Independent event probability

- If A and B are two independent variable the probability of them will be

- $P(A \cap B) = P(A) . P(B)$
- $P(B|A) = P(B), \qquad (1)$
- For A,B independent substitute 1 In 3
- $P(A \cap B) = P(B)P(A)$

Example

- <u>Example:</u> Suppose that we have a fuse box containing 20 fuses, of which 5 are defective. if 2 fuses are taken from the box in succession without replacing the first, what is the probability that both fuses are defective? where the two variable are independent?

- <u>Solution</u>: Let A be the event that the first fuse is defective and B the second fuse is defective.
- $P\ (\ A\ \ \cap B\ )$ is the event that A occurs, then B occurs after A occurred.
- The probability of the first fuse is defective is
- $P\ (A) = \frac{5}{20} = \frac{1}{4}\ ,\ P\ (B) = \frac{4}{19}$
- The probability of the second is defective given that the first fuse was defective is
- $P(A \cap B) = P(A)P(B) \rightarrow independant$
- $= \left(\frac{1}{4}\right)\left(\frac{4}{19}\right) = \frac{1}{19}$

Theorem of Total Probability

If the events $B_1, B_2, \ldots, B_k$ constitute a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1, 2, \ldots, k$, then for any event A of S,

$$P(A) = \sum_{i=1}^{k} P(B_i \cap A) = \sum_{i=1}^{k} P(B_i)P(A|B_i).$$

In a certain assembly plant, three machines, B_1, B_2, and B_3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective?

General Rule law القانون العام للاحتمال

P (A) $= P\,[(\,B_1 \cap A) \cup (B_2\ \cap A\,) \cup \cdots \qquad (\,B_k \cap A\,)]$

$(\,B_1)\,P(A|B_1) + P(B_2\,)\,P(A|B_2\,) + \cdots \qquad P(\,B_k)\,P(A|B_k)$

$= \sum_{i=1}^{k} P(\,Bi)\,P(A|Bi)$ **,**

and that is the "Rule of elimination", 2)

Theorem of Total Probability

If the events $B_1, B_2, \ldots, B_k$ constitute a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1, 2, \ldots, k$, then for any event A of S,

$$P(A) = \sum_{i=1}^{k} P(B_i \cap A) = \sum_{i=1}^{k} P(B_i)P(A|B_i).$$

In a certain assembly plant, three machines, B_1, B_2, and B_3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective?

Consider the following events:

A: the product is defective,

B_1: the product is made by machine B_1,

B_2: the product is made by machine B_2,

B_3: the product is made by machine B_3.

Applying the rule of elimination, we can write

$$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3).$$

$$P(B_1)P(A|B_1) = (0.3)(0.02) = 0.006,$$
$$P(B_2)P(A|B_2) = (0.45)(0.03) = 0.0135,$$
$$P(B_3)P(A|B_3) = (0.25)(0.02) = 0.005,$$

$$P(A) = 0.006 + 0.0135 + 0.005 = 0.0245.$$

(Bayes' Rule) If the events $B_1, B_2, \ldots, B_k$ constitute a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1, 2, \ldots, k$, then for any event A in S such that $P(A) \neq 0$,

$$P(B_r|A) = \frac{P(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r)P(A|B_r)}{\sum_{i=1}^{k} P(B_i)P(A|B_i)} \quad \text{for } r = 1, 2, \ldots, k.$$

Suppose a random product is selected, what is the probability that it is made by machine B_1? We are not asking for the probability of getting defective product as solved by rule of domination, instead we ask for the source machine that produced that defective product. Bayes' rule solves such type of problems it is known that,

$$P(B_r|A) = \frac{P(B_r \cap A)}{P(A)}$$, substituting the P(A) from equation (2)

$$P(B_r|A) = \frac{P(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)}$$, we know that $P(B \cap A) = P(B)\, P(A|B)$

$$= \frac{P(B_r)P(A|B_r)}{\sum_{i=1}^{k} P(B_i)P(A|B_i)},$$ Bayes' Rule

Example: Referring to example 1, if a product was chosen randomly and found to defective; what is the probability that it was made by machine B3?

Answer: Using Bayes' rule

$$P(B_3|A) = \frac{P(B_3)P(A|B_3)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)}$$

$$= \frac{0.005}{0.006 + 0.0135 + 0.005} = \frac{0.005}{0.0245} = \frac{10}{49} \ \#$$

Activities: Using Bay's Rule solve the following:

The piece is manufactured in a factory and directed to test its validity by two workers. If the probability of dealing by the first worker is 0 .6 and second worker is 0.4. If it valid and chosen by first worker is 0.94. and by second worker is 0.98. What is the probability that the piece chosen by first worker.

Question1: Let $S = \{E1, E2, . . ., E6\}$; $A = \{E1, E3, E5\}$; $B = \{E1, E2, E3\}$; $C = \{E2, E4, E6\}$; $D = \{E6\}$.

Suppose that all elementary events are equally likely.

(i) What does it mean that all elementary events are equally likely?

(ii) Use the complementation rule to find $P(A^c)$.

(iii) Find $P(A|B)$ and $P(B|A)$

(iv) Find $P(D)$ and $P(D|C)$

(v) Find $P(A \cap B)$ and $P(A \cup B)$.

Question 2: Suppose S={1, 2, 3, 4, 5, 6}

A={1, 2, 3, 4, 5 }   B={3, 4, 5, 6 }

Find Probability of A ∩ B

Question 3: A coin is tossed twice. What is the probability that at least 1 head occurs?

Question4: Khalid passes Math with probability ¼ and passes English with probability 2/3, if he passes both courses with probability ¾ what is the probability that

a) pass at least one course

b) pass Math and fail English

c) fail both courses

Question5: complete the following formulas

P(B| A) = ………………, P(A) > 0, where A, B are events, P is the probability

P(S) = ……………… where S is the sample space and P is the probability

if P(A|B) = P(B|A) then both events A, B are said to be ………………

Question6: Use event relationships to fill in the blanks in table below. Show your answers under the table.

P(A)	P(B)	Conditions for Events for A and B	P(A ∩ B)	P(A U B)	P(A | B)
0.3	0.4	………..	0.12	………..	………..
0.3	0.4	………..	………..	0.7	………..
0.1	0.5	Mutually exclusive	………..	………..	………..
0.2	0.5	Independent	………..	………..	………..

Question7: Complete the following list of equations

In case of two dependent events, $P(M \cup E) = P(M) + P(E) -$ ....

The conditional probability of two dependent events

P(B|A) = ...

P(A) + P(A') = ...

In case of 3 independent events A, B, and C the probability $P(M \cup E \cup F)=$

Multiple Choice Questions:

1. Two events, A and B, are said to be independent if:

a. P(A U B) = P(A).P(B)

b. P(A U B) = P(A) + P(B)

c. P(A | B) = P(B)

d. P(B | A) = P(A)

2) suppose that the probability of event A is 0.2 and the probability of event B is 0.4. Also, suppose that the two events are independent. Then P(A|B) is:

A. P(A)=0.2

B. P(A)/P(B)=0.2/0.4=1⁄2

C. P(A)×P(B)=(0.2)(0.4)=0.08

D. None of the above.

3). If $P(A) = 0.8$, $P(B) = 0.3$ and $P(A|B) = 0.6$, what is $P(A \cap B)$?

a. 0.18

b. 0.24

c. 0.03

d. 0.30

4) $P(A \cap B) = \emptyset$ represents:

- Independent events.
- Mutually exclusive events.
- Conditional events.
- Dependent events.

5) The following formula: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ represents:

- ❑ the conditional probability.
- ❑ the additive rule.
- ❑ independence.
- ❑ the multiplication rule.

6) Suppose that a sample space S consists of four simple events: A, B, C, and D. That is

S = {A, B, C, D}. If P(A) = .4, P(B) = .1, P(C) = .2, what is P(D)?

- 0.7
- 0.1
- 0.3
- 1

7) An event A will occur with probability 0.5. An event B will occur with probability 0.6. The probability that both A and B will occur is 0.1. The conditional probability of A given B is:

A) Cannot be determined from the information given.

B) is 0.167.

C) is 0.200.

D) is 0.833.

8) If three fair coins are tossed, what is the probability of getting at least two heads?

[1] 2/3 [2] 1/2 [3] 3/8 [4] 1/8