MATH203 Calculus

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Power Series

Definition

If x is a variable, then an intinite series of the form
$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_2 x + \cdots + a_n x^n + \ldots;\ a_i \in \mathbb{R}$ is called a power series in x or. $\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_2(x-c) + \cdots + a_n(x-c)^n + \ldots;\ c \in \mathbb{R}$
is called a power series in $(x-c)$

Remarks:

1. We can check the convergence or divergence of a power series $\sum_{n=0}^{\infty} a_n x^n$ for different values of x.
2. Every power series in x converges if $x = 0$.
3. To find all other values of x for which $\sum_{n=0}^{\infty} a_n x^n$ is convergent, we often use **the absolute ratio test**.

Interval of convergence

After finding values of x which are convergent in the interval, say (a,b), this is called the interval of convergence for the power series $\sum_{n=0}^{\infty} a_n x^n$.

Radius of convergence

Half of the length of interval of convergence is called the radius of convergence of the the power series $\sum_{n=0}^{\infty} a_n x^n$.

Theorem

Every power series $\sum_{n=0}^{\infty} a_n x^n$ satisfies one of the following:

1. The series converges only when $x = 0$ and this convergence is absolute.
2. The series converges for all x, and this convergence is absolute.
3. There is a number $R >$ such that the series converges absolutely when $x < R$ and diverges when $x > R$. Note that the series may converge or diverge depending on the particular series.

Examples

Find the interval of convergence and radius of convergence of the following series:

(1): $\sum_{n=1}^{\infty} \frac{n}{3^n} x^n$ (2): $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ (3): $\sum_{n=0}^{\infty} (n!) x^n$

(4): $\sum_{n=0}^{\infty} (2x)^n \frac{1}{n}$ (5): $\sum_{n=0}^{\infty} x^n \frac{1}{\sqrt{n}}$ (6): $\sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1} (x-3)^n$

Solution:

Examples

Determine whether the series is absolute convergent, conditionally convergent or divergent

(1): $\sum_{n=1}^{\infty}(-1)^n(1+e^{-n})$ (2): $\sum_{n=1}^{\infty}(-1)^n\frac{n}{\ln n}$

(3): $\sum_{n=1}^{\infty}(-1)^n\frac{n^2+3}{(2n-5)^2}$ (4): $\sum_{n=1}^{\infty}\frac{n!}{(-5)^n}$

Solution:

Examples

Find the interval of convergence and radius of convergence of the following series:

(1): $\sum_{n=0}^{\infty} \frac{1}{n+4} x^n$ (2): $\sum_{n=0}^{\infty} \frac{x^n n^2}{2^n}$

(3): $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{n}} x^n$ (4): $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{10^n} (x-4)^n$

Solution: