Usually W_t is given and the other forces are desired. In this case, it is not difficult to discover that

$$\begin{aligned} W_r &= W_t \tan\phi_t \\ W_a &= W_t \tan\psi \\ W &= \frac{W_t}{\cos\phi_n \cos\psi} \end{aligned} \tag{13–40}$$

EXAMPLE 13–9

In Fig. 13–38 a 1-hp electric motor runs at 1800 rev/min in the clockwise direction, as viewed from the positive x axis. Keyed to the motor shaft is an 18-tooth helical pinion having a normal pressure angle of 20°, a helix angle of 30°, and a normal diametral pitch of 12 teeth/in. The hand of the helix is shown in the figure. Make a three-dimensional sketch of the motor shaft and pinion, and show the forces acting on the pinion and the bearing reactions at A and B. The thrust should be taken out at A.

Solution

From Eq. (13–19) we find

$$\phi_t = \tan^{-1}\frac{\tan\phi_n}{\cos\psi} = \tan^{-1}\frac{\tan 20^\circ}{\cos 30^\circ} = 22.8^\circ$$

Also, $P_t = P_n \cos\psi = 12\cos 30^\circ = 10.39$ teeth/in. Therefore the pitch diameter of the pinion is $d_p = 18/10.39 = 1.732$ in. The pitch-line velocity is

$$V = \frac{\pi d n}{12} = \frac{\pi(1.732)(1800)}{12} = 816 \text{ ft/min}$$

The transmitted load is

$$W_t = \frac{33\,000H}{V} = \frac{(33\,000)(1)}{816} = 40.4 \text{ lbf}$$

From Eq. (13–40) we find

$$\begin{aligned} W_r &= W_t \tan\phi_t = (40.4)\tan 22.8^\circ = 17.0 \text{ lbf} \\ W_a &= W_t \tan\psi = (40.4)\tan 30^\circ = 23.3 \text{ lbf} \\ W &= \frac{W_t}{\cos\phi_n \cos\psi} = \frac{40.4}{\cos 20^\circ \cos 30^\circ} = 49.6 \text{ lbf} \end{aligned}$$

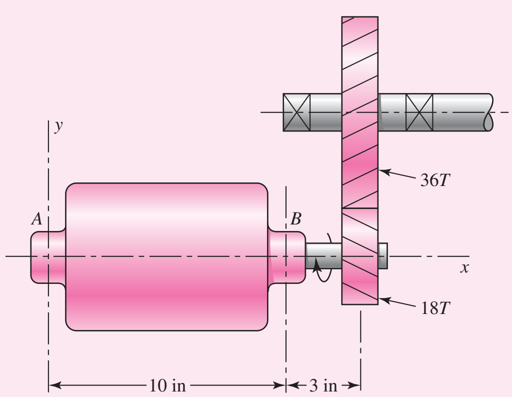
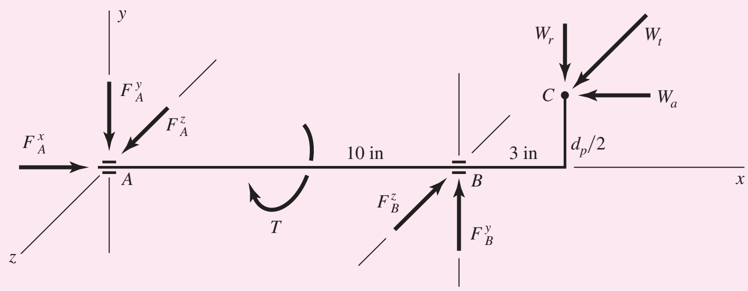


Figure 13–38

The motor and gear train of Ex. 13–9.

Figure 13–39

Free body diagram of motor shaft of Ex. 13–9.



These three forces, W_r in the $-y$ direction, W_a in the $-x$ direction, and W_t in the $+z$ direction, are shown acting at point C in Fig. 13–39. We assume bearing reactions at A and B as shown. Then $F_A^x = W_a = 23.3$ lbf. Taking moments about the z axis,

$$-(17.0)(13) + (23.3)\left(\frac{1.732}{2}\right) + 10F_B^y = 0$$

or $F_B^y = 20.1$ lbf. Summing forces in the y direction then gives $F_A^y = 3.1$ lbf. Taking moments about the y axis, next

$$10F_B^z - (40.4)(13) = 0$$

or $F_B^z = 52.5$ lbf. Summing forces in the z direction and solving gives $F_A^z = 12.1$ lbf. Also, the torque is $T = W_t d_p/2 = (40.4)(1.732/2) = 35$ lbf · in.

For comparison, solve the problem again using vectors. The force at C is

$$\mathbf{W} = -23.3\mathbf{i} - 17.0\mathbf{j} + 40.4\mathbf{k} \text{ lbf}$$

Position vectors to B and C from origin A are

$$\mathbf{R}_B = 10\,\mathbf{i} \qquad \mathbf{R}_C = 13\mathbf{i} + 0.866\mathbf{j}$$

Taking moments about A, we have

$$\mathbf{R}_B \times \mathbf{F}_B + \mathbf{T} + \mathbf{R}_C \times \mathbf{W} = \mathbf{0}$$

Using the directions assumed in Fig. 13–39 and substituting values gives

$$10\,\mathbf{i} \times \left(F_B^y\mathbf{j} - F_B^z\mathbf{k}\right) - T\,\mathbf{i} + (13\mathbf{i} + 0.866\mathbf{j}) \times (-23.3\mathbf{i} - 17.0\mathbf{j} + 40.4\mathbf{k}) = \mathbf{0}$$

When the cross products are formed, we get

$$\left(10F_B^y\mathbf{k} + 10F_B^z\mathbf{j}\right) - T\,\mathbf{i} + (35\mathbf{i} - 525\mathbf{j} - 201\mathbf{k}) = \mathbf{0}$$

whence $T = 35$ lbf · in, $F_B^y = 20.1$ lbf, and $F_B^z = 52.5$ lbf.

Next,

$$\mathbf{F}_A = -\mathbf{F}_B - \mathbf{W}, \text{ and so } \mathbf{F}_A = 23.3\mathbf{i} - 3.1\mathbf{j} + 12.1\mathbf{k} \text{ lbf.}$$

13–17 Force Analysis—Worm Gearing

If friction is neglected, then the only force exerted by the gear will be the force W, shown in Fig. 13–40, having the three orthogonal components W^x, W^y, and W^z. From the geometry of the figure, we see that