Taibah University
College of Engineering

Course title:

Structural Analysis

Code: AE 321
Total credit hours = 3
Pre-requisites: GE201 Statics ;
AE212 Architectural Design (1)

Lecturer: Dr. Ahmed Kamal

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Approximate Analysis of Statically Indeterminate Structures

AE 321: Structural Analysis Dr. Ahmed Kamal

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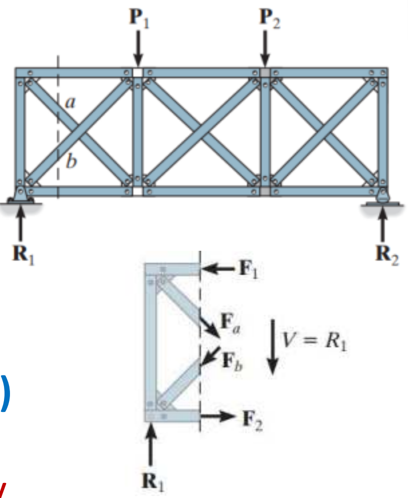
1. Use of Approximate Methods

- When a *model* is used to represent any structure, the analysis of it must satisfy *both* the conditions of:
  1) equilibrium and
  2) compatibility of displacement at the joints.
- The *material's modulus of elasticity and the size and shape of the members* are needed for *statically indeterminate analysis of a* structure.
- For analysis, a simpler model of the structure must be developed, one that is statically determinate.

- Once this model is specified, the analysis of it is called an ***approximate analysis***. a preliminary design of the members of a structure can be made.
- An approximate analysis also provides insight as to a structure's behavior under load and is beneficial when checking a more exact analysis or when time, money, or capability are not available for performing the more exact analysis.

2. Approximate Analysis of Trusses

In the case shown, it will be noticed that if a diagonal is removed from each of the three panels, it will render the truss statically determinate.



$b + r > 2j$ **$16 + 3 > 8(2)$**

Hence, the truss is statically indeterminate to the ***third degree***.

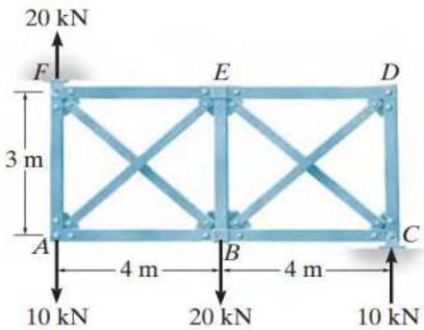
Therefore we must make three assumptions regarding the bar forces *in order to reduce the truss to one that is statically determinate.*

Method 1: If the diagonals are intentionally designed to be long and slender, it is reasonable to assume that they cannot support a compressive force; otherwise, they may easily buckle. Hence the panel shear is resisted entirely by the tension diagonal, whereas the compressive diagonal is assumed to be a zero-force member.

Method 2: If the diagonal members are intended to be constructed from large rolled sections such as angles or channels, they may be equally ***capable of supporting a tensile and compressive force.*** Here we will assume that the tension and compression diagonals each carry half the panel shear.

EXAMPLE

Determine (approximately) the forces in the members of the truss shown in Figure. *The diagonals are to be designed to support both tensile and compressive forces,* and therefore each is assumed to carry half the panel shear. The support reactions have been computed.

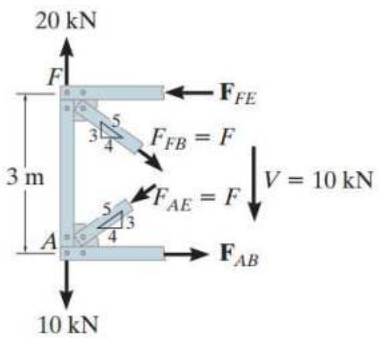


SOLUTION

b + r > 2j   11 + 3 > 6(2)

Statically indeterminate to the ***second*** degree.

- The two assumptions require the tensile and compressive diagonals to carry equal forces,



$$F_{FB} = F_{AE} = F.$$

$$+\uparrow \Sigma F_y = 0;$$

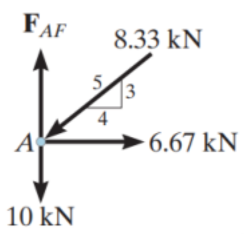
$$20 - 10 - 2\left(\tfrac{3}{5}\right)F = 0 \qquad F = 8.33 \text{ kN}$$

so that

$$F_{FB} = 8.33 \text{ kN (T)} \quad \textit{Ans.}$$
$$F_{AE} = 8.33 \text{ kN (C)} \quad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad -8.33\left(\tfrac{4}{5}\right)(3) + F_{FE}(3) = 0 \quad F_{FE} = 6.67 \text{ kN (C)} \quad \textit{Ans.}$$
$$\circlearrowleft + \Sigma M_F = 0; \quad -8.33\left(\tfrac{4}{5}\right)(3) + F_{AB}(3) = 0 \quad F_{AB} = 6.67 \text{ kN (T)} \quad \textit{Ans.}$$

From joint *A*

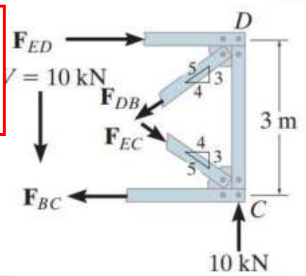


$$+\uparrow \Sigma F_y = 0; \quad F_{AF} - 8.33\left(\tfrac{3}{5}\right) - 10 = 0$$

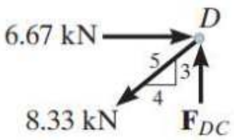
$$F_{AF} = 15 \text{ kN (T)} \quad \textit{Ans.}$$

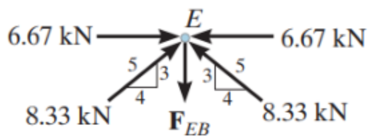
A vertical section through the right panel

$$F_{DB} = 8.33\ \text{kN (T)}, \quad F_{ED} = 6.67\ \text{kN (C)}$$
$$F_{EC} = 8.33\ \text{kN (C)}, \quad F_{BC} = 6.67\ \text{kN (T)}$$



joints D and E,

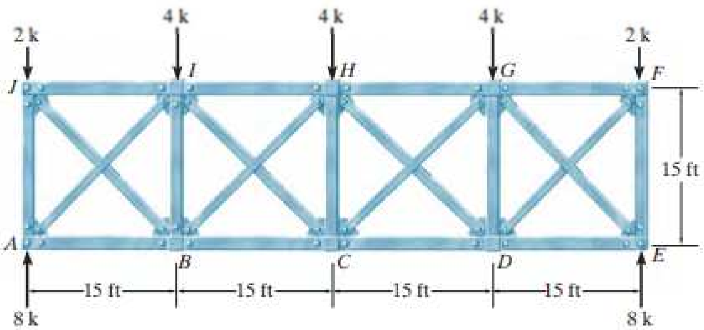




$$F_{DC} = 5\ \text{kN (C)}$$
$$F_{EB} = 10\ \text{kN (T)}$$

EXAMPLE

Cross bracing is used to provide lateral support for this bridge deck due to the wind and unbalanced traffic loads. Determine (approximately) the forces in the members of this truss. *Assume the diagonals are slender and therefore will not support a compressive force.*



SOLUTION

b + r > 2j   21 + 3 > 10(2)

Statically indeterminate to the **fourth** degree.

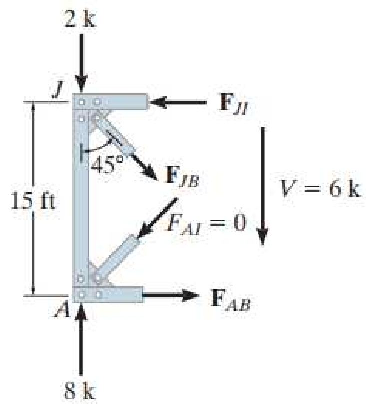
The four assumptions to be used require that each compression diagonal sustain zero force.

$$+\uparrow \Sigma F_y = 0;$$

$$F_{AI} = 0$$

$$8 - 2 - F_{JB}\cos 45^\circ = 0$$

$$F_{JB} = 8.49\text{ k (T)}$$



$$\circlearrowleft + \Sigma M_A = 0; \quad -8.49\sin 45^\circ(15) + F_{JI}(15) = 0$$

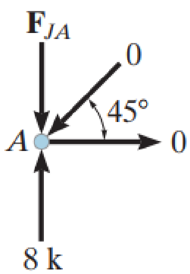
$$F_{JI} = 6\text{ k (C)}$$

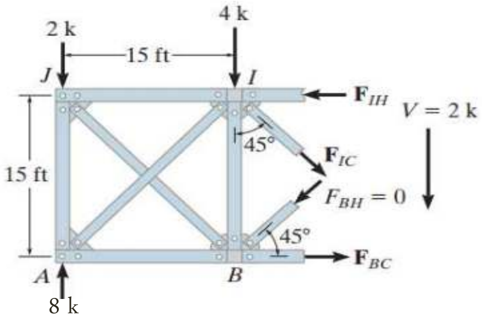
$$\circlearrowleft + \Sigma M_J = 0; \quad -F_{AB}(15) = 0$$

$$F_{AB} = 0$$

From joint A,

$$F_{JA} = 8\text{ k (C)}$$





$$F_{BH} = 0 \qquad \textit{Ans.}$$

$+\uparrow \Sigma F_y = 0; \qquad 8 - 2 - 4 - F_{IC} \cos 45° = 0$

$$F_{IC} = 2.83 \text{ k (T)} \qquad \textit{Ans.}$$

$\circlearrowleft + \Sigma M_B = 0; \quad -8(15) + 2(15) - 2.83 \sin 45°(15) + F_{IH}(15) = 0$

$$F_{IH} = 8 \text{ k (C)} \qquad \textit{Ans.}$$

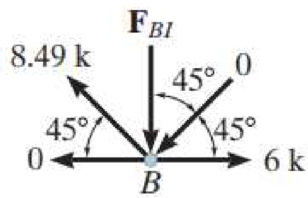
$\circlearrowleft + \Sigma M_I = 0; \qquad -8(15) + 2(15) + F_{BC}(15) = 0$

$$F_{BC} = 6 \text{ k (T)} \qquad \textit{Ans.}$$

From joint B, $\qquad +\uparrow \Sigma F_y = 0;$

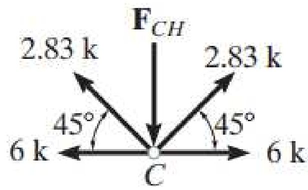
$$8.49 \sin 45° - F_{BI} = 0$$

$$F_{BI} = 6 \text{ k (C)}$$



The forces in the other members can be determined by **symmetry**,

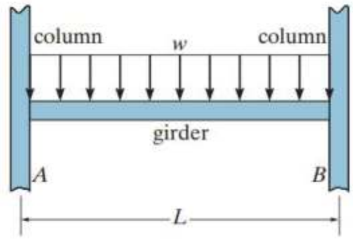
except F_{CH}; however, from joint C,



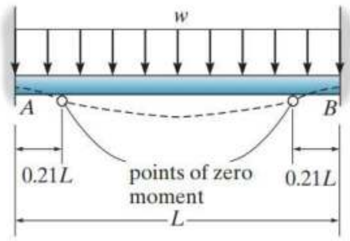
$+\uparrow \Sigma F_y = 0; \qquad 2(2.83 \sin 45°) - F_{CH} = 0$

$$F_{CH} = 4 \text{ k (C)}$$

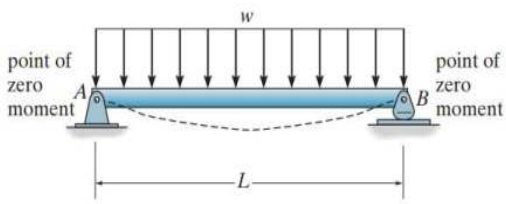
3. Approximate Analysis for Vertical Loads on Building Frames:



Typical building frame



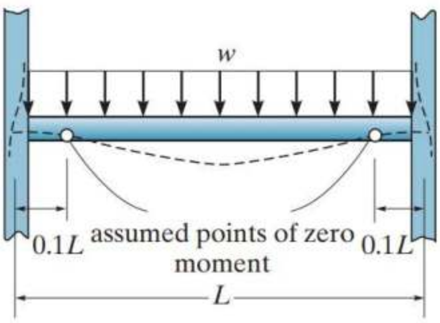
Fixed supported



simply supported

In reality, however, the columns will provide some flexibility at the supports, and therefore we will assume that zero moment occurs at ***the average point*** between the two extremes.

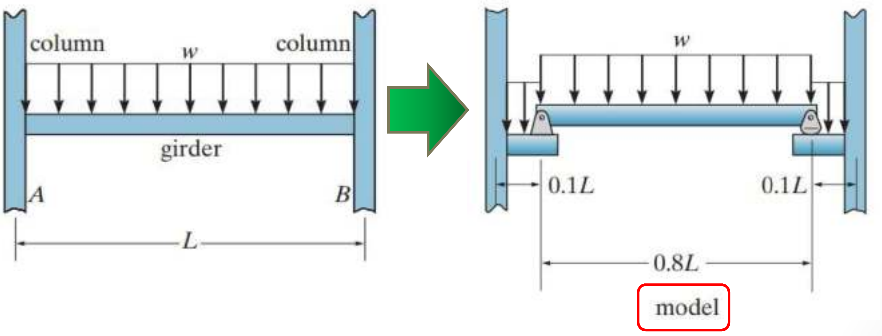
$(0.21L + 0)/2 \approx 0.1L$ from each support



approximate case

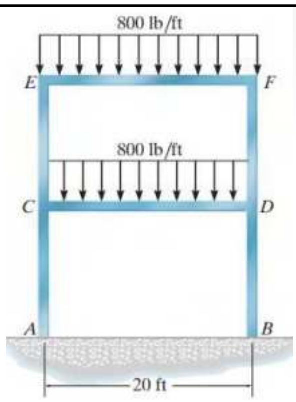
The following three assumptions are incorporated in this model:

1. There is zero moment in the girder, $0.1L$ from the left support.
2. There is zero moment in the girder, $0.1L$ from the right support.
3. The girder does not support an axial force.

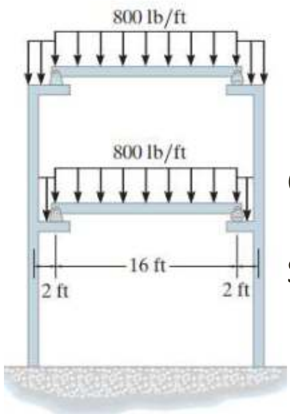


EXAMPLE

Determine (approximately) the moment at the joints E and C caused by members EF and CD of the building bent in Figure.



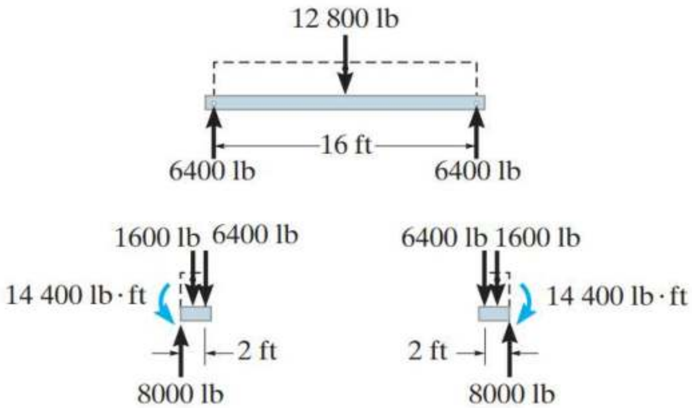
SOLUTION



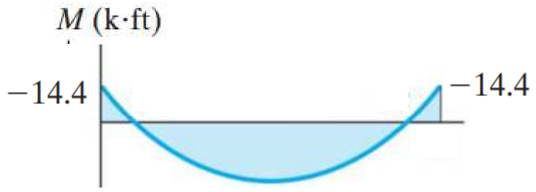
Model

Cantilevered span length = 0.1 L = 2 ft

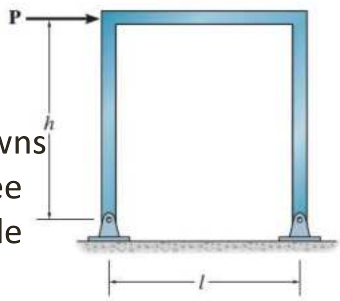
Simple Beam length = 16 ft



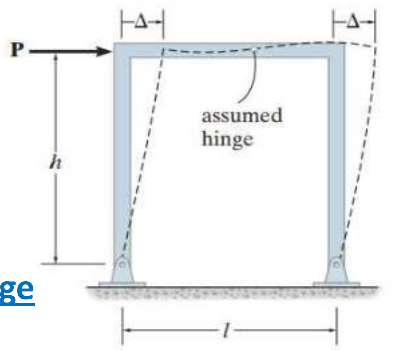
$$M = 1600(1) + 6400(2) = 14\,400 \text{ lb} \cdot \text{ft} = 14.4 \text{ k} \cdot \text{ft}$$



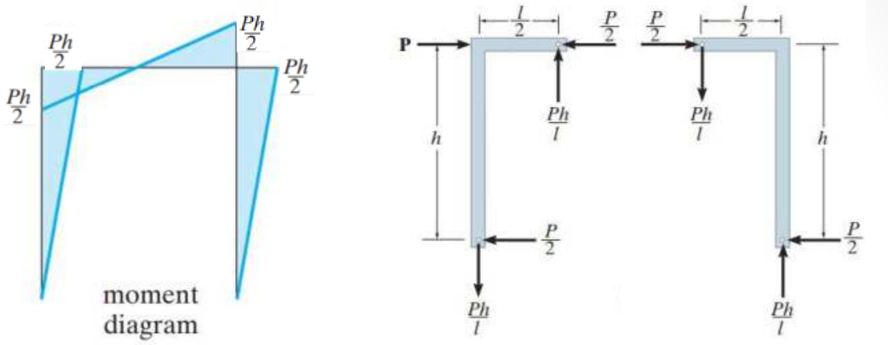
4. Approximate Analysis for Portal Frames



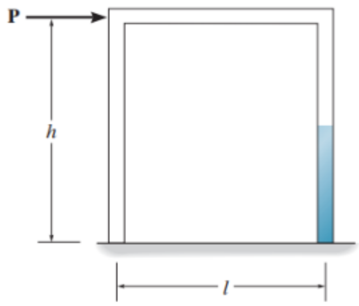
a) Pin Supported: Since **four** unknowns exist at the supports but only three equilibrium equations are available for solution, this structure is ***statically indeterminate to the first degree.***

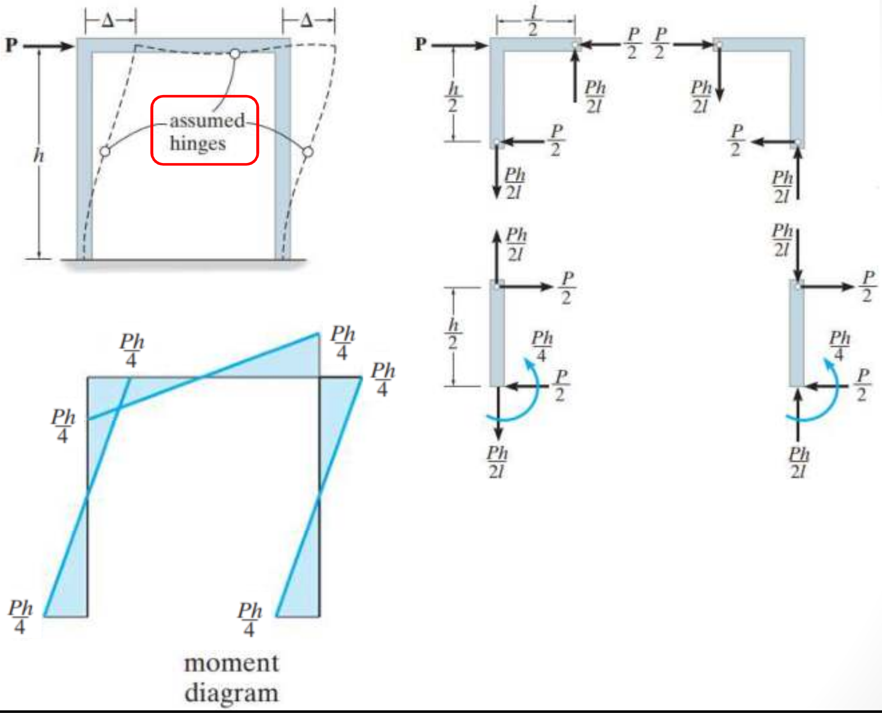


Where ***a point of inflection*** is located approximately at the girder's midpoint. Since the moment in the girder is zero at this point. we can assume **a hinge exists there .**



b) Fixed Supported: Portals with two fixed supports are statically indeterminate to the **third degree** since there are a total of six unknowns at the supports.





c) **Partial Fixity:** the points of inflection on the columns lie somewhere between the case of having a pin-supported portal "inflection points" are at the supports, and a fixed-supported portal, where the inflection points are at the center of the columns. Many engineers arbitrarily define the location at ***h/3***, *and therefore place hinges at these points, and also at the center of the girder.*