

Chapter 3

Vectors

3.1 Coordinate Systems

3.2 Vector and Scalar Quantities

3.3 Some Properties of Vectors

3.4 Components of a Vector and Unit Vectors



Vectors



- Vector quantities
 - Physical quantities that have both numerical and directional properties
- Mathematical operations of vectors in this chapter
 - Addition
 - Subtraction

Coordinate Systems

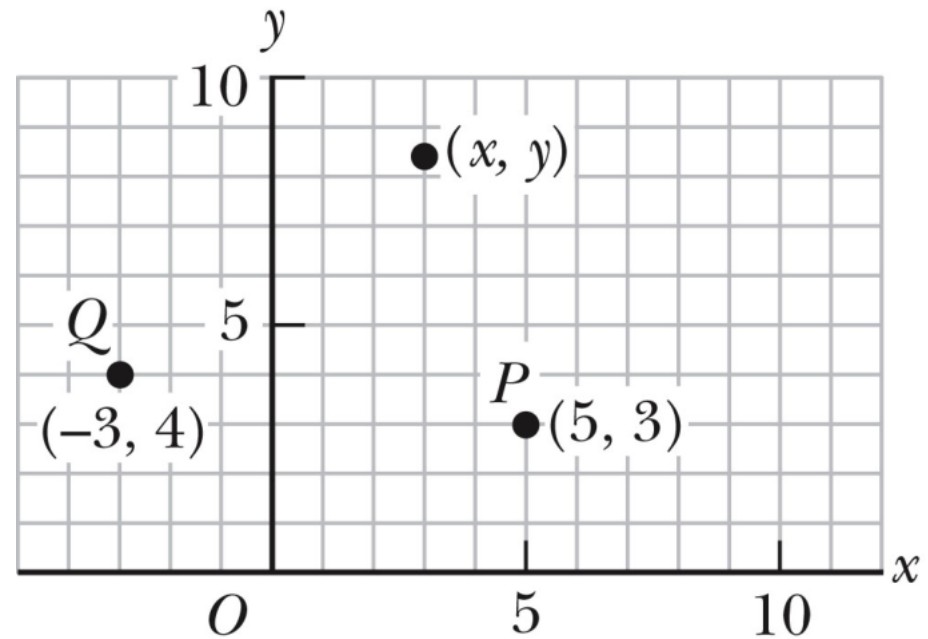


- Used to describe the position of a point in space
- Common coordinate systems are:
 - Cartesian
 - Polar

Cartesian Coordinate System



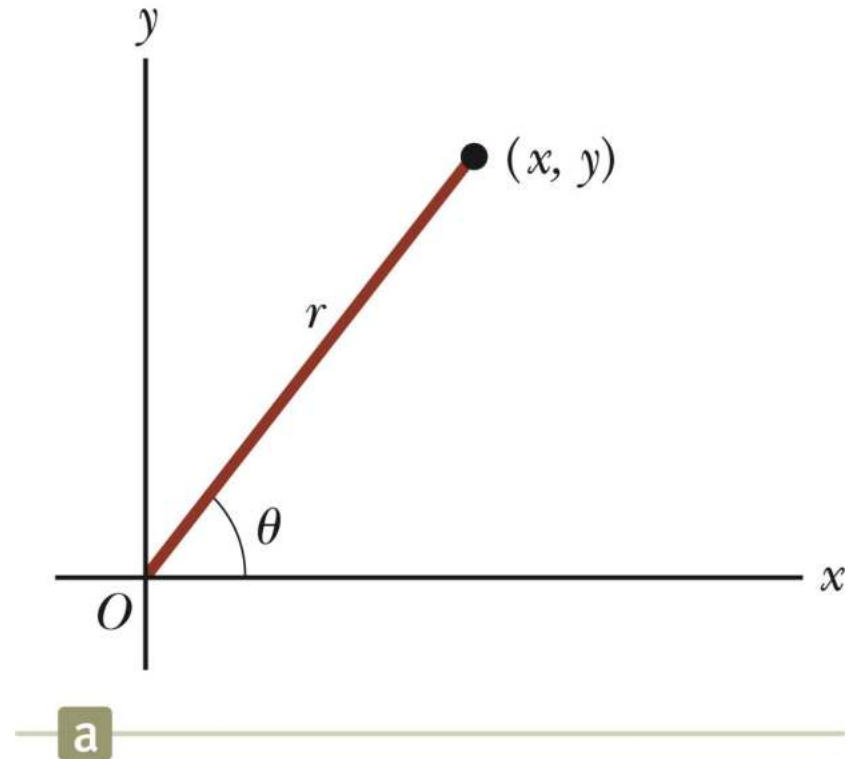
- Also called rectangular coordinate system
- x - and y - axes intersect at the origin
- Points are labeled (x, y)



Polar Coordinate System



- Origin and reference line are noted
- Point is distance r from the origin in the direction of angle θ , from reference line
 - The reference line is often the x-axis.
- Points are labeled (r, θ)



Polar to Cartesian Coordinates

- Based on forming a right triangle from r and θ
- $x = r \cos \theta$
- $y = r \sin \theta$
- If the Cartesian coordinates are known:

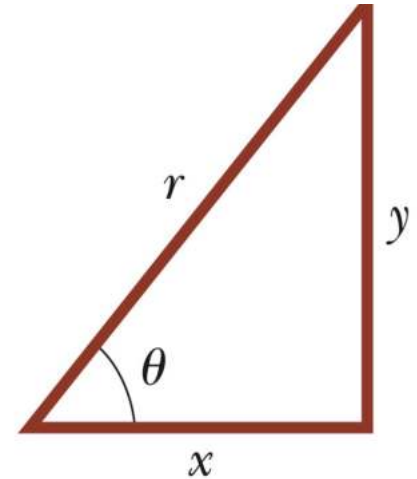
$$\tan \theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$



b

Example 3.1



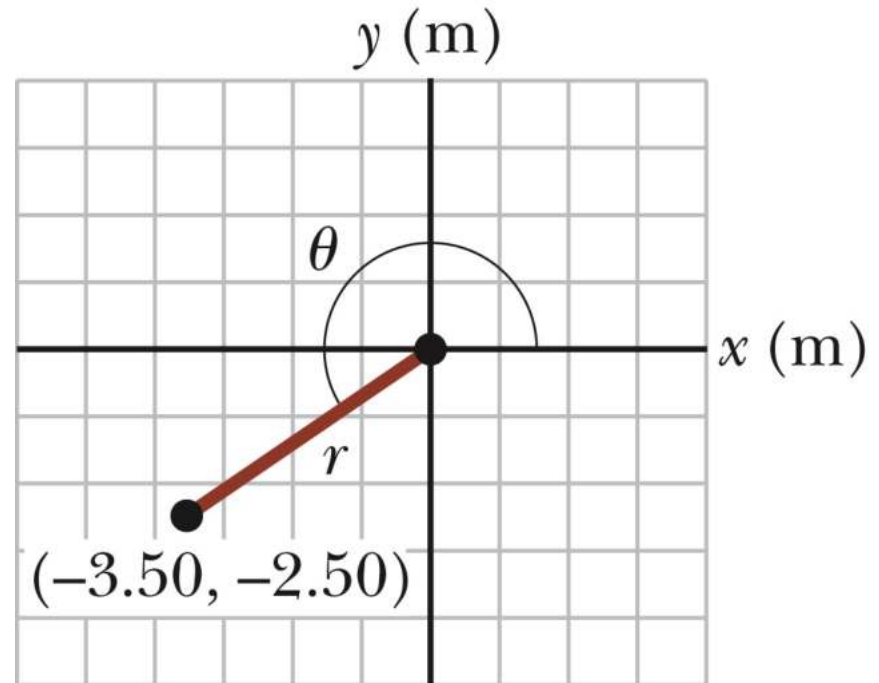
•The Cartesian coordinates of a point in the xy plane are $(x,y) = (-3.50, -2.50)$ m, as shown in the figure. Find the polar coordinates of this point.

Solution: From Equation 3.4,

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-3.50 \text{ m})^2 + (-2.50 \text{ m})^2} \\ &= 4.30 \text{ m} \end{aligned}$$

and from Equation 3.3,

$$\begin{aligned} \tan \theta &= \frac{y}{x} = \frac{-2.50 \text{ m}}{-3.50 \text{ m}} = 0.714 \\ \theta &= 216^\circ \quad (\text{signs give quadrant}) \end{aligned}$$



Vectors and Scalars

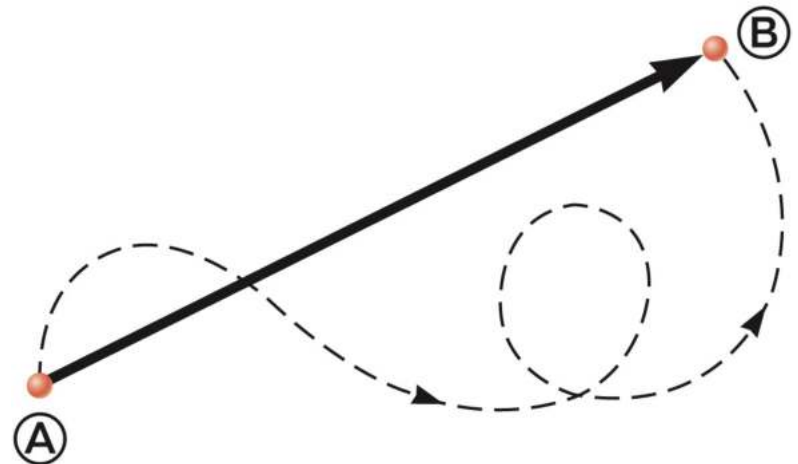


- A ***scalar quantity*** is completely specified by a single value with an appropriate unit and has no direction.
 - Many are always positive
 - Some may be positive or negative
 - Rules for ordinary arithmetic are used to manipulate scalar quantities.
- A ***vector quantity*** is completely described by a number and appropriate units plus a direction.

Vector Example



- A particle travels from A to B along the path shown by the broken line.
 - This is the **distance** traveled and is a scalar.
- The **displacement** is the solid line from A to B
 - The displacement is independent of the path taken between the two points.
 - Displacement is a vector.





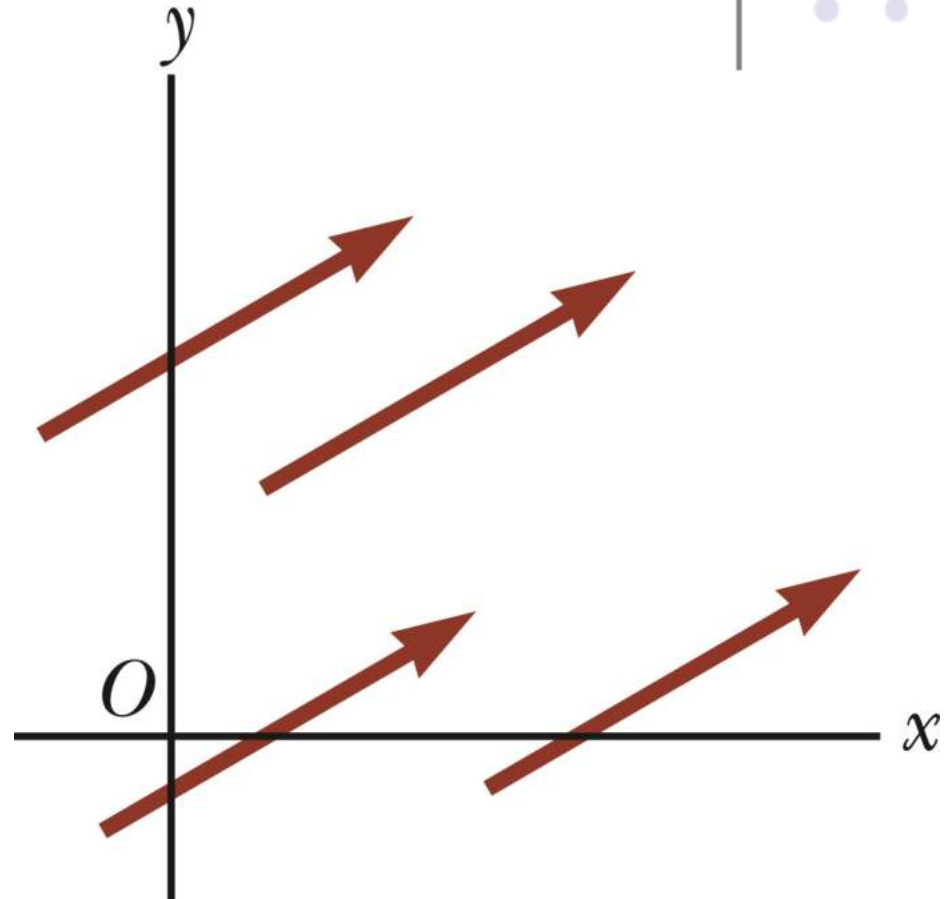
Vector Notation

- Text uses bold with arrow to denote a vector: $\vec{\mathbf{A}}$
- Also used for printing is simple bold print: \mathbf{A}
- When dealing with just the magnitude of a vector in print, an italic letter will be used: A or $|A|$ or \vec{A}
 - The magnitude of the vector has physical units.
 - The magnitude of a vector is always a positive number.
- When handwritten, use an arrow: \vec{A}

Equality of Two Vectors



- Two vectors are *equal* if they have the same magnitude and the same direction.
- $\vec{A} = \vec{B}$ if $A = B$ and they point along parallel lines
- All of the vectors shown are equal.
- Allows a vector to be moved to a position parallel to itself



Adding Vectors



- Vector addition is very different from adding scalar quantities.
- When adding vectors, their directions must be taken into account.
- Units must be the same
- Graphical Methods
 - Use scale drawings
- Algebraic Methods
 - More convenient

Adding Vectors Graphically

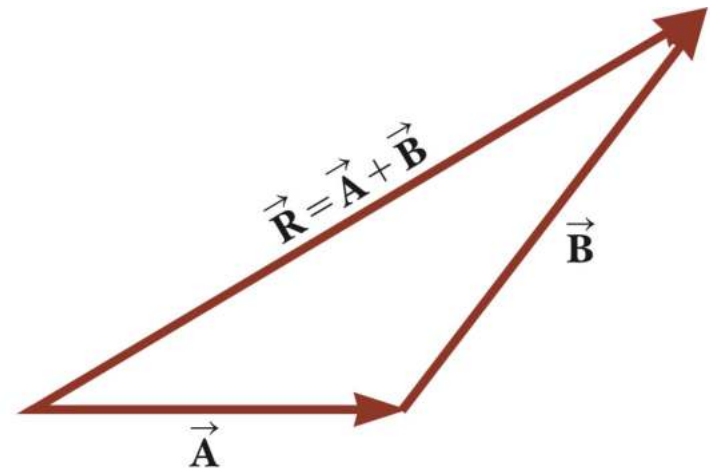


- Choose a scale.
- Draw the first vector, \vec{A} , with the appropriate length and in the direction specified, with respect to a coordinate system.
- Draw the next vector with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector \vec{A} and parallel to the coordinate system used for \vec{A} .



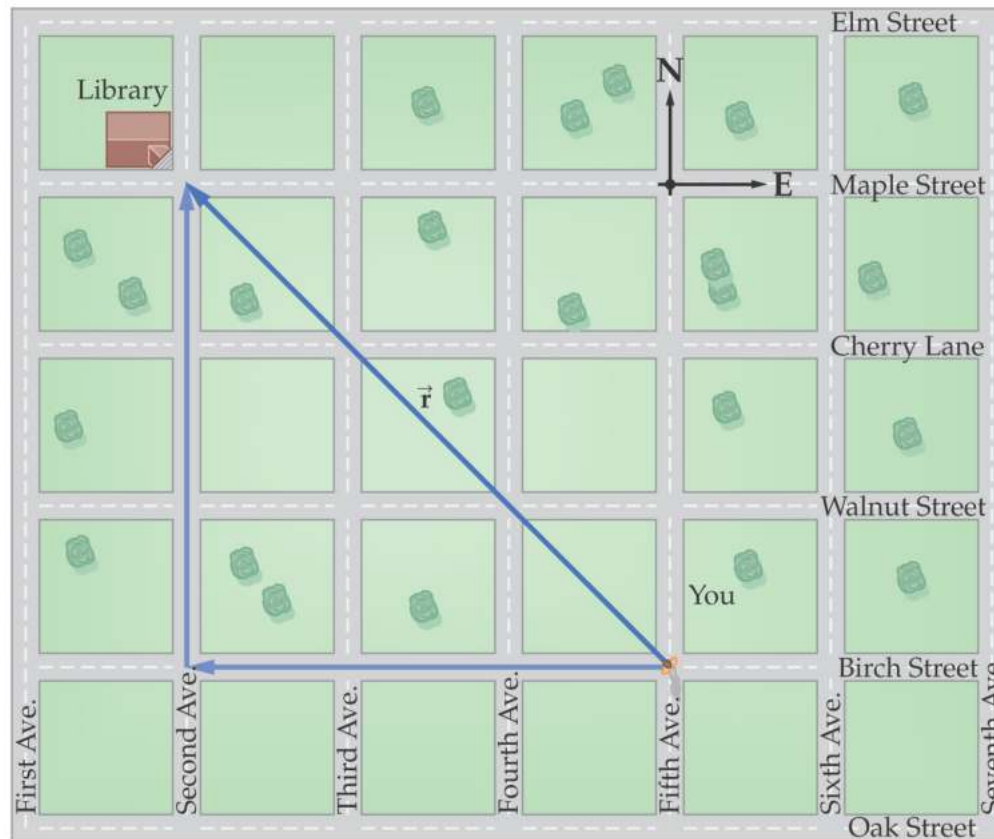
Adding Vectors Graphically, cont

- Continue drawing the vectors “tip-to-tail” or “head-to-tail”.
- The resultant is drawn from the origin of the first vector to the end of the last vector.
- Measure the length of the resultant and its angle.
 - Use the scale factor to convert length to actual magnitude.



The Components of a Vector

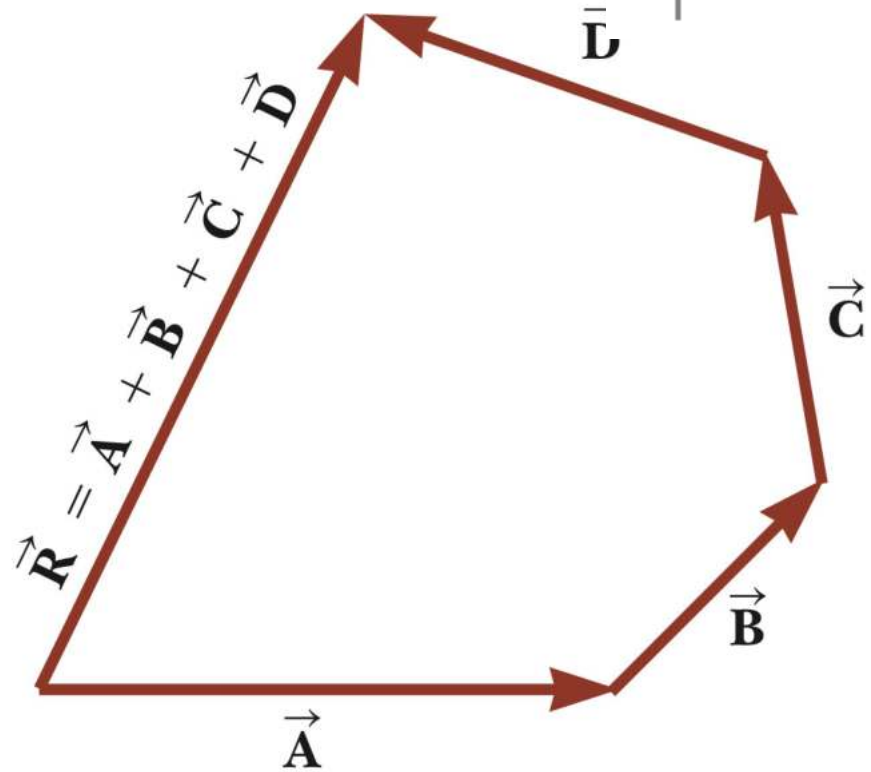
Even though you know how far and in which direction the library is, you may not be able to walk there in a straight line:





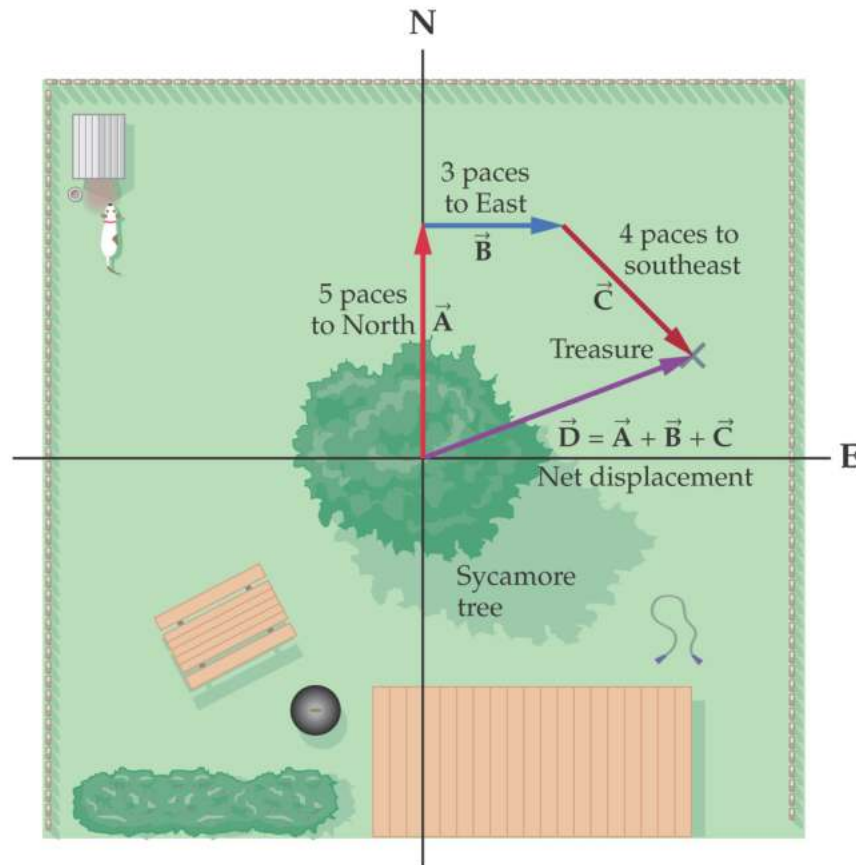
Adding Vectors Graphically, final

- When you have many vectors, just keep repeating the process until all are included.
- The resultant is still drawn from the tail of the first vector to the tip of the last vector.



Adding and Subtracting Vectors

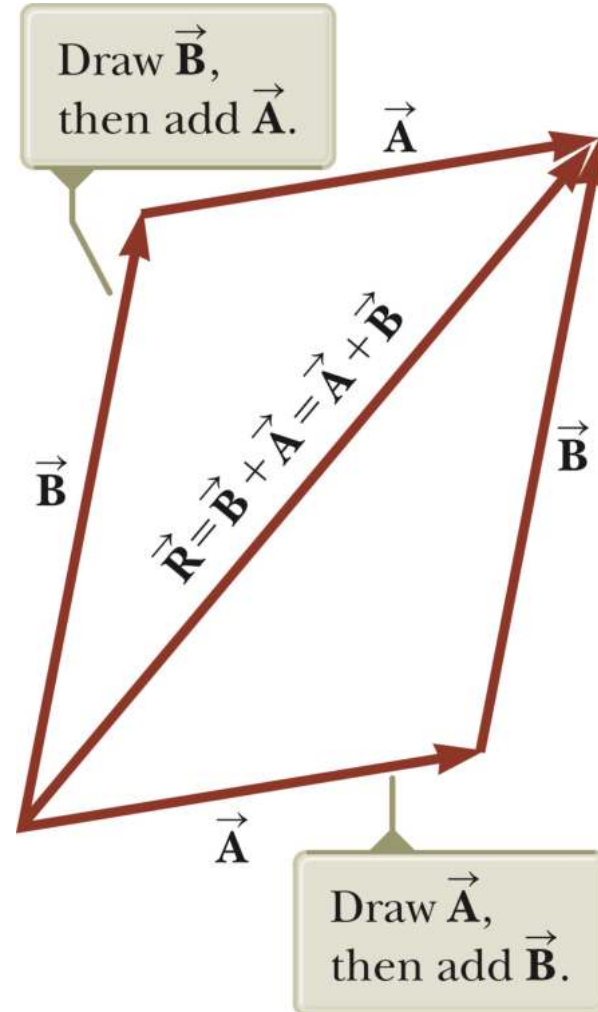
Adding vectors graphically: Place the tail of the second at the head of the first. The sum points from the tail of the first to the head of the



Adding Vectors, Rules



- When two vectors are added, the sum is independent of the order of the addition.
 - This is the *Commutative Law of Addition*.
- $\vec{A} + \vec{B} = \vec{B} + \vec{A}$



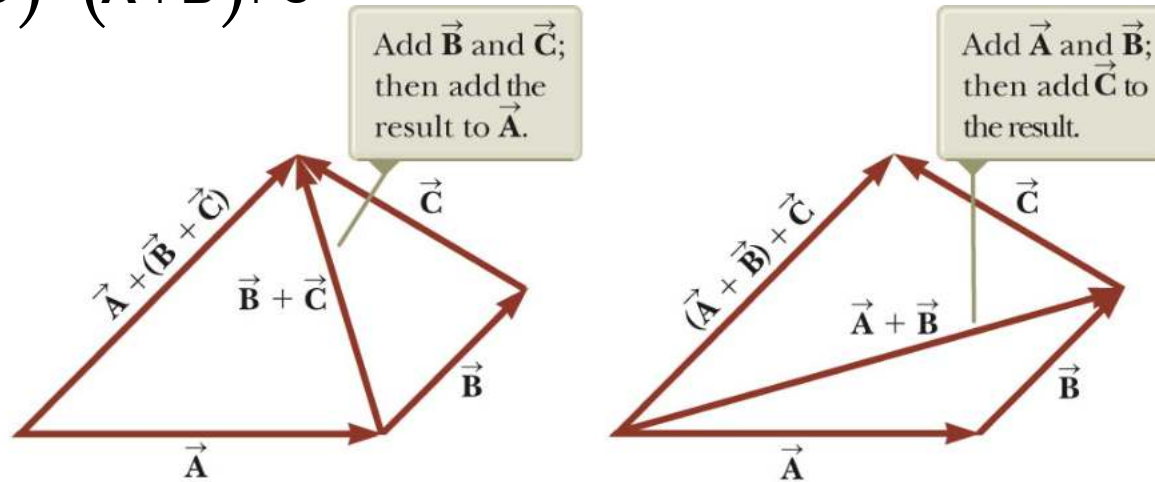


Adding Vectors, Rules cont.

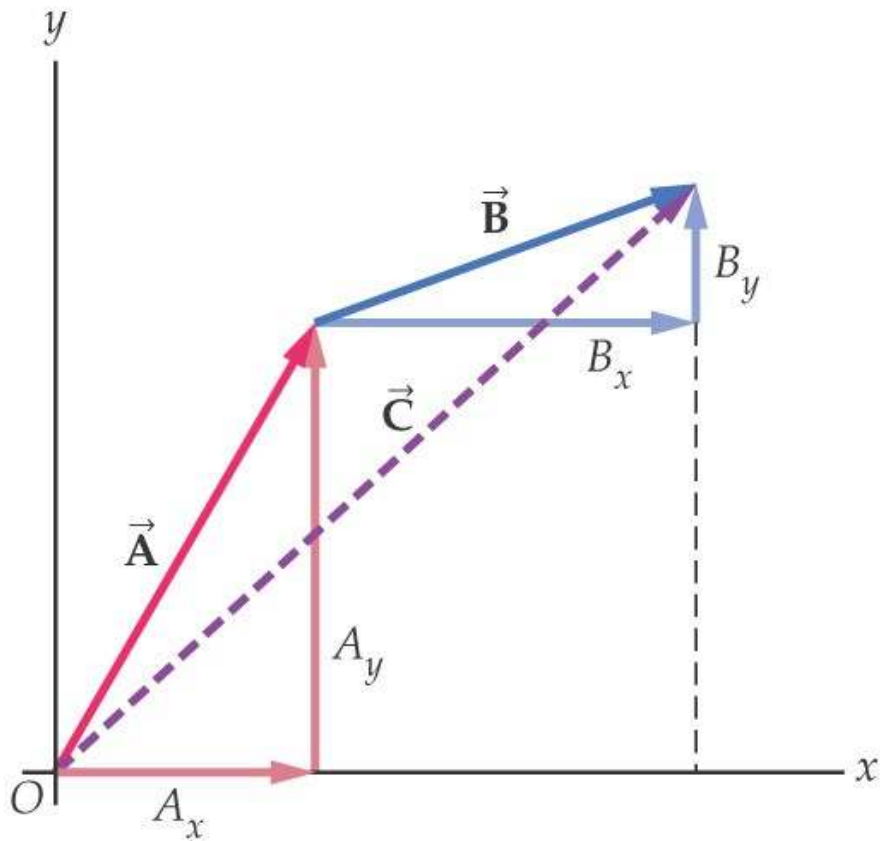
- When adding three or more vectors, their sum is independent of the way in which the individual vectors are grouped.

- This is called the *Associative Property of Addition*.

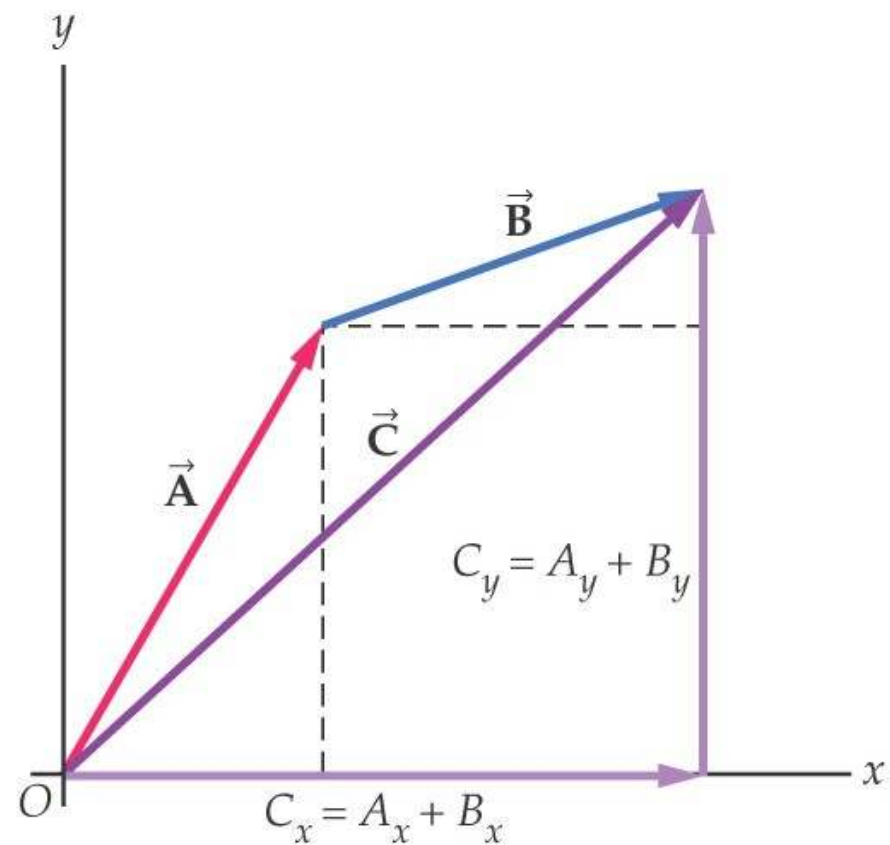
- $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$



Adding and Subtracting Vectors



(a)



(b)

Adding Vectors, Rules final



- When adding vectors, all of the vectors must have the same units.
- All of the vectors must be of the same type of quantity.
 - For example, you cannot add a displacement to a velocity.

Negative of a Vector

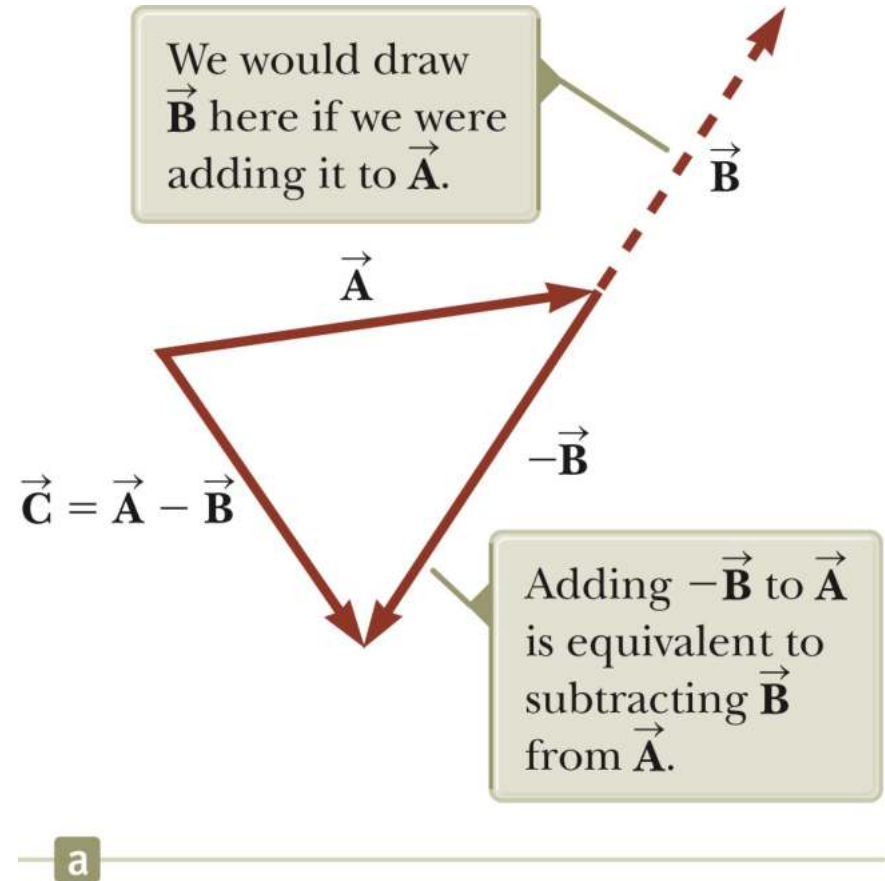


- The negative of a vector is defined as the vector that, when added to the original vector, gives a resultant of zero.
 - Represented as $-\vec{A}$
 - $\vec{A} + (-\vec{A}) = 0$
- The negative of the vector will have the same magnitude, but point in the opposite direction.



Subtracting Vectors

- Special case of vector addition: $\vec{A} - \vec{B}$
- If $\vec{A} + (-\vec{B})$, then use
- Continue with standard vector addition procedure.



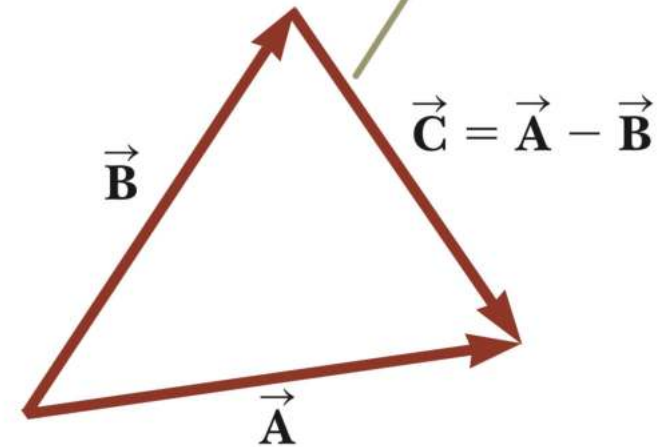
Subtracting Vectors, Method 2



- Another way to look at subtraction is to find the vector that, added to the second vector gives you the first vector.

- $\vec{A} + (-\vec{B}) = \vec{C}$
 - As shown, the resultant vector points from the tip of the second to the tip of the first.

Vector $\vec{C} = \vec{A} - \vec{B}$ is the vector we must add to \vec{B} to obtain \vec{A} .



Multiplying or Dividing a Vector by a Scalar



- The result of the multiplication or division of a vector by a scalar is a vector.
- The magnitude of the vector is multiplied or divided by the scalar.
- If the scalar is positive, the direction of the result is the same as of the original vector.
- If the scalar is negative, the direction of the result is opposite that of the original vector.

Component Method of Adding Vectors

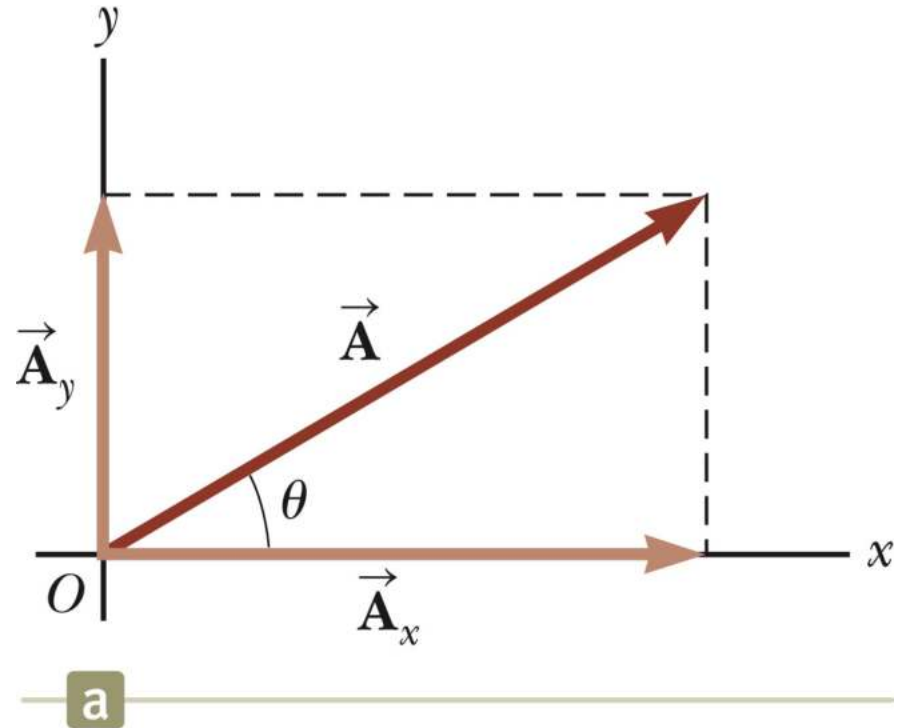


- Graphical addition is not recommended when:
 - High accuracy is required
 - If you have a three-dimensional problem
- Component method is an alternative method
 - It uses projections of vectors along coordinate axes

Components of a Vector, Introduction



- A **component** is a projection of a vector along an axis.
 - Any vector can be completely described by its components.
- It is useful to use **rectangular components**.
 - These are the projections of the vector along the x- and y-axes.



Vector Component Terminology

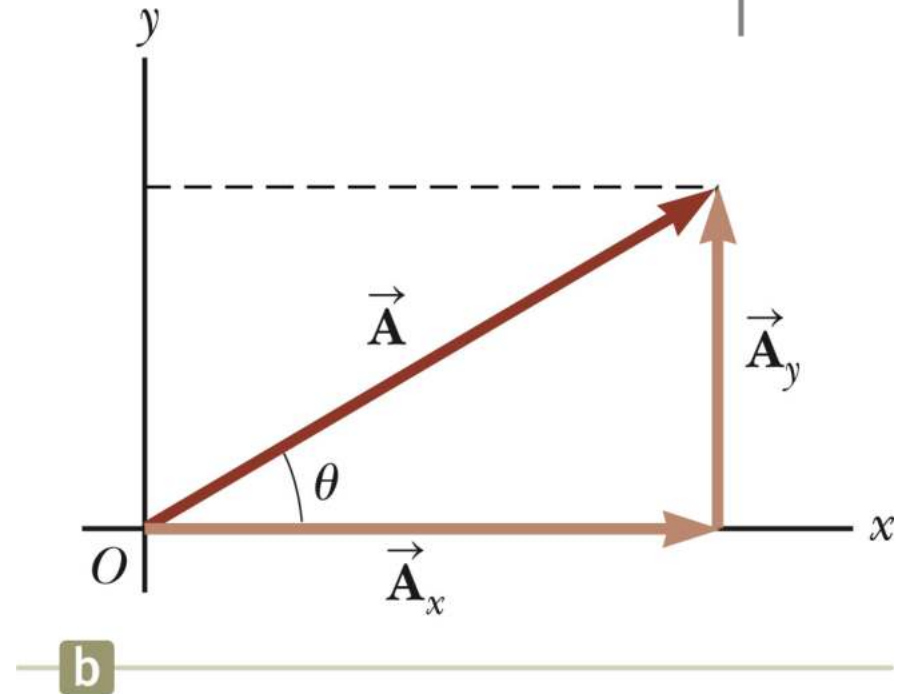


- $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ are the *component vectors* of $\vec{\mathbf{A}}$.
 - They are vectors and follow all the rules for vectors.
- A_x and A_y are scalars, and will be referred to as the *components* of $\vec{\mathbf{A}}$.

Components of a Vector



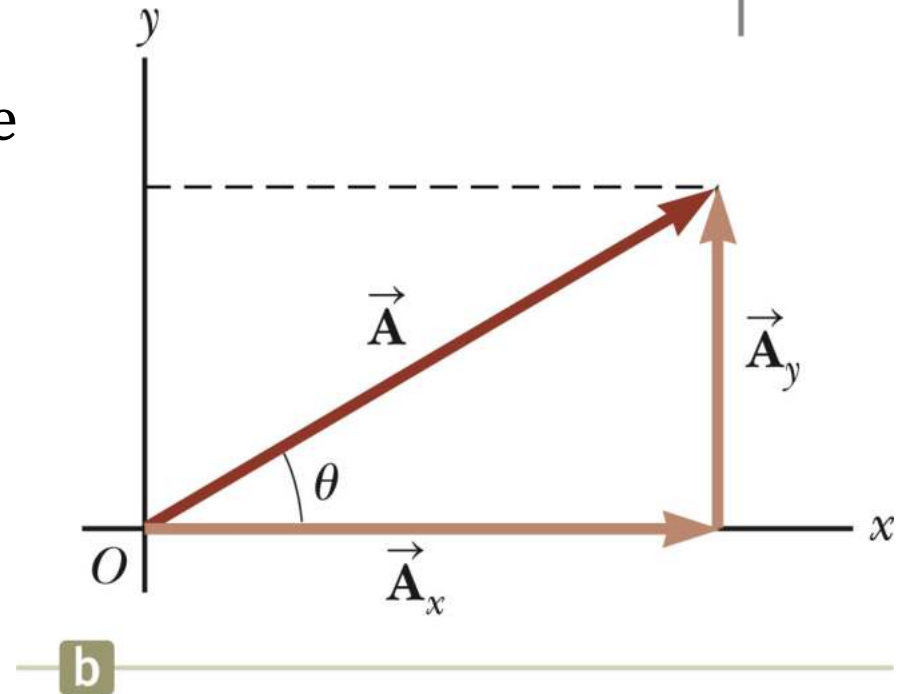
- Assume you are given a vector $\vec{\mathbf{A}}$
- It can be expressed in terms of two other vectors, $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$
- These three vectors form a right triangle.
- $$\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$$



Components of a Vector, 2



- The y -component is moved to the end of the x -component.
- This is due to the fact that any vector can be moved parallel to itself without being affected.
 - This completes the triangle.



Components of a Vector, 3



- The x-component of a vector is the projection along the x-axis.

$$A_x = A \cos \theta$$

- The y-component of a vector is the projection along the y-axis.

$$A_y = A \sin \theta$$

- This assumes the angle θ is measured with respect to the x-axis.

- If not, do not use these equations, use the sides of the triangle directly.

Components of a Vector, 4

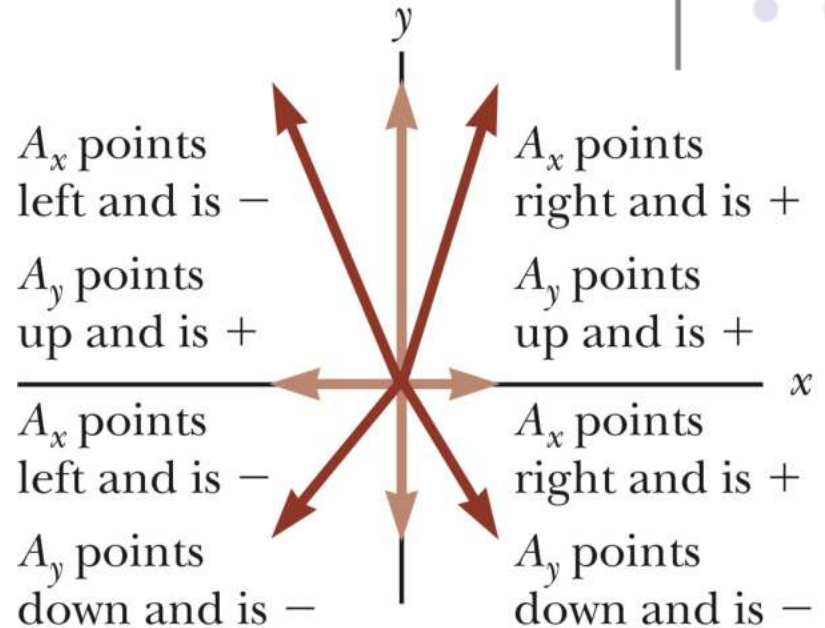


- The components are the legs of the right triangle whose hypotenuse is the length of A .
 - $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1} \frac{A_y}{A_x}$
 - May still have to find θ with respect to the positive x -axis
- In a problem, a vector may be specified by its components or its magnitude and direction.

Components of a Vector, final



- The components can be positive or negative and will have the same units as the original vector.
- The signs of the components will depend on the angle.





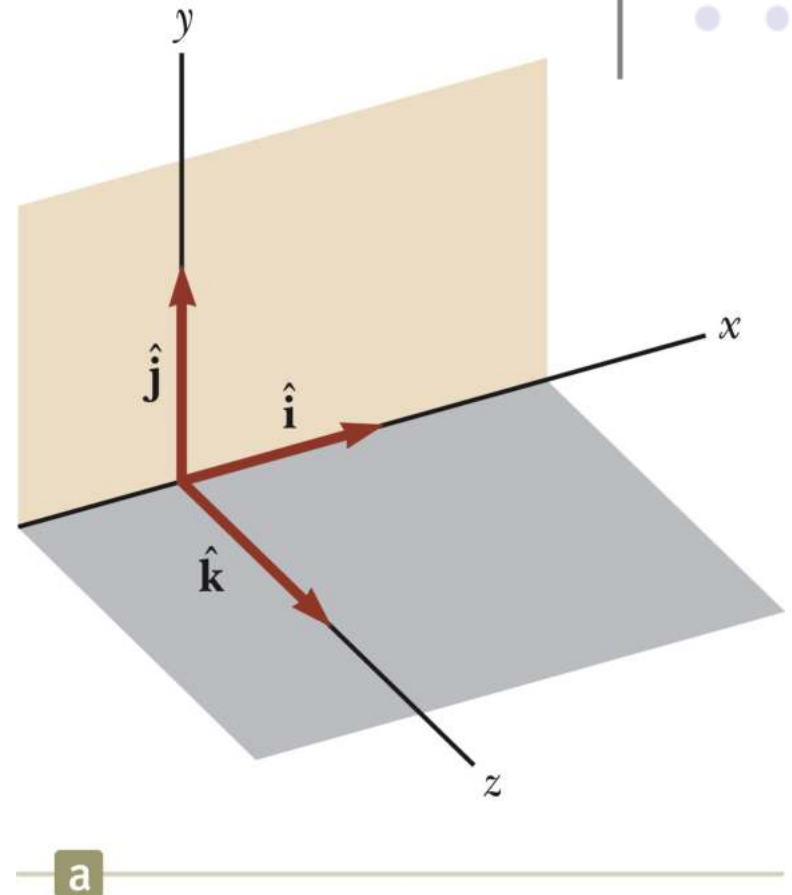
Unit Vectors

- A *unit vector* is a dimensionless vector with a magnitude of exactly 1.
- Unit vectors are used to specify a direction and have no other physical significance.



Unit Vectors, cont.

- The symbols \hat{i} , \hat{j} , and \hat{k} represent unit vectors
- They form a set of mutually perpendicular vectors in a right-handed coordinate system $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$
- The magnitude of each unit vector is 1

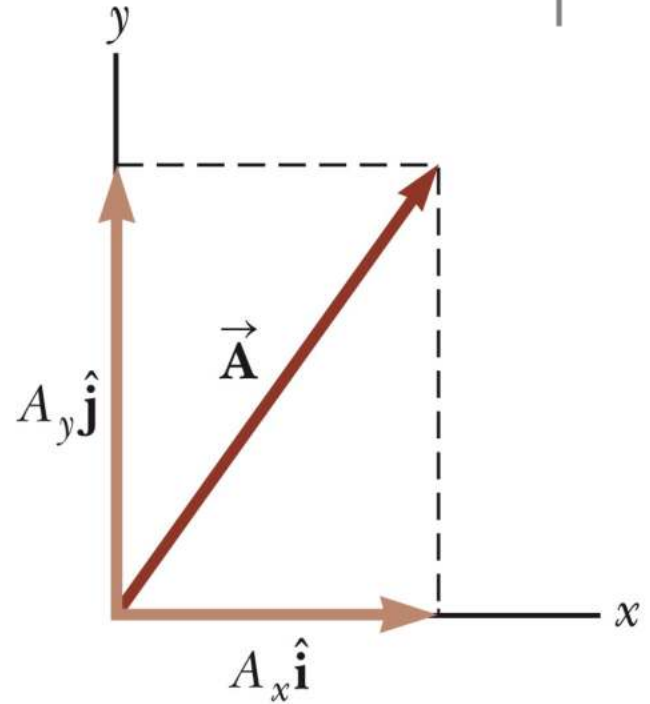


Unit Vectors in Vector Notation



- \mathbf{A}_x is the same as $A_x \hat{\mathbf{i}}$ and \mathbf{A}_y is the same as $A_y \hat{\mathbf{j}}$ etc.
- The complete vector can be expressed as:

$$\vec{\mathbf{A}} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}$$



b

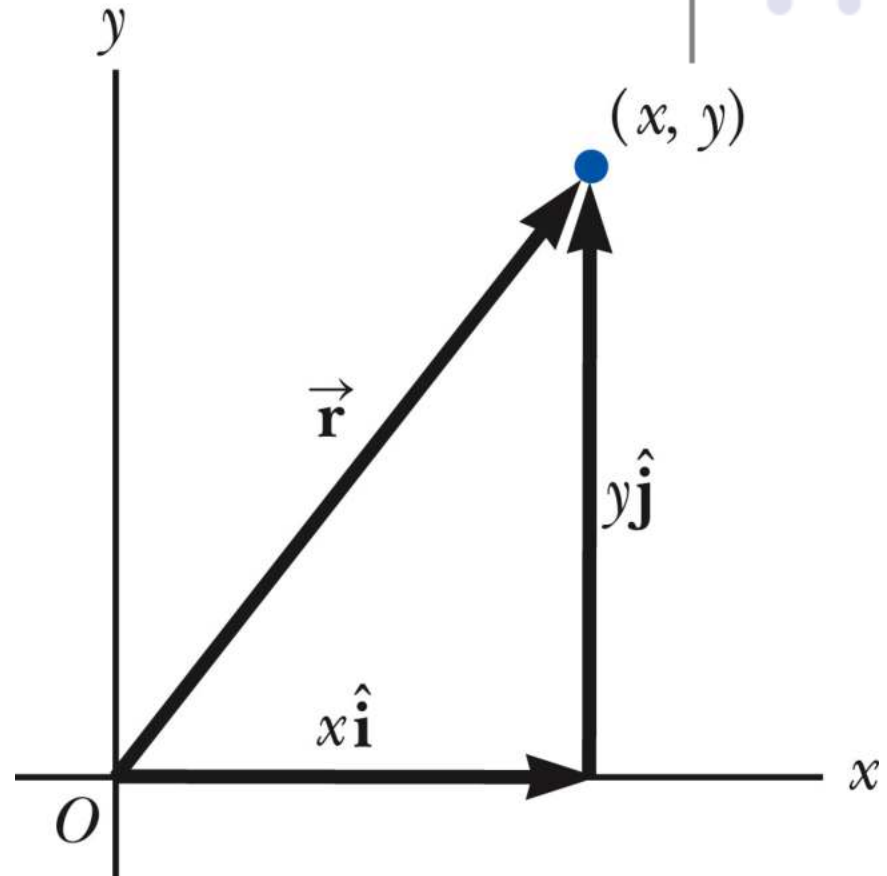
Position Vector, Example



- A point lies in the xy plane and has Cartesian coordinates of (x, y) .
- The point can be specified by the position vector.

$$\hat{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

- This gives the components of the vector and its coordinates.



Adding Vectors Using Unit Vectors



- Using $\vec{R} = \vec{A} + \vec{B}$

- Then

$$\vec{R} = (A_x \hat{i} + A_y \hat{j}) + (B_x \hat{i} + B_y \hat{j})$$

$$\vec{R} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}$$

$$\vec{R} = R_x \hat{i} + R_y \hat{j}$$

$$R = \sqrt{R_x^2 + R_y^2} \quad \theta = \tan^{-1} \frac{R_y}{R_x}$$

- So $R_x = A_x + B_x$ and $R_y = A_y + B_y$

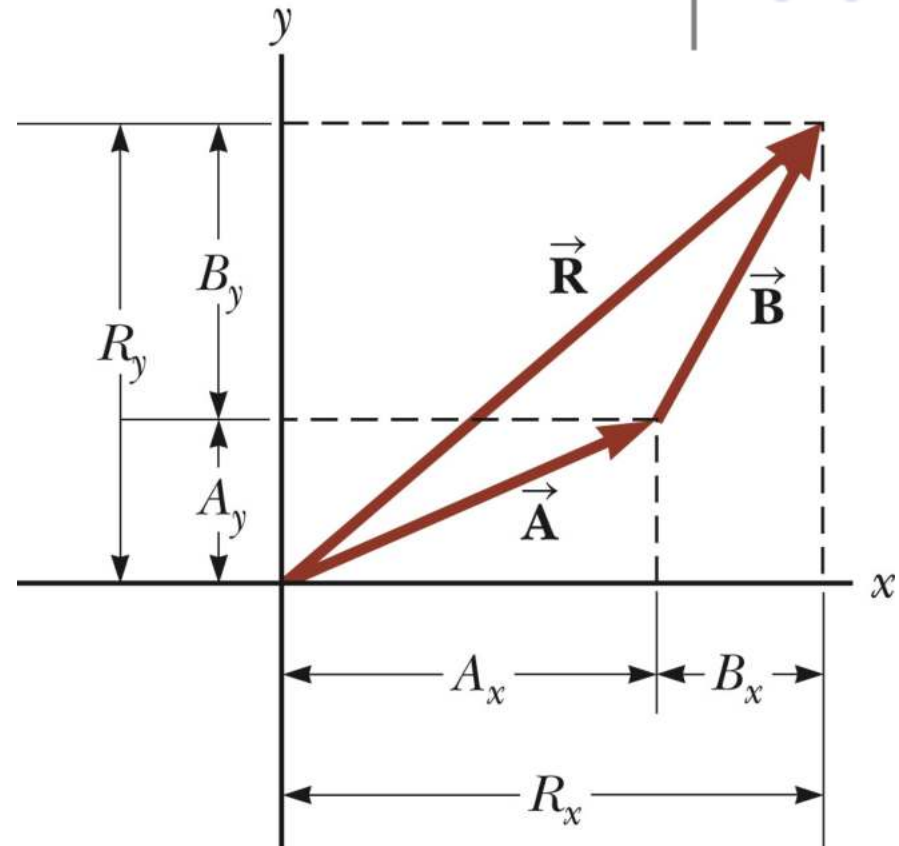
Adding Vectors with Unit Vectors



- Note the relationships among the components of the resultant and the components of the original vectors.

- $R_x = A_x + B_x$

- $R_y = A_y + B_y$





Three-Dimensional Extension

- Using $\vec{R} = \vec{A} + \vec{B}$

- Then

$$\vec{R} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) + (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\vec{R} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j} + (A_z + B_z) \hat{k}$$

$$\vec{R} = R_x \hat{i} + R_y \hat{j} + R_z \hat{k}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \quad \theta_x = \cos^{-1} \frac{R_x}{R}, \text{ etc.}$$

- So $R_x = A_x + B_x$, $R_y = A_y + B_y$ and $R_z = A_z + B_z$



Adding Three or More Vectors

- The same method can be extended to adding three or more vectors.
- Assume

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$$

- And

$$\begin{aligned}\vec{\mathbf{R}} = & (A_x + B_x + C_x)\hat{\mathbf{i}} + (A_y + B_y + C_y)\hat{\mathbf{j}} \\ & + (A_z + B_z + C_z)\hat{\mathbf{k}}\end{aligned}$$

Example



Example 3.3

The Sum of Two Vectors

Find the sum of two displacement vectors \vec{A} and \vec{B} lying in the xy plane and given by

$$\vec{A} = (2.0\hat{i} + 2.0\hat{j}) \text{ m} \quad \text{and} \quad \vec{B} = (2.0\hat{i} - 4.0\hat{j}) \text{ m}$$

SOLUTION

Conceptualize You can conceptualize the situation by drawing the vectors on graph paper. Draw an approximation of the expected resultant vector.

Categorize We categorize this example as a simple substitution problem. Comparing this expression for \vec{A} with the general expression $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$, we see that $A_x = 2.0 \text{ m}$, $A_y = 2.0 \text{ m}$, and $A_z = 0$. Likewise, $B_x = 2.0 \text{ m}$, $B_y = -4.0 \text{ m}$, and $B_z = 0$. We can use a two-dimensional approach because there are no z components.

Use Equation 3.14 to obtain the resultant vector \vec{R} :

$$\vec{R} = \vec{A} + \vec{B} = (2.0 + 2.0)\hat{i} \text{ m} + (2.0 - 4.0)\hat{j} \text{ m}$$

Evaluate the components of \vec{R} :

$$R_x = 4.0 \text{ m} \quad R_y = -2.0 \text{ m}$$

Use Equation 3.16 to find the magnitude of \vec{R} :

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0 \text{ m})^2 + (-2.0 \text{ m})^2} = \sqrt{20} \text{ m} = 4.5 \text{ m}$$

Find the direction of \vec{R} from Equation 3.17:

$$\tan \theta = \frac{R_y}{R_x} = \frac{-2.0 \text{ m}}{4.0 \text{ m}} = -0.50$$

Your calculator likely gives the answer -27° for $\theta = \tan^{-1}(-0.50)$. This answer is correct if we interpret it to mean 27° clockwise from the x axis. Our standard form has been to quote the angles measured counterclockwise from the $+x$ axis, and that angle for this vector is $\theta = 333^\circ$.



Example 3.4 The Resultant Displacement

A particle undergoes three consecutive displacements: $\Delta\vec{r}_1 = (15\hat{i} + 30\hat{j} + 12\hat{k})$ cm, $\Delta\vec{r}_2 = (23\hat{i} - 14\hat{j} - 5.0\hat{k})$ cm, and $\Delta\vec{r}_3 = (-13\hat{i} + 15\hat{j})$ cm. Find unit-vector notation for the resultant displacement and its magnitude.

SOLUTION

► 3.4 continued

Categorize Despite the difficulty in conceptualizing in three dimensions, we can categorize this problem as a substitution problem because of the careful bookkeeping methods that we have developed for vectors. The mathematical manipulation keeps track of this motion along the three perpendicular axes in an organized, compact way, as we see below.

To find the resultant displacement, add the three vectors:

$$\begin{aligned}\Delta\vec{r} &= \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 \\ &= (15 + 23 - 13)\hat{i} \text{ cm} + (30 - 14 + 15)\hat{j} \text{ cm} + (12 - 5.0 + 0)\hat{k} \text{ cm} \\ &= (25\hat{i} + 31\hat{j} + 7.0\hat{k}) \text{ cm}\end{aligned}$$

Find the magnitude of the resultant vector:

$$\begin{aligned}R &= \sqrt{R_x^2 + R_y^2 + R_z^2} \\ &= \sqrt{(25 \text{ cm})^2 + (31 \text{ cm})^2 + (7.0 \text{ cm})^2} = 40 \text{ cm}\end{aligned}$$

Example 3.5

Taking a Hike

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(A) Determine the components of the hiker's displacement for each day.

SOLUTION

Conceptualize We conceptualize the problem by drawing a sketch as in Figure 3.17. If we denote the displacement vectors on the first and second days by \vec{A} and \vec{B} , respectively, and use the car as the origin of coordinates, we obtain the vectors shown in Figure 3.17. The sketch allows us to estimate the resultant vector as shown.

Categorize Having drawn the resultant \vec{R} , we can now categorize this problem as one we've solved before: an addition of two vectors. You should now have a hint of the power of categorization in that many new problems are very similar to problems we have already solved if we are careful to conceptualize them. Once we have drawn the displacement vectors and categorized the problem, this problem is no longer about a hiker, a walk, a car, a tent, or a tower. It is a problem about vector addition, one that we have already solved.

Analyze Displacement \vec{A} has a magnitude of 25.0 km and is directed 45.0° below the positive x axis.

Find the components of \vec{A} using Equations 3.8 and 3.9:

$$A_x = A \cos(-45.0^\circ) = (25.0 \text{ km})(0.707) = 17.7 \text{ km}$$

$$A_y = A \sin(-45.0^\circ) = (25.0 \text{ km})(-0.707) = -17.7 \text{ km}$$

The negative value of A_y indicates that the hiker walks in the negative y direction on the first day. The signs of A_x and A_y also are evident from Figure 3.17.

Find the components of \vec{B} using Equations 3.8 and 3.9:

$$B_x = B \cos 60.0^\circ = (40.0 \text{ km})(0.500) = 20.0 \text{ km}$$

$$B_y = B \sin 60.0^\circ = (40.0 \text{ km})(0.866) = 34.6 \text{ km}$$

(B) Determine the components of the hiker's resultant displacement \vec{R} for the trip. Find an expression for \vec{R} in terms of unit vectors.

SOLUTION

Use Equation 3.15 to find the components of the resultant displacement $\vec{R} = \vec{A} + \vec{B}$:

$$R_x = A_x + B_x = 17.7 \text{ km} + 20.0 \text{ km} = 37.7 \text{ km}$$

$$R_y = A_y + B_y = -17.7 \text{ km} + 34.6 \text{ km} = 17.0 \text{ km}$$

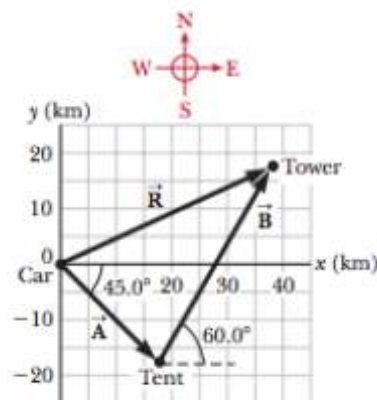


Figure 3.17 (Example 3.5) The total displacement of the hiker is the vector $\vec{R} = \vec{A} + \vec{B}$.



3.5 continued

Write the total displacement in unit-vector form:

$$\vec{\mathbf{R}} = (37.7\hat{\mathbf{i}} + 17.0\hat{\mathbf{j}}) \text{ km}$$

Finalize Looking at the graphical representation in Figure 3.17, we estimate the position of the tower to be about (38 km, 17 km), which is consistent with the components of $\vec{\mathbf{R}}$ in our result for the final position of the hiker. Also, both components of $\vec{\mathbf{R}}$ are positive, putting the final position in the first quadrant of the coordinate system, which is also consistent with Figure 3.17.

WHAT IF? After reaching the tower, the hiker wishes to return to her car along a single straight line. What are the components of the vector representing this hike? What should the direction of the hike be?

Answer The desired vector $\vec{\mathbf{R}}_{\text{car}}$ is the negative of vector $\vec{\mathbf{R}}$:

$$\vec{\mathbf{R}}_{\text{car}} = -\vec{\mathbf{R}} = (-37.7\hat{\mathbf{i}} - 17.0\hat{\mathbf{j}}) \text{ km}$$

The direction is found by calculating the angle that the vector makes with the x axis:

$$\tan \theta = \frac{R_{\text{car},y}}{R_{\text{car},x}} = \frac{-17.0 \text{ km}}{-37.7 \text{ km}} = 0.450$$

which gives an angle of $\theta = 204.2^\circ$, or 24.2° south of west.

Example 3.5 – Taking a Hike



- A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

Example 3.5 – Taking a Hike

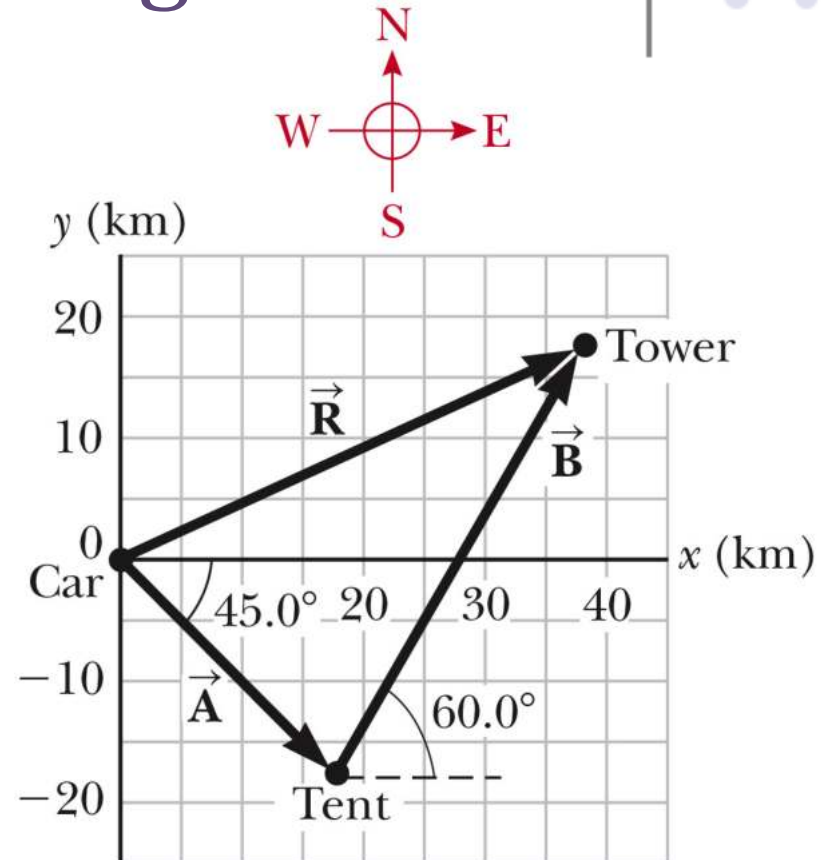


- A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

Example 3.5 – Solution, Conceptualize and Categorize



- *Conceptualize* the problem by drawing a sketch as in the figure.
- Denote the displacement vectors on the first and second days by $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ respectively.
- Use the car as the origin of coordinates.
- The vectors are shown in the figure.
- Drawing the resultant $\vec{\mathbf{R}}$, we can now *categorize* this problem as an addition of two vectors.



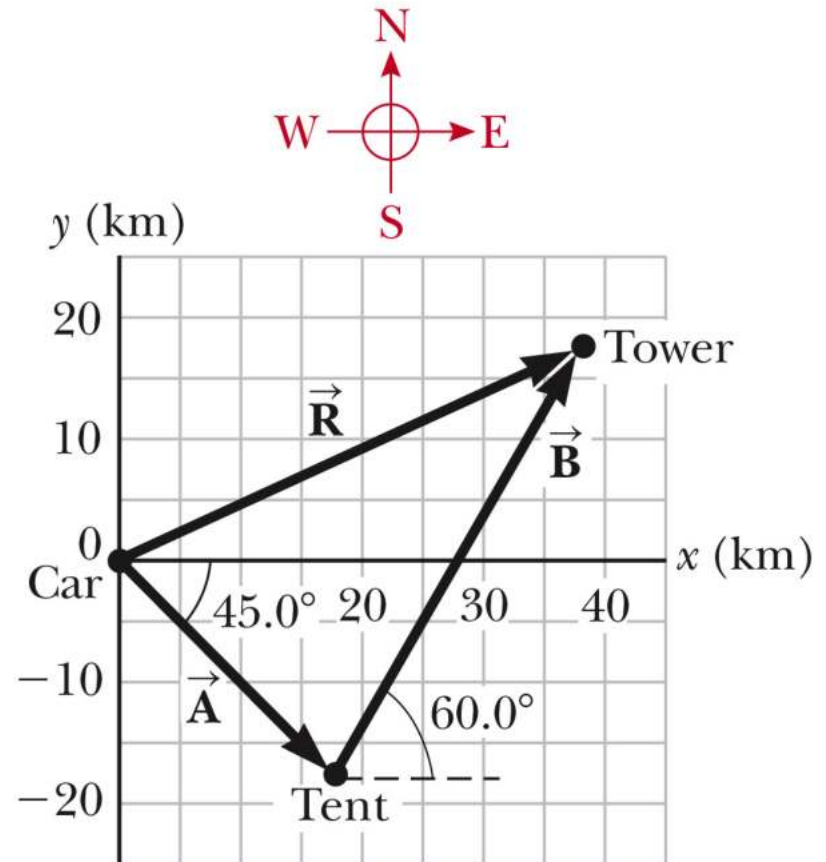


Example 3.5 – Solution, Analysis

- Analyze this problem by using our new knowledge of vector components.
- The first displacement has a magnitude of 25.0 km and is directed 45.0° below the positive x axis.
- Its components are:

$$A_x = A \cos(-45.0^\circ) = (25.0 \text{ km})(0.707) = 17.7 \text{ km}$$

$$A_y = A \sin(-45.0^\circ) = (25.0 \text{ km})(-0.707) = -17.7 \text{ km}$$



Example 3.5 – Solution, Analysis 2

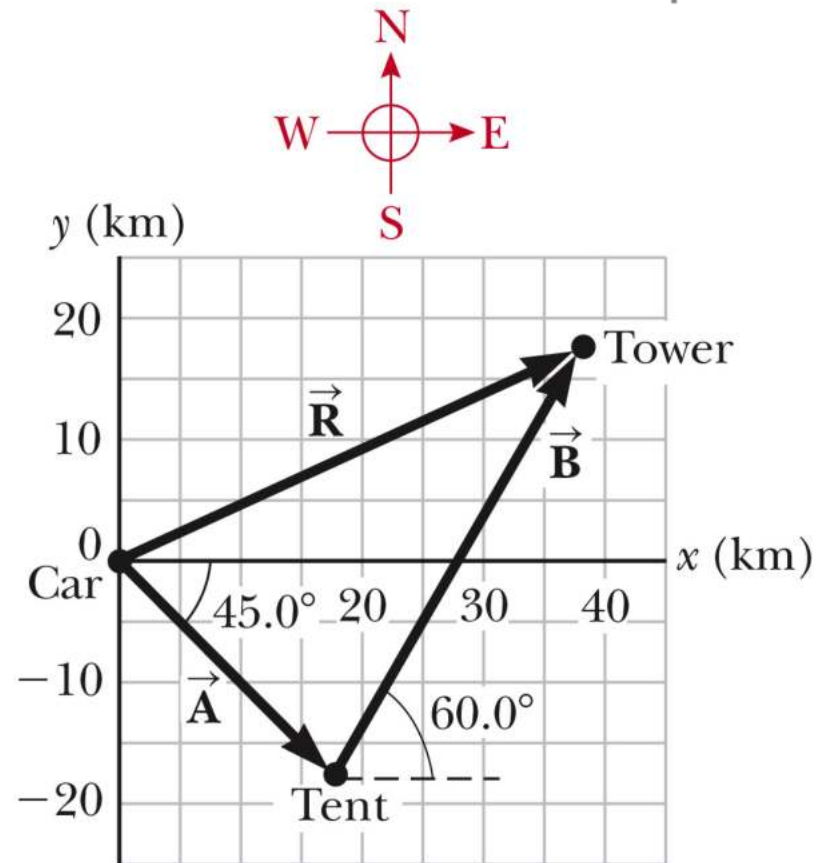


- The second displacement has a magnitude of 40.0 km and is 60.0° north of east.

- Its components are:

$$B_x = B \cos 60.0^\circ = (40.0 \text{ km})(0.500) = 20.0 \text{ km}$$

$$B_y = B \sin 60.0^\circ = (40.0 \text{ km})(0.866) = 34.6 \text{ km}$$



Example 3.5 – Solution, Analysis 3



- The negative value of A_y indicates that the hiker walks in the negative y direction on the first day.
- The signs of A_x and A_y also are evident from the figure.
- The signs of the components of B are also confirmed by the diagram.

Example 3.5 – Analysis, 4



- Determine the components of the hiker's resultant displacement for the trip.

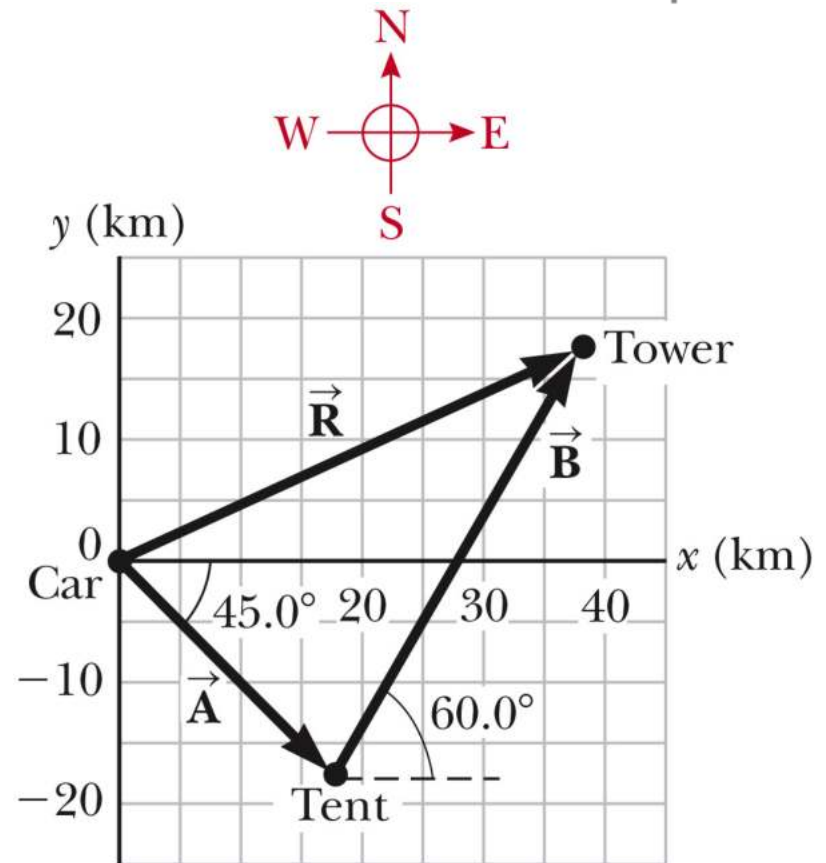
- Find an expression for the resultant in terms of unit vectors.

- The resultant displacement for the trip has components given by

- $R_x = A_x + B_x = 17.7 \text{ km} + 20.0 \text{ km} = 37.7 \text{ km}$
- $R_y = A_y + B_y = -17.7 \text{ km} + 34.6 \text{ km} = 16.9 \text{ km}$

- In unit vector form

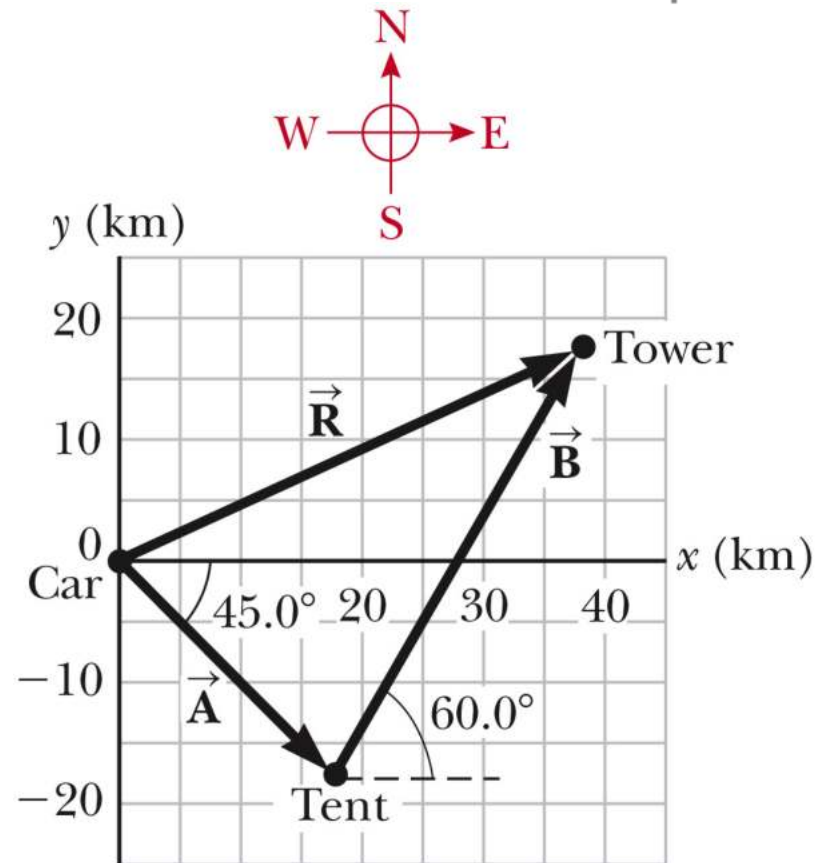
$$\vec{R} = (37.7\hat{i} + 16.9\hat{j}) \text{ km}$$





Example 3.5 – Solution, Finalize

- The resultant vector has a magnitude of 41.3 km and is directed 24.1° north of east.
- The units of \vec{R} are km, which is reasonable for a displacement.
- From the graphical representation, estimate that the final position of the hiker is at about (38 km, 17 km) which is consistent with the components of the resultant.



Example 3.5 – Solution, Finalize, cont.



- Both components of the resultant are positive, putting the final position in the first quadrant of the coordinate system.
 - This is also consistent with the figure.

Homework

1. What is the magnitude of the vector $(10\mathbf{i} - 10\mathbf{k})$ m/s? (a) 0 (b) 10 m/s (c) 210 m/s (d) 10 (e) 14.1 m/s.
2. The magnitudes of two vectors A and B are $A = 5$ units and $B = 2$ units. Find the largest and smallest values possible for the resultant vector $R = A + B$.
3. The magnitude of vector A is 8 km, and the magnitude of B is 6 km. Which of the following are possible values for the magnitude of $A + B$? Choose all possible answers. (a) 10 km (b) 8 km (c) 2 km (d) 0 (e) 22 km
4. What is the y component of the vector $(13\mathbf{i} - 8\mathbf{k})$ m/s? (a) 3 m/s (b) 28 m/s (c) 0 (d) 8 m/s (e) none of those answers
5. Can the magnitude of a vector have a negative value?
6. Yes or no: Is each of the following quantities a vector? (a) force (b) temperature (c) the volume of water in a can (d) the ratings of a TV show (e) the height of a building (f) the velocity of a sports car (g) the age of the Universe
7. Two points in the xy plane have Cartesian coordinates (2.00, 24.00) m and (23.00, 3.00) m. Determine (a) the distance between these points and (b) their polar coordinates.
8. A force F_1 of magnitude 6.00 units acts on an object at the origin in a direction in a 30.0° above the positive x axis (Fig. P3.10). A second force F_2 of magnitude 5.00 units acts on the object in the direction of the positive y axis. Find graphically the magnitude and direction of the resultant force $F_1 + F_2$.
9. Vector A has a magnitude of 8.00 units and makes an angle of 45.0° with the positive x axis. Vector B also has a magnitude of 8.00 units and is directed along the negative x axis. Using graphical methods, find (a) the vector sum $A + B$ and (b) the vector difference $A - B$, $B - A$ and $A - 2B$
10. A vector has an x component of -25.0 units and a y component of 40.0 units. Find the magnitude and direction of this vector.
11. The helicopter view in Fig. P3.29 shows two people pulling on a stubborn mule. Find (a) the single force that is equivalent to the two forces shown, and (b) the force that a third person would have to exert on the mule to make the resultant force equal to zero. The forces are measured in units of newtons (abbreviated N).
12. A point is located in a polar coordinate system by the coordinates $r = 2.50$ m and $\theta = 35.0^\circ$. Find the cartesian coordinates of this point, assuming the two coordinate systems have the same origin.
13. Find the horizontal and vertical components of the 100-m displacement of a superhero who flies from the top of a tall building following the path shown in Figure P3.52.

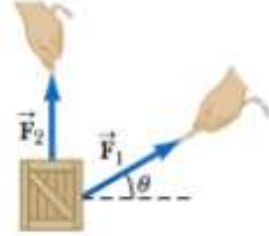


Figure P3.10

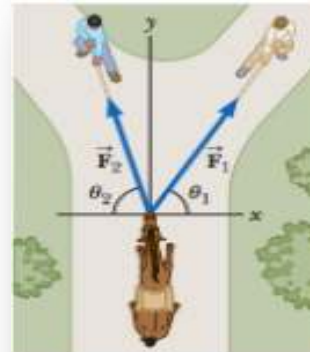


Figure P3.29

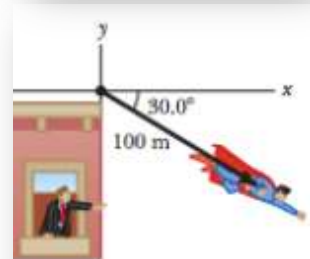


Figure P3.52