The inverse trignometric functions

$$\int \tan^3 x \, dx$$

$$= \int \tan^2 x \tan x \, dx$$

$$= \int (\sec^2 x - 1) \tan x \, dx$$

$$= \int (\tan x)' \sec^2 x \, dx - \int \tan x \, dx$$

$$= \frac{1}{2} \tan^2 x + \ln|\cos x| + c$$

$$\int \cot^3 x \, dx$$

Exer $\int \csc^3 x \, dx$

$\vdots$

Inverse trignometric functions

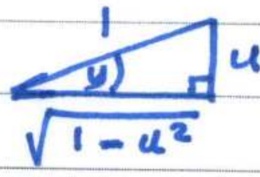
$\sin 30^\circ = \frac{1}{2}$ $\qquad$ $\arcsin \frac{1}{2} = 30^\circ = \frac{\pi}{6}$

$\sin \frac{\pi}{6} = \frac{1}{2}$

$\arcsin\left(\sin \frac{\pi}{6}\right) = \arcsin\left(\frac{1}{2}\right)$

$\frac{\pi}{6} = \arcsin \frac{1}{2}$

$u = \sin y \Rightarrow y = \arcsin u$



$\frac{du}{dx} = \cos y \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{du/dx}{\cos y} = \frac{u'}{\sqrt{1-u^2}}$

$\int \sec^3 x \, dx$

$= \int \sec^2 x \, \sec x \, dx$

$= \int (1 + \tan^2 x) \sec x \, dx$

$= \int \sec x \, dx + \int \tan^2 x \sec x \, dx$

$= \ln|\sec x + \tan x| + \int (\sec^2 x - 1) \sec x \, dx$

$\int \cancel{\sec^3 x} \, dx = \ln|\cancel{\sec x + \tan x}| + \int \cancel{\sec^3 x} \, dx$
$- \int \cancel{\sec x} \, dx$

$\int \sec^3 x \, dx = \int \overset{②}{\sec^2 x} \; \overset{①}{\sec x} \, dx$

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$= \sec x \tan x - \int \sec x \tan^2 x \, dx$

$\int \sec^3 x \, dx = \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$
$\int \sec^3 x \, dx = \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$
$2 \int \sec^3 x \, dx = \sec x \tan x + \ln|\sec x + \tan x| + c$
$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + c$

$$y = \arcsin u \Rightarrow \frac{dy}{dx} = \frac{u'}{\sqrt{1-u^2}}$$

$f(x)$	$f'(x)$
$\arcsin u$	$\frac{u'}{\sqrt{1-u^2}}$
$\arctan u$	$\frac{u'}{1+u^2}$
$\text{arcsec}\, u$	$\frac{u'}{u\sqrt{u^2-1}}$

تحفظ حفظاً جيداً

$\arccos u$ $\quad \frac{-u'}{\sqrt{1-u^2}}$

$\text{arccot}\, u$ $\quad \frac{-u'}{1+u^2}$

$\text{arccsc}\, u$ $\quad \frac{-u'}{u\sqrt{u^2-1}}$

EX 1

$$\frac{d}{dx}\left(\arcsin \underbrace{3x^2}_{u}\right) = \frac{6x}{\sqrt{1-9x^4}}$$

$$\frac{d}{dx}\arctan(3x-1) = \frac{3}{1+(3x-1)^2}$$