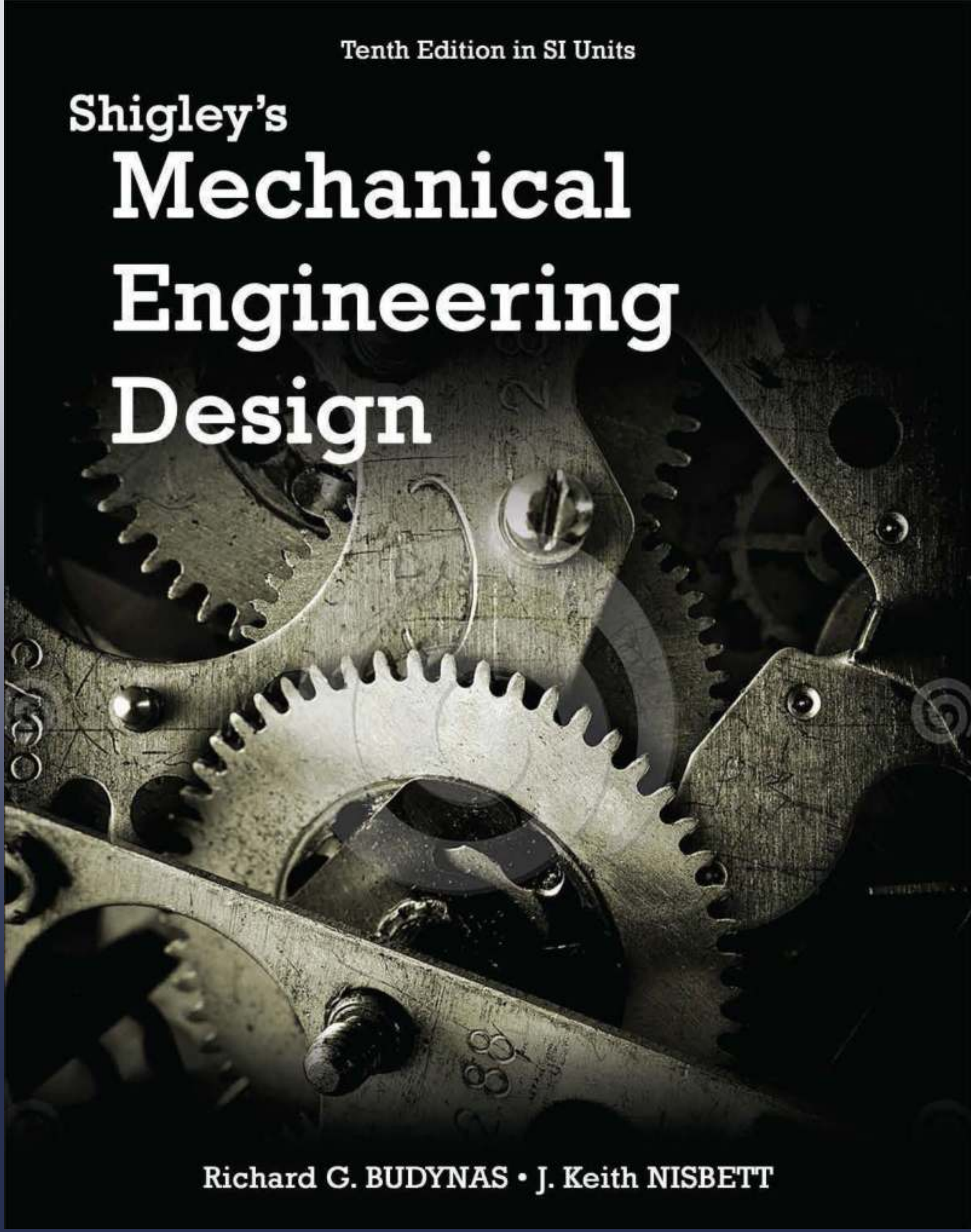
Mc Graw Hill Education

Lecture Slides

Chapter 6

Fatigue Failure Resulting from Variable Loading



Introduction to Fatigue in Metals

- Loading produces stresses that are variable, repeated, alternating, or fluctuating
- Maximum stresses well below yield strength
- Failure occurs after many stress cycles
- Failure is by sudden ultimate fracture
- No visible warning in advance of failure

Stages of Fatigue Failure

- *Stage I* – Initiation of micro-crack due to cyclic plastic deformation
- *Stage II* – Progresses to macro-crack that repeatedly opens and closes, creating bands called *beach marks*
- *Stage III* – Crack has propagated far enough that remaining material is insufficient to carry the load, and fails by simple ultimate failure

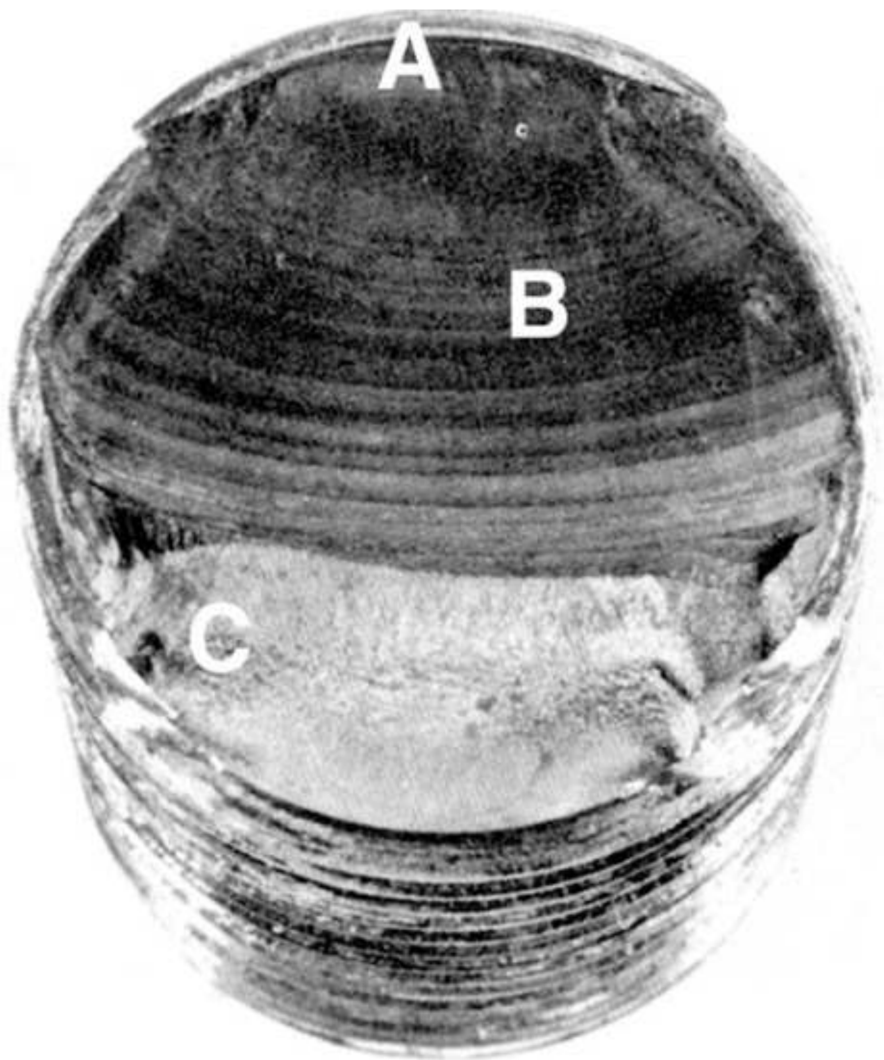


Fig. 6–1

S-N Diagram

- Number of cycles to failure at varying stress levels is plotted on log-log scale
- For steels, a knee occurs near 10^6 cycles
- Strength corresponding to the knee is called *endurance limit* S_e

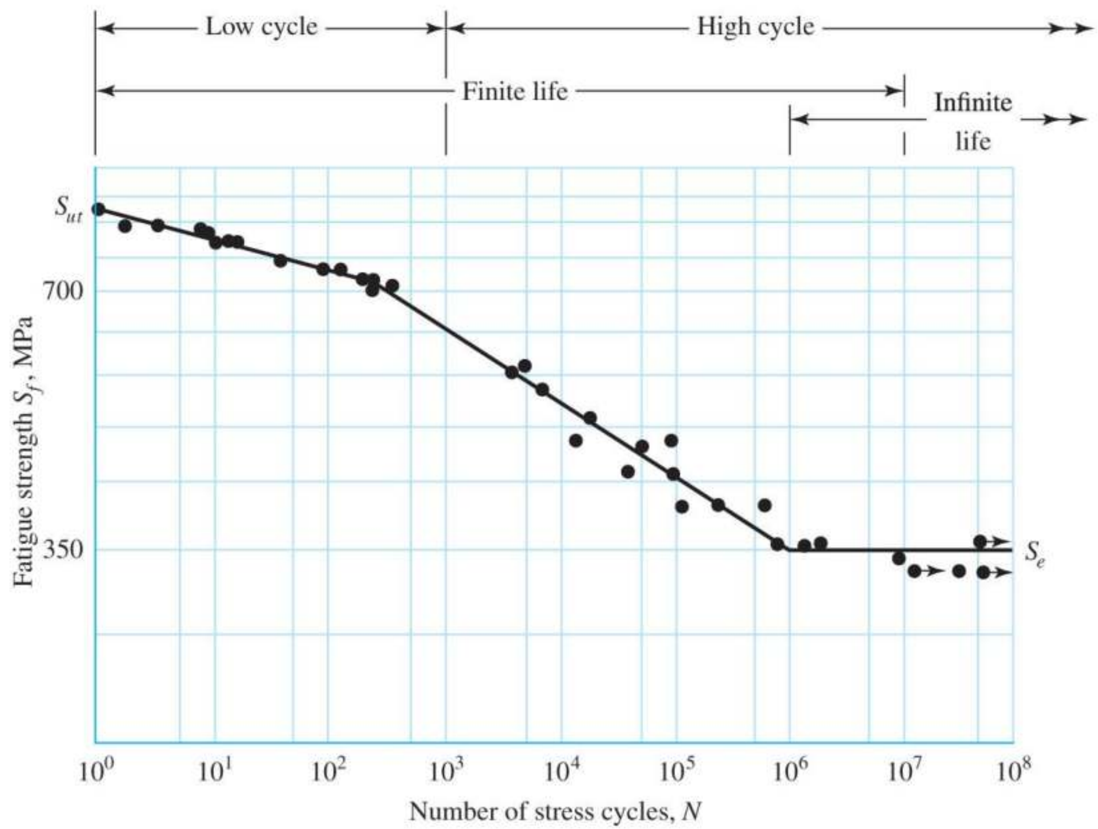


Fig. 6–10

Endurance Limit Modifying Factors

- Endurance limit S'_e is for carefully prepared and tested specimen
- If warranted, S_e is obtained from testing of actual parts
- When testing of actual parts is not practical, a set of *Marin factors* are used to adjust the endurance limit

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \qquad (6\text{–}18)$$

k_a = surface condition modification factor

k_b = size modification factor

k_c = load modification factor

k_d = temperature modification factor

k_e = reliability factor[13]

k_f = miscellaneous-effects modification factor

S'_e = rotary-beam test specimen endurance limit

S_e = endurance limit at the critical location of a machine part in the geometry and condition of use

Surface Factor k_a

- Stresses tend to be high at the surface
- Surface finish has an impact on initiation of cracks at localized stress concentrations
- Surface factor is a function of ultimate strength. Higher strengths are more sensitive to rough surfaces.

$$k_a = aS_{ut}^b \qquad (6\text{–}19)$$

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	–0.085
Machined or cold-drawn	2.70	4.51	–0.265
Hot-rolled	14.4	57.7	–0.718
As-forged	39.9	272.	–0.995

From C.J. Noll and C. Lipson, "Allowable Working Stresses," *Society for Experimental Stress Analysis*, vol. 3, no. 2, 1946 p. 29. Reproduced by O.J. Horger (ed.) *Metals Engineering Design ASME Handbook*, McGraw-Hill, New York. Copyright © 1953 by The McGraw-Hill Companies, Inc. Reprinted by permission.

Example 6–3

A steel has a minimum ultimate strength of 520 MPa and a machined surface. Estimate k_a.

Solution From Table 6–2, $a = 4.51$ and $b = -0.265$. Then, from Eq. (6–19)

Answer

$$k_a = 4.51(520)^{-0.265} = 0.860$$

Size Factor k_b

- Larger parts have greater surface area at high stress levels
- Likelihood of crack initiation is higher
- Size factor is obtained from experimental data with wide scatter
- For bending and torsion loads, the trend of the size factor data is given by

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \quad (6\text{–}20)$$

- Applies only for round, rotating diameter
- For axial load, there is no size effect, so $k_b = 1$

Loading Factor k_c

- Accounts for changes in endurance limit for different types of fatigue loading.
- Only to be used for single load types. Use Combination Loading method (Sec. 6–14) when more than one load type is present.

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \tag{6–26}$$

Temperature Factor k_d

- Endurance limit appears to maintain same relation to ultimate strength for elevated temperatures as at room temperature
- This relation is summarized in Table 6–4

Table 6–4

Effect of Operating Temperature on the Tensile Strength of Steel.* (S_T = tensile strength at operating temperature; S_{RT} = tensile strength at room temperature; $0.099 \leq \hat{\sigma} \leq 0.110$)

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

*Data source: Fig. 2–9.

Temperature Factor k_d

- If ultimate strength is known for operating temperature, then just use that strength. Let $k_d = 1$ and proceed as usual.
- If ultimate strength is known only at room temperature, then use Table 6–4 to estimate ultimate strength at operating temperature. With that strength, let $k_d = 1$ and proceed as usual.
- Alternatively, use ultimate strength at room temperature and apply temperature factor from Table 6–4 to the endurance limit.

$$k_d = \frac{S_T}{S_{RT}} \tag{6–28}$$

- A fourth-order polynomial curve fit of the underlying data of Table 6–4 can be used in place of the table, if desired.

$$\begin{aligned} k_d = {} & 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 \\ & + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4 \end{aligned} \tag{6–27}$$

Reliability Factor k_e

- From Fig. 6–17, $S'_e = 0.5\ S_{ut}$ is typical of the data and represents 50% reliability.
- Reliability factor adjusts to other reliabilities.
- *Only* adjusts Fig. 6–17 assumption. *Does not* imply overall reliability.

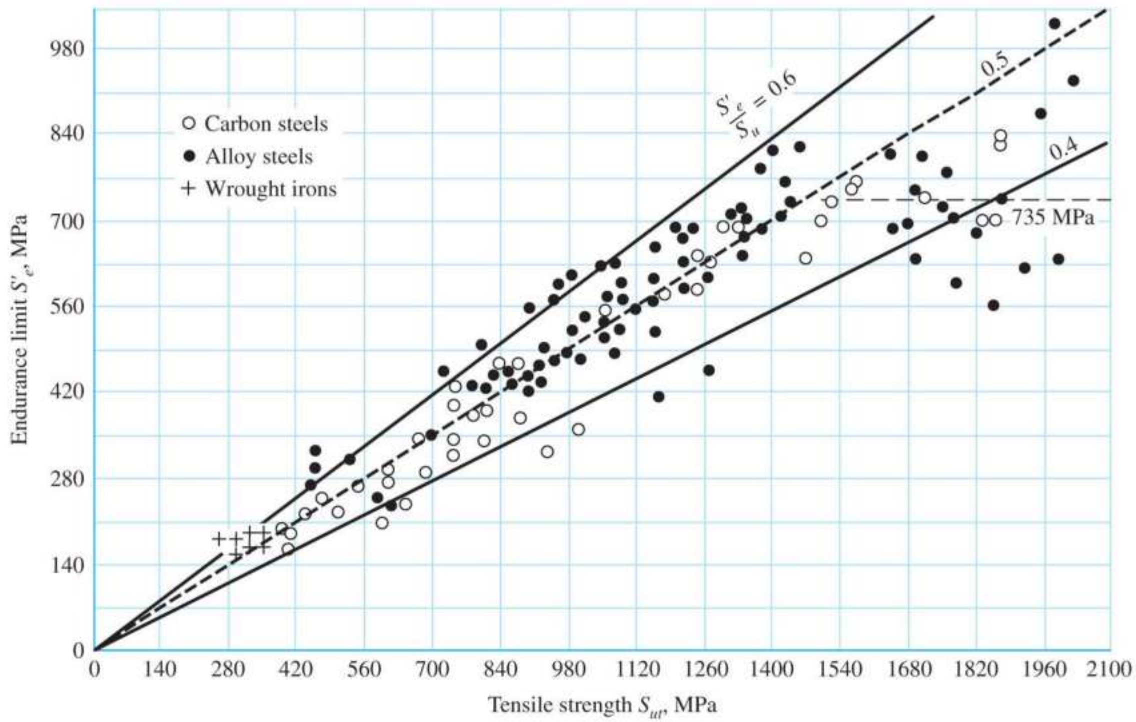


Fig. 6–17

Reliability Factor k_e

- Simply obtain k_e for desired reliability from Table 6–5.

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Table 6–5

Miscellaneous-Effects Factor k_f

- Reminder to consider other possible factors.
  - Residual stresses
  - Directional characteristics from cold working
  - Case hardening
  - Corrosion
  - Surface conditioning, e.g. electrolytic plating and metal spraying
  - Cyclic Frequency
  - Frettage Corrosion
- Limited data is available.
- May require research or testing.

Stress Concentration and Notch Sensitivity

- For dynamic loading, stress concentration effects must be applied.
- Obtain K_t as usual (e.g. Appendix A–15)
- For fatigue, some materials are not fully sensitive to K_t so a reduced value can be used.
- Define K_f as the *fatigue stress-concentration factor*.
- Define q as *notch sensitivity*, ranging from 0 (not sensitive) to 1 (fully sensitive).

$$q = \frac{K_f - 1}{K_t - 1} \qquad \text{or} \qquad q_{\text{shear}} = \frac{K_{fs} - 1}{K_{ts} - 1} \tag{6–31}$$

- For $q = 0$, $K_f = 1$
- For $q = 1$, $K_f = K_t$

Notch Sensitivity

- Obtain q for bending or axial loading from Fig. 6–20.
- Then get K_f from Eq. (6–32): $K_f = 1 + q(K_t - 1)$

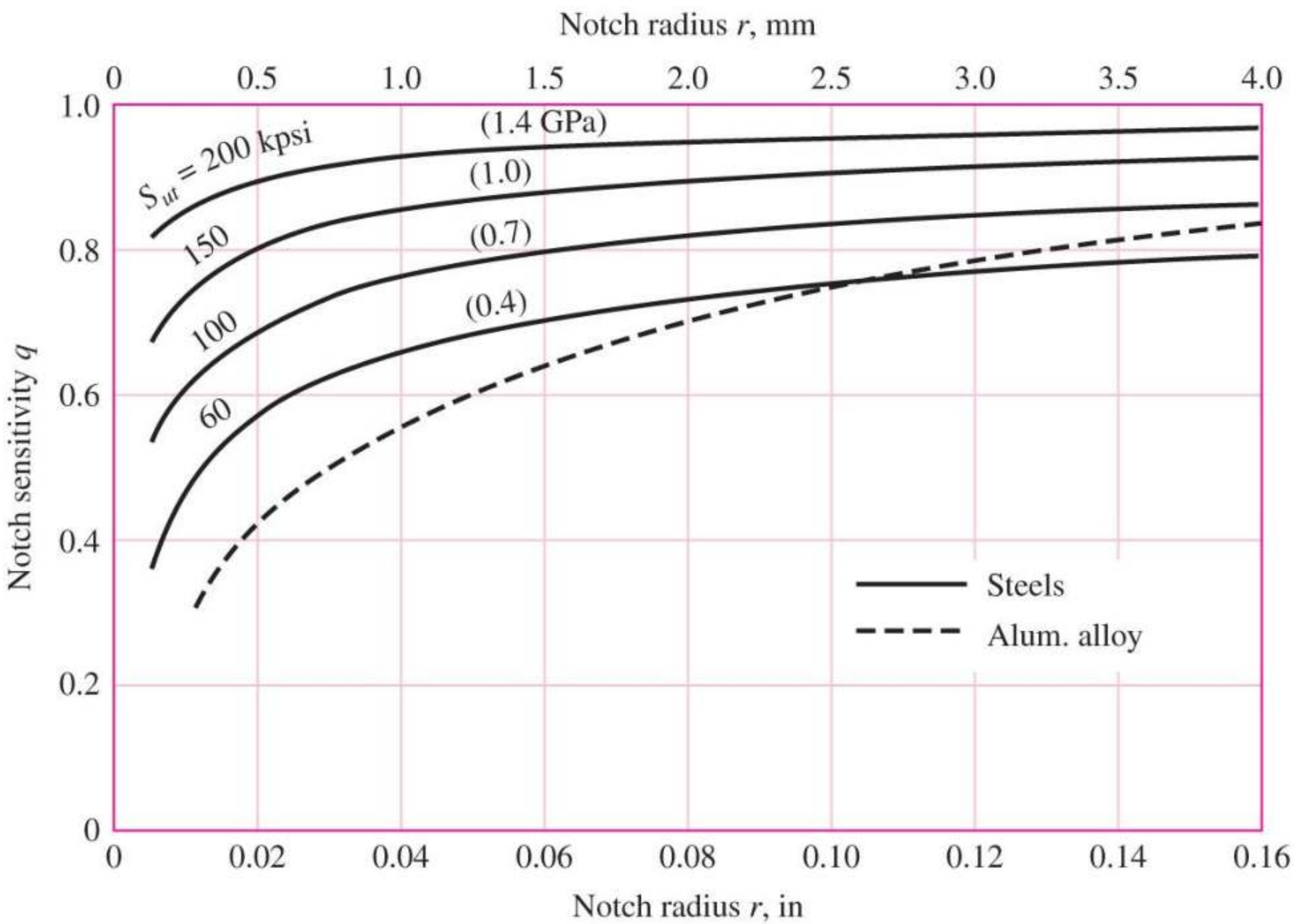


Fig. 6–20

Notch Sensitivity

- Obtain q_s for torsional loading from Fig. 6–21.
- Then get K_{fs} from Eq. (6–32): $K_{fs} = 1 + q_s(K_{ts} - 1)$

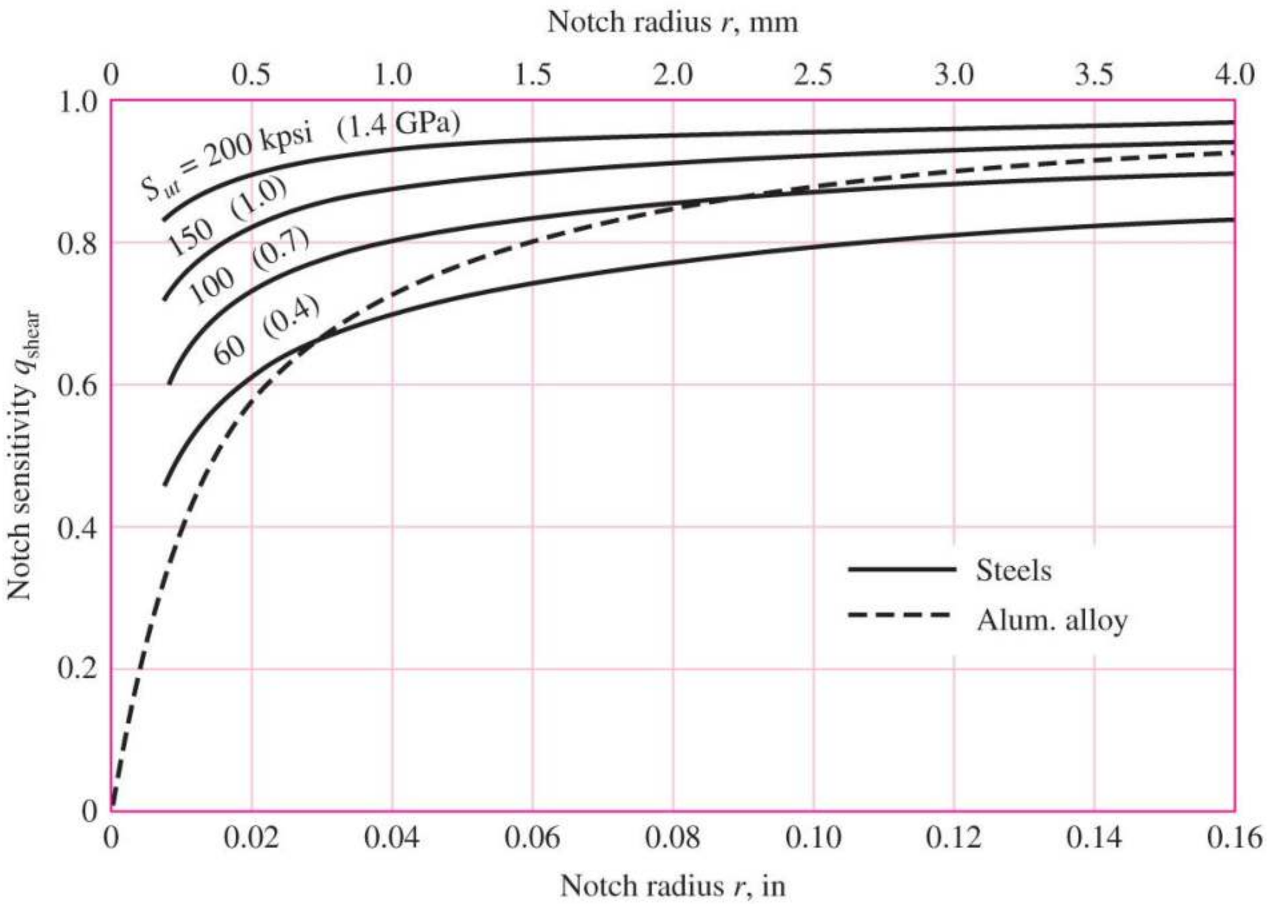


Fig. 6–21

Example 6–6

A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate K_f using:
(*a*) Figure 6–20.
(*b*) Equations (6–33) and (6–35).

Solution

From Fig. A–15–9, using $D/d = 38/32 = 1.1875$, $r/d = 3/32 = 0.093\,75$, we read the graph to find $K_t = 1.65$.
(*a*) From Fig. 6–20, for $S_{ut} = 690$ MPa and $r = 3$ mm, $q = 0.84$. Thus, from Eq. (6–32)

$$K_f = 1 + q(K_t - 1) = 1 + 0.84(1.65 - 1) = 1.55 \qquad \text{Answer}$$

(*b*) From Eq. (6–35*a*) with $S_{ut} = 690\text{ MPa} = 100\text{ kpsi}$, $\sqrt{a} = 0.0622\sqrt{\text{in}} = 0.313\sqrt{\text{mm}}$. Substituting this into Eq. (6–33) with $r = 3$ mm gives

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} = 1 + \frac{1.65 - 1}{1 + \dfrac{0.313}{\sqrt{3}}} = 1.55 \qquad \text{Answer}$$

Application of Fatigue Stress Concentration Factor

- Use K_f as a multiplier to increase the nominal stress.
- Some designers (and previous editions of textbook) sometimes applied $1/K_f$ as a Marin factor to reduce S_e.
- For infinite life, either method is equivalent, since

Fatigue factor of safety

$$n_f = \frac{S_e}{K_f \sigma} = \frac{(1/K_f) S_e}{\sigma}$$

- For finite life, increasing stress is more conservative. Decreasing S_e applies more to high cycle than low cycle.

Example 6–7

For the step-shaft of Ex. 6–6, it is determined that the fully corrected endurance limit is $S_e = 280$ MPa. Consider the shaft undergoes a fully reversing nominal stress in the fillet of $(\sigma_{\text{rev}})_{\text{nom}} = 260$ MPa. Estimate the number of cycles to failure.

Solution

From Ex. 6–6, $K_f = 1.55$, and the ultimate strength is $S_{ut} = 690$ MPa $= 100$ kpsi. The maximum reversing stress is

$$(\sigma_{\text{rev}})_{\text{max}} = K_f(\sigma_{\text{rev}})_{\text{nom}} = 1.55(260) = 403 \text{ MPa}$$

From Fig. 6–18, $f = 0.845$. From Eqs. (6–14), (6–15), and (6–16)

$$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.845(690)]^2}{280} = 1214 \text{ MPa}$$

$$b = -\frac{1}{3}\log\frac{f\,S_{ut}}{S_e} = -\frac{1}{3}\log\left[\frac{0.845(690)}{280}\right] = -0.1062$$

$$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{403}{1214}\right)^{1/-0.1062} = 32.3(10^3) \text{ cycles} \quad \text{Answer}$$

Example 6–8

A 1015 hot-rolled steel bar has been machined to a diameter of 25 mm. It is to be placed in reversed axial loading for 70 000 cycles to failure in an operating environment of 300°C. Using ASTM minimum properties, and a reliability of 99 percent, estimate the endurance limit and fatigue strength at 70 000 cycles.

Solution

From Table A–20, $S_{ut} = 340$ MPa at 20°C. Since the rotating-beam specimen endurance limit is not known at room temperature, we determine the ultimate strength at the elevated temperature first, using Table 6–4. From Table 6–4,

$$\left(\frac{S_T}{S_{RT}}\right)_{300^\circ} = 0.975$$

The ultimate strength at 300°C is then

$$(S_{ut})_{300^\circ} = (S_T / S_{RT})_{300^\circ}\,(S_{ut})_{20^\circ} = 0.975(340) = 331.5 \text{ MPa}$$

Example 6–8 (continued)

The rotating-beam specimen endurance limit at 300°C is then estimated from Eq. (6–8) as

$$S'_e = 0.5(331.5) = 165.8 \text{ MPa}$$

Next, we determine the Marin factors. For the machined surface, Eq. (6–19) with Table 6–2 gives

$$k_a = aS_{ut}^b = 4.51(331.5^{-0.265}) = 0.969$$

For axial loading, from Eq. (6–21), the size factor $k_b = 1$, and from Eq. (6–26) the loading factor is $k_c = 0.85$. The temperature factor $k_d = 1$, since we accounted for the temperature in modifying the ultimate strength and consequently the endurance limit. For 99 percent reliability, from Table 6–5, $k_e = 0.814$. Finally, since no other conditions were given, the miscellaneous factor is $k_f = 1$. The endurance limit for the part is estimated by Eq. (6–18) as

Example 6–8 (continued)

Answer

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$
$$= 0.969(1)(0.85)(1)(0.814)(1)165.8 = 111 \text{ MPa}$$

For the fatigue strength at 70 000 cycles we need to construct the *S-N* equation. From p. 285, since $S_{ut} = 331.5 < 490$ MPa, then $f = 0.9$. From Eq. (6–14)

$$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.9(331.5)]^2}{111} = 801.929$$

and Eq. (6–15)

$$b = -\frac{1}{3}\log\left(\frac{f\ S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{0.9(331.5)}{111}\right] = -0.1431$$

Finally, for the fatigue strength at 70 000 cycles, Eq. (6–13) gives

Answer

$$S_f = a\ N^b = 891(70\ 000)^{-0.1431} = 162.51 \text{ MPa}$$

Example 6–9

Figure 6–22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part.

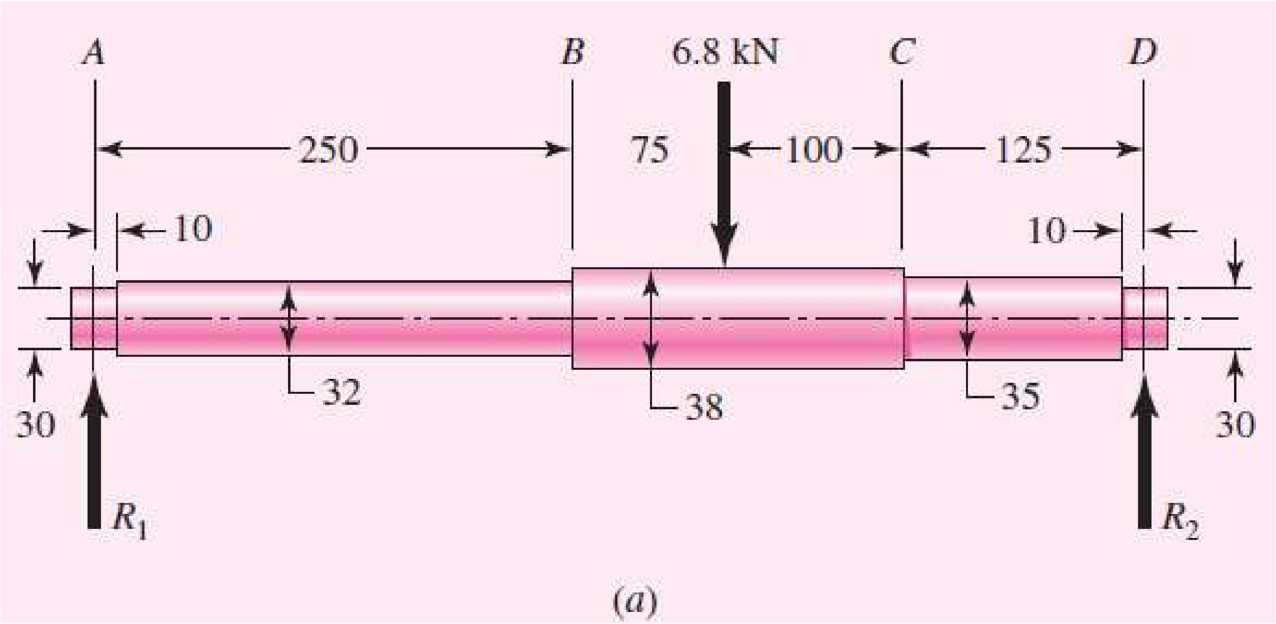


Fig. 6–22

Example 6–9 (continued)

From Fig. 6–22*b* we learn that failure will probably occur at *B* rather than at *C* or at the point of maximum moment. Point *B* has a smaller cross section, a higher bending moment, and a higher stress-concentration factor than *C*, and the location of maximum moment has a larger size and no stress-concentration factor.

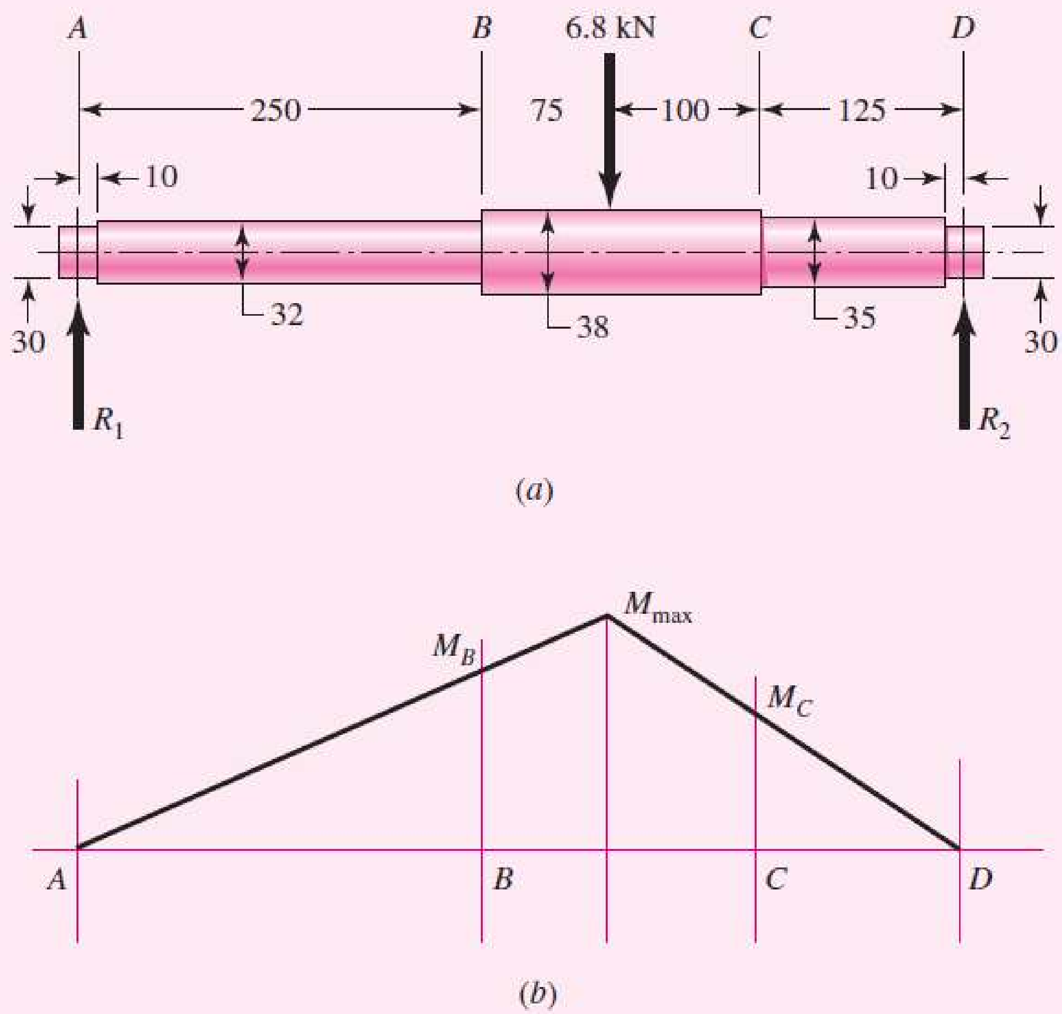


Fig. 6–22

Example 6–9 (continued)

We shall solve the problem by first estimating the strength at point B, since the strength will be different elsewhere, and comparing this strength with the stress at the same point.

From Table A–20 we find $S_{ut} = 690$ MPa and $S_y = 580$ MPa. The endurance limit S'_e is estimated as

$$S'_e = 0.5(690) = 345 \text{ MPa}$$

From Eq. (6–19) and Table 6–2,

$$k_a = 4.51(690)^{-0.265} = 0.798$$

From Eq. (6–20),

$$k_b = (32/7.62)^{-0.107} = 0.858$$

Since $k_c = k_d = k_e = k_f = 1$,

$$S_e = 0.798(0.858)345 = 236 \text{ MPa}$$

Example 6–9 (continued)

To find the geometric stress-concentration factor K_t we enter Fig. A–15–9 with $D/d = 38/32 = 1.1875$ and $r/d = 3/32 = 0.093\,75$ and read $K_t \doteq 1.65$. Substituting $S_{ut} = 690/6.89 = 100$ kpsi into Eq. (6–35*a*) yields $\sqrt{a} = 0.0622\sqrt{\text{in}} = 0.313\sqrt{\text{mm}}$. Substituting this into Eq. (6–33) gives

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} = 1 + \frac{1.65 - 1}{1 + 0.313/\sqrt{3}} = 1.55$$

The next step is to estimate the bending stress at point B. The bending moment is

$$M_B = R_1 x = \frac{225F}{550}250 = \frac{225(6.8)}{550}250 = 695.5\ \text{N}\cdot\text{m}$$

Just to the left of B the section modulus is $I/c = \pi d^3/32 = \pi 32^3/32 = 3.217\,(10^3)\ \text{mm}^3$. The reversing bending stress is, assuming infinite life,

$$\sigma_{\text{rev}} = K_f\frac{M_B}{I/c} = 1.55\frac{695.5}{3.217}(10)^{-6} = 335.1(10^6)\ \text{Pa} = 335.1\ \text{MPa}$$

This stress is greater than S_e and less than S_y. This means we have both finite life and no yielding on the first cycle.

Example 6–9 (continued)

For finite life, we will need to use Eq. (6–16). The ultimate strength, $S_{ut} = 690$ MPa = 100 kpsi. From Fig. 6–18, $f = 0.844$. From Eq. (6–14)

$$a = \frac{(f\ S_{ut})^2}{S_e} = \frac{[0.844(690)]^2}{236} = 1437 \text{ MPa}$$

and from Eq. (6–15)

$$b = -\frac{1}{3}\log\left(\frac{f\ S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{0.844(690)}{236}\right] = -0.1308$$

From Eq. (6–16),

$$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{335.1}{1437}\right)^{-1/0.1308} = 68(10^3) \text{ cycles}$$ Answer

Characterizing Fluctuating Stresses

- The *S-N* diagram is applicable for *completely reversed* stresses
- Other fluctuating stresses exist
- Sinusoidal loading patterns are common, but not necessary

Fluctuating Stresses

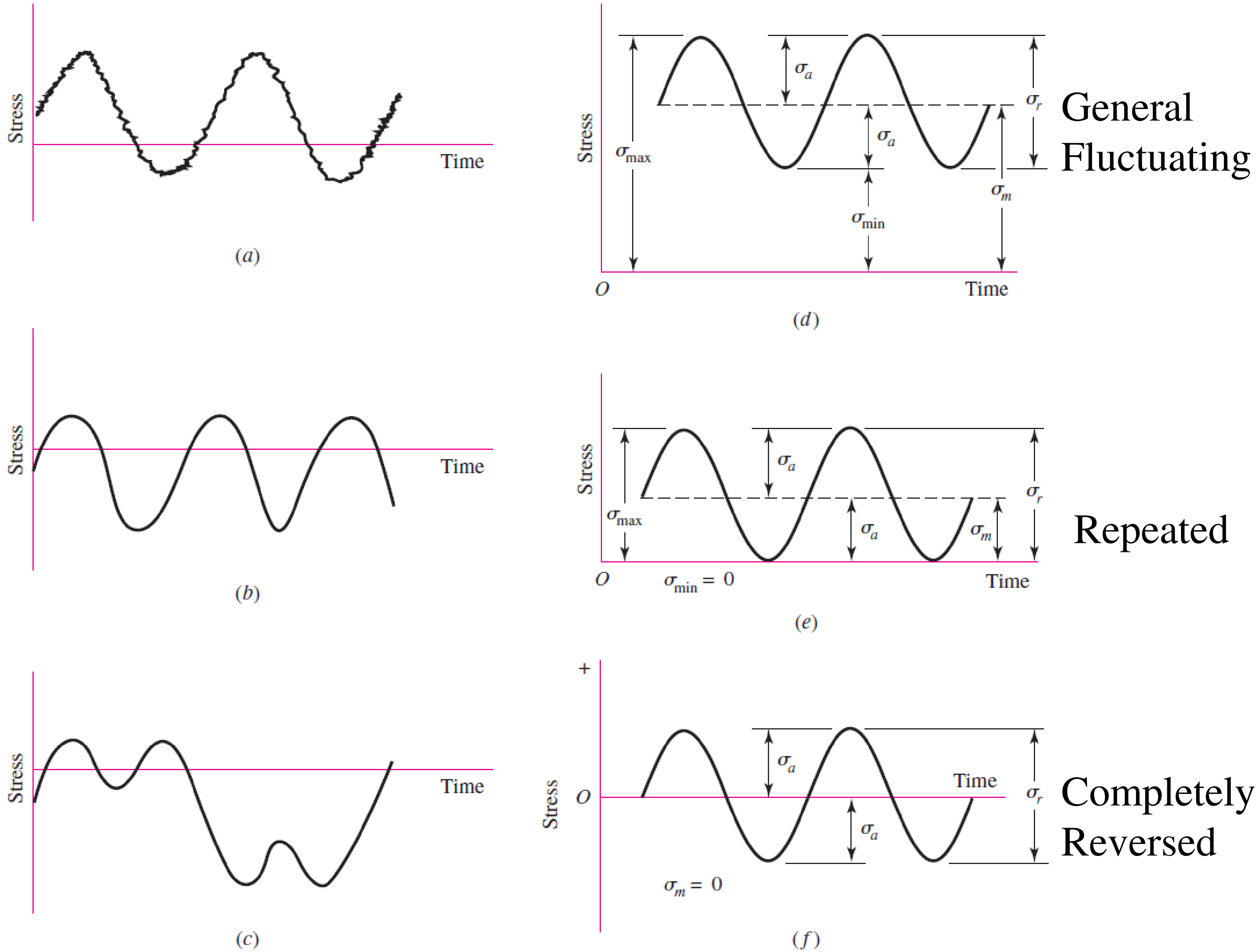
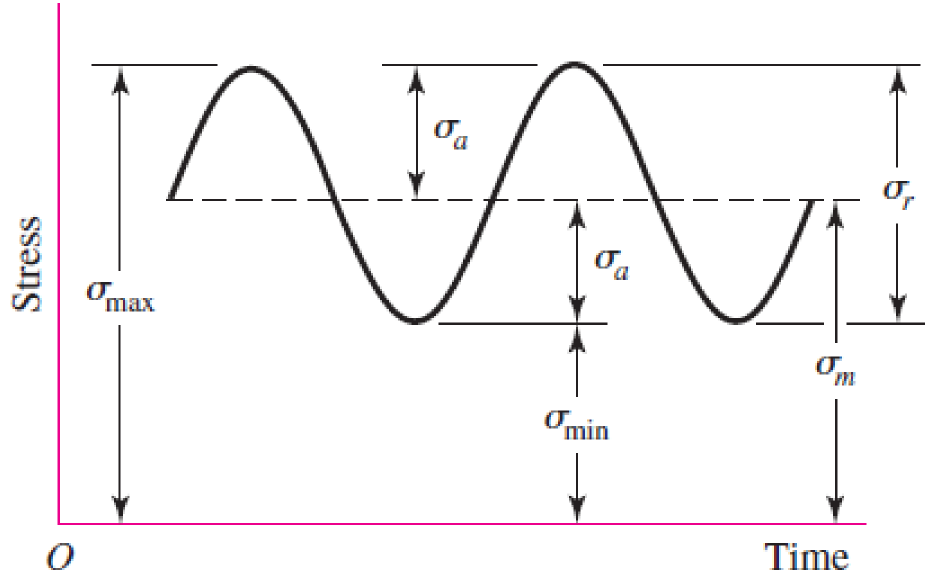


Fig. 6–23

Characterizing Fluctuating Stresses

- Fluctuating stresses can often be characterized simply by the minimum and maximum stresses, σ_{min} and σ_{max}
- Define σ_m as *midrange* steady component of stress (sometimes called *mean* stress) and σ_a as amplitude of *alternating* component of stress



$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = \left| \frac{\sigma_{max} - \sigma_{min}}{2} \right| \qquad (6\text{–}36)$$

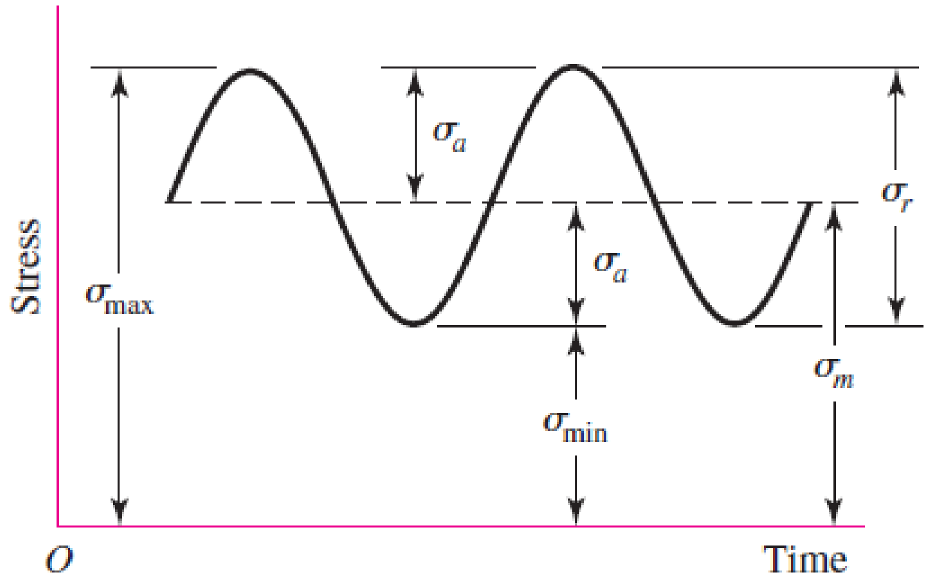
Characterizing Fluctuating Stresses

- Other useful definitions include *stress ratio*

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \qquad (6\text{–}37)$$

and *amplitude ratio*

$$A = \frac{\sigma_a}{\sigma_m} \qquad (6\text{–}38)$$



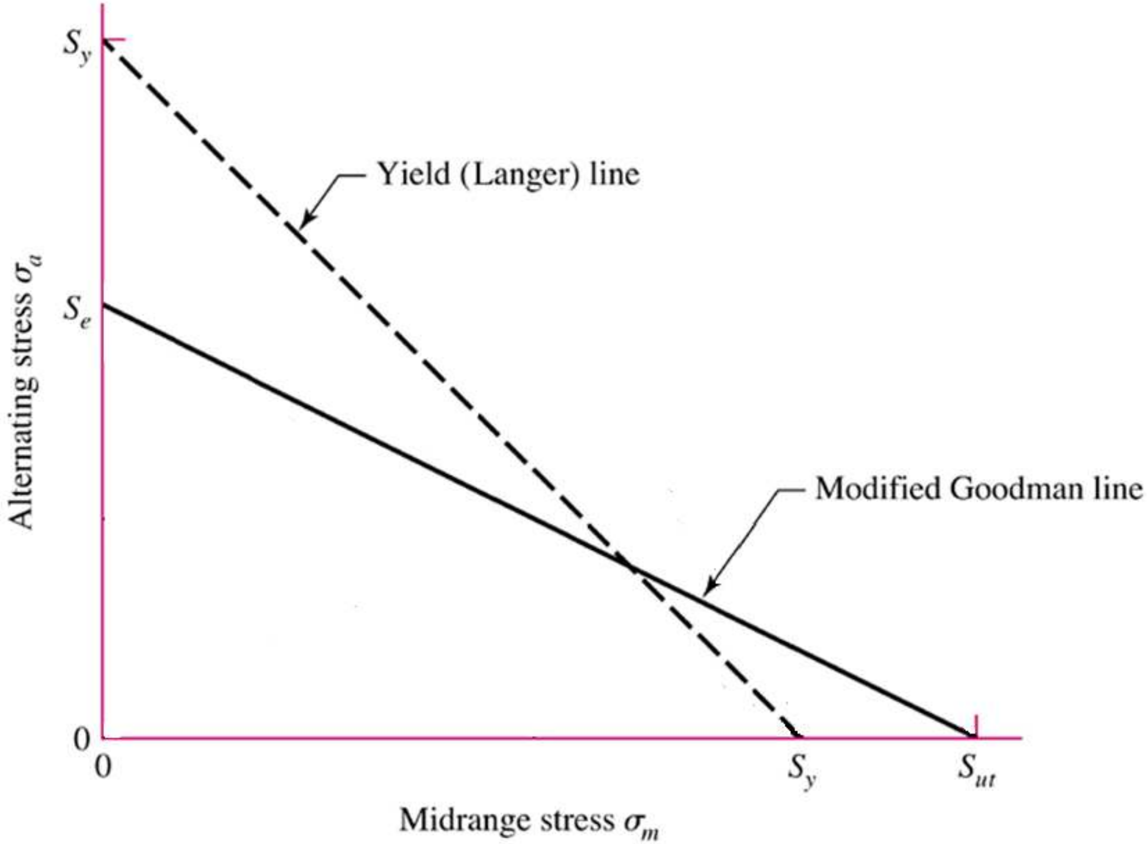
Application of K_f for Fluctuating Stresses

- For fluctuating loads at points with stress concentration, the best approach is to design to avoid all localized plastic strain.
- In this case, K_f should be applied to both alternating and midrange stress components.
- When localized strain does occur, some methods (e.g. *nominal mean stress* method and *residual stress* method) recommend only applying K_f to the alternating stress.
- The *Dowling method* recommends applying K_f to the alternating stress and K_{fm} to the mid-range stress, where K_{fm} is

$$
\begin{aligned}
K_{fm} &= K_f & K_f|\sigma_{\max,o}| &< S_y \\
K_{fm} &= \frac{S_y - K_f\sigma_{ao}}{|\sigma_{mo}|} & K_f|\sigma_{\max,o}| &> S_y \qquad (6\text{–}39)\\
K_{fm} &= 0 & K_f|\sigma_{\max,o} - \sigma_{\min,o}| &> 2S_y
\end{aligned}
$$

Plot of Alternating vs Midrange Stress

- Probably most common and simple to use is the plot of σ_a vs σ_m
- Has gradually usurped the name of Goodman or Modified Goodman diagram
- Modified Goodman line from S_e to S_{ut} is one simple representation of the limiting boundary for infinite life



Commonly Used Failure Criteria

- Five commonly used failure criteria are shown
- Gerber passes through the data
- ASME-elliptic passes through data and incorporates rough yielding check

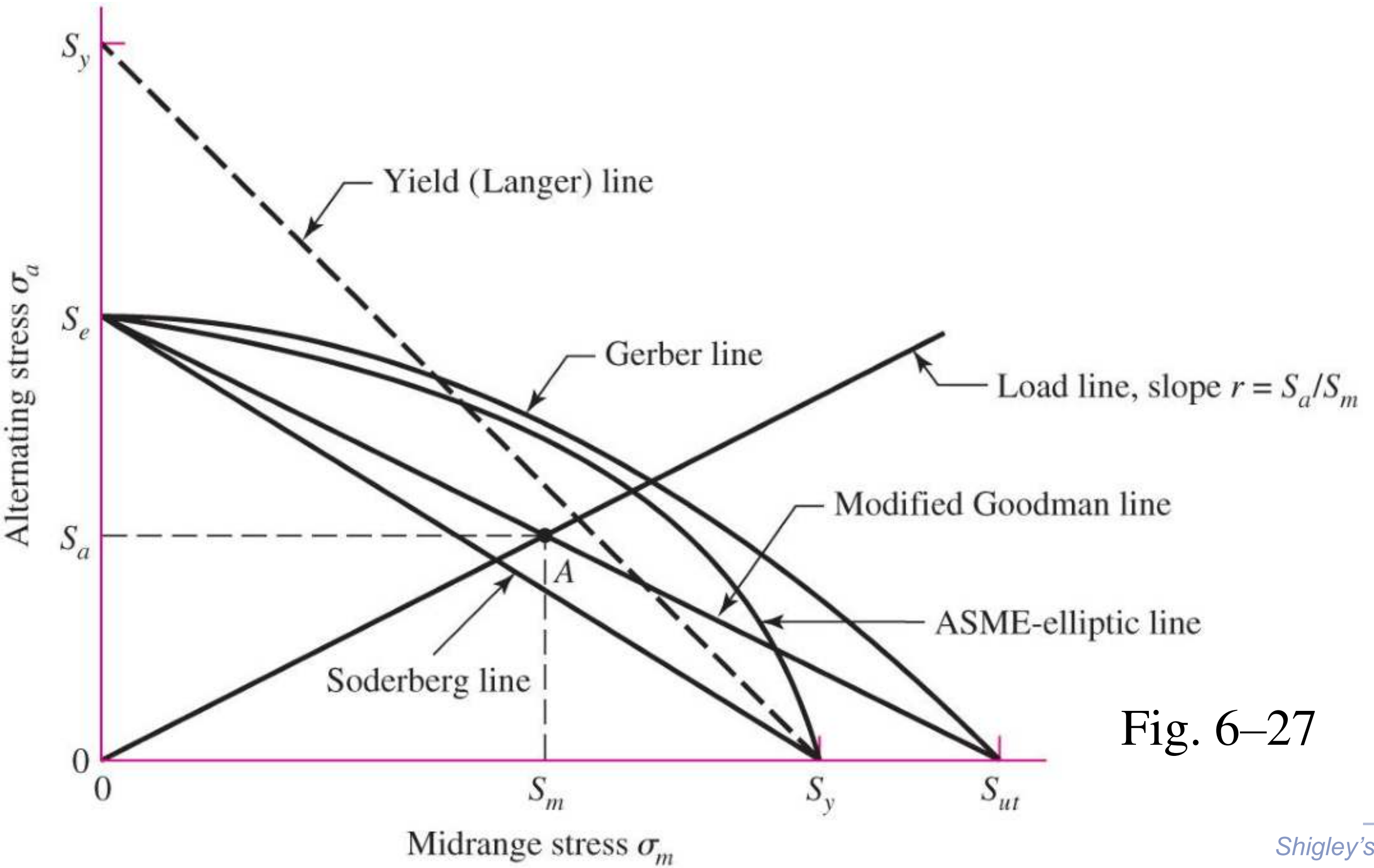


Fig. 6–27

Commonly Used Failure Criteria

- Modified Goodman is linear, so simple to use for design. It is more conservative than Gerber.
- Soderberg provides a very conservative single check of both fatigue and yielding.

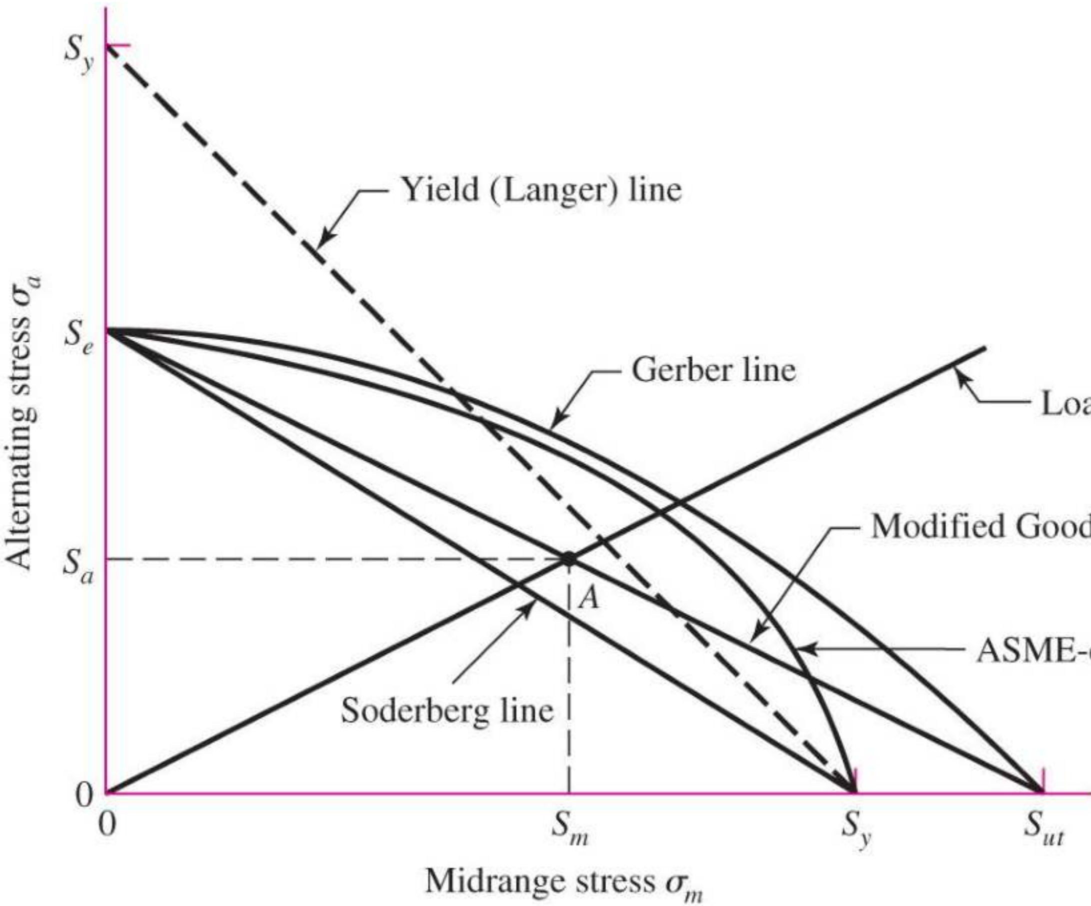


Fig. 6–27

Commonly Used Failure Criteria

- Langer line represents standard yield check.
- It is equivalent to comparing maximum stress to yield strength.

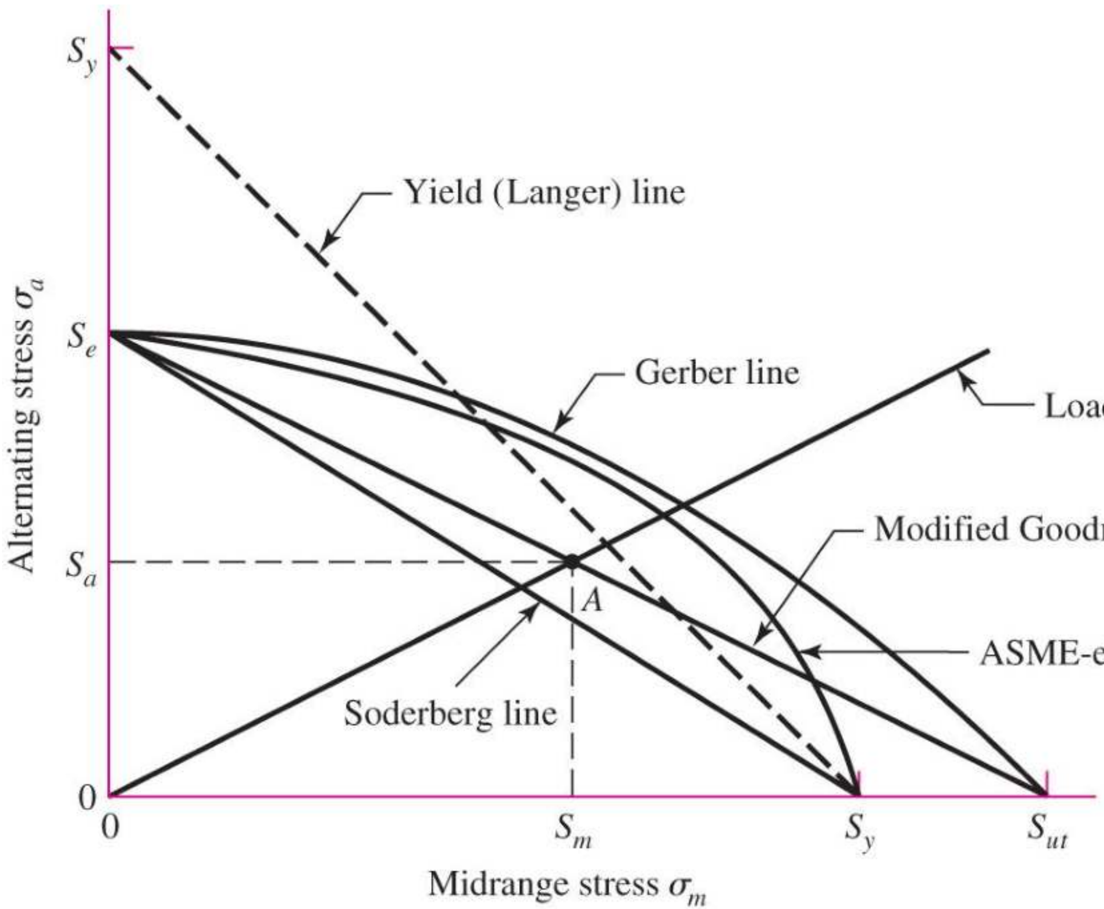


Fig. 6–27

Equations for Commonly Used Failure Criteria

- Intersecting a constant slope load line with each failure criteria produces design equations
- *n* is the design factor or factor of safety for infinite fatigue life

Soderberg $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n} \qquad (6\text{–}45)$$

mod-Goodman $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \qquad (6\text{–}46)$$

Gerber $$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1 \qquad (6\text{–}47)$$

ASME-elliptic $$\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1 \qquad (6\text{–}48)$$

Summarizing Table for Modified Goodman

Table 6–6

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Modified Goodman and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$	$S_a = \frac{r S_e S_{ut}}{r S_{ut} + S_e}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1 + r}$
$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$	$S_m = \frac{(S_y - S_e) S_{ut}}{S_{ut} - S_e}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m, r_{\text{crit}} = S_a / S_m$

Fatigue factor of safety

$$n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$$

Summarizing Table for Gerber

Table 6–7

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Gerber and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_a = \frac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{rS_{ut}}\right)^2}\right]$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{rS_y}{1+r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1+r}$
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_m = \frac{S_{ut}^2}{2S_e}\left[1 - \sqrt{1 + \left(\frac{2S_e}{S_{ut}}\right)^2\left(1 - \frac{S_y}{S_e}\right)}\right]$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{crit} = S_a/S_m$

Fatigue factor of safety

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \qquad \sigma_m > 0$$

Example 6–10

A 40-mm-diameter bar has been machined from an AISI 1050 cold-drawn bar. This part is to withstand a fluctuating tensile load varying from 0 to 70 kN. Because of the ends, and the fillet radius, a fatigue stress-concentration factor K_f is 1.85 for 10^6 or larger life. Find S_a and S_m and the factor of safety guarding against fatigue and first-cycle yielding, using (*a*) the Gerber fatigue line and (*b*) the ASME-elliptic fatigue line.

Solution

We begin with some preliminaries. From Table A–20, $S_{ut} = 690$ MPa and $S_y = 580$ MPa. Note that $F_a = F_m = 35$ kN. The Marin factors are, deterministically,

$k_a = 4.51(690)^{-0.265} = 0.798$: Eq. (6–19), Table 6–2, p. 296

$k_b = 1$ (axial loading, see k_c)

$k_c = 0.85$: Eq. (6–26), p. 290

$k_d = k_e = k_f = 1$

$S_e = 0.798(1)0.850(1)(1)(1)0.5(690) = 234$ MPa: Eqs. (6–8), (6–18), p. 290, p. 295

Example 6–10 (continued)

The nominal axial stress components σ_{ao} and σ_{mo} are

$$\sigma_{ao} = \frac{4F_a}{\pi d^2} = \frac{4(35000)}{\pi\, 0.04^2} = 27.9 \text{ MPa} \quad \sigma_{mo} = \frac{4F_m}{\pi d^2} = \frac{4(35000)}{\pi\, 0.04^2} = 27.9 \text{ MPa}$$

Applying K_f to both components σ_{ao} and σ_{mo} constitutes a prescription of no notch yielding:

$$\sigma_a = K_f \sigma_{ao} = 1.85(27.9) = 51.6 \text{ MPa} = \sigma_m$$

(*a*) Let us calculate the factors of safety first. From the bottom panel from Table 6–7 the factor of safety for fatigue is

Answer

$$n_f = \frac{1}{2}\left(\frac{690}{51.6}\right)^2 \left(\frac{51.6}{234}\right) \left\{-1 + \sqrt{1 + \left[\frac{2(51.6)234}{690(51.6)}\right]^2}\right\} = 4.11$$

From Eq. (6–49) the factor of safety guarding against first-cycle yield is

Answer

$$n_y = \frac{S_y}{\sigma_a + \sigma_m} = \frac{580}{51.6 + 51.6} = 5.62$$

Example 6–10 (continued)

Thus, we see that fatigue will occur first and the factor of safety is 4.13. This can be seen in Fig. 6–28 where the load line intersects the Gerber fatigue curve first at point *B*. If the plots are created to true scale it would be seen that $n_f = OB/OA$.

From the first panel of Table 6–7, $r = \sigma_a/\sigma_m = 1$,

Answer

$$S_a = \frac{(1)^2\,690^2}{2(234)}\left\{-1+\sqrt{1+\left[\frac{2(234)}{(1)690}\right]^2}\right\} = 211.9 \text{ MPa}$$

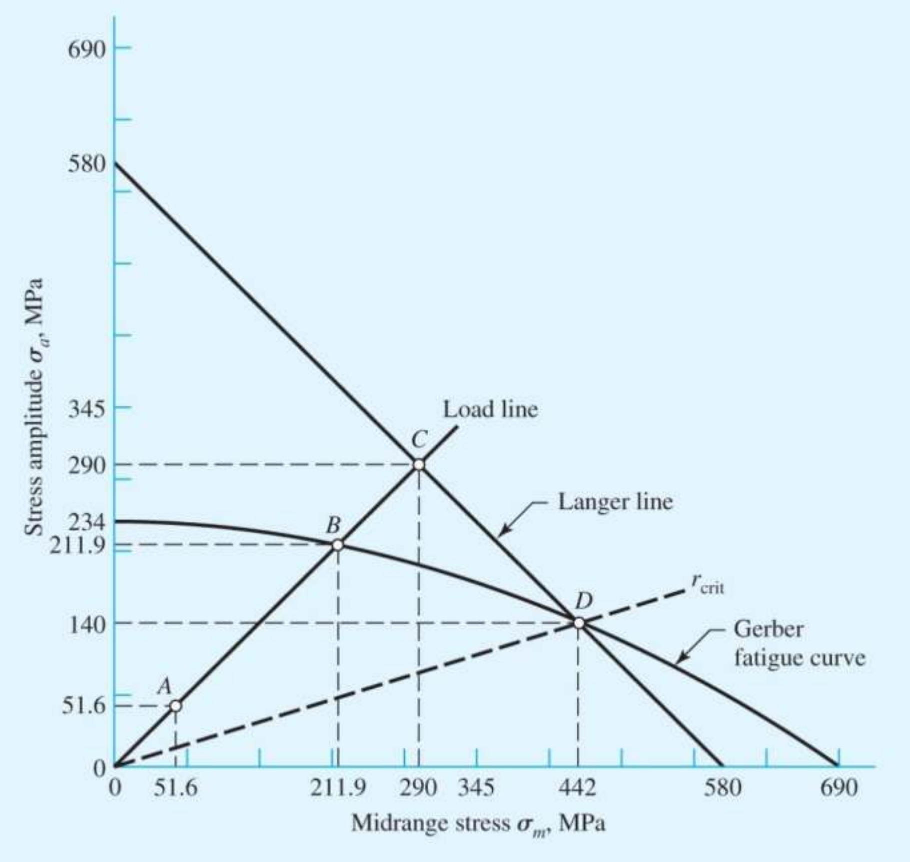


Fig. 6–28

Example 6–10 (continued)

Answer

$$S_m = \frac{S_a}{r} = \frac{211.9}{1} = 211.9 \text{ MPa}$$

As a check on the previous result, $n_f = OB/OA = S_a/\sigma_a = S_m/\sigma_m = 211.9/51.6 = 4.12$ and we see total agreement.

We could have detected that fatigue failure would occur first without drawing Fig. 6–28 by calculating r_{crit}. From the third row third column panel of Table 6–7, the intersection point between fatigue and first-cycle yield is

$$S_m = \frac{690^2}{2(234)}\left[1 - \sqrt{1 + \left(\frac{2(234)}{690}\right)^2\left(1 - \frac{580}{234}\right)}\right] = 442 \text{ MPa}$$

$$S_a = S_y - S_m = 580 - 442 = 138 \text{ MPa}$$

The critical slope is thus

$$r_{\text{crit}} = \frac{S_a}{S_m} = \frac{138}{442} = 0.312$$

which is less than the actual load line of $r = 1$. This indicates that fatigue occurs before first-cycle-yield.

(*b*) Repeating the same procedure for the ASME-elliptic line, for fatigue

Answer

$$n_f = \sqrt{\frac{1}{(51.6/234)^2 + (51.6/580)^2}} = 4.21$$

Example 6–10 (continued)

Again, this is less than $n_y = 5.62$ and fatigue is predicted to occur first. From the first row second column panel of Table 6–8, with $r = 1$, we obtain the coordinates S_a and S_m of point B in Fig. 6–29 as

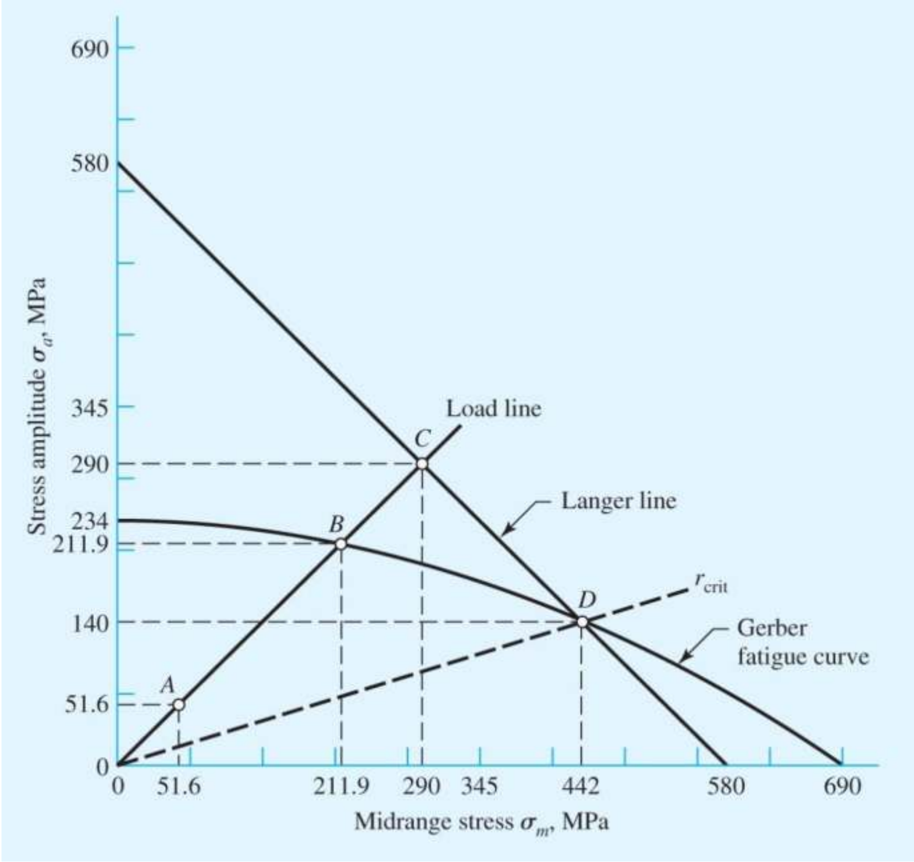


Fig. 6–28

Example 6–10 (continued)

Answer

$$S_a = \sqrt{\frac{(1)^2 234^2 (580)^2}{234^2 + (1)^2 580^2}} = 217 \text{ MPa}, \quad S_m = \frac{S_a}{r} = \frac{217}{1} = 217 \text{ MPa}$$

To verify the fatigue factor of safety, $n_f = S_a/\sigma_a = 217/51.6 = 4.21$.

As before, let us calculate r_{crit}. From the third row second column panel of Table 6–8,

$$S_a = \frac{2(580)\,234^2}{234^2 + 580^2} = 162 \text{ MPa}, \; S_m = S_y - S_a = 580 - 162 = 418 \text{ MPa}$$

$$r_{crit} = \frac{S_a}{S_m} = \frac{162}{418} = 0.388$$

which again is less than $r = 1$, verifying that fatigue occurs first with $n_f = 4.21$.

The Gerber and the ASME-elliptic fatigue failure criteria are very close to each other and are used interchangeably. The ANSI/ASME Standard B106.1M–1985 uses ASME-elliptic for shafting.

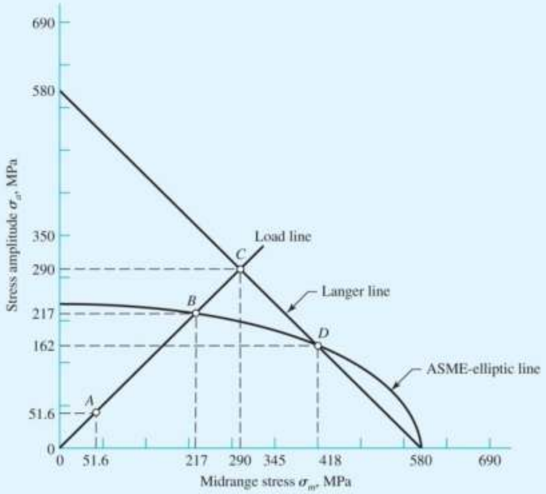


Fig. 6–29