$$\frac{d}{dx} \arcsin u = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} \arctan u = \frac{u'}{1+u^2}$$

$$\frac{d}{dx} \operatorname{arcsec} u = \frac{u'}{u\sqrt{u^2-1}}$$

Inverse hyperpolic fu$\underline{n}$

$$\frac{d}{dx} \operatorname{arcsinh} u = \frac{u'}{\sqrt{1+u^2}}$$

$$\frac{d}{dx} \operatorname{arctanh} u = \frac{u'}{1-u^2}$$

$$\frac{d}{dx} \operatorname{arcsech} u = \frac{u'}{-u\sqrt{1-u^2}}$$

Ex: find $\frac{d}{dx}(x^2 \cdot \operatorname{arcsinh} 4x)$

$$= x^2 \frac{4}{\sqrt{1-16x^2}} + 2x \operatorname{arcsinh} 4x$$

$$\frac{d}{dx}\left(\operatorname{arctanh}^3 x^2\right)$$

$$= \frac{d}{dx}\left(\operatorname{arctanh} x^2\right)^3$$

$$= 3\left(\operatorname{arctanh} x^2\right)^2 \frac{2x}{1-x^4}$$

$$= \frac{6x}{1-x^4} \operatorname{arctanh}^2 x^2$$

Find the derivatives

$$f(x) = \left(2 + \operatorname{arcsech} x^3\right)^4$$

الحل

$$f'(x) = 4\left(2 + \operatorname{arcsech} x^3\right)^3 \frac{3x^2}{-x^3\sqrt{1-x^6}}$$

$$= \frac{-12\left(2+\operatorname{arcsech} x^3\right)^3}{x\sqrt{1-x^6}}$$

قواعد

$$\int \frac{u'}{\sqrt{1+u^2}} = \operatorname{arcsinh} u + c$$

$$\int \frac{u'}{1-u^2} = \operatorname{arctanh} u + c$$

$$\int \frac{u'}{u\sqrt{1-u^2}} = -\operatorname{arcsech} u + c$$

$$\int \frac{3}{\sqrt{1+16x^2}}\, dx$$

$$= \frac{3}{4}\int \frac{4}{\sqrt{1+(4x)^2}}\, dx = \frac{3}{4}\operatorname{arcsinh} 4x + c$$

$$\int \frac{1}{x\sqrt{1-4x^2}}\, dx$$

$$= -\int \frac{2}{-2x\sqrt{1-(2x)^2}}\, dx = \operatorname{arcsech} 2x + c$$

$$\int \frac{5x}{4-9x^4}\,dx$$

$$= \frac{1}{4}\int \frac{5x}{1-\frac{9}{4}x^4}\,dx$$

$$= \frac{1}{4}\cdot\frac{5}{3}\int \frac{3x}{1-\left(\frac{3}{2}x^2\right)^2}\,dx \qquad u = \frac{3}{2}x^2,\quad u' = 3x$$

$$= \frac{5}{12}\operatorname{arctanh}\frac{3}{2}x^2 + C$$

Exer

Find

$$\frac{d}{dx}\left(e^{x}\arctan x^3\right), \quad \frac{d}{dx}\left(\operatorname{arcsinh}\sqrt{x}\right)$$

$$\int \frac{x}{\sqrt{1+x^4}}\,dx, \quad \int \frac{1}{1-x}\cdot\frac{1}{\sqrt{x}}\,dx$$