$$\int \cos x\,dx = \sin x + c \quad \checkmark$$

$$\int \sin x\,dx = -\cos x + c \quad \checkmark$$

$$\int ① ②\,dx = ① \int ②\,dx - \int ①' \int ②\,dx$$

تكامل الثانية . مشتقة الأولى × تكامل الثانية . (الأولى) = (الثانية)(الأولى) ∫ −

$$\int u\,dv = u\,v - \int v\,du$$

(مشتقة الأولى × تكامل الثانية) (تكامل الثانية الأولى)

Ex

$$\int \underset{①}{x}\ \underset{②}{\sin x}\,dx$$

$$= x(-\cos x) - \int 1(-\cos x)\,dx$$

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + c$$

التحقق — المشتقة $= -(-x\sin x + \cos x) + \cos x$

$= x\sin x$

∴ التكامل صحيح

EX: ② ①

$$\int \underline{x^2}\ \underline{\cos x}\ dx$$

$$= (\cos x)\frac{x^3}{3} - \int (-\sin x)\frac{x^3}{3}\,dx$$

$$= \frac{1}{3}x^3\cos x + \frac{1}{3}\int x^3 \sin x\,dx$$

① ②

$$\int x^2 \cos x\ dx$$

$$= x^2 \sin x - \int 2x \sin x\ dx$$

$$= x^2 \sin x - 2\int \underset{\text{①}}{\underline{x}}\ \underset{\text{②}}{\underline{\sin x}}\ dx$$

$$= x^2 \sin x - 2\left[x(-\cos x) - \int -\cos x\,dx\right]$$

$$= x^2 \sin x + 2x\cos x - 2\int \cos x\,dx$$

$$= x^2 \sin x + 2x\cos x - 2\sin x + C$$

Exer

$$\int x^2 \sin x\,dx = \text{- - - - - -}$$

$$= \text{- - - - - -}$$

EX: $\int x e^{x^2} dx$ (① x^2, ② e^{x^2})

$= x(\ ?\ ) \cdots\cdots X$

$\frac{1}{2}\int e^{x^2} 2x\ dx = \frac{1}{2} e^{x^2} + C$

EX: $\int x e^{x} dx$ (① x, ② e^x)

$= x e^x - \int e^x dx = x e^x - e^x + C$

EX: $\int x^2 e^x dx$ (① x^2, ② e^x)

$= x^2 e^x - \int 2x e^x dx$

$= x^2 e^x - 2\int x e^x dx$ (① x, ② e^x)

$= x^2 e^x - 2\left[x e^x - \int e^x dx\right]$

$= x^2 e^x - 2x e^x + 2e^x + C$

Ex: ① ②
$$\int x \ln x \, dx$$

$$= x(\times) - \cdots\cdots$$

② ①
$$\int x \ln x \, dx$$

$$= (\ln x)\frac{x^2}{2} - \int \frac{1}{x}\frac{x^2}{2}\,dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\,dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

Exer: ② ①
$$\int x^2 \ln x \, dx$$

Exer ② ①
$$\int x^5 \ln x \, dx$$

$$= \frac{x^6}{6}\ln x - \int \frac{1}{x}\frac{x^6}{6}\,dx$$

$$= \frac{1}{6}x^6 \ln x - \frac{1}{6}\int x^5\,dx \quad = \frac{1}{6}x^6 \ln x - \frac{1}{36}x^6 + C$$

Ex:

$$\int \ln x \, dx$$

$$= \int (\ln x)^{①} \cdot 1^{②} \, dx$$

$$= (\ln x)\, x - \int \frac{1}{x} \cdot x \, dx$$

$$= x \ln x - \int 1 \, dx$$

$$= x \ln x - x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

Ex:

$$\int \tan x \, dx = -\int \frac{-\sin x}{\cos x} \, dx = -\ln|\cos x| + C$$

$$= -\ln|\sec x|^{-1} + C$$

$$\left.\begin{aligned} \int \tan x \, dx &= \ln|\sec x| + C \\ \int \tan x \, dx &= -\ln|\cos x| + C \end{aligned}\right\}$$

$$\sin^2 x + \cos^2 x = 1 \qquad \div \cos^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$ قاعدة مهمة ثانوي

$$\tan^2 x = \sec^2 x - 1$$

Ex:

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx$$

$$= \tan x - x + C$$

$$\cot^2 x + 1 = \csc^2 x$$

$$\cot^2 x = \csc^2 x - 1$$

Ex:

$$\int \cot^2 x \, dx = \int (\csc^2 x - 1) \, dx$$

$$\int \cot^2 x \, dx = -\cot x - x + C$$

$\int \sin^2 x \, dx$?! $\int \cos^2 x \, dx$?!

$$\int \sin^2 x\,dx = \int (\sin x)^2\,dx \quad ?! \Rightarrow$$

Ex:

$$\int \sin^2 x \cos x\,dx$$

$$= \int (\sin x)^2 \cos x\,dx = \frac{(\sin x)^3}{3} + C$$

$$= \frac{1}{3}\sin^3 x + C$$

$$\int \sin^2 x\,dx \quad ?! \quad \int \cos^2 x\,dx \quad ?!$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= (1 - \sin^2 x) - \sin^2 x$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$2\sin^2 x = 1 - \cos 2x$$

قاعدة

$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$$

$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$$

Ex:

$$\int \sin^2 x\,dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$$

$$= \int \frac{1}{2}\,dx - \frac{1}{2}\frac{1}{(2)}\int (2)\cos 2x\,dx$$

$$= \frac{1}{2}x - \frac{1}{4}\sin 2x + C$$

Exer

$$\int \cos^2 x\,dx$$

$$= \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

$$\int \sec x\,dx \quad ?!$$

$$\int \csc x\,dx \quad ?!$$