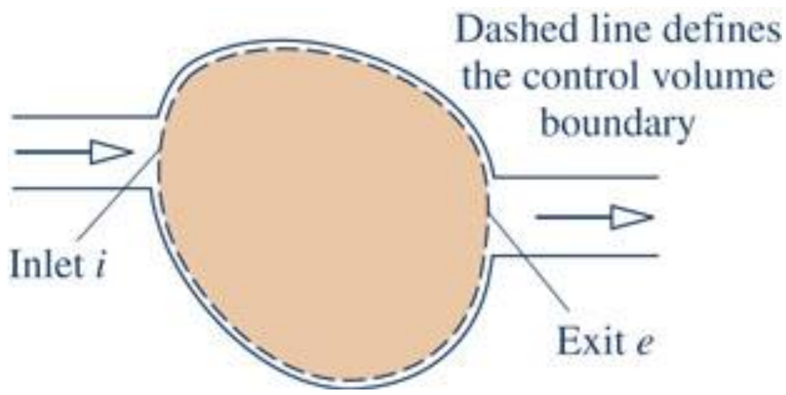
Conservation of Mass Principle

Conservation of mass: Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.

The conservation of mass principle for a control volume:

$$\begin{bmatrix} \text{time } \textit{rate of change} \text{ of} \\ \text{mass contained within the} \\ \text{control volume } \textit{at time t} \end{bmatrix} = \begin{bmatrix} \text{time } \textit{rate of flow} \text{ of} \\ \text{mass } \textit{in} \text{ across} \\ \text{inlet } \textit{i at time t} \end{bmatrix} - \begin{bmatrix} \text{time } \textit{rate of flow} \\ \text{of mass } \textit{out} \text{ across} \\ \text{exit } \textit{e at time t} \end{bmatrix}$$

$$\frac{dm_{\mathrm{cv}}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$



For multiple inlets and outlets:

$$\frac{dm_{\mathrm{cv}}}{dt} = \sum_{in} \dot{m}_{in} - \sum_{out} \dot{m}_{out}$$

For steady-state control volume

$$\sum_{in} \dot{m}_{in} = \sum_{out} \dot{m}_{out}$$

For single stream:

$$\dot{m}_{in} = \dot{m}_{out} \quad \rightarrow \quad \rho_{in} \mathrm{A}_{in} \mathrm{V}_{in} = \rho_{out} \mathrm{A}_{out} \mathrm{V}_{out}$$

Conservation of Mass Principle

For a process involving an incompressible substance,

$$v_{in} = v_{out} = v \quad \text{or} \quad \rho_{in} = \rho_{out} = \rho$$

Therefore,

$$\dot{m}_{in} = \dot{m}_{out} \quad \rightarrow \quad \rho_{in} A_{in} V_{in} = \rho_{out} A_{out} V_{out} \quad \rightarrow \quad \dot{V}_{in} = \dot{V}_{out}$$

For multiple inlets and outlets:

$$\sum_{in} \dot{m}_{in} = \sum_{out} \dot{m}_{out} \quad \Rightarrow \quad \sum_{in} \dot{V}_{in} = \sum_{out} \dot{V}_{out}$$

During a steady-flow process, volume flow rates are *not necessarily* conserved although mass flow rates are *always* conserved.

Steady-flow process: *A process during which a fluid flows through a control volume steadily.* → Mass and energy content of the control volume remain constant. Properties are constant at a particular location.

Energy Balance

The law of conservation of energy for a control volume:

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work and mass}} = \underbrace{dE_{sys}/dt}_{\text{Rate of change in internal, kinetic, potential etc. energies}} = 0 \quad \text{(steady-state)}$$

$$\dot{Q}_{in} + \dot{W}_{in} + \dot{m}_{in}\left(u_{in} + \frac{V_{in}^2}{2} + gz_{in}\right) = \dot{Q}_{out} + \dot{W}_{out} + \dot{m}_{out}\left(u_{out} + \frac{V_{out}^2}{2} + gz_{out}\right)$$

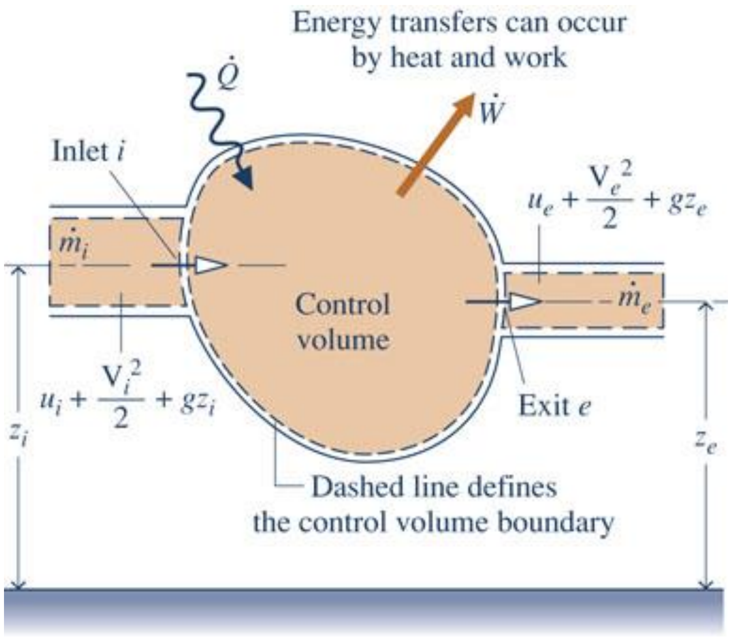
The expression for work is

$$\dot{W} = \dot{W}_{cv} + \dot{m}(pv)$$

where

$\dot{W}_{cv} \equiv$ work associated with rotating shafts, displacement of the boundary, and electrical effects.

$(pv) \equiv$ is the flow work. Required to push mass in/out of control volume. Needed for maintaining a continuous flow through the control volume.



Energy Balance

$$\dot{Q}_{in} + \dot{W}_{cv,in} + \dot{m}_{in}(\underline{u_{in} + (pv)_{in}} + \frac{V_{in}^2}{2} + gz_{in}) = \dot{Q}_{out} + \dot{W}_{cv,out} + \dot{m}_{out}(\underline{u_{out} + (pv)_{out}} + \frac{V_{out}^2}{2} + gz_{out})$$

$$\dot{Q}_{in} + \dot{W}_{cv,in} + \dot{m}_{in}(h_{in} + \frac{V_{in}^2}{2} + gz_{in}) = \dot{Q}_{out} + \dot{W}_{cv,out} + \dot{m}_{out}(h_{out} + \frac{V_{out}^2}{2} + gz_{out})$$

For multiple inlets and outlets:

$$\dot{Q}_{in} + \dot{W}_{cv,in} + \sum_{in} \dot{m}_{in}(h_{in} + \frac{V_{in}^2}{2} + gz_{in}) = \dot{Q}_{out} + \dot{W}_{cv,out} + \sum_{out} \dot{m}_{out}(h_{out} + \frac{V_{out}^2}{2} + gz_{out})$$

Energy balance relations with sign conventions (i.e., heat input and work output are positive):

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m}_{out}(h_{out} + \frac{V_{out}^2}{2} + gz_{out}) - \sum_{in} \dot{m}_{in}(h_{in} + \frac{V_{in}^2}{2} + gz_{in})$$

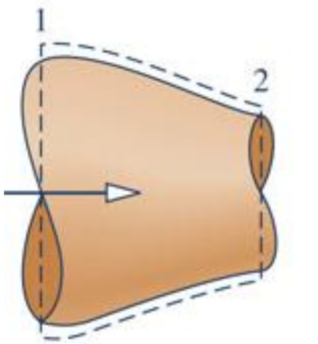
$$\dot{Q} - \dot{W} = \dot{m}\left[(h_{out} - h_{in}) + \left(\frac{V_{out}^2 - V_{in}^2}{2}\right) + g(z_{out} - z_{in})\right]$$

$$q - w = (h_{out} - h_{in}) + \left(\frac{V_{out}^2 - V_{in}^2}{2}\right) + g(z_{out} - z_{in})$$

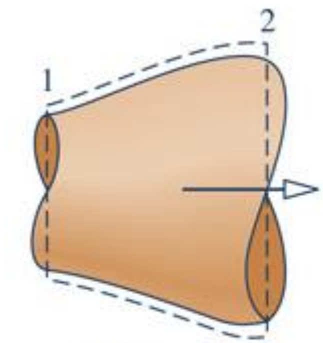
V_{in} (m/s)	V_{out} (m/s)	Δke (kJ/kg)
0	45	1
500	502	1

At very high velocities, even small changes in velocities can cause significant changes in the fluid kinetic energy.

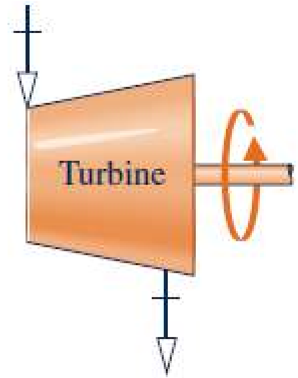
Examples of Steady-flow Devices

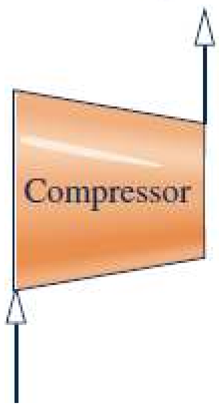


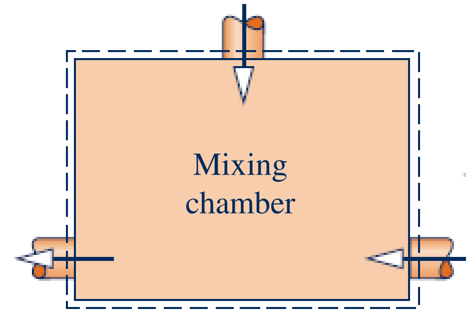
Nozzle

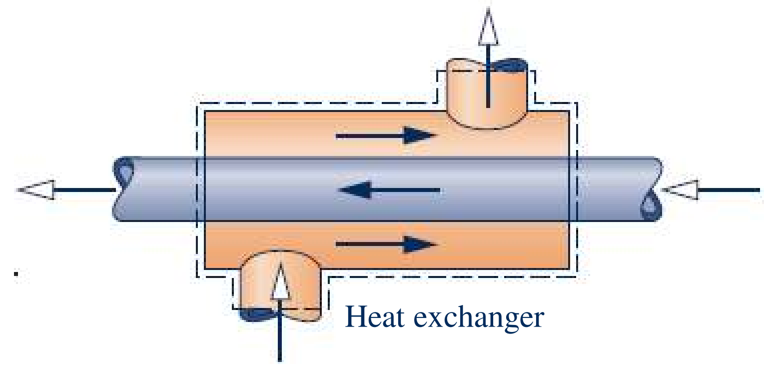


Diffuser

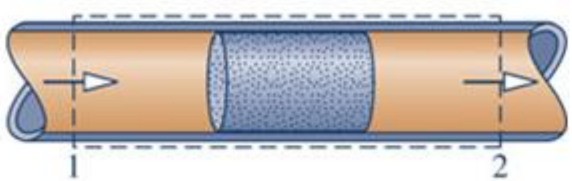








Heat exchanger



Throttle valve

Steady-flow Devices: Nozzle & Diffuser

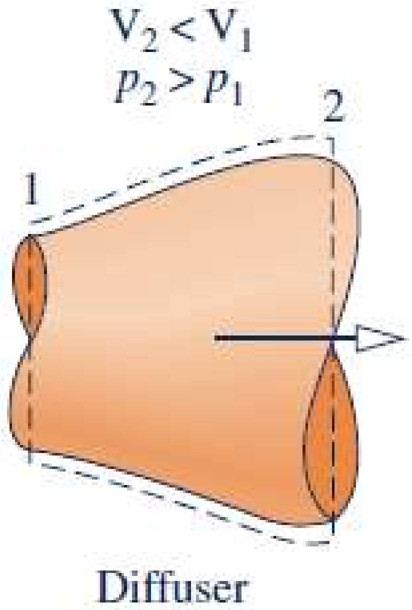
Nozzles and diffusers are commonly utilized in jet engines, rockets, spacecraft, and even garden hoses.

A **nozzle** is a device that *increases the velocity of a fluid* at the expense of pressure.

A **diffuser** is a device that *increases the pressure of a fluid* by slowing it down.

The cross-sectional area of a nozzle decreases in the flow direction for subsonic flows and increases for supersonic flows. The reverse is true for diffusers.

$$\dot{Q}_{in} + \dot{W}_{cv,in} + \sum_{in} \dot{m}_{in}\left(h_{in} + \frac{V_{in}^2}{2} + gz_{in}\right) = \dot{Q}_{out} + \dot{W}_{cv,out} + \sum_{out} \dot{m}_{out}\left(h_{out} + \frac{V_{out}^2}{2} + gz_{out}\right)$$

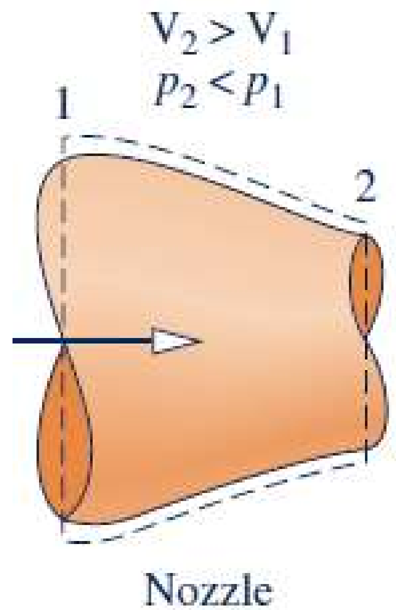


Energy balance for a nozzle or diffuser:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$

(since $\dot{Q} \cong 0$, $\dot{W} = 0$, and $\Delta \text{pe} \cong 0$)



Problem

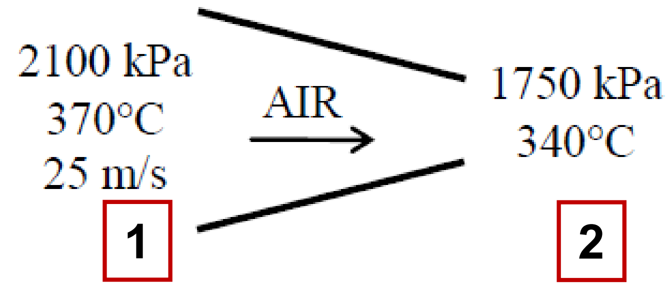
Air, assumed as an ideal gas, is accelerated in an adiabatic nozzle with the conditions shown in the figure. *Assuming constant specific heat, determine the velocity at the exit.*

System: Open, Air(Ideal+Const. sp. Heat), 1-in, 1-out, V_2=?

in
P_1 = 2100 kPa
T_1 = 370 °C
V_1 = 25 m/s

SSSF adiabtic →

out
P_2 = 1750 kPa
T_2 = 340 °C
$\dot{m}_2 = \dot{m}_1 = \dot{m}$



$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{sys} \Rightarrow \dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + V_1^2/2) = \dot{m}(h_2 + V_2^2/2) \Rightarrow h_1 + V_1^2/2 = h_2 + V_2^2/2$$

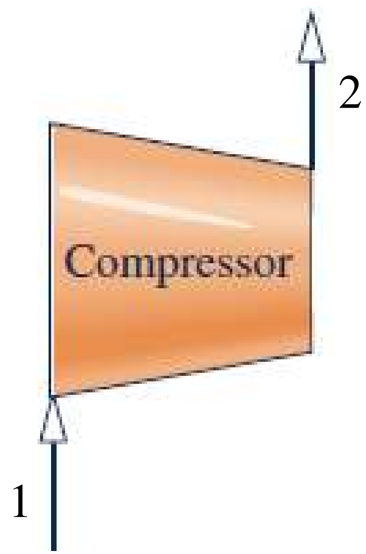
$$V_2 = \left[V_1^2 + 2(h_1 - h_2)\right]^{0.5} = \left[V_1^2 + 2c_p(T_1 - T_2)\right]^{0.5}$$

T_{avg} = 628 K
c_p = 1.057 kJ/kg.K
(Table A-20)

$$= \left[(25\ \text{m/s})^2 + 2(1.057\ \text{kJ/kg}\cdot\text{K})(370-340)\text{K}\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)\right]^{0.5} = \boxed{253.1\ m/s}$$

Steady-flow Devices: Turbine & Compressor

Compressors, as well as **pumps** and **fans**, are devices used to increase the pressure of a fluid. Work is supplied to these devices from an external source through a rotating shaft.

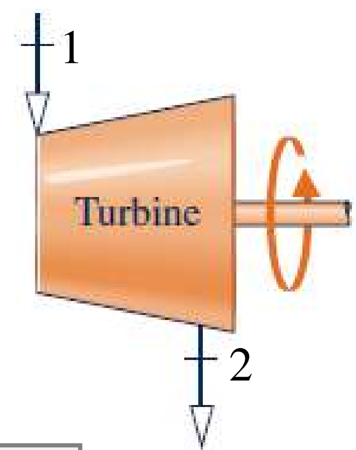


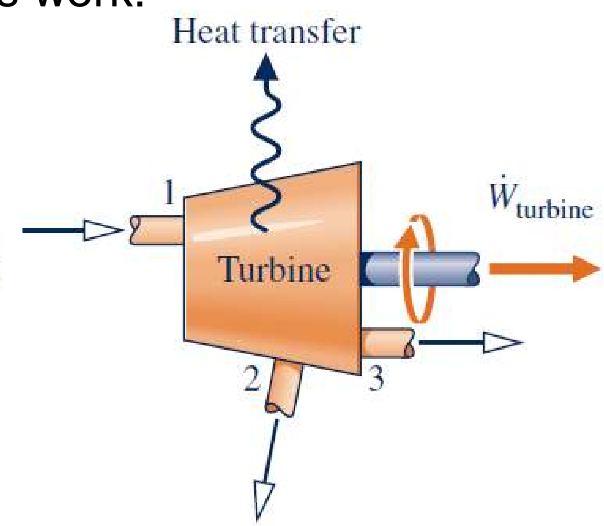
$$\dot{W}_{in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2$$

$$(\text{since } \Delta ke = \Delta pe \cong 0)$$

Turbine drives the electric generator in steam, gas or hydroelectric power plants.

As the fluid passes through the turbine, work is done against the blades attached to the shaft resulting in its rotation and the turbine produces work.





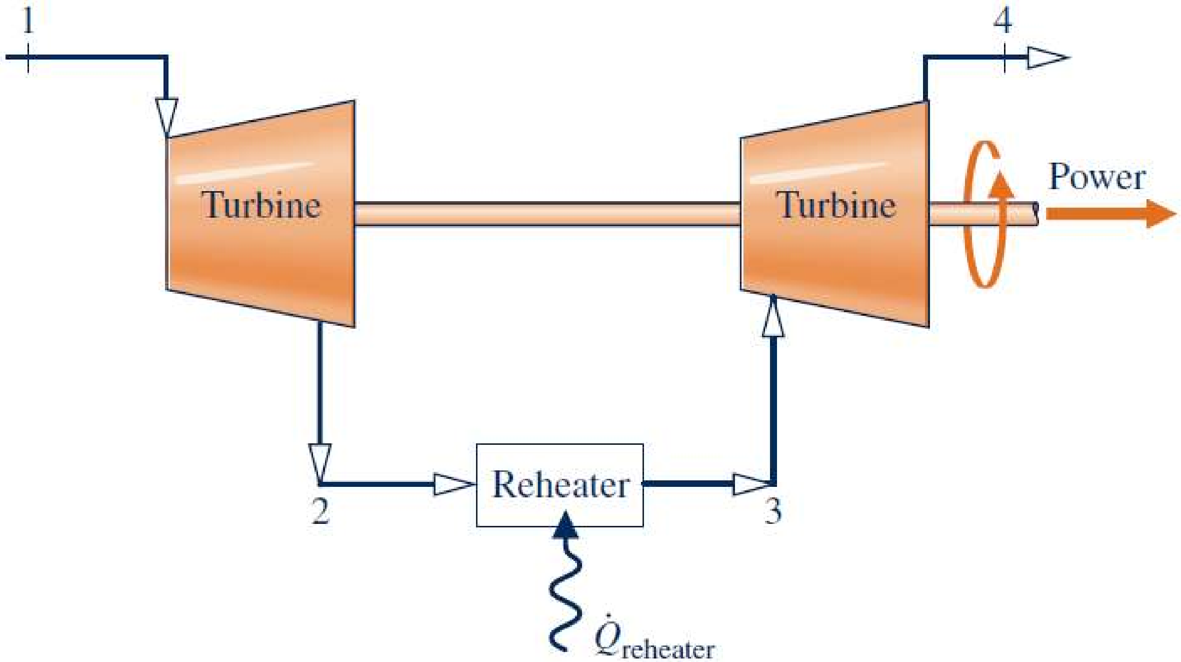
$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{W}_{out} + \dot{Q}_{out} + \dot{m}h_2$$

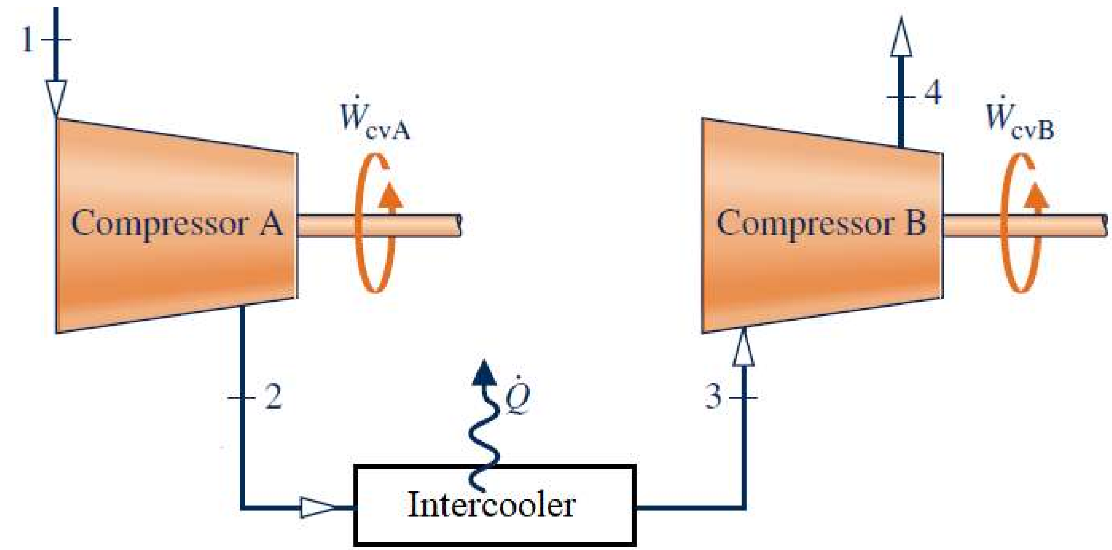
$$(\text{since } \Delta ke = \Delta pe \cong 0)$$

Steady-flow Devices: Turbine & Compressor

We may have more than one turbine in series, decreasing pressure in steps



We may have more than one compressor in series, increasing pressure in steps



Problem

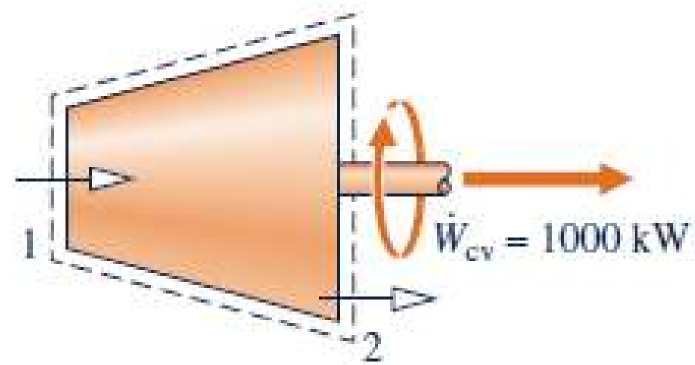
Steam enters a turbine operating at steady state with a mass flow rate of 4600 kg/h. The turbine develops a power output of 1000 kW. At the inlet, the pressure is 6 MPa, the temperature is 400 °C, and the velocity is 10 m/s. At the exit, the pressure is 10 kPa, the quality is 90%, and the velocity is 30 m/s. *Calculate the rate of heat transfer between the turbine and surroundings, in kW.*

System: Open, Steam, 1-in, 1-out, $\dot{Q}$=?

in
P_1 = 6 MPa
T_1 = 400 °C
$\dot{m}_1$ = 4600 kg/h
V_1 = 10 m/s

SSSF →

out
P_2 = 10 kPa
x_2 = 0.9
$\dot{m}_2 = \dot{m}_1 = \dot{m}$
V_2 = 30 m/s



$$\dot{Q} - \dot{W} = \cancel{\sum_{out}} \dot{m}_{out}\left(h_{out} + \frac{V_{out}^2}{2} + \cancel{gz_{out}}\right) - \cancel{\sum_{in}} \dot{m}_{in}\left(h_{in} + \frac{V_{in}^2}{2} + \cancel{gz_{in}}\right)$$

$$\dot{Q} - \dot{W} = \dot{m}\left[\left(h_2 + \frac{V_2^2}{2}\right) - \left(h_1 + \frac{V_1^2}{2}\right)\right] = \dot{m}\left[(h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2}\right)\right]$$

$$h_1 = 3177.2\ kJ/kg \ ; \ h_2 = h_f + x_2 h_{fg} = 191.83 + 0.9(2392.8) = 2345.4\ kJ/kg$$

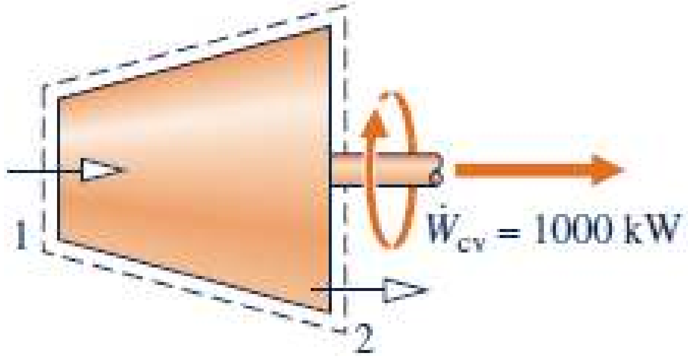
Problem

$$\dot{Q} - \dot{W} = \dot{m}\left[(h_2 - h_1) + (\frac{V_2^2 - V_1^2}{2})\right]$$

$$\dot{Q} - 1000 = \frac{4600}{3600}\left[(2345.4 - 3177.2) + (\frac{30^2 - 10^2}{2(1000)})\right] \qquad \left[\because \frac{m^2}{s^2} \equiv \frac{J}{kg}\right]$$

$$\dot{Q} = 1000 + 1.278\left[(-831.8) + (0.4)\right]$$

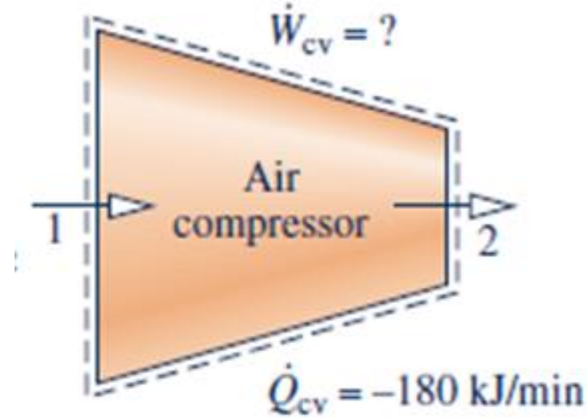
$$\dot{Q} = \boxed{-62.3\ kW}$$



Note: For Δke=0, heat transfer would be -62.9 kW

Problem

Air enters a compressor operating at steady state at a pressure of 100 kPa, a temperature of 290 K, and a velocity of 6 m/s through an inlet with an area of 0.1 m^2. At the exit, the pressure is 700 kPa, the temperature is 450 K, and the velocity is 2 m/s. Heat transfer from the compressor to its surroundings occurs at a rate of 180 kJ/min. *Employing the ideal gas model with variable specific heat, calculate the power input to the compressor, in kW.*



System: Open, Air, 1-in, 1-out, $\dot{W}$=?

in	SSSF	out
P_1 = 100 kPa		P_2 = 700 kPa
T_1 = 290 K	→	T_2 = 450 K
A_1 = 0.1 m^2		$\dot{m}_2 = \dot{m}_1 = \dot{m}$
V_1 = 6 m/s		V_2 = 2 m/s

$$\dot{Q} - \dot{W} = \cancel{\sum_{out}} \dot{m}_{out}\left(h_{out} + \frac{V_{out}^2}{2} + \cancel{gz_{out}}\right) - \cancel{\sum_{in}} \dot{m}_{in}\left(h_{in} + \frac{V_{in}^2}{2} + \cancel{gz_{in}}\right)$$

$$\dot{Q} - \dot{W} = \dot{m}\left[\left(h_2 + \frac{V_2^2}{2}\right) - \left(h_1 + \frac{V_1^2}{2}\right)\right] \qquad \Rightarrow \dot{W} = \dot{Q} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\right]$$

Problem

$$\dot{m} = \frac{A_1V_1}{v_1} = \frac{A_1V_1}{(RT_1/P_1)} = \frac{0.1(6)}{(0.287(290)/100)} = 0.72\ kg\ /\ s$$

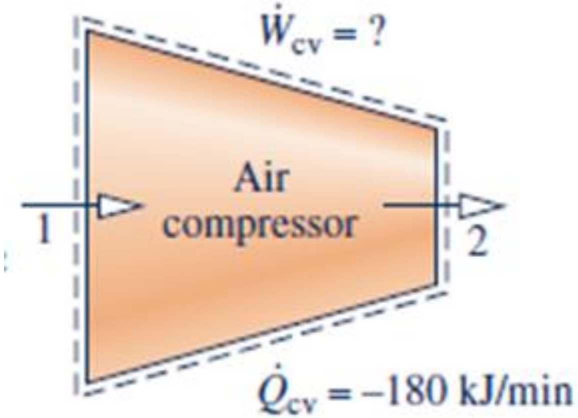
Using **Tables A-22**, we get

$$h_1(@\,290K) = 290.16\ kJ\ /\ kg\ ;\quad h_2(@\,450K) = 451.8\ kJ\ /\ kg$$

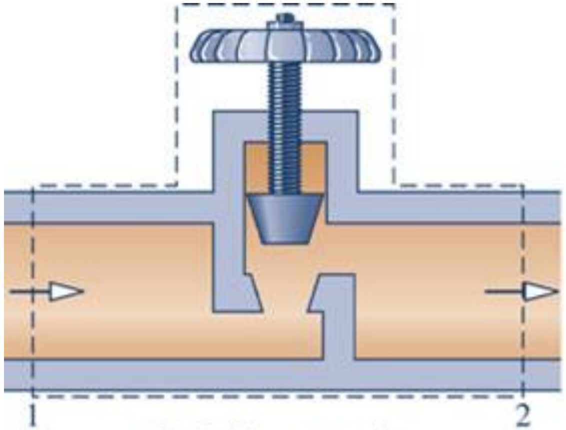
Now, $$\dot{W} = \frac{-180}{60} + 0.72\left[(290.16 - 451.8) + \left(\frac{6^2 - 2^2}{2(1000)}\right)\right]$$

$$\dot{W} = -3 + 0.72\left[-161.64 + 0.02\right]$$

$$\dot{W} = \boxed{-119.4\ kW}$$



Steady-flow Devices: Throttle valve

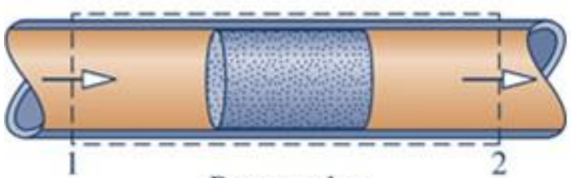


Partially open valve

Throttling valves are *any kind of flow-restricting devices* that cause a significant pressure drop in the fluid.

What is the difference between a turbine and a throttling valve?

The pressure drop in the fluid is often accompanied by a *large drop in temperature*, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications.

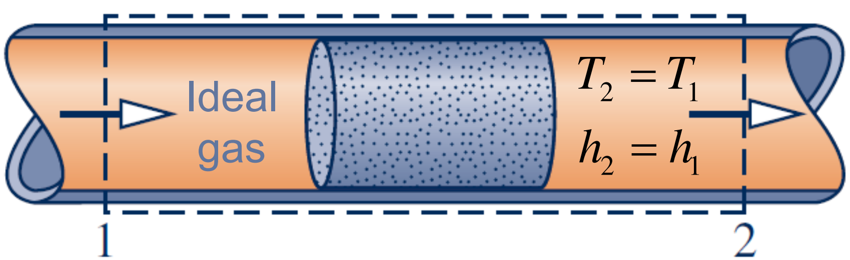


Porous plug

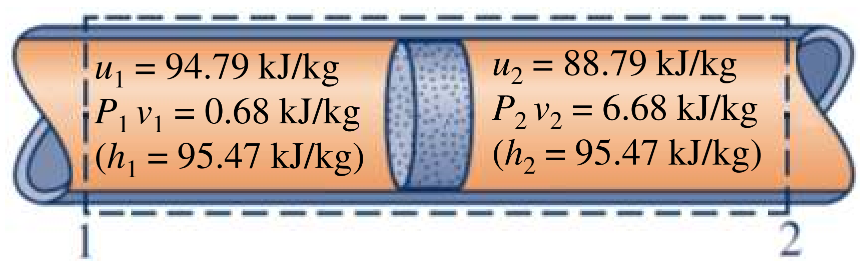
Energy balance

$$h_2 \cong h_1$$

$$u_1 + P_1 v_1 = u_2 + P_2 v_2$$



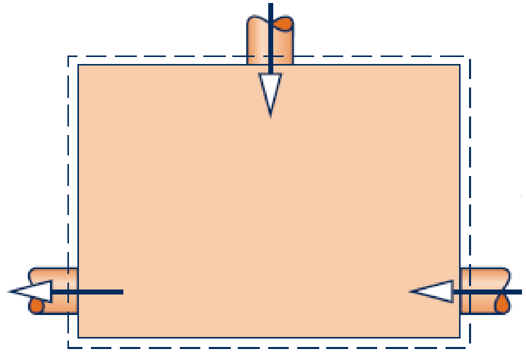
The temperature of an ideal gas does not change during a throttling (h = constant) process since $h = h(T)$.



During a throttling process, the enthalpy of a fluid remains constant. But internal and flow energies may be converted to each other.

Steady-flow Devices: Mixing Chamber

In engineering applications, the section where the mixing process takes place is commonly referred to as a mixing chamber.

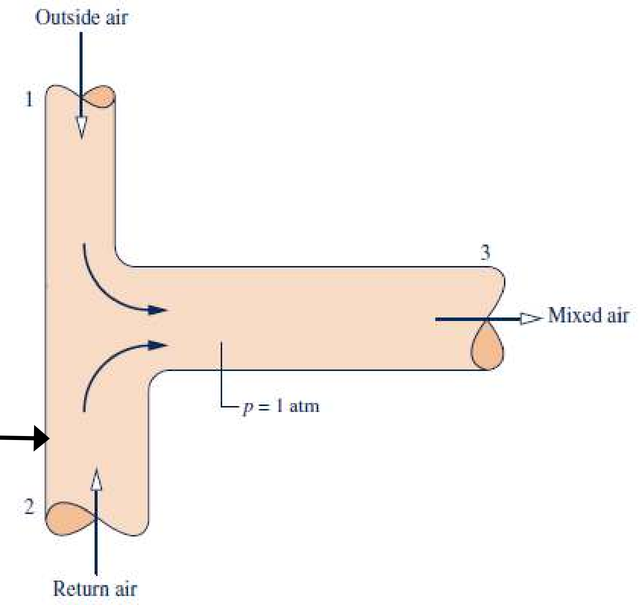


$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i\right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + g z_e\right)$$

$$\sum_i \dot{m}_i h_i = \sum_e \dot{m}_e h_e$$

$$(\text{since } \dot{Q} \cong 0,\ \dot{W} = 0,\ \Delta\text{ke} = \Delta\text{pe} \cong 0)$$

Write the energy balance for the mixing process shown

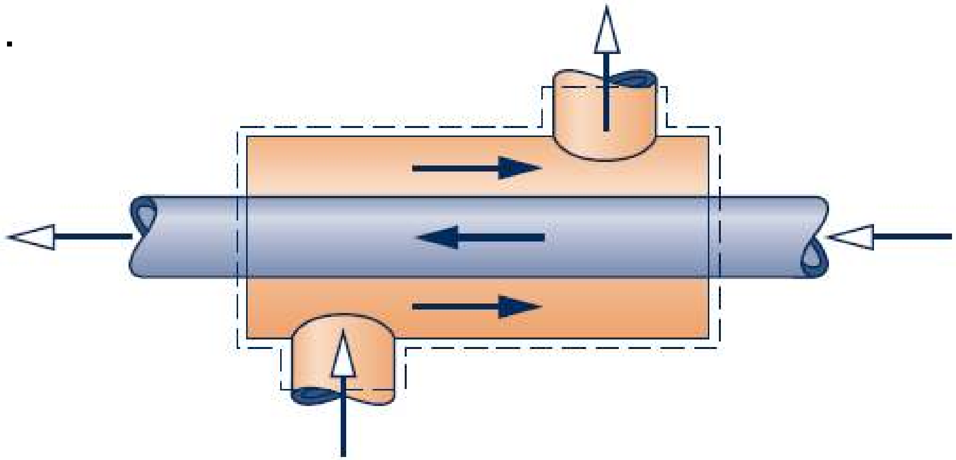


Steady-flow Devices: Heat Exchanger

Heat exchangers are devices where two moving fluid streams exchange heat without mixing.

$$\sum_i \dot{m}_i h_i = \sum_e \dot{m}_e h_e$$

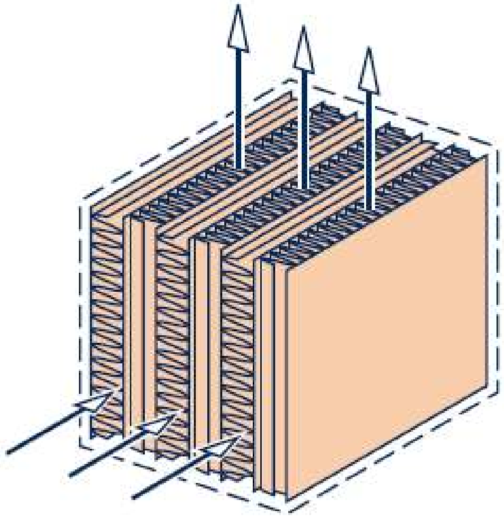
$$(\text{since } \dot{Q} \cong 0,\ \dot{W} = 0,\ \Delta\text{ke} = \Delta\text{pe} \cong 0)$$



Counterflow heat exchanger

Alternative scenarios:

1. Heat loss/gain from surrounding is taken into account
2. CV is one of the tubes ($\dot{Q}_{cv} = \dot{Q}_{in}$ or $\dot{Q}_{out}$)



Cross-flow heat exchanger

Common heat exchanger names:
Boiler, Condenser, Evaporator

Problems

A thin-walled double-pipe counter-flow heat exchanger is used to cool oil ($c_{p,o}$= 2.20 kJ/kg·°C) from 150 to 40°C at a rate of 2 kg/s by water ($c_{p,o}$= 4.18 kJ/kg·°C) that enters at 22°C at a rate of 1.5 kg/s. *Determine the rate of heat transfer in the heat exchanger and the exit temperature of water.*

System: Open, Oil+Water, 1-in(each), 1-out(each), $\dot{Q}$=?, $T_{w,e}$=?

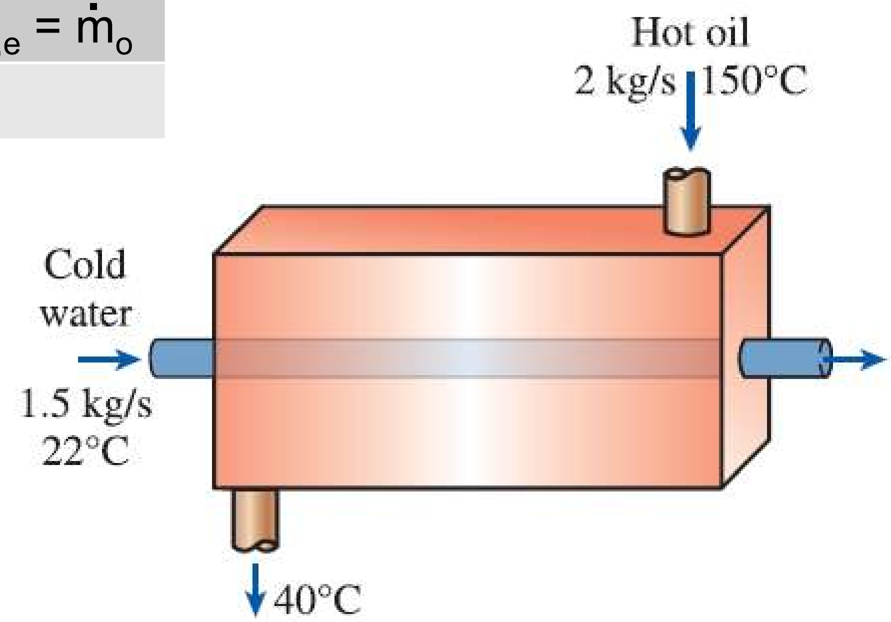
in		out
$T_{o,i}$ = 150 °C		$T_{o,e}$ = 40 °C
$T_{w,i}$ = 22 °C	SSSF adiabatic →	$\dot{m}_{w,i} = \dot{m}_{w,e} = \dot{m}_w$
$\dot{m}_{o,i}$ = 2 kg/s		$\dot{m}_{o,i} = \dot{m}_{o,e} = \dot{m}_o$
$\dot{m}_{w,i}$ = 1.5 kg/s		

Taking the oil tube as the CV,

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_o h_{o,i} = \dot{Q}_{out} + \dot{m}_o h_{o,e} \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{Q}_{out} = \dot{m}_o (h_{o,i} - h_{o,e}) = \dot{m}_o c_{p,o} (T_{o,i} - T_{o,e})$$



Problems

$$\dot{Q}_{out} = 2(2.2)(150 - 40) = \boxed{484\ kW}$$

Now, taking the water tube as the CV,

$$\dot{E}_{in} = \dot{E}_{out}$$

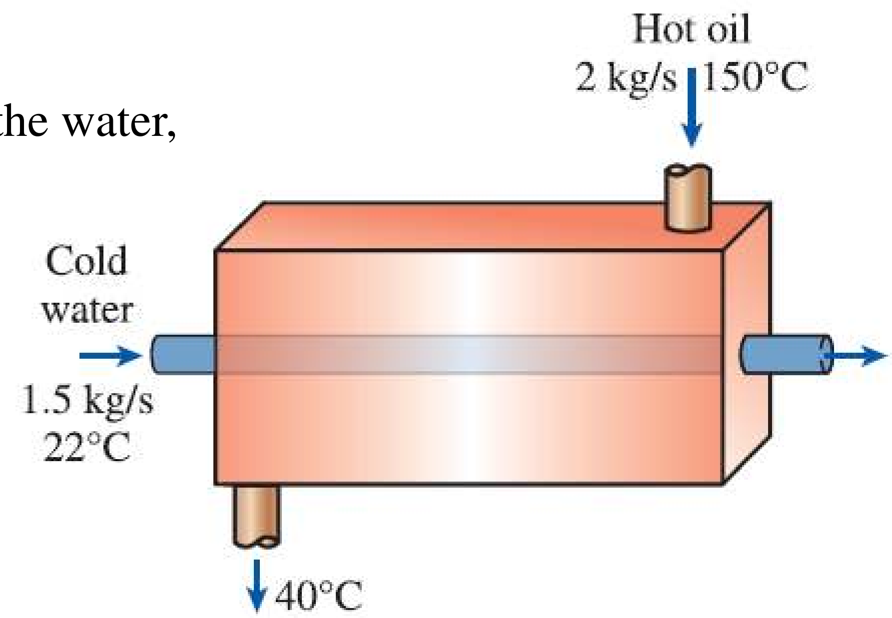
$$\dot{Q}_{in} + \dot{m}_w h_{w,i} = \dot{m}_w h_{w,e}$$

$$\dot{Q}_{in} = \dot{m}_w (h_{w,e} - h_{w,i}) = \dot{m}_w c_{p,w} (T_{w,e} - T_{w,i})$$

$$T_{w,e} = T_{w,i} + \frac{\dot{Q}_{in}}{\dot{m}_w c_{p,w}}$$

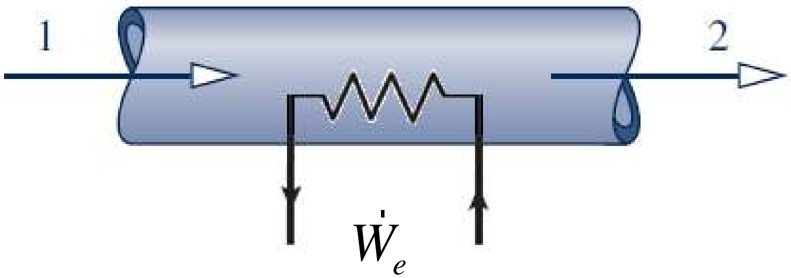
Noting that the heat lost by the oil is gained by the water,

$$T_{w,e} = 22 + \frac{484}{1.5(4.18)} = \boxed{99.2\ ^{\circ}C}$$



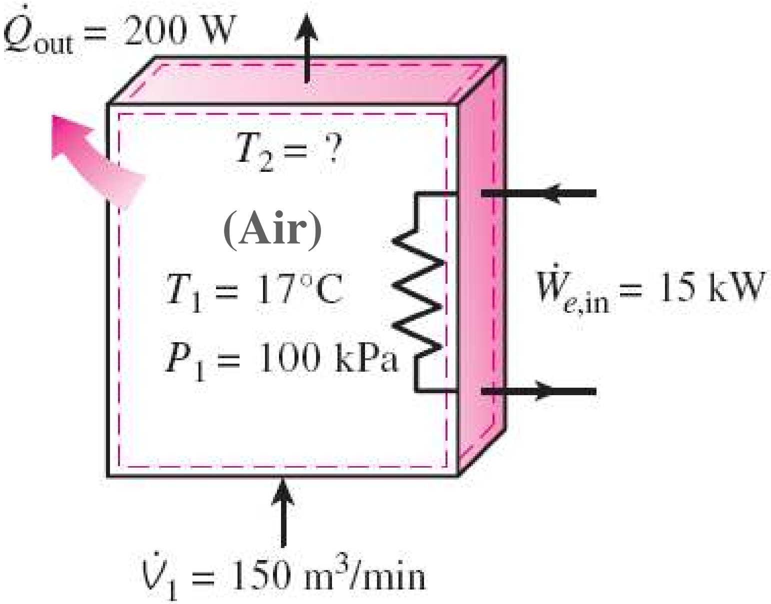
Steady-flow Devices: Pipe & Duct Flow

The transport of liquids or gases in pipes and ducts is of great importance in many engineering applications. Flow through a pipe or a duct usually satisfies the steady-flow conditions.



Pipe or duct flow may involve more than one form of work at the same time.

Heat losses from a hot fluid flowing through an uninsulated pipe or duct to the cooler environment may be very significant.



Energy balance for figure assuming ideal gas, constant specific heat case

$$\dot{E}_{in} = \dot{E}_{out}$$

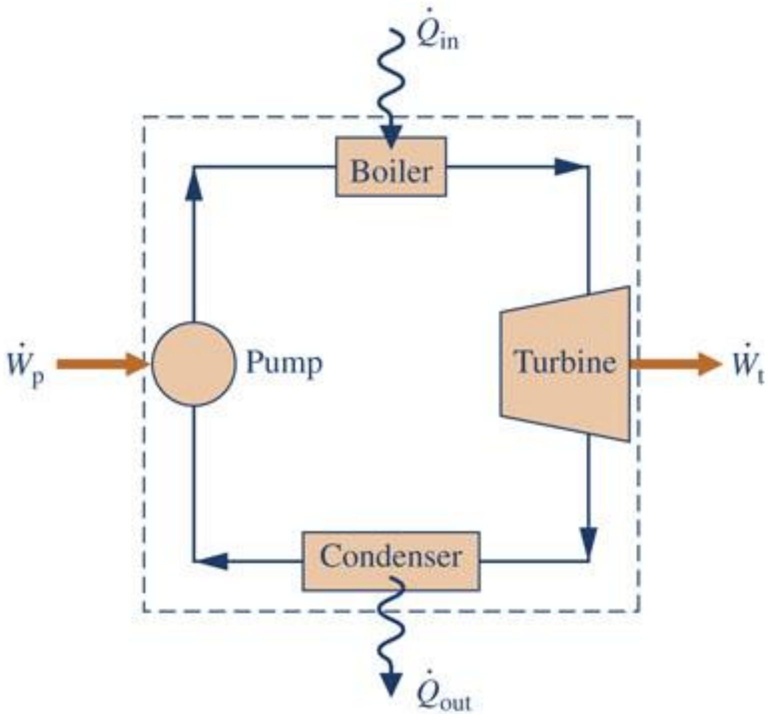
$$\dot{W}_{e,in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2$$

$$\dot{W}_{e,in} - \dot{Q}_{out} = \dot{m}c_p(T_2 - T_1)$$

Steady-flow Devices: System Integration

► Engineers creatively combine components to achieve some overall objective, subject to constraints such as minimum total cost. This engineering activity is called **system integration**.

► The **simple power plant** of figure shown provides an illustration.



For a **cycle** (SSSF),

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\Rightarrow \dot{Q}_{in} + \dot{W}_{in} = \dot{Q}_{out} + \dot{W}_{out}$$

or $$\dot{Q}_{net,in} = \dot{W}_{net,out}$$