CHAPTER THREE

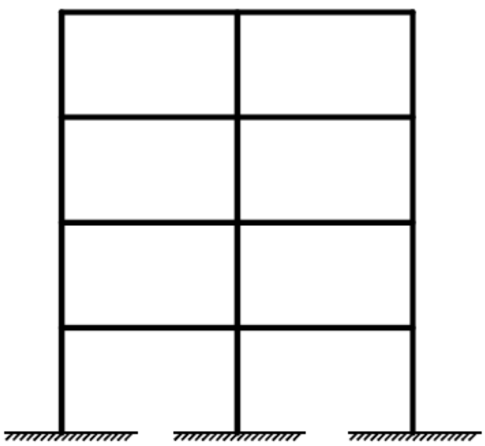
ANALYSIS OF PLANE FRAMES

A frame is a structure composed of straight-line members. The members may be connected by rigid joints, pin-connected joints and semi-rigid joints.

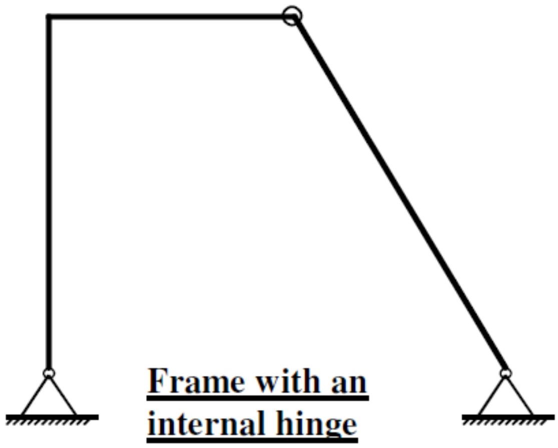
If all the joints are pins (which transmit no bending moments), the frame is commonly called a truss. Rigid joints are capable of transmitting both forces and bending moments.

A rigid frame is one in which some or all of its joints are rigid. Rigid frames are usually statically indeterminate. Our study will be confined to determinate plane frames. In a plane frame, all the members and loading must be in the same plane.

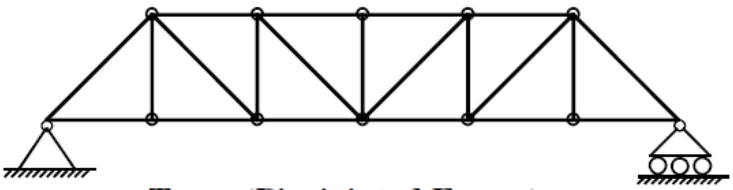
A frame is completely analyzed when its support reactions, and the variations in axial forces, shear forces and bending moments along all its members are found.



Rigid Frame



Frame with an internal hinge



Truss (Pin-jointed Frame)

Rigid Joints

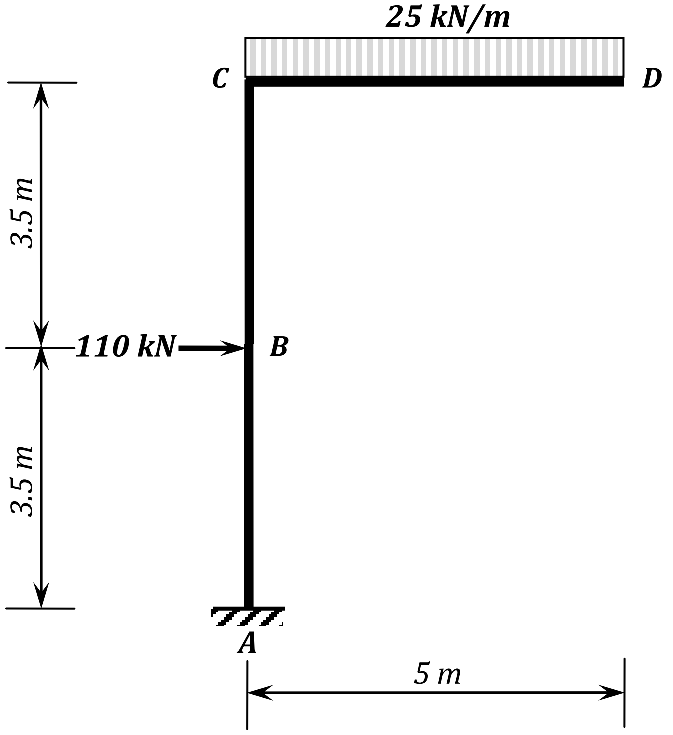
At rigid joints, the ends of connected members must not only move together vertically and horizontally but must all rotate by the same amount. A rigid joint preserves the angle between members connected to it.

In steel structures, rigid and pin-joints consist of welded connections and simple bolted connections respectively. There is another type of connection in between of these two extremes and it is called a semi-rigid connection (the connection is neither rigid nor pinned, it is in between these two extremes). In reinforced concrete structures, beams and columns are usually formed together resulting in substantially rigid joints.

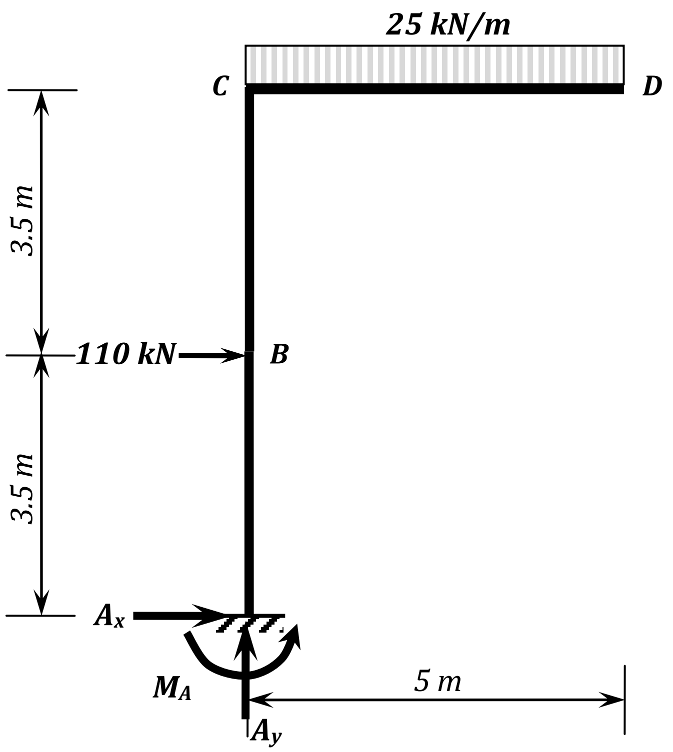
Example 1:

For the plane frame shown in Figure below, draw:

1. *The axial load diagram.*
2. *The shear force diagram.*
3. *the bending moment diagram.*



Solution:



Calculate support reactions:

By taking moment about support A:

$$M_A = -110(3.5) - \frac{25(5)^2}{2} = -697.5\ kN.m\ (hog)$$

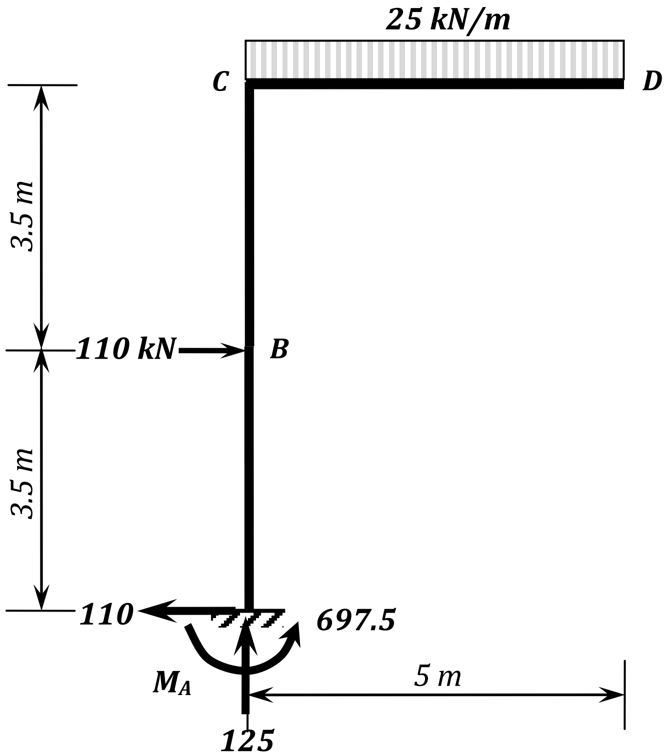
By taking the vertical balance $(\sum F_Y = 0)$*:*

$$A_y = 25(5) = 125\ kN$$

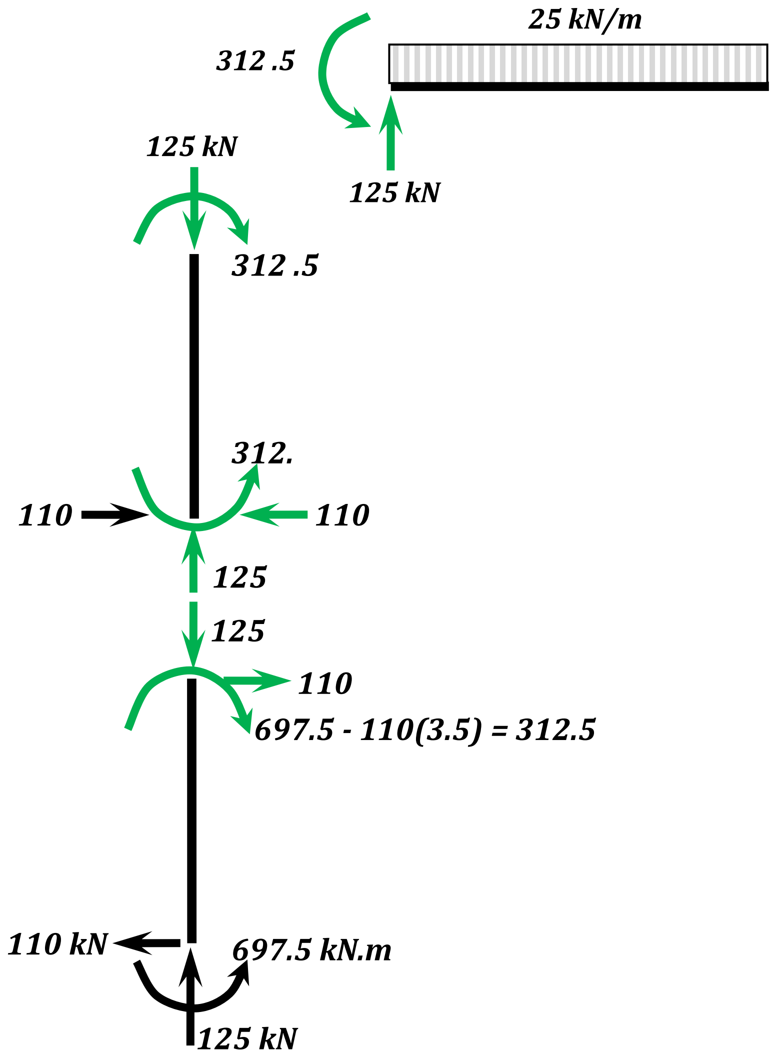
By taking the horizontal balance$(\sum F_X = 0)$*:*

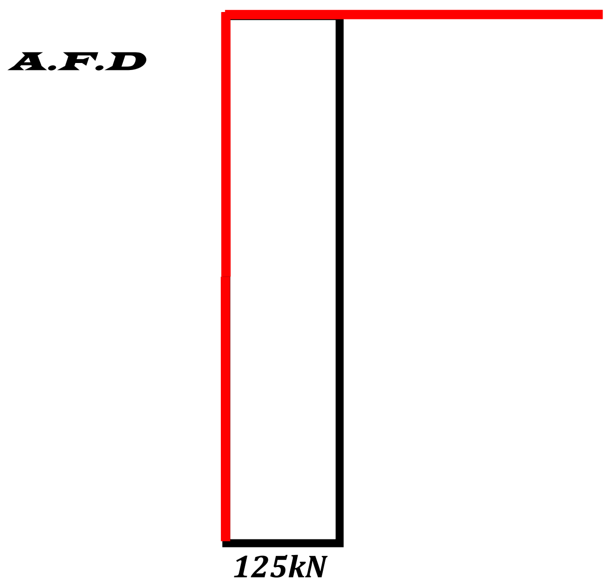
$$A_y + 110 = 0$$

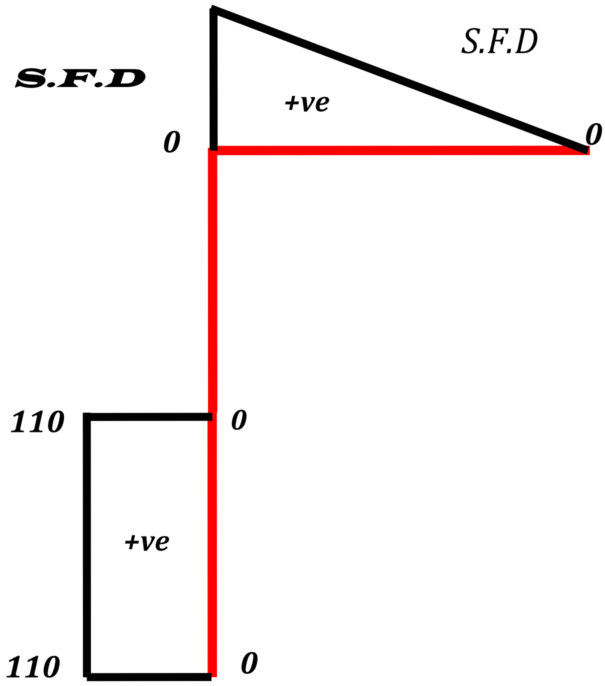
$$A_y = -110\ kN$$

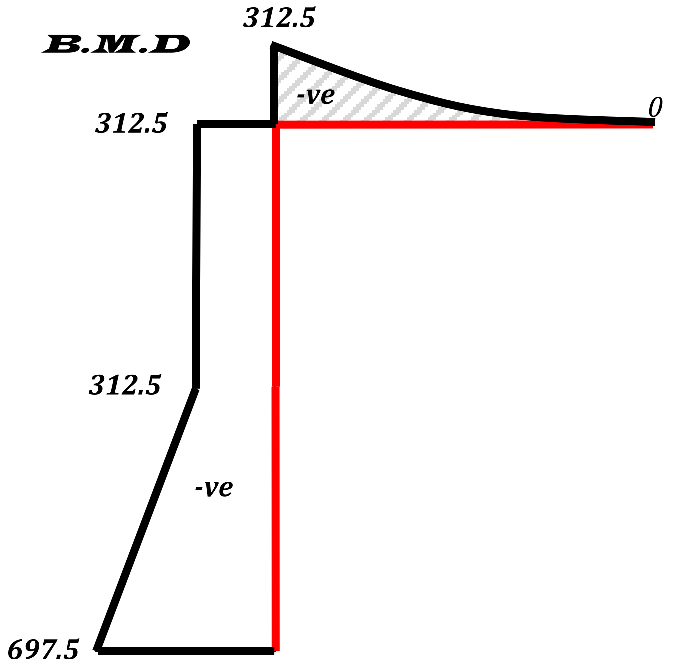


Take balance portion by portion:





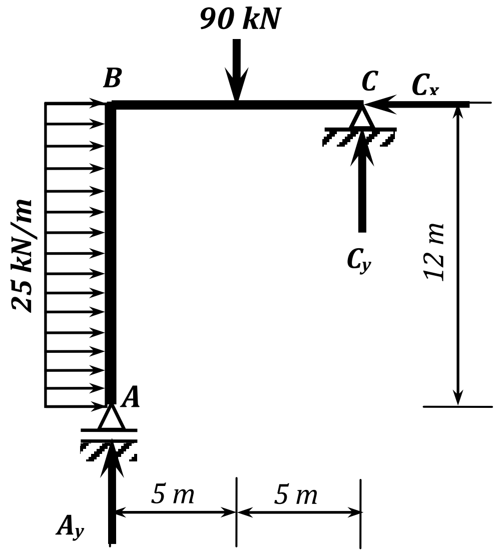

S.F.D
S.F.D
+ve
0
0
110
0
+ve
110
0


312.5
B.M.D
-ve
0
312.5
312.5
-ve
697.5

Example 2:

For the plane frame shown in Figure below, draw:

1. *The axial load diagram.*
2. *The shear force diagram.*
3. *the bending moment diagram.*



First find the reactions:

By taking moment about C $(\sum M_C = 0)$*:*

$$A_y(10) - 25(12)\left(\frac{12}{2}\right) - 90(5) = 0$$

$$A_y(10) = 1{,}800 + 450$$

$$A_y = \frac{2{,}250}{10} = 225\ kN$$

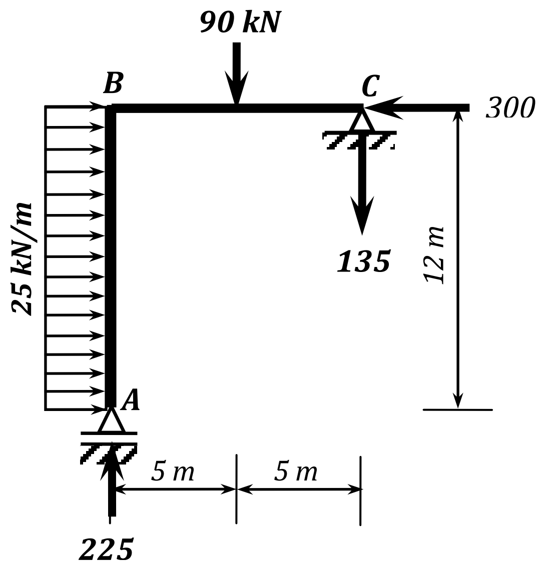
By taking the vertical balance: $(\sum F_y = 0)$*:*

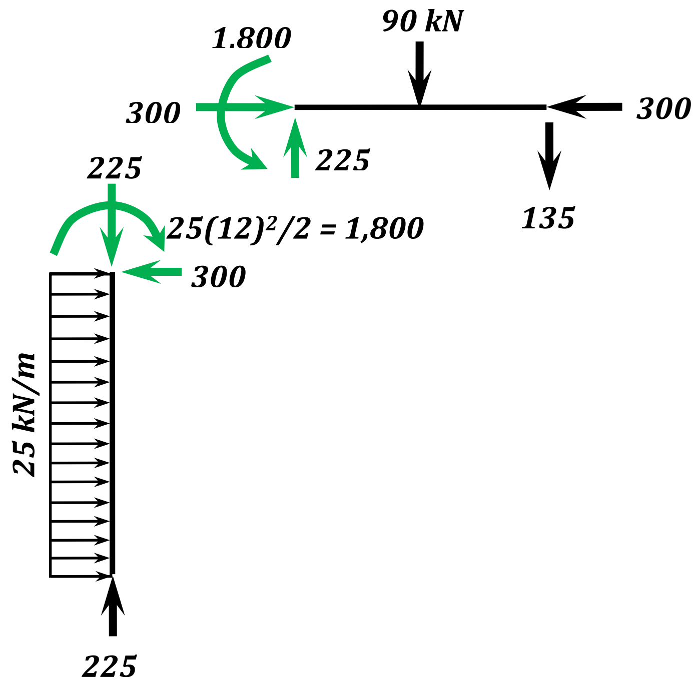
$$A_y + C_y = 90$$

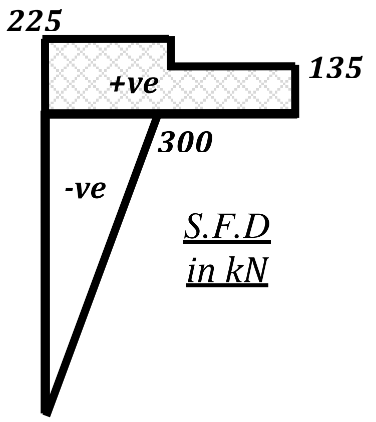
$$C_y = 90 - 225 = -135\ kN$$

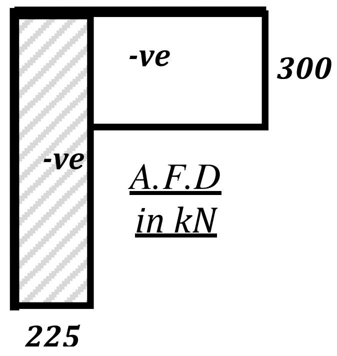
By taking the horizontal balance: $(\sum F_x = 0)$*:*

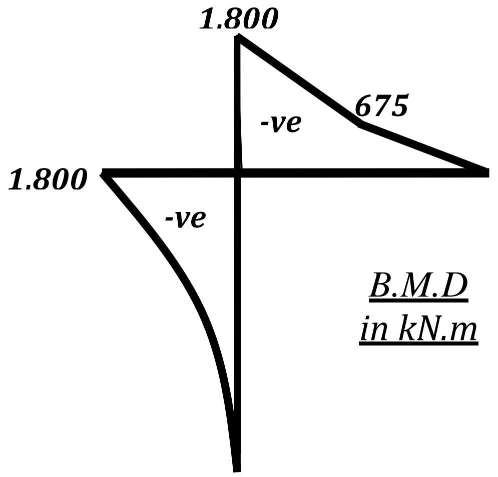
$$C_x = 25(12) = 300\ kN$$


90 kN
B
C
300
25 kN/m
135
12 m
A
5 m
5 m
225


1.800
90 kN
300
300
225
225
25(12)²/2 = 1,800
135
300
25 kN/m
225

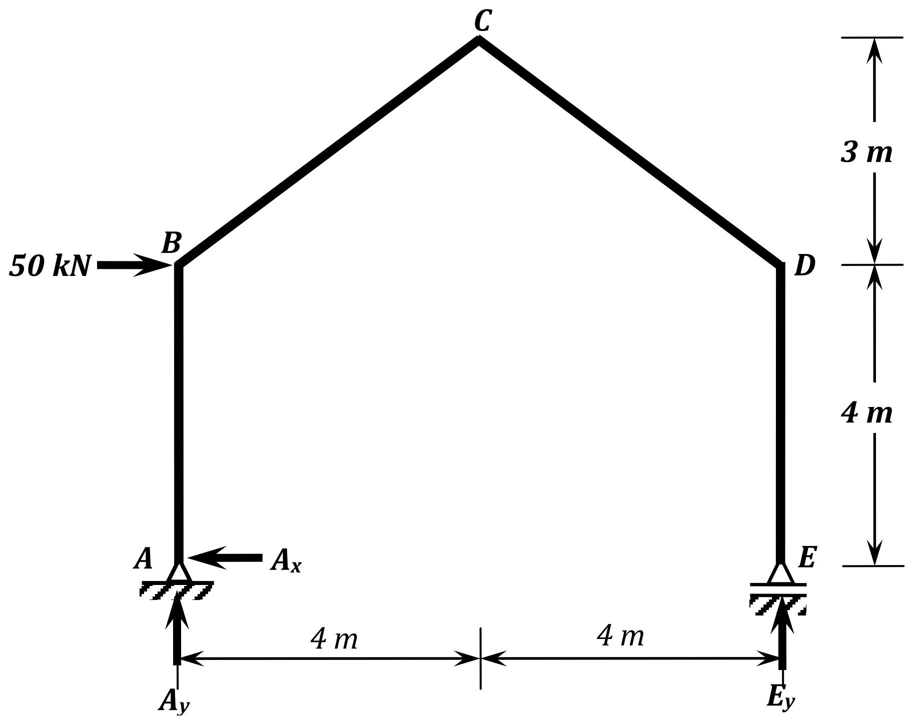

225
+ve
135
300
-ve
S.F.D
in kN


-ve
300
-ve
A.F.D
in kN
225


1.800
675
-ve
1.800
-ve
B.M.D
in kN.m

Example 3:

Draw the axial force, shear force & bending moment for the plane frame shown in Figure below.



Solution:

The support reactions:

By taking the horizontal balance ($\sum F_X = 0$):

$$A_x = 50\ kN$$

By taking moment about support E:

$$A_y(8) + 50(4) = 0$$

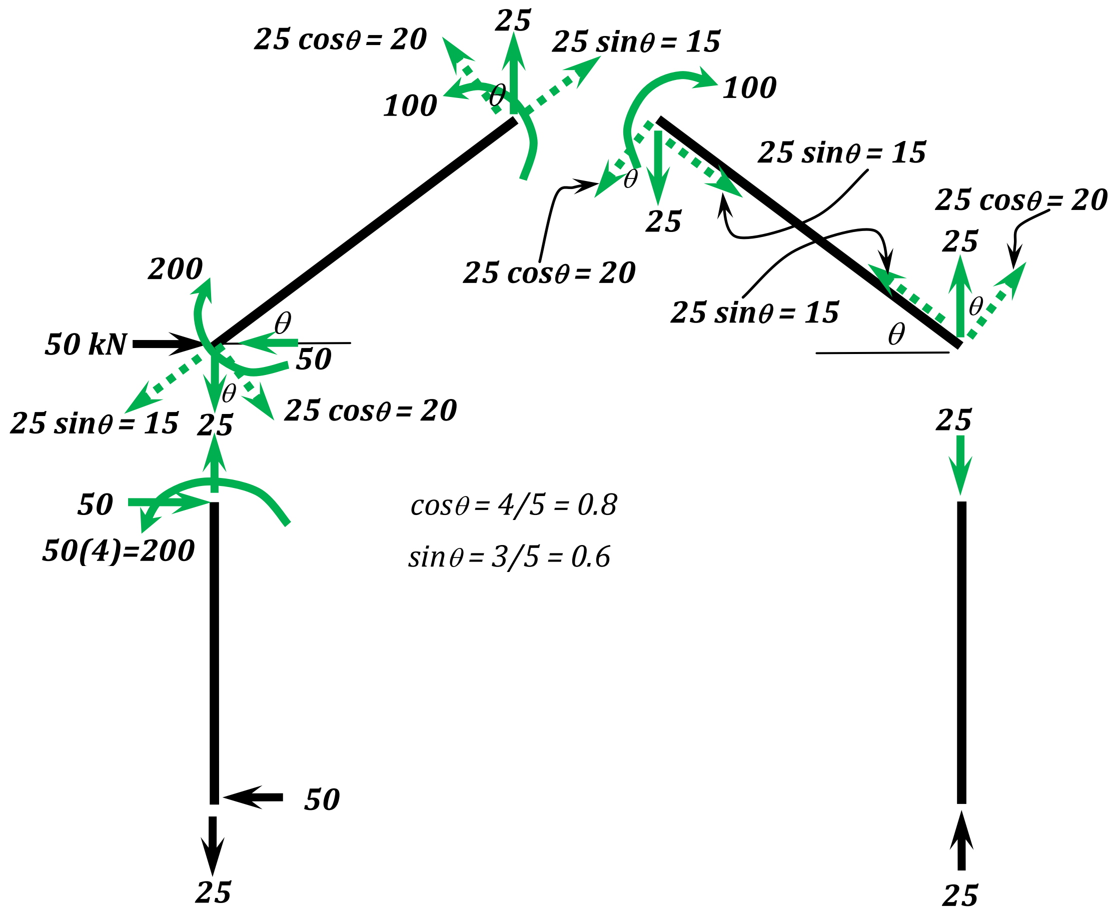
$$A_y = -\frac{200}{8} = -25\ kN$$

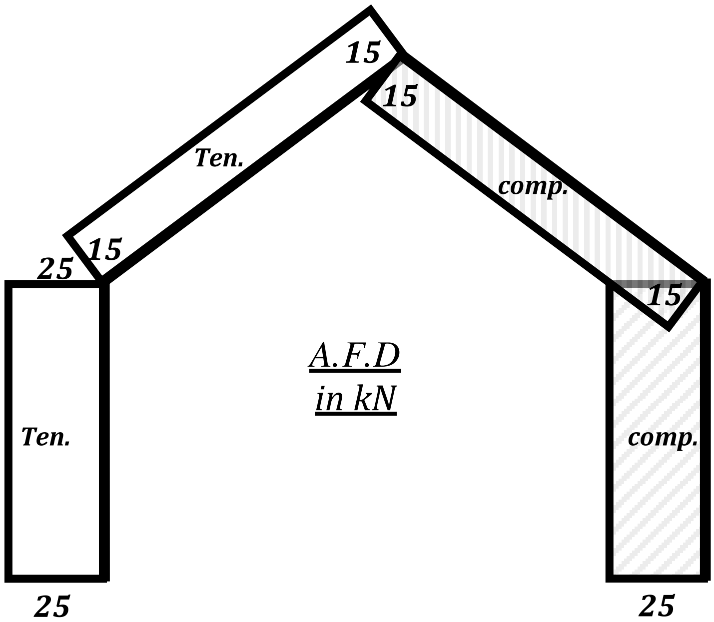
By taking the vertical balance ($\sum F_Y = 0$):

$$A_y + E_y = 0$$

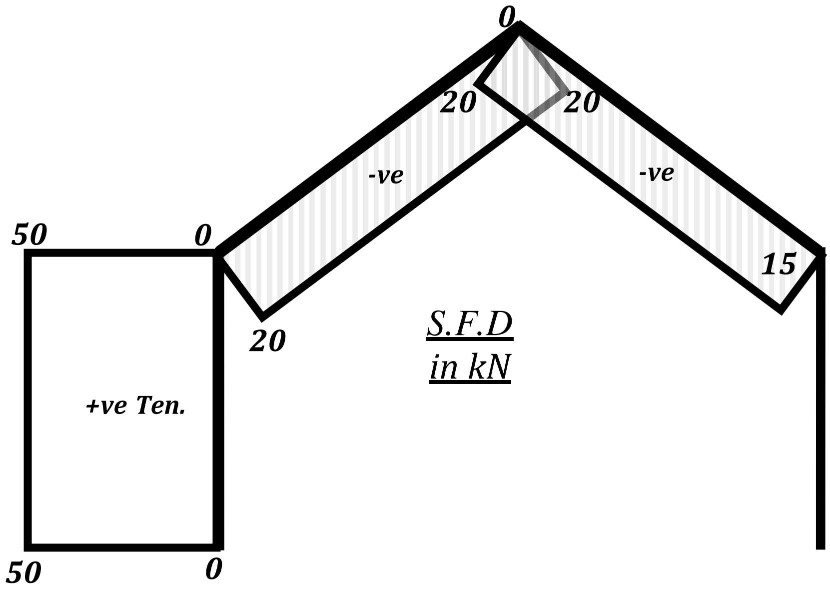
$$E_y = 25\ kN$$

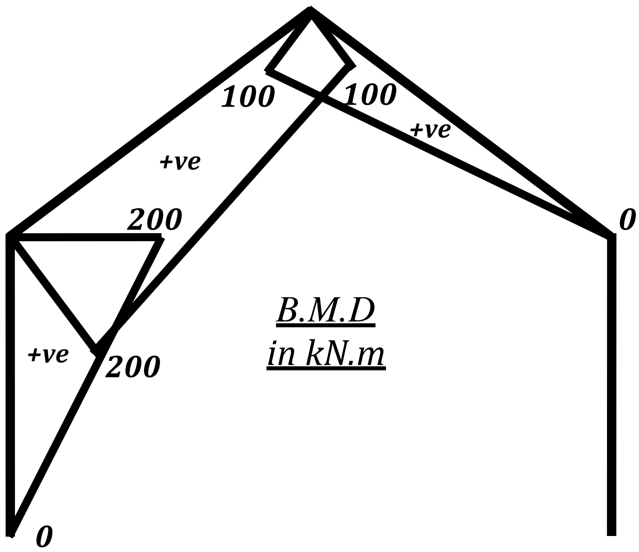
Calculate end forces part by part:





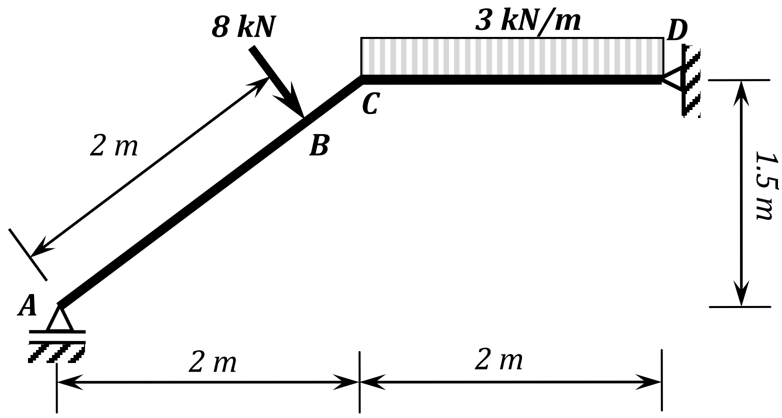
A.F.D
in kN


0
20
20
-ve
-ve
50
0
15
20
S.F.D
in kN
+ve Ten.
50
0


100
100
+ve
+ve
200
0
B.M.D
in kN.m
+ve
200
0

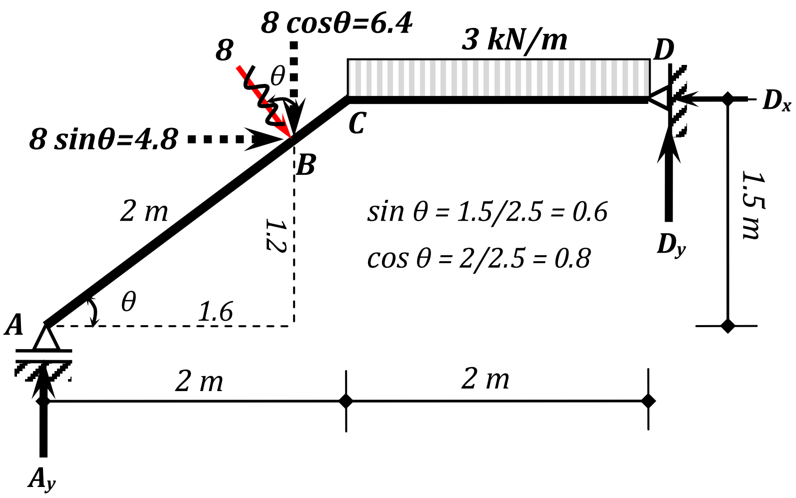
Example 4:

Draw the axial force, shear force & bending moment for the frame shown in Figure below.



Solution:

The support reactions:



Calculate support reactions:

By taking moment about support D:

$$A_y(4) - 4.8(1.5 - 1.2) - 6.4(4 - 1.6) - 3\frac{(2)^2}{2} = 0$$

$$A_y = \frac{22.8}{4} = 5.7\ kN$$

By taking the vertical balance ($\sum F_Y = 0$)*:*

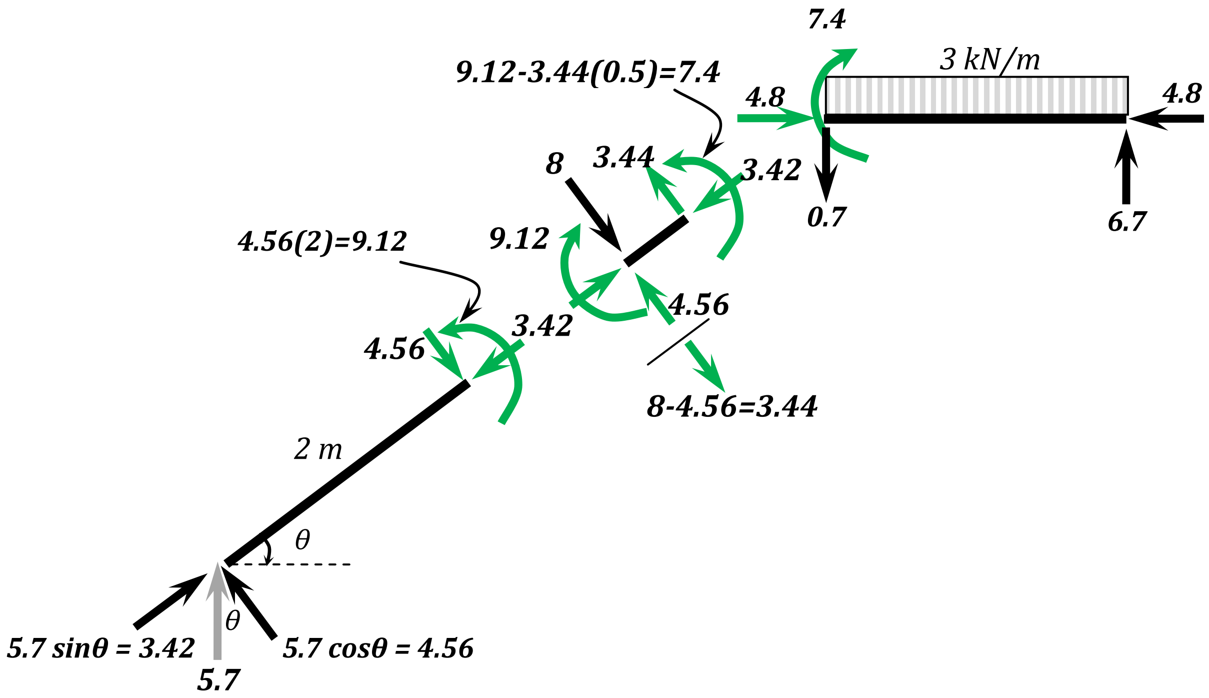
$$A_y + D_y = 6.4 + 3(2)$$

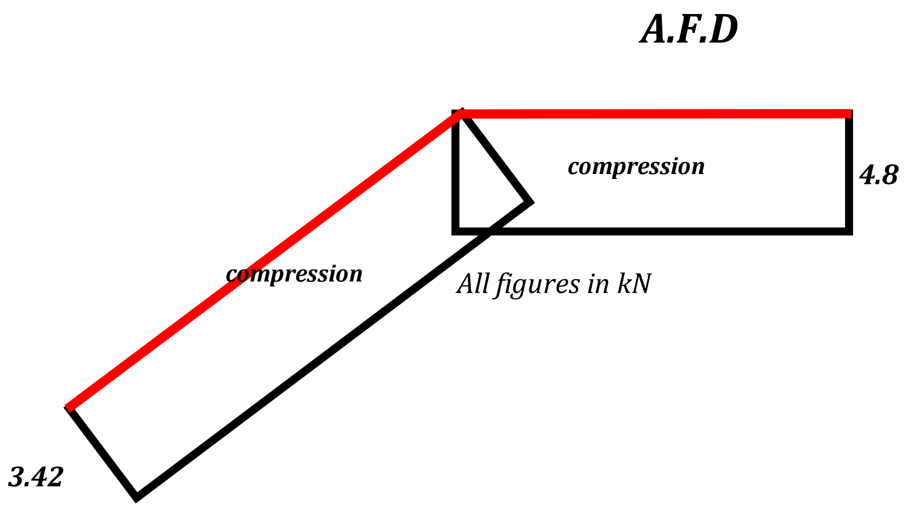
$$D_y = 12.4 - 5.7 = 6.7\ kN$$

By taking the horizontal balance ($\sum F_X = 0$):

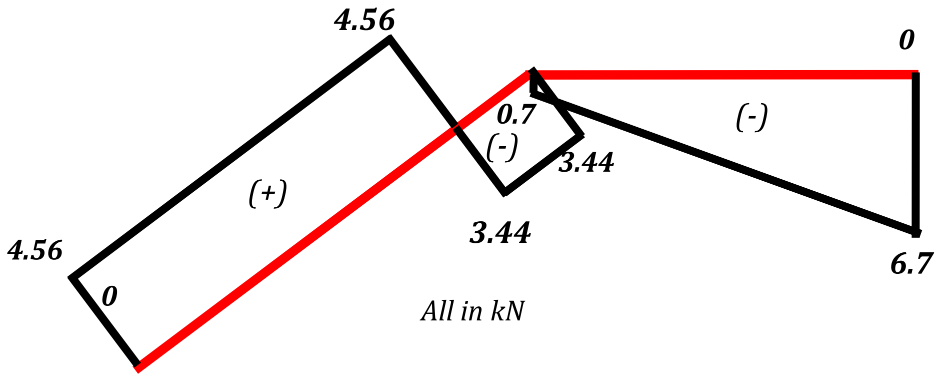
$$D_x = 4.8\ kN$$

Calculate end forces part by part:



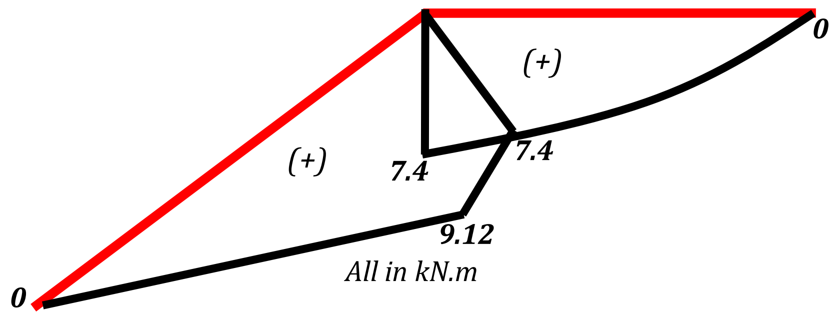


S.F.D



All in kN

B.M.D



All in kN.m