المؤسسة العامة للتدريب التقني والمهني
Technical and Vocational Training Corporation

Course of Electromechanical Energy Conversion

Chapter I: **Magnetic circuits**

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1. INTRODUCTION TO MAGNETIC CIRCUITS

- The complete, detailed solution for magnetic fields in most situations of practical engineering interest involves the solution of Maxwell's equations along with various constitutive relationships which describe material properties. Although in practice exact solutions are often unattainable, various simplifying assumptions permit the attainment of useful engineering solutions.!

- We begin with the assumption that the frequencies and sizes involved are such that the displacement-current term in Maxwell's equations can be neglected. This term accounts for magnetic fields being produced in space by time-varying electric fields and is associated with electromagnetic radiation. Neglecting this term results in the magneto-quasistatic form of the relevant Maxwell's equations which relate magnetic fields to the currents which produce them.

$$\oint_C H.dl = \int_S J.da \qquad (1.1)$$

$$\oint_S B.da = 0 \qquad (1.2)$$

Equation 1.1 states that the line integral of the tangential component of the magnetic field intensity ***H*** around a closed contour ***C*** is equal to the total current passing through any surface ***S*** linking that contour. From Eq. 1.1 we see that the source of ***H*** is the current density ***J***.

Equation 1.2 states that the magnetic flux density ***B*** is conserved, i.e., that no net flux enters or leaves a closed surface (this is equivalent to saying that there exist no monopole charge sources of magnetic fields). From these equations we see that the magnetic field quantities can be determined solely from the instantaneous values of the source currents and that time variations of the magnetic fields follow directly from time variations of the sources.

A second simplifying assumption involves the concept of the magnetic circuit. The general solution for the magnetic field intensity ***H*** and the magnetic flux density ***B*** in a structure of complex geometry is extremely difficult. However, a three-dimensional field problem can often be reduced to what is essentially a one dimensional circuit equivalent, yielding solutions of acceptable engineering accuracy.

A magnetic circuit consists of a structure composed for the most part of high permeability magnetic material. The presence of high-permeability material tends to cause magnetic flux to be confined to the paths defined by the structure, much as currents are confined to the conductors of an electric circuit.

A simple example of a magnetic circuit is shown in Fig. 1.1. The core is assumed to be composed of magnetic material whose permeability is much greater than that of the surrounding air ($\mu >> \mu_0$). The core is of uniform cross section and is excited by a winding of ***N*** turns carrying a current of ***i*** amperes. This winding produces a magnetic field in the core, as shown in the figure.

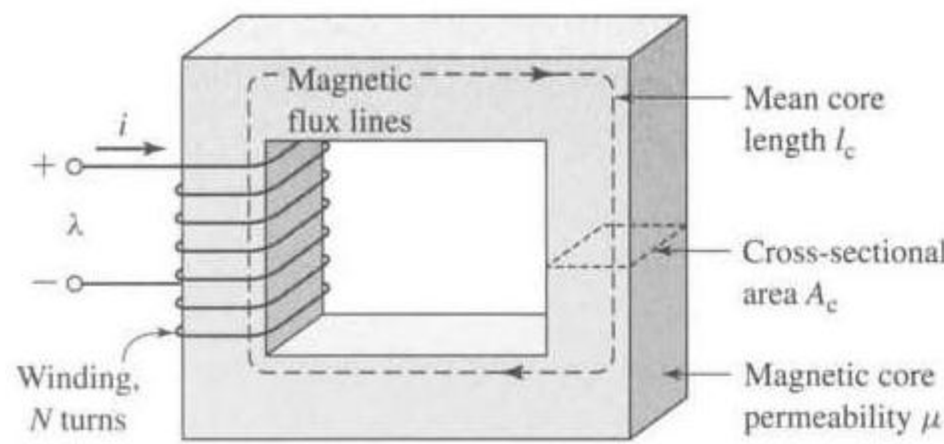

Figure 1.1 Simple magnetic circuit.

Because of the high permeability of the magnetic core, an exact solution would show that the magnetic flux is confined almost entirely to the core, the field lines follow the path defined by the core, and the flux density is essentially uniform over a cross section because the cross-sectional area is uniform. The magnetic field can be visualized in terms of flux lines which form closed loops interlinked with the winding.

As applied to the magnetic circuit of Fig. 1.1, the source of the magnetic field in the core is the ampere-turn product ***N i***. In magnetic circuit terminology ***N i*** is the magnetomotive force (mmf) ***F*** acting on the magnetic circuit. Although Fig. 1.1 shows only a single coil, transformers and most rotating machines have at least two windings, and ***N i*** must be replaced by the algebraic sum of the ampere-turns of all the windings.

The magnetic flux $\boldsymbol{\phi}$ crossing a surface ***S*** is the surface integral of the normal component of ***B***; thus

$$\phi = \int_S B.da \qquad (1.3)$$

In SI units, the unit of $\boldsymbol{\phi}$ is the weber (***Wb***).

Equation 1.2 states that the net magnetic flux entering or leaving a closed surface (equal to the surface integral of B over that closed surface) is zero. This is equivalent to saying that all the flux which enters the surface enclosing a volume must leave that volume over some other portion of that surface because magnetic flux lines form closed loops.

These facts can be used to justify the assumption that the magnetic flux density is uniform across the cross section of a magnetic circuit such as the core of Fig. 1.1. In this case Eq. 1.3 reduces to the simple scalar equation

$$\phi_c = B_c A_c \qquad (1.4)$$

where $\boldsymbol{\phi_c}$ = flux in core

$\boldsymbol{B_c}$ = flux density in core

$\boldsymbol{A_c}$ = cross-sectional area of core

From Eq. 1.1, the relationship between the mmf acting on a magnetic circuit and the magnetic field intensity in that circuit is

$$F = Ni = \oint H.dl \qquad (1.5)$$

The direction of $\boldsymbol{H_c}$ in the core can be found from the right-hand rule, which can be stated in two equivalent ways. (1) Imagine a current-carrying conductor held in the right hand with the thumb pointing in the direction of current flow; the fingers then point in the direction of the magnetic field created by that current. (2) Equivalently, if the coil in Fig. 1.1 is grasped in the right hand (figuratively speaking) with the fingers pointing in the direction of the current, the thumb will point in the direction of the magnetic fields.

The relationship between the magnetic field intensity $\boldsymbol{H}$ and the magnetic flux density $\boldsymbol{B}$ is a property of the material in which the field exists. It is common to assume a linear relationship; thus

$$B = \mu H \tag{I.7}$$

where $\boldsymbol{\mu}$ is known as the magnetic permeability. In SI units, $\boldsymbol{H}$ is measured in units of amperes per meter; $\boldsymbol{B}$ is in webers per square meter; also known as teslas (**T**), and $\boldsymbol{\mu}$ is in webers per ampere-turn-meter; or equivalently henrys per meter. In SI units the permeability of free space is $\boldsymbol{\mu_0 = 4\pi \times 10^{-7}}$ henrys per meter. The permeability of linear magnetic material can be expressed in terms of $\boldsymbol{\mu_r}$, its value relative to that of free space, or $\boldsymbol{\mu = \mu_r \mu_0}$. Typical values of $\boldsymbol{\mu_r}$ range from 2000 to 80,000 for materials used in transformers and rotating machines. For the present we assume that $\boldsymbol{\mu_r}$ is a known constant, although it actually varies appreciably with the magnitude of the magnetic flux density.

Transformers are wound on closed cores like that of Fig. 1.1. However, energy conversion devices which incorporate a moving element must have air gaps in their magnetic circuits. A magnetic circuit with an air gap is shown in Fig. 1.2. When the air-gap length $\boldsymbol{g}$ is much smaller than the dimensions of the adjacent core faces, the magnetic flux $\boldsymbol{\phi}$ will follow the path defined by the core and the air gap and the techniques of magnetic-circuit analysis can be used. If the air-gap length becomes excessively large, the flux will be observed to "leak out" of the sides of the air gap and the techniques of magnetic-circuit analysis will no longer be strictly applicable.

Thus, provided the air-gap length $\boldsymbol{g}$ is sufficiently small, the configuration of Fig. 1.2 can be analyzed as a magnetic circuit with two series components: a magnetic core of permeability $\boldsymbol{\mu}$, cross-sectional area $\boldsymbol{A_c}$, and mean length $\boldsymbol{l_c}$, and an air gap of permeability $\boldsymbol{\mu_0}$, cross-sectional area $\boldsymbol{A_g}$, and length $\boldsymbol{g}$.

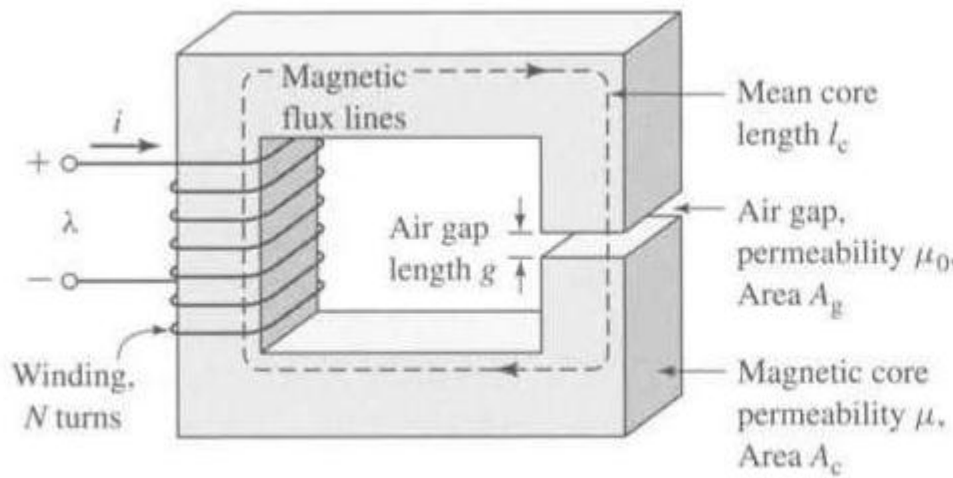

Figure 1.2 Magnetic circuit with air gap.

In the core the flux density can be assumed uniform; thus

$$B_c = \frac{\phi}{A_c} \tag{1.8}$$

and in the air gap

$$B_g = \frac{\phi}{A_g} \tag{1.9}$$

where $\boldsymbol{\phi}$ = the flux in the magnetic circuit.

Application of Eq. 1.5 to this magnetic circuit yields

$$F = H_c l_c + H_g g \tag{1.10}$$

and using the linear B -H relationship of Eq. 1.7 gives

$$F = \frac{B_c}{\mu} l_c + \frac{B_g}{\mu_0} g \tag{1.11}$$

Here the $\boldsymbol{F = N\,i}$ is the mmf applied to the magnetic circuit. From Eq. 1.10 we see that a portion of the mmf, $\boldsymbol{F_c = H_c\,l_c}$, is required to produce magnetic field in the core while the remainder, $\boldsymbol{F_g = H_g\,g}$, produces magnetic field in the air gap.

For practical magnetic materials, $\boldsymbol{B_c}$ and $\boldsymbol{H_c}$ are not simply related by a known constant permeability $\boldsymbol{\mu}$ as described by Eq. 1.7. In fact, $\boldsymbol{B_c}$ is often a nonlinear, multivalued function of $\boldsymbol{H_c}$. Thus, although Eq. 1.10 continues to hold, it does not lead directly to a simple expression relating the mmf and the flux densities, such as that of Eq. 1.11. Instead the specifics of the nonlinear $\boldsymbol{B_c}$-$\boldsymbol{H_c}$ relation must be used, either graphically or analytically. However, in many cases, the concept of constant material permeability gives results of acceptable engineering accuracy and is frequently used.

From Eqs. 1.8 and 1.9, Eq. 1.11 can be rewritten in terms of the total flux $\boldsymbol{\phi}$ as

$$F = \phi\left(\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g}\right) \qquad (1.12)$$

The terms that multiply the flux in this equation are known as the reluctance ***R*** of the core and air gap, respectively,

$$R_c = \frac{l_c}{\mu A_c} \qquad (1.13)$$

$$R_g = \frac{g}{\mu_0 A_g} \qquad (1.14)$$

and thus $$F = \phi\left(R_c + R_g\right) \qquad (1.15)$$

Finally, Eq. 1.15 can be inverted to solve for the flux

$$\phi = \frac{F}{R_c + R_g} \qquad (1.16)$$

or

$$\phi = \frac{F}{\dfrac{l_c}{\mu A_c} + \dfrac{g}{\mu_0 A_g}} \qquad (1.17)$$

In general, for any magnetic circuit of total reluctance $\boldsymbol{R_{tot}}$, the flux can be found as

$$\phi = \frac{F}{R_{tot}} \qquad (1.18)$$

The term which multiplies the mmf is known as the permeance ***P*** and is the inverse of the reluctance; thus, for example, the total permeance of a magnetic circuit is

$$P_{tot} = \frac{1}{R_{tot}} \qquad (1.19)$$

Note that Eqs. 1.15 and 1.16 are analogous to the relationships between the current and voltage in an electric circuit. This analogy is illustrated in Fig. 1.3. Figure 1.3a shows an electric circuit in which a voltage ***V*** drives a current ***i*** through resistors $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$. Figure 1.3b shows the schematic equivalent representation of the magnetic circuit of Fig. 1.2. Here we see that the mmf ***F*** (analogous to voltage in the electric circuit) drives a flux $\boldsymbol{\phi}$ (analogous to the current in the electric circuit) through the combination of the reluctances of the core $\boldsymbol{R_c}$ and the air gap $\boldsymbol{R_g}$. This analogy between the solution of electric and magnetic circuits can often be exploited to produce simple solutions for the fluxes in magnetic 'circuits of considerable complexity.

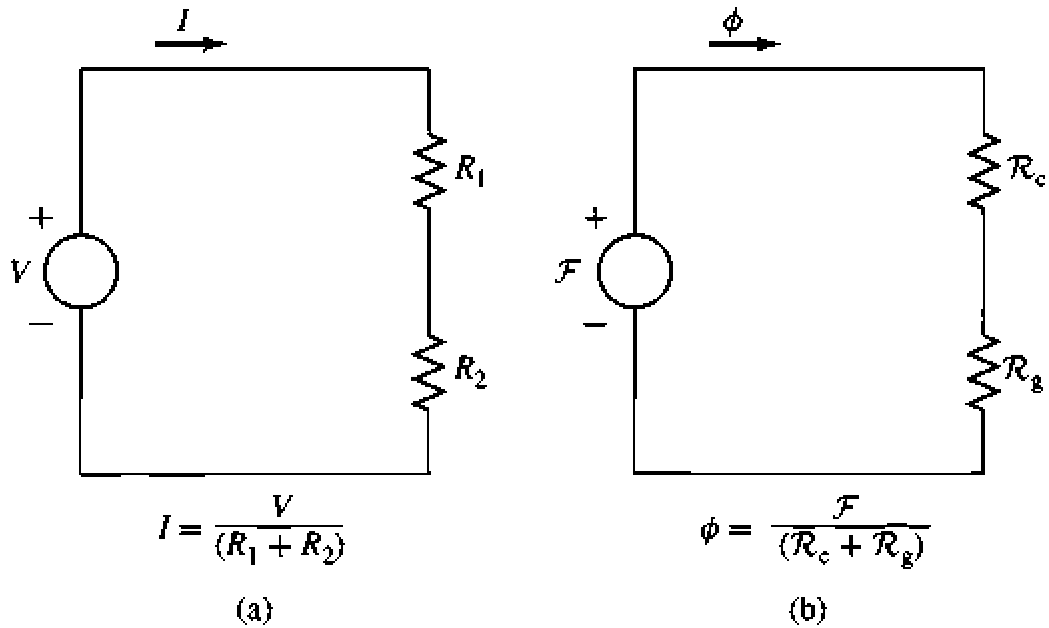

Figure 1.3 Analogy between electric and magnetic circuits. (a) Electric circuit, (b) magnetic circuit.

The fraction of the mmf required to drive flux through each portion of the magnetic circuit, commonly referred to as the mmf drop across that portion of the magnetic circuit, varies in proportion to its reluctance (directly analogous to the voltage drop across a resistive element in an electric circuit). From Eq. 1.13 we see that high material permeability can result in low core reluctance, which can often be made much smaller than that of the air gap; i.e., for $(\mu A_c / l_c) >> (\mu_0 A_g / g)$, $R_c << R_g$ and thus $R_{tot} \approx R_g$. In this case, the reluctance of the core can be neglected and the flux and hence B can be found from Eq. 1.16 in terms of F and the air-gap properties alone:

$$\phi \approx \frac{F}{R_g} = \frac{F\mu_0 A_g}{g} = Ni\,\frac{\mu_0 A_g}{g} \qquad (1.20)$$

Practical magnetic materials have permeabilities which are not constant but vary with the flux level. From Eqs. 1.13 to 1.16 we see that as long as this permeability remains sufficiently large, its variation will not significantly affect the performance of the magnetic circuit.

In practical systems, the magnetic field lines "fringe" outward somewhat as they cross the air gap, as illustrated in Fig. 1.4. Provided this fringing effect is not excessive, the magnetic-circuit concept remains applicable. The effect of these *fringing fields* is to increase the effective cross-sectional area $\boldsymbol{A_g}$ of the air gap. Various empirical methods have been developed to account for this effect. A correction for such fringing fields in short air gaps can be made by adding the gap length to each of the two dimensions making up its cross-sectional area. In this course the effect of fringing fields is usually ignored. If fringing is neglected, $\boldsymbol{A_g = A_c}$.

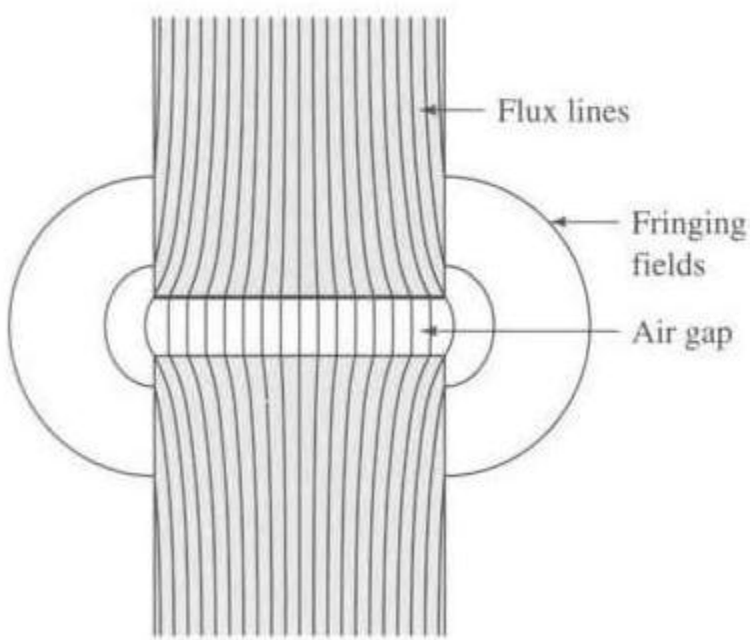

Figure 1.4 Air-gap fringing fields.

In general, magnetic circuits can consist of multiple elements in series and parallel. To complete the analogy between electric and magnetic circuits, we can generalize Eq. 1.5 as

$$F = \oint H dl = \sum_k F_k = \sum_k H_k l_k \qquad (1.21)$$

where ***F*** is the mmf (total ampere-turns) acting to drive flux through a closed loop of a magnetic circuit,

$$F = \int_S J.da \qquad (1.22)$$

and $\boldsymbol{F_k = H_k l_k}$ is the mmf drop across the k'th element of that loop. This is directly analogous to Kirchoff's voltage law for electric circuits.

$$V = \sum_k R_k i_k \qquad (1.23)$$

where ***V*** is the source voltage driving current around a loop and $\boldsymbol{R_k i_k}$ is the voltage drop across the k'th resistive element of that loop.

Similarly, the analogy to Kirchoff's current law

$$\sum_n i_n = 0 \qquad (1.24)$$

which says that the sum of currents into a node in an electric circuit equals zero is

$$\sum_n \phi_n = 0 \qquad (1.25)$$

which states that the sum of the flux into a node in a magnetic circuit is zero.

Exercise 1.

The magnetic circuit shown in Fig. 1.2 has dimensions $\boldsymbol{A_c = A_g}$ = 9 cm^2 , $\boldsymbol{g}$ = 0.050 cm, $\boldsymbol{l_c}$ = 30 cm, and $\boldsymbol{N}$ = 500 turns. Assume the value $\boldsymbol{\mu_r}$ = 70,000 for core material.

(a) Find the reluctances $\boldsymbol{R_c}$ and $\boldsymbol{R_g}$.

For the condition that the magnetic circuit is operating with $\boldsymbol{B_c}$ = 1.0 T, find

(b) the flux $\boldsymbol{\phi}$

(c) the current $\boldsymbol{i}$.

(d) Find the flux $\boldsymbol{\phi}$ and the current $\boldsymbol{i}$ if the number of turns is doubled to $\boldsymbol{N}$ = 1000

(e) Find the flux $\boldsymbol{\phi}$ and the current $\boldsymbol{i}$ if the number of turns is $\boldsymbol{N}$ = 500 turns while the gap is reduced to 0.040 cm.

Exercise 2.

The magnetic structure of a synchronous machine is shown schematically in Fig. 1.5. Assuming that rotor and stator iron have infinite permeability (***μ→∞***). Consider ***g*** = l cm, and $\boldsymbol{A_g}$ = 2000 cm^2.

1) find the air-gap flux $\boldsymbol{\phi}$ and the flux density $\boldsymbol{B_g}$ if ***i*** = 10 A and ***N*** = 1000 turns.
2) Now consider the air-gap flux density to be $\boldsymbol{B_g}$ = 0.9 T. Find the air-gap flux $\boldsymbol{\phi}$ and, for a coil of N = 500 turns, the current required to produce this level of air-gap flux.

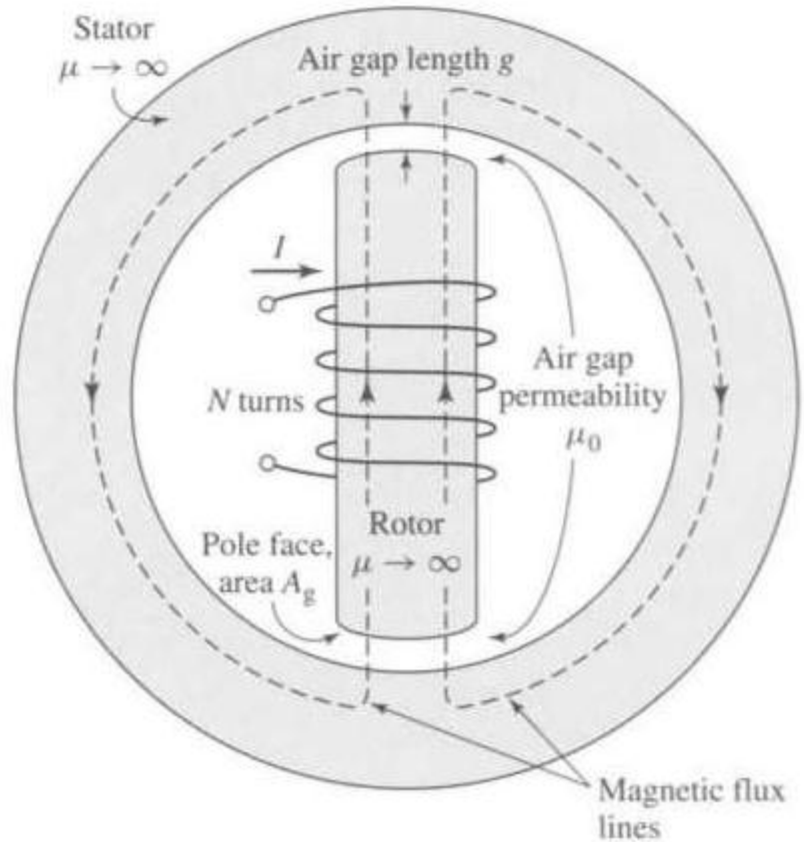

Figure 1.5 Simple synchronous machine.

2. FLUX LINKAGE and INDUCTANCE ENERGY

When a magnetic field varies with time, an electric field ***E*** is produced in space as determined by Faraday's law:

$$\oint_C E.ds = -\frac{d}{dt}\int_S B.da \qquad (1.26)$$

In magnetic structures with windings of high electrical conductivity, such as in Fig. 1.2, it can be shown that the ***E*** field in the wire is extremely small and can be neglected, so that the left-hand side of Eq. 1.26 reduces to the negative of the induced voltage ***e*** at the winding terminals. In addition, the flux on the right-hand side of Eq. 1.26 is dominated by the core flux $\boldsymbol{\phi}$. Since the winding (and hence the contour ***C***) links the core flux N times, Eq. 1.26 reduces to

$$e = N\frac{d\varphi}{dt} = \frac{d\lambda}{dt} \qquad (1.27)$$

where $\boldsymbol{\lambda}$ is the ***flux linkage*** of the winding and is defined as

$$\lambda = N\varphi \tag{1.28}$$

Flux linkage is measured in **units of webers** (or equivalently **weber-turns**). The symbol ***φ*** is used to indicate the instantaneous value of a time-varying flux.

Note that the direction of the induced voltage ***e*** is defined by Eq. 1.26 so that if the winding terminals were short-circuited, a current would flow in such a direction as to oppose the change of flux linkage.

For a magnetic circuit composed of magnetic material of constant magnetic permeability or which includes a dominating air gap, the relationship between ***ϕ*** and ***i*** will be linear and we can define the ***inductance L*** as

$$L = \frac{\lambda}{i} \tag{1.29}$$

Substitution of Eqs. 1.5, 1.18 and 1.28 into Eq. 1.29 gives

$$L = \frac{N^2}{R_{tot}} \tag{1.30}$$

For example, from Eq. 1.20, under the assumption that the reluctance of the core is negligible as compared to that of the air gap, the inductance of the winding in Fig. 1.2 is equal to

$$L = \frac{N^2}{\dfrac{g}{\mu_0 A_g}} = \frac{N^2 \mu_0 A_g}{g} \qquad (1.31)$$

Inductance is measured in henrys (**H**) or weber-turns per ampere.

Equation 1.31 shows the dimensional form of expressions for inductance; inductance is proportional to the square of the number of turns, to a magnetic permeability, and to a cross sectional area and is inversely proportional to a length.
It must be emphasized that, strictly speaking, the concept of inductance requires a linear relationship between flux and mmf. Thus, it cannot be rigorously applied in situations where the nonlinear characteristics of magnetic materials, as is discussed earlier, dominate the performance of the magnetic system. However, in many situations of practical interest, the reluctance of the system is dominated by that of an air gap (which is of course linear) and the nonlinear effects of the magnetic material can be ignored. In other cases it may be perfectly acceptable to assume an average value of magnetic permeability for the core material and to calculate a corresponding average inductance which can be used for calculations of reasonable engineering accuracy.

Exercise 3.

The magnetic circuit of Fig. 1.6a consists of an *500* -turn winding on a magnetic core of infinite permeability with two parallel air gaps of lengths $\boldsymbol{g_1}$ *=0,1cm* and $\boldsymbol{g_2}$ *=0,01cm* and areas $\boldsymbol{A_1}$ =5cm^2 and $\boldsymbol{A_2}$ *=10cm^2*, respectively. Find

(a) the inductance of the winding
(b) the flux density $\boldsymbol{B_1}$ in gap 1 when the winding is carrying a current ***i=2A***. Neglect fringing effects at the air gap.

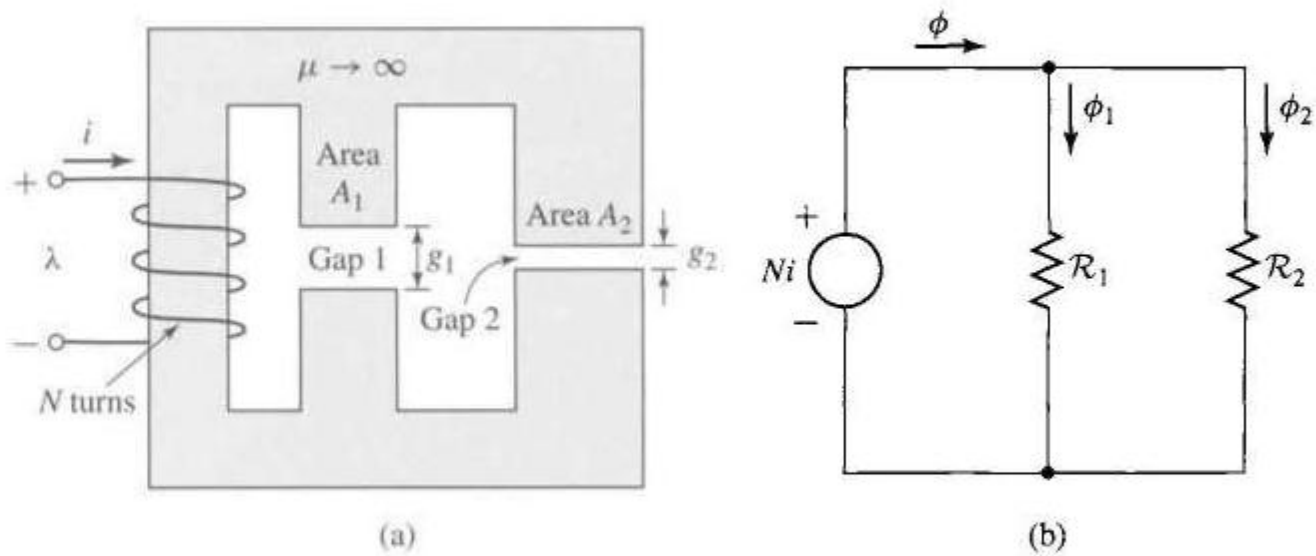

Figure 1.6 (a) Magnetic circuit and (b) equivalent circuit for Example 1.3.

Exercise 4.

Consider a magnetic circuit of Fig. 1.2 which has dimensions $\boldsymbol{A_c = A_g}$ = 9 cm^2 , $\boldsymbol{g}$ = 0.050 cm, $\boldsymbol{l_c}$ = 30 cm, and $\boldsymbol{N}$ = 500 turns with the relative permeability of the core material is assumed to be $\boldsymbol{\mu_r}$ = 70000 at a flux density of 1.0 T.

1) Calculate the inductance of the winding.

2) In a practical device, the core would be constructed from electrical steel such as M-5 electrical steel. This material is highly nonlinear and its relative permeability (defined for the purposes of this example as the ratio B / H) varies from a value of approximately $\boldsymbol{\mu_r}$ = 72,300 at a flux density of B = 1.0 T to a value of on the order of $\boldsymbol{\mu_r}$ = 2900 as the flux density is raised to 1.8 T.

(a) Calculate the inductance under the assumption that the relative permeability of the core steel is 72,300.

(b) Calculate the inductance under the assumption that the relative permeability is equal to 2900 .

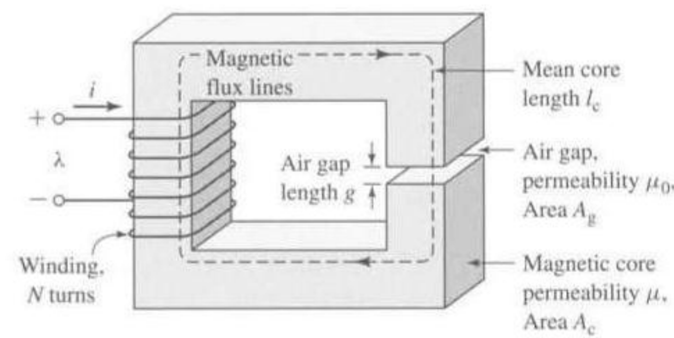

Figure 1.2 Magnetic circuit with air gap.

3. ENERGY

Figure 1.8 shows a magnetic circuit with an air gap and two windings. In this case note that the mmf acting on the magnetic circuit is given by the total ampere-turns acting on the magnetic circuit (i.e., the net ampere turns of both windings) and that the reference directions for the currents have been chosen to produce flux in the same direction. The total mmf is therefore

$$F = N_1 i_1 + N_2 i_2 \tag{1.32}$$

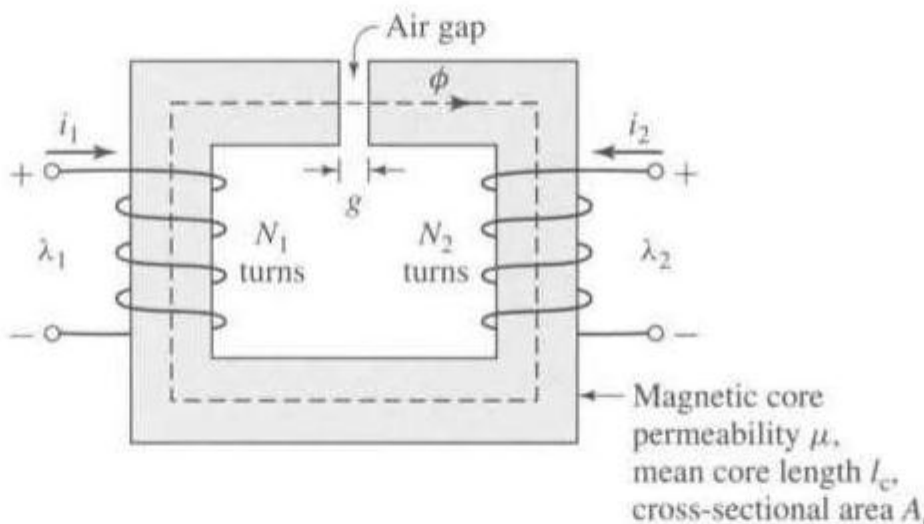

Figure 1.8 Magnetic circuit with two windings.

From Eq. 1.20, with the reluctance of the core neglected and assuming that $\mathbf{A_c = A_g}$, the core flux $\boldsymbol{\phi}$ is

$$\phi = (N_1 i_1 + N_2 i_2)\frac{\mu_0 A_c}{g} \qquad (1.33)$$

In Eq. 1.33, $\boldsymbol{\phi}$ is the resultant core flux produced by the total mmf of the two windings. It is this resultant $\boldsymbol{\phi}$ which determines the operating point of the core material.

If Eq. 1.33 is broken up into terms attributable to the individual currents, the resultant flux linkages of coil 1 can be expressed as

$$\lambda_1 = N_1\varphi = N_1^2\left(\frac{\mu_0 A_c}{g}\right)i_1 + N_1 N_2\left(\frac{\mu_0 A_c}{g}\right)i_2 \qquad (1.34)$$

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \qquad (1.35)$$

where

$$L_{11} = N_1^2\left(\frac{\mu_0 A_c}{g}\right) \qquad (1.36)$$

is the self-inductance of coil 1 and $\boldsymbol{L_{11}\, i_1}$ is the flux linkage of coil 1 due to its own current $\boldsymbol{i_1}$.

The mutual inductance between coils 1 and 2 is

$$L_{12} = N_1 N_2 \left(\frac{\mu_0 A_c}{g} \right) \qquad (1.37)$$

and $\boldsymbol{L_{12} i_2}$ is the flux linkage of coil 1 due to current $\boldsymbol{i_2}$ in the other coil. Similarly, the flux linkage of coil 2 is

$$\lambda_2 = N_2 \phi = N_1 N_2 \left(\frac{\mu_0 A_c}{g} \right) i_1 + N_2^2 \left(\frac{\mu_0 A_c}{g} \right) i_2 \qquad (1.38)$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2 \qquad (1.39)$$

where $\boldsymbol{L_{21} = L_{l2}}$ is the mutual inductance and

$$L_{22} = N_2^2 \left(\frac{\mu_0 A_c}{g} \right) \qquad (1.40)$$

is the self-inductance of coil 2.

It is important to note that the resolution of the resultant flux linkages into the components produced by $\boldsymbol{i_1}$ and $\boldsymbol{i_2}$ is based on superposition of the individual effects and therefore implies a linear flux-mmf relationship (characteristic of materials of constant permeability).

Substitution of Eq. 1.29 in Eq. 1.27 yields

$$e = \frac{d}{dt}(Li) \tag{1.41}$$

for a magnetic circuit with a single winding. For a static magnetic circuit, the inductance is fixed (assuming that material nonlinearities do not cause the inductance to vary), and this equation reduces to the familiar circuit-theory form

$$e = L\frac{di}{dt} \tag{1.42}$$

However, in electromechanical energy conversion devices, inductances are often time varying, and Eq. 1.41 must be written as

$$e = L\frac{di}{dt} + i\frac{dL}{dt} \tag{1.43}$$

Note that in situations with multiple windings, the total flux linkage of each winding must be used in Eq. 1.27 to find the winding-terminal voltage.

The power at the terminals of a winding on a magnetic circuit is a measure of the rate of energy flow into the circuit through that particular winding. The power, ***p***, is determined from the product of the voltage and the current

$$p = ie = i\frac{d\lambda}{dt} \tag{1.44}$$

and its unit is watts (W), or joules per second. Thus the change in magnetic stored energy ΔW in the magnetic circuit in the time interval t_1 to t_2 is

$$\Delta W = \int_{t_1}^{t_2} p.dt = \int_{t_1}^{t_2} i.d\lambda \qquad (1.45)$$

In SI units, the magnetic stored energy W is measured in joules (*J*).

For a single-winding system of constant inductance, the change in magnetic stored energy as the flux level is changed from λ_1 to λ_2 can be written as

$$\Delta W = \int_{\lambda_1}^{\lambda_2} i.d\lambda = \int_{\lambda_1}^{\lambda_2} \frac{\lambda}{L}.d\lambda = \frac{1}{2L}\left(\lambda_2^2 - \lambda_1^2\right) \qquad (1.46)$$

The total magnetic stored energy at any given value of λ can be found from setting λ_1 equal to zero:

$$W = \frac{1}{2L}\lambda^2 = \frac{L}{2}i^2 \qquad (1.47)$$

Exercise 5,

Consider a magnetic circuit of Fig. 1.2 which has dimensions $A_c = A_g$ = 9 cm^2 , g = 0.050 cm, l_c = 30 cm, and N = 500 turns with the relative permeability of the core material is assumed to be μ_r = 70000. Find

(a) the inductance L,
(b) the magnetic stored energy W for B_c = 1.0 T,
(c) the induced voltage e for a 60-Hz time-varying core flux of the form B_c = 1.0 sinωt T.
(d) Repeat b) and c) for B_c = 0.8 T and assuming the core flux varies at 50 Hz instead of 60 Hz.

4. **AC EXCITATION**

In ac power systems, the waveforms of voltage and flux closely approximate sinusoidal functions of time. We use as our model a closed-core magnetic circuit, i.e., with no air gap, such as that shown in Fig. 1.1. The magnetic path length is $\boldsymbol{l_c}$, and the cross-sectional area is $\boldsymbol{A_c}$ throughout the length of the core. We further assume a sinusoidal variation of the core flux $\boldsymbol{\varphi}$(t); thus

$$\varphi(t) = \phi_{\max} \sin \omega t = A_c B_{\max} \sin \omega t \qquad (1.48)$$

where $\boldsymbol{\phi_{max}}$ = amplitude of core flux $\boldsymbol{\varphi}$ in webers
$\boldsymbol{B_{max}}$ = amplitude of flux density $\boldsymbol{B_c}$ in teslas
$\boldsymbol{\omega}$ = angular frequency = $\boldsymbol{2\pi f}$
$\boldsymbol{f}$ = frequency in Hz

From Eq. 1.27, the voltage induced in the ***N***-turns winding is

$$e(t) = \omega N \phi_{\max} \cos \omega t = E_{\max} \cos \omega t \qquad (1.49)$$

where

$$E_{\max} = \omega N \phi_{\max} = 2\pi f N A_c B_{\max} \qquad (1.50)$$

In steady-state ac operation, we are usually more interested in the root-mean square or ***rms*** values of voltages and currents than in instantaneous or maximum values. In general, the ***rms*** value of a periodic function of time, ***f(t)***, of period ***T*** is defined as

$$F_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)dt} \qquad \textbf{(1.51)}$$

From Eq. 1.51, the ***rms*** value of a sine wave can be shown to be ***0.707*** times its peak value. Thus the ***rms*** value of the induced voltage is

$$E_{rms} = \frac{2\pi}{\sqrt{2}} fNA_c B_{\max} = \sqrt{2}\pi fNA_c B_{\max} \qquad \textbf{(1.52)}$$

To produce magnetic flux in the core requires current in the exciting winding known as the exciting current, i_φ. The nonlinear magnetic properties of the core require that the waveform of the exciting current differs from the sinusoidal waveform of the flux. A curve of the exciting current as a function of time can be found graphically from the magnetic characteristics of the core material, as illustrated in Fig. 1.11a.

Since B_c and H_c are related to φ and i_φ, by known geometric constants, the ac hysteresis loop of Fig. 1.11b has been drawn in terms of $\varphi = B_c A_c$ and $i_\varphi = H_c l_c/N$.
Sine waves of induced voltage, ***e***, and flux, φ, in accordance with Eqs. 1.48 and 1.49, are shown in Fig. 1,11a.

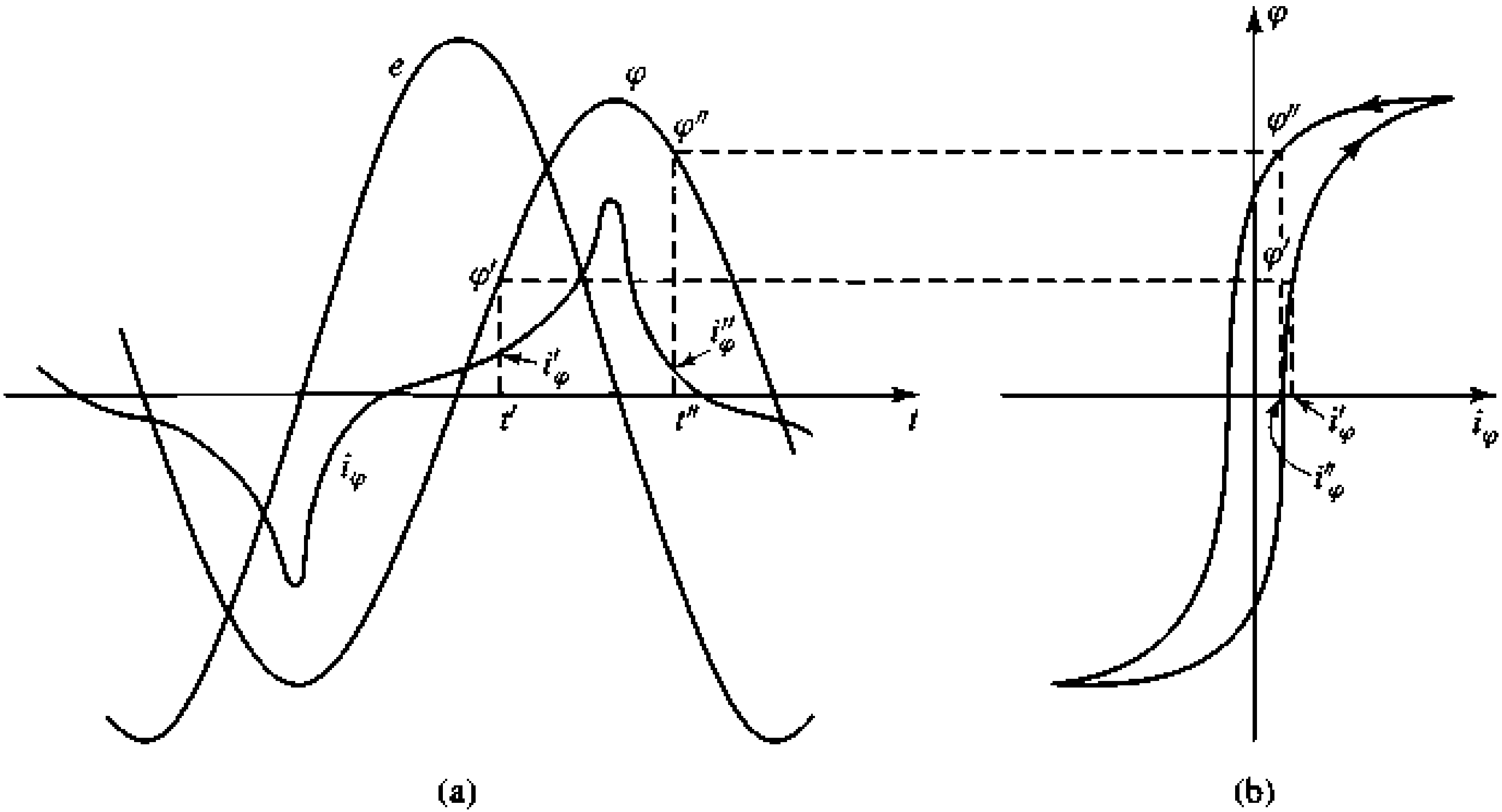

Figure 1.11 Excitation phenomena. (a) Voltage, flux, and exciting current; (b) corresponding hysteresis loop.

At any given time, the value of i_φ, corresponding to the given value of flux can be found directly from the hysteresis loop. For example, at time t' the flux is φ' and the current is i'_φ; at time t'' the corresponding values are φ'' and i''_φ. Notice that since the hysteresis loop is multivalued, it is necessary to be careful to pick the rising-flux values (φ' in the figure) from the rising-flux portion of the hysteresis loop; similarly the falling-flux portion of the hysteresis loop must be selected for the falling-flux values (φ'' in the figure). Notice that, because the hysteresis loop "flattens out" due to saturation effects, the waveform of the exciting current is sharply peaked. Its ***rms*** value $I_{\varphi,rms}$ is defined by Eq, 1.51, where T is the period of a cycle. It is related to the corresponding ***rms*** value $H_{c,rms}$ of H_c by the relationship

$$I_{\varphi,rms} = \frac{l_c H_{c,rms}}{N} \qquad (1.53)$$

The ac excitation characteristics of core materials are often described in terms of rms voltamperes rather than a magnetization curve relating B and H. From Eqs. 1.52 and 1.53, the rms voltamperes required to excite the core of Fig. 1.1 to a specified flux density is equal to

$$E_{rms} I_{\varphi,rms} = \sqrt{2}\pi f N A_c B_{\max} \frac{l_c H_{rms}}{N} = \sqrt{2}\pi f A_c l_c B_{\max} H_{rms} \qquad (1.54)$$

In Eq. 1.54, the product $A_c l_c$ can be seen to be equal to the volume of the core and hence the rms exciting voltamperes required to excite the core with sinusoidal can be seen to be proportional to the frequency of excitation, the core volume and the product of the peak flux density and the rms magnetic field intensity.

For a magnetic material of mass density ρ_c, the mass of the core is $A_c l_c \rho_c$ and the exciting rms voltamperes per unit mass, P_a, can be expressed as

$$P_a = \frac{E_{rms} I_{\varphi,rms}}{mass} = \frac{\sqrt{2}\pi f}{\rho_c} B_{\max} H_{rms} \qquad (1.55)$$

Note that, normalized in this fashion, the rms exciting voltamperes can be seen to be a property of the material alone. In addition, note that they depend only on B_{max} because H_{rms} is a unique function of B_{max} as determined by the shape of the material hysteresis loop at any given frequency f. As a result, the ac excitation requirements for a magnetic material are often supplied by manufacturers in terms of rms voltamperes per unit weight as determined by laboratory tests on closed-core samples of the material. These results are illustrated in Fig. 1.12 for M-5 grain-oriented electrical steel.

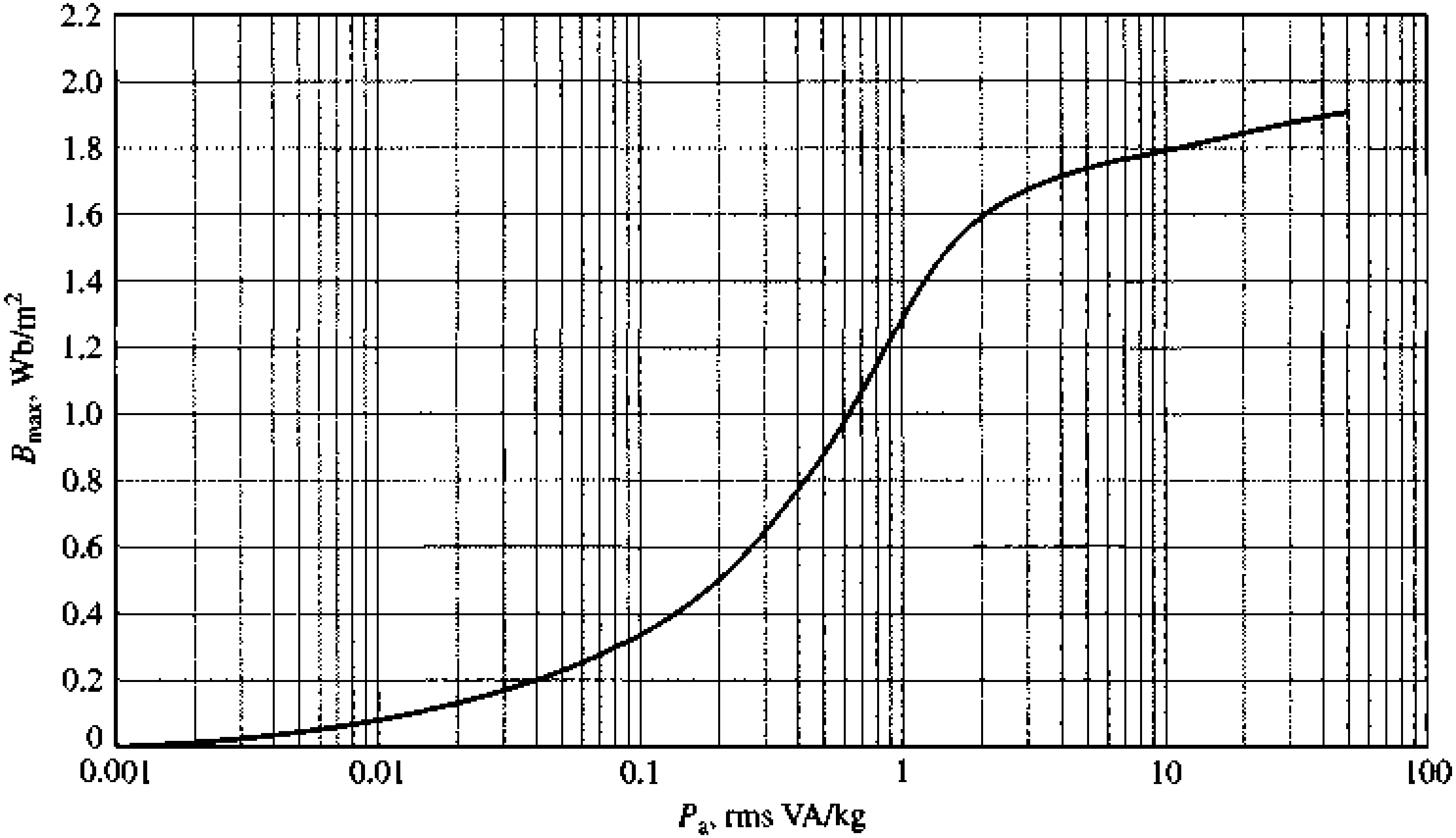

Figure 1.12 Exciting rms voltamperes per kilogram at 60 Hz for M-5 grain-oriented electrical steel 0.012 in thick. (*Armco Inc.*)

The exciting current supplies the mmf required to produce the core flux and the power input associated with the energy in the magnetic field in the core. Part of this energy is dissipated as losses and results in heating of the core. The rest appears as reactive power associated with energy storage in the magnetic field. This reactive power is not dissipated in the core; it is cyclically supplied and absorbed by the excitation source.

Two loss mechanisms are associated with time-varying fluxes in magnetic materials. The first is ohmic heating $\boldsymbol{RI^2}$, associated with induced currents in the core material. From Faraday's law (Eq. 1.26) we see that time-varying magnetic fields give rise to electric fields. In magnetic materials these electric fields result in induced currents, commonly referred to as *eddy currents,* which circulate in the core material and oppose changes in flux density in the material. To counteract the corresponding demagnetizing effect, the current in the exciting winding must increase. Thus the resultant "dynamic" *B-H* loop under ac operation is somewhat "fatter" than the hysteresis loop for slowly varying conditions, and this effect increases as the excitation frequency is increased. It is for this reason that the characteristics of electrical steels vary with frequency and hence manufacturers typically supply characteristics over the expected operating frequency range of a particular electrical steel. Note for example that the exciting rms voltamperes of Fig. 1.12 are specified at a frequency of 60 Hz.

To reduce the effects of eddy currents, magnetic structures are usually built of thin sheets of laminations of the magnetic material. These laminations, which are aligned in the direction of the field lines, are insulated from each other by an oxide layer on their surfaces or by a thin coat of insulating enamel or varnish. This greatly reduces the magnitude of the eddy currents since the layers of insulation interrupt the current paths; the thinner the laminations, the lower the losses. In general, eddy-current loss tends to increase as the square of the excitation frequency and also as the square of the peak flux density.

The second loss mechanism is due to the hysteretic nature of magnetic material. In a magnetic circuit like that of Fig. 1.1 or the transformer of Fig. 2.4, a time-varying excitation will cause the magnetic material to undergo a cyclic variation described by a hysteresis loop such as that shown in Fig. 1.13.

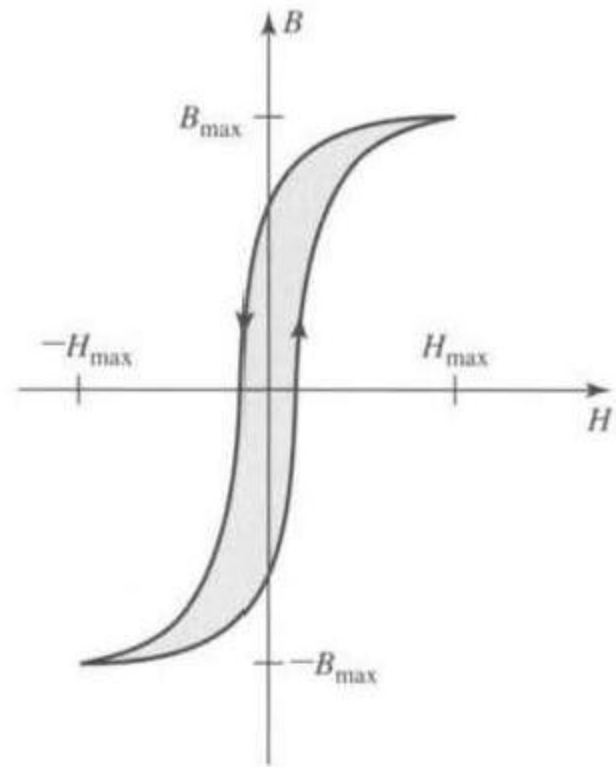

Figure 1.13 Hysteresis loop; hysteresis loss is proportional to the loop area (shaded).

Equation 1.45 can be used to calculate the energy input ***W*** to the magnetic core of Fig. 1.1 as the material undergoes a single cycle

$$W = \oint i_{\varphi} d\lambda = \oint \frac{H_c l_c}{N} A_c N.dB_c = A_c l_c \oint H_c dB_c \qquad (1.56)$$

Recognizing that $\boldsymbol{A_c l_c}$ is the volume of the core and that the integral is the area of the ac hysteresis loop, we see that each time the magnetic material undergoes a cycle, there is a net energy input into the material. This energy is required to move around the magnetic dipoles in the material and is dissipated as heat in the material. Thus for a given flux level, the corresponding hysteresis losses are proportional to the area of the hysteresis loop and to the total volume of material. Since there is an energy loss per cycle, hysteresis power loss is proportional to the frequency of the applied excitation.

In general, these losses depend on the metallurgy of the material as well as the flux density and frequency. Information on core loss is typically presented in graphical form. It is plotted in terms of watts per unit weight as a function of flux density; often a family of curves for different frequencies are given. Figure 1.14 shows the core loss $\boldsymbol{P_c}$ for M-5 grain-oriented electrical steel at 60 Hz.

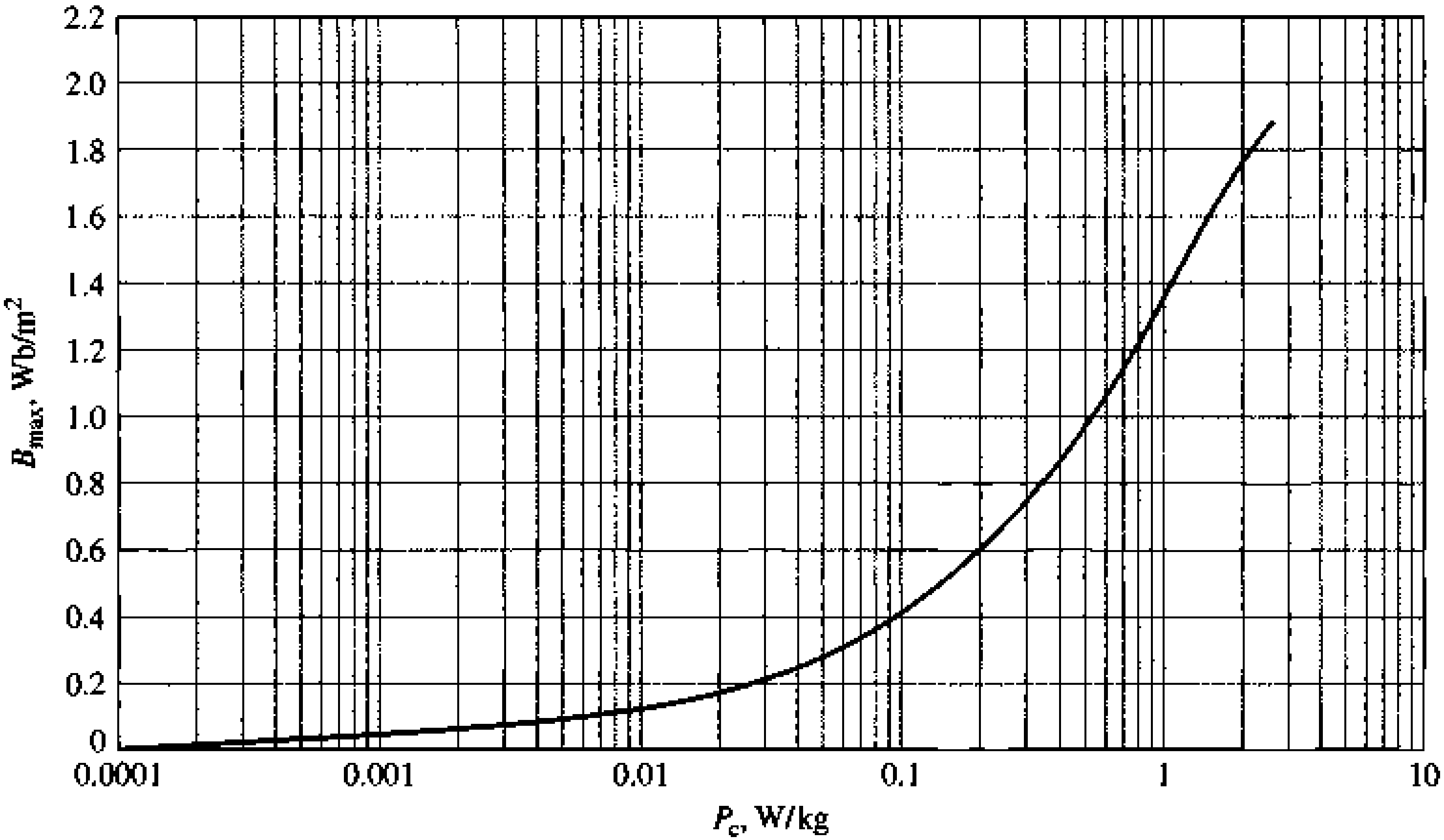

Figure 1.14 Core loss at 60 Hz in watts per kilogram for M-5 grain-oriented electrical steel 0.012 in thick. (*Armco Inc.*)

Nearly all transformers and certain sections of electric machines use sheet-steel material that has highly favorable directions of magnetization along which the core loss is low and the permeability is high. This material is termed grain-oriented steel. The reason for this property lies in the atomic structure of a crystal of the silicon-iron alloy, which is a body-centered cube; each cube has an atom at each corner as well as one in the center of the cube. In the cube, the easiest axis of magnetization is the cube edge; the diagonal across the cube face is more difficult, and the diagonal through the cube is the most difficult. By suitable manufacturing techniques most of the crystalline cube edges are aligned in the rolling direction to make it the favorable direction of magnetization. The behavior in this direction is superior in core loss and permeability to *nonoriented steels* in which the crystals are randomly oriented to produce a material with characteristics which are uniform in all directions. As a result, oriented steels can be operated at higher flux densities than the nonoriented grades.

Nonoriented electrical steels are used in applications where the flux does not follow a path which can be oriented with the rolling direction or where low cost is of importance. In these steels the losses are somewhat higher and the permeability is very much lower than in grain-oriented steels.

Exercise 6,

The magnetic core in Fig. 1.15 is made from laminations of M-5 grain-oriented electrical steel. The winding is excited with a 60-Hz voltage to produce a flux density in the steel of
B = 1.5 sin ωt T, where w = 377 rad/sec. The steel occupies 0.94 of the core cross sectional area. The mass-density of the steel is 7.65 g/cm^3. Find
(a) the applied voltage,
(b) The peak current if H_{max}=36 A.turns/m,
(c) the rms exciting current,
(d) the core loss.

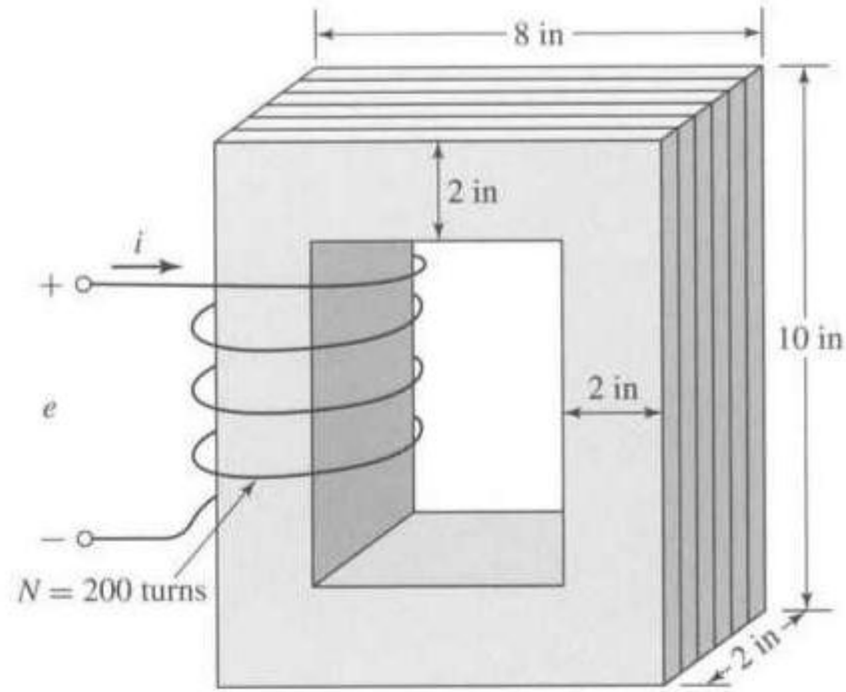

Figure 1.15 Laminated steel core with winding for Example 1.8.

Exercise 7,

Repeat exercise 6 for a 60-Hz voltage of B = 1.0 sin ωt T and H_{max}=12 A.turns/m.

4. PERMANENT MAGNETS

Figure 1.16a shows the second quadrant of a hysteresis loop for Alnico 5, a typical permanent-magnet material, while Fig. 1.16b shows the second quadrant of a hysteresis loop for M-5 steel. Notice that the curves are similar in nature. The hysteresis loop of Alnico 5 is characterized by a large value of *residual flux density* or *remanent magnetization,* B_r, (approximately 1.22 T) as well as a large value of *coercivity,* H_c, (approximately -49 *kA l m).*

The remanent magnetization, B_r, corresponds to the flux density which would remain in a closed magnetic structure, such as that of Fig. 1.1, made of this material, if the applied mmf (and hence the magnetic field intensity *H)* were reduced to zero. However, although the M-5 electrical steel also has a large value of remanent magnetization (approximately 1.4 T), it has a much smaller value of coercivity (approximately *-6 A l m,* smaller by a factor of over 7500).

The coercivity H_c corresponds to the value of magnetic field intensity (which is proportional to the mmf) required to reduce the material flux density to zero.

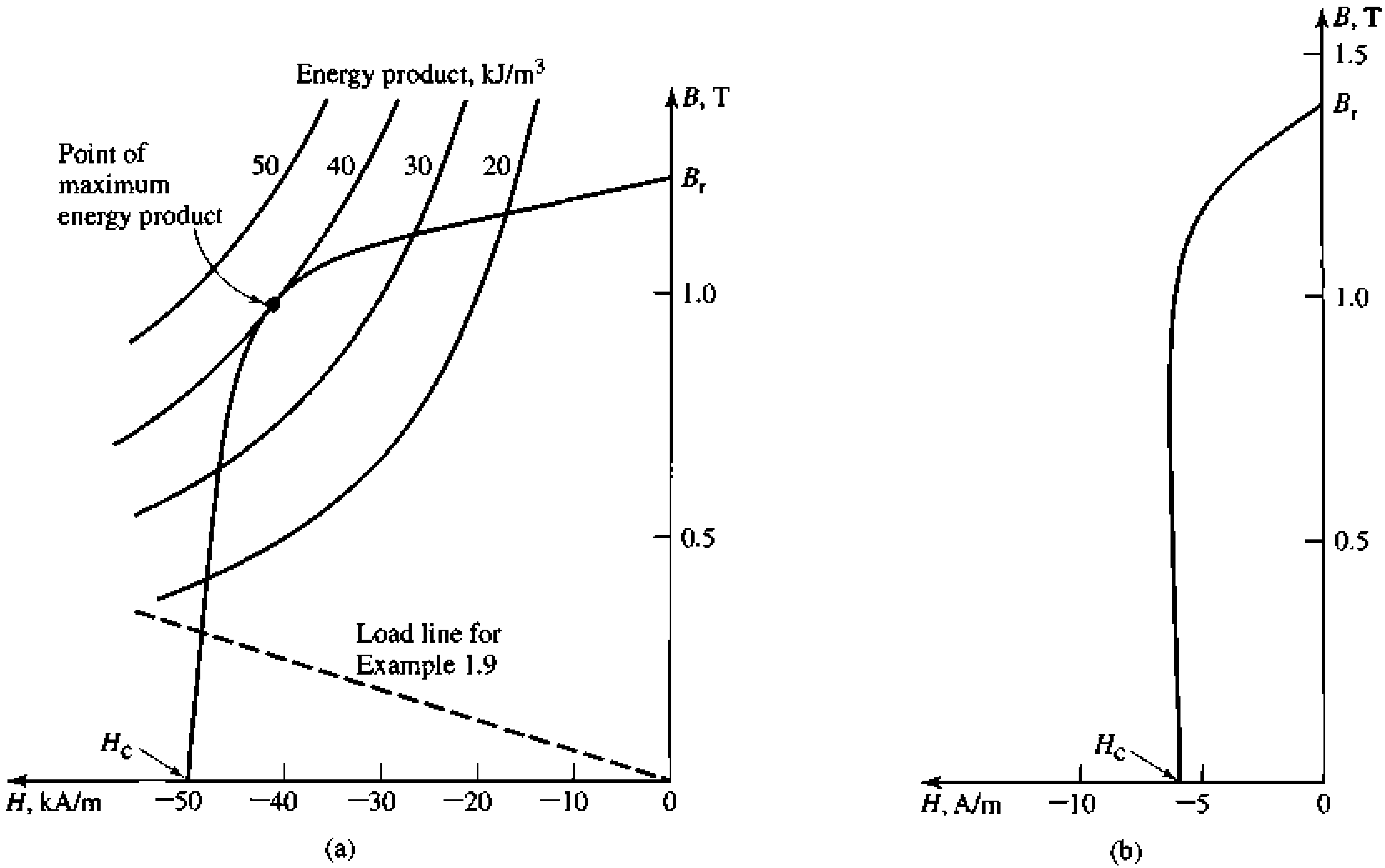

Figure 1.16 (a) Second quadrant of hysteresis loop for Alnico 5; **(b)** second quadrant of hysteresis loop for M-5 electrical steel. (Armco Inc.)

The significance of remanent magnetization is that it can produce magnetic flux in a magnetic circuit in the absence of external excitation (such as winding currents). This is a familiar phenomenon widely used in devices such as loudspeakers and permanent magnet motors.

From Fig. 1.16, it would appear that both Alnico 5 and M-5 electrical steel would be useful in producing flux in unexcited magnetic circuits since they both have large values of remanent magnetization.

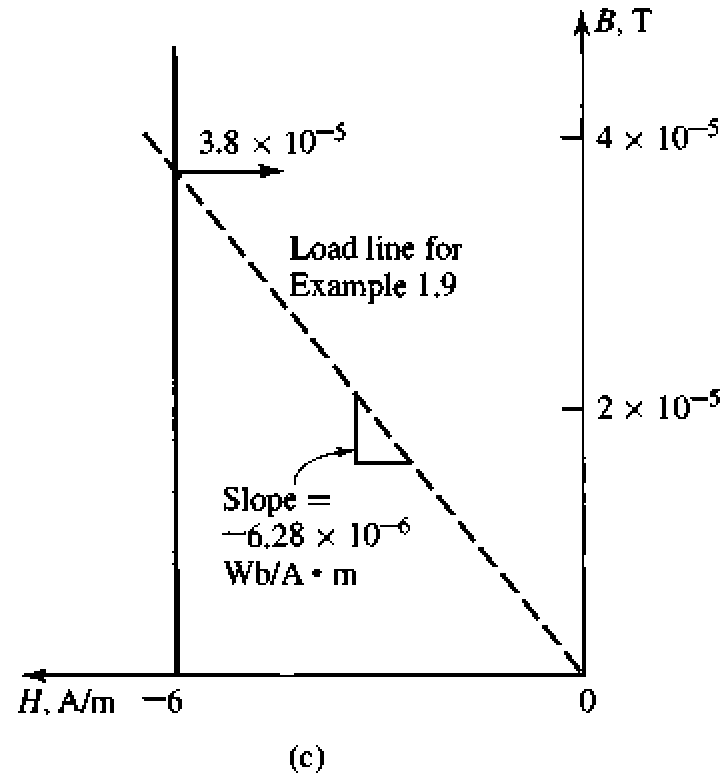

Figure 1.16 (c) hysteresis loop for M-5 electrical steel expanded for small B. (Armco Inc.)

Exercise 8,

As shown in Fig. 1.17, a magnetic circuit consists of a core of high permeability ($\mu \rightarrow \infty$), an air gap of length g = 0.2 cm, and a section of magnetic material of length l_m = 1.0 cm. The cross-sectional area of the core and gap is equal to $A_m = A_g$ = 4 cm^2

Calculate the flux density B_g in the air gap if the magnetic material is

(a) Alnico 5
(b) M-5 electrical steel.

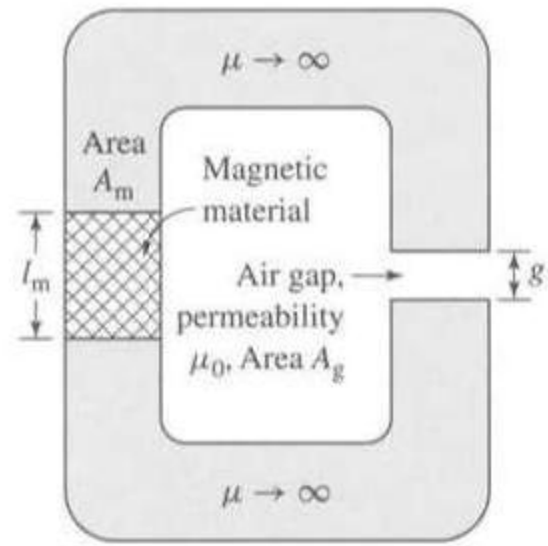

Figure 1.17 Magnetic circuit