College of Engineering
Industrial Engineering

Balanced Incomplete Block Design

BIBD

BIBD Model Adequacy

Example

Randomized Incomplete Block Design

These are randomized block designs in which every treatment is not present in every block.

The experimenter is not be able to run all the treatment combinations in each block because:

- Shortages of Experimental Apparatus or Facilities
- The Physical Size of the Block

Balanced Incomplete Block Design (BIBD)

An incomplete block design in which any two treatments appear together an equal number of times. This happens when all treatment comparisons are equally important and the treatment combinations used in each block should be selected in a balanced manner.

Balanced Incomplete Block Design

Suppose that there are **a** treatments and that each block can hold exactly **k** where ($k < a$) treatments. A balanced incomplete block design may be constructed by taking binomial coefficient of a and k blocks and assigning a different combination of treatments to each block.

Assume that there are **a** treatments and **b** blocks. In addition, assume that each block contains **k** treatments, that each treatment occurs **r** times in the design (or is replicated r times), and that there are $\mathbf{N = a\,r = bk}$ total observations. Furthermore, the number of times each pair of treatments appears in the same block is

$$\lambda = \frac{r(k-1)}{a-1}$$

If $\mathbf{a = b}$, the design is said to be **symmetric.**

Balanced Incomplete Block Design

The statistical model for the BIBD

$$y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}$$

where y_{ij} is the ith observation in the jth block, μ is the overall mean, τ_i is the effect of the ith treatment, β_j is the effect of the jth block, and ϵ_{ij} is the NID $(0, \sigma^2)$ random error

The BIBD ANOVA Table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments (adjusted)	$\dfrac{k \sum Q_i^2}{\lambda a}$	$a - 1$	$\dfrac{SS_{\text{Treatments(adjusted)}}}{a-1}$	$F_0 = \dfrac{MS_{\text{Treatments(adjusted)}}}{MS_E}$
Blocks	$\dfrac{1}{k} \sum y_{.j}^2 - \dfrac{y_{..}^2}{N}$	$b - 1$	$\dfrac{SS_{\text{Blocks}}}{b-1}$	
Error	SS_E (by subtraction)	$N - a - b + 1$	$\dfrac{SS_E}{N-a-b+1}$	
Total	$\sum \sum y_{ij}^2 - \dfrac{y_{..}^2}{N}$	$N - 1$		

Balanced Incomplete Block Design

$$SS_T = \sum_i \sum_j y_{ij}^2 - \frac{y_{..}^2}{N}$$

$$SS_T = SS_{\text{Treatments(adjusted)}} + SS_{\text{Blocks}} + SS_E$$

$$SS_{\text{Blocks}} = \frac{1}{k} \sum_{j=1}^{b} y_{.j}^2 - \frac{y_{..}^2}{N}$$

$$SS_{\text{Treatments(adjusted)}} = \frac{k \sum_{i=1}^{a} Q_i^2}{\lambda a}$$

$$Q_i = y_{i.} - \frac{1}{k} \sum_{j=1}^{b} n_{ij} y_{.j} \qquad i = 1, 2, \ldots, a$$

Rejection Criteria

$$F_0 > F_{\alpha,(a-1),(N-a-b+1)}$$

BIBD Model Adequacy

Example

A chemical engineer thinks that the time of reaction for a chemical process is a function of the type of catalyst employed. Four catalysts are currently being investigated. The experimental procedure consists of selecting a batch of raw material, loading the pilot plant, applying each catalyst in a separate run of the pilot plant, and observing the reaction time. Because variations in the batches of raw material may affect the performance of the catalysts, the engineer decides to use batches of raw material as blocks. However, each batch is only large enough to permit three catalysts to be run. Therefore, a randomized incomplete block design must be used. The balanced incomplete block design for this experiment, along with the observations recorded in the following table.

Teratment (Catalyst)	Block (Batch of Raw Material) 1	2	3	4	$y_{i.}$
1	73	74	—	71	218
2	—	75	67	72	214
3	73	75	68	—	216
4	75	—	72	75	222
$y_{.j}$	221	224	207	218	$870 = y_{..}$

BIBD Model Adequacy

Example

a = 4, b = 4, k = 3 , λ = 2 : N = 12

$$SS_T = \sum_i \sum_j y_{ij}^2 - \frac{y_{..}^2}{12}$$

$$= 63{,}156 - \frac{(870)^2}{12} = 81.00$$

$$SS_{\text{Blocks}} = \frac{1}{3} \sum_{j=1}^{4} y_{.j}^2 - \frac{y_{..}^2}{12}$$

$$= \frac{1}{3}\left[(221)^2 + (207)^2 + (224)^2 + (218)^2\right] - \frac{(870)^2}{12} = 55.00$$

$$Q_1 = (218) - \tfrac{1}{3}(221 + 224 + 218) = -9/3$$

$$Q_2 = (214) - \tfrac{1}{3}(207 + 224 + 218) = -7/3$$

$$Q_3 = (216) - \tfrac{1}{3}(221 + 207 + 224) = -4/3$$

$$Q_4 = (222) - \tfrac{1}{3}(221 + 207 + 218) = 20/3$$

BIBD Model Adequacy

$$SS_{\text{Treatments(adjusted)}} = \frac{k \sum_{i=1}^{4} Q_i^2}{\lambda a}$$

$$= \frac{3[(-9/3)^2 + (-7/3)^2 + (-4/3)^2 + (20/3)^2]}{(2)(4)} = 22.75$$

$$SS_E = SS_T - SS_{\text{Treatments(adjusted)}} - SS_{\text{Blocks}}$$

$$= 81.00 - 22.75 - 55.00 = 3.25$$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Treatments (adjusted for blocks)	22.75	3	7.58	11.66	0.0107
Blocks	55.00	3	—		
Error	3.25	5	0.65		
Total	81.00	11			

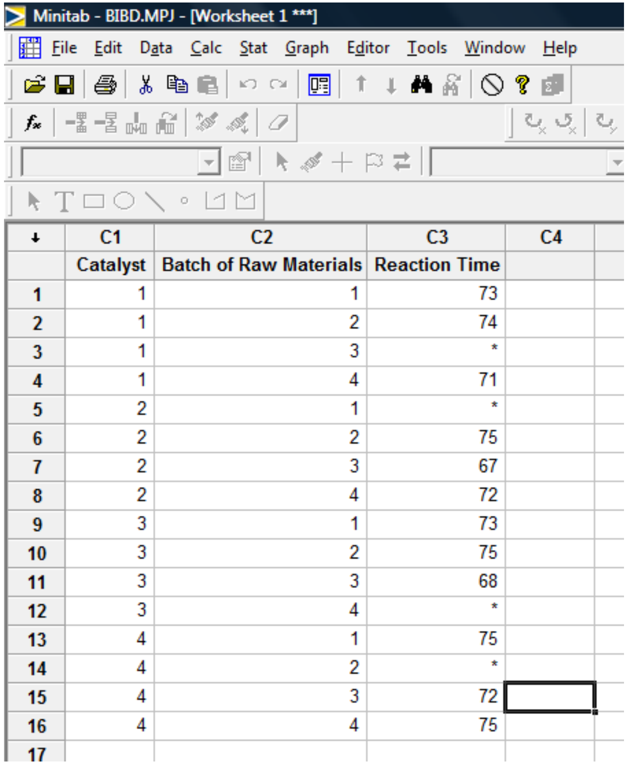
$$F_{\alpha,(a-1),(N-a-b+1)} = F_{0.05,3,5} = 5.41$$

BIBD Model Adequacy

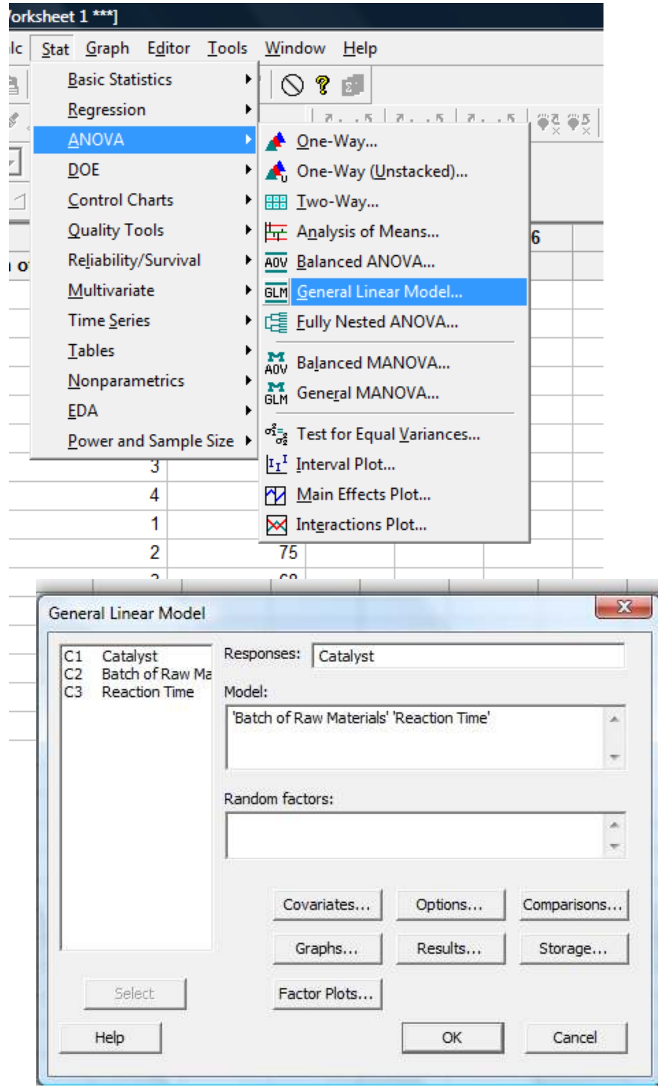
BIBD

BIBD Model Adequacy

Example



	C1 Catalyst	C2 Batch of Raw Materials	C3 Reaction Time	C4
1	1	1	73	
2	1	2	74	
3	1	3	*	
4	1	4	71	
5	2	1	*	
6	2	2	75	
7	2	3	67	
8	2	4	72	
9	3	1	73	
10	3	2	75	
11	3	3	68	
12	3	4	*	
13	4	1	75	
14	4	2	*	
15	4	3	72	
16	4	4	75	
17				



BIBD Model Adequacy

General Linear Model: Reaction Time versus Catalyst, Batch of Raw Mat

Factor	Type	Levels	Values
Catalyst	fixed	4	1, 2, 3, 4
Batch of Raw Materials	fixed	4	1, 2, 3, 4

Analysis of Variance for Reaction Time, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Catalyst	3	11.667	22.750	7.583	11.67	0.011
Batch of Raw Materials	3	66.083	66.083	22.028	33.89	0.001
Error	5	3.250	3.250	0.650		
Total	11	81.000				

BIBD Model Adequacy

An engineer is studying the mileage performance characteristics of five types of gasoline additives. In the road test he wishes to use cars as blocks; however, because of a time constraint, he must use an incomplete block design. He runs the balanced design with the five blocks that follow. Analyze the data from this experiment (use α = 0.05) and draw conclusions

	Car				
Additive	1	2	3	4	5
1		17	14	13	12
2	14	14		13	10
3	12		13	12	9
4	13	11	11	12	
5	11	12	10		8

BIBD Model Adequacy

$a = 5, b = 5, k = 4, \lambda = 3 : N = 20$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments (adjusted)	$\frac{k \sum Q_i^2}{\lambda a}$	$a - 1$	$\frac{SS_{\text{Treatments(adjusted)}}}{a - 1}$	$F_0 = \frac{MS_{\text{Treatments(adjusted)}}}{MS_E}$
Blocks	$\frac{1}{k} \sum y_{.j}^2 - \frac{y_{..}^2}{N}$	$b - 1$	$\frac{SS_{\text{Blocks}}}{b - 1}$	
Error	SS_E (by subtraction)	$N - a - b + 1$	$\frac{SS_E}{N - a - b + 1}$	
Total	$\sum \sum y_{ij}^2 - \frac{y_{..}^2}{N}$	$N - 1$		

BIBD Model Adequacy

Example

General Linear Model: Milage Performance versus Additive, Car

Factor	Type	Levels	Values
Additive	fixed	5	1, 2, 3, 4, 5
Car	fixed	5	1, 2, 3, 4, 5

Analysis of Variance for Milage Performance, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Additive	4	31.7000	35.7333	8.9333	9.81	0.001
Car	4	35.2333	35.2333	8.8083	9.67	0.001
Error	11	10.0167	10.0167	0.9106		
Total	19	76.9500				

S = 0.954257   R-Sq = 86.98%   R-Sq(adj) = 77.52%

Unusual Observations for Milage Performance

Obs	Milage Performance	Fit	SE Fit	Residual	St Resid
17	11.0000	12.5167	0.6401	-1.5167	-2.14 R

R denotes an observation with a large standardized residual.

$$F_{\alpha,(a-1),(N-a-b+1)} = F_{0.05,4,11} = 3.36$$