Week 14

Chapter 6

Question 6.2: An engineer is interested in the effects of cutting speed (A), tool geometry (B), and cutting angle (c) on the life (in hours) of a machine tool. Two levels of each factor are chosen, and three replicates of 2^3 factorial design are run. The results are as follows:

A	B	C	Treatment Combination	Replicate I	Replicate II	Replicate III
-	-	-	(1)	22	31	25
+	-	-	a	32	43	29
-	+	-	b	35	34	50
+	+	-	ab	55	47	46
-	-	+	c	44	45	38
+	-	+	ac	40	37	36
-	+	+	bc	60	50	54
+	+	+	abc	39	41	47

a) Estimate the factor effects. Which effects appear to be large?
b) Use the analysis of variance to confirm your conclusion for part (a).
c) Write down a regression model for predicting tool life (in hours) based on the results of this experiment.
d) Analyze the residuals. Are there any obvious problem?
e) On the basis of an analysis of main effect and interaction plots, what coded factor levels of A, B, and C would you recommend using?

Answer: By Minitab:

Factors are:

Cutting speed (A),
Tool geometry (B),
Cutting speed (C),
and Tool Life (in hours) is the response.

2^3 factorial design are run

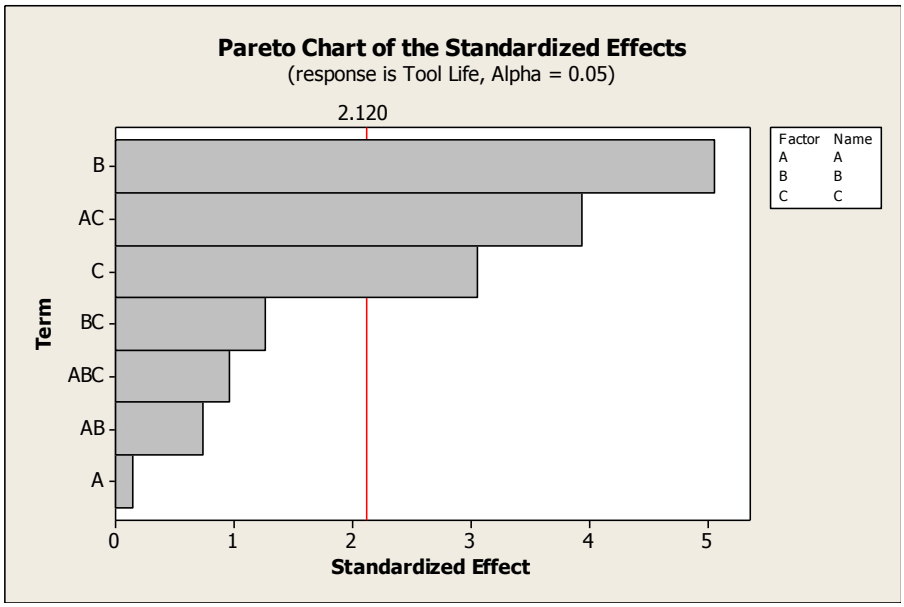
a) Estimate the factor effects. Which effects appear to be large?

Factorial Fit: Life Hours versus Cutting Angle, Tool Geometry, ..

Estimated Effects and Coefficients for Life Hours (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		40.833	1.121	36.42	0.000
Cutting Angle	6.833	3.417	1.121	3.05	0.008
Tool Geometry	11.333	5.667	1.121	5.05	0.000
Cutting Speed	0.333	0.167	1.121	0.15	0.884
Cutting Angle*Tool Geometry	-2.833	-1.417	1.121	-1.26	0.224
Cutting Angle*Cutting Speed	-8.833	-4.417	1.121	-3.94	0.001
Tool Geometry*Cutting Speed	-1.667	-0.833	1.121	-0.74	0.468
Cutting Angle*Tool Geometry* Cutting Speed	-2.167	-1.083	1.121	-0.97	0.348

S = 5.49242 PRESS = 1086
R-Sq = 76.96% R-Sq(pred) = 48.17% R-Sq(adj) = 66.89%



From the Minitab we find that, the factors B, C and AC appear to have large effects on Tool Life.

b) Use the analysis of variance to confirm your conclusion for part (a).

Analysis of Variance for Life Hours (coded units)

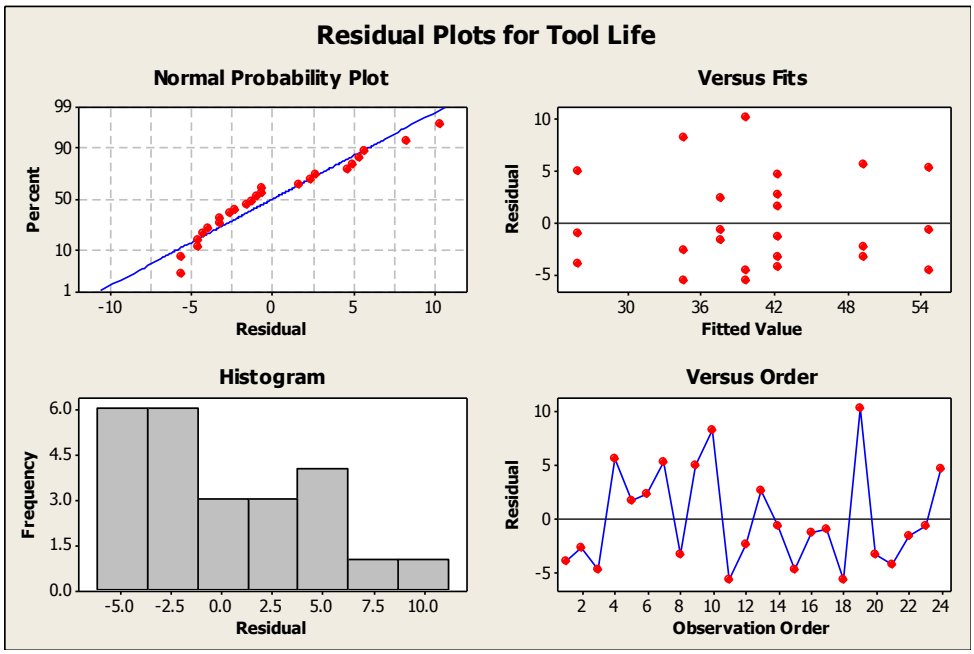
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	1051.50	1051.50	350.500	11.62	0.000
Cutting Speed	1	0.67	0.67	0.667	0.02	0.884
Tool Geometry	1	770.67	770.67	770.667	25.55	0.000
Cutting Angle	1	280.17	280.17	280.167	9.29	0.008
2-Way Interactions	3	533.00	533.00	177.667	5.89	0.007
Cutting Speed*Tool Geometry	1	16.67	16.67	16.667	0.55	0.468
Cutting Speed*Cutting Angle	1	468.17	468.17	468.167	15.52	0.001
Tool Geometry*Cutting Angle	1	48.17	48.17	48.167	1.60	0.224
3-Way Interactions	1	28.17	28.17	28.167	0.93	0.348
Cutting Speed*Tool Geometry*Cutting Angle	1	28.17	28.17	28.167	0.93	0.348
Residual Error	16	482.67	482.67	30.167		
Pure Error	16	482.67	482.67	30.167		
Total	23	2095.33				

Unusual Observations for Life Hours

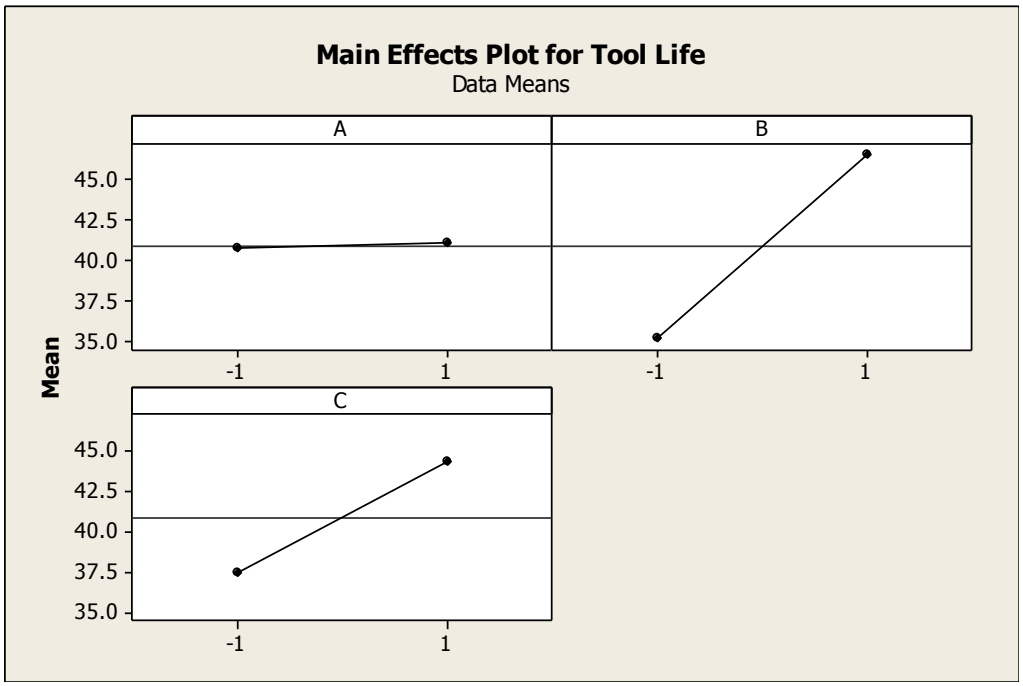
Obs	StdOrder	Life Hours	Fit	SE Fit	Residual	St Resid
9	9	50.0000	39.6667	3.1710	10.3333	2.30R

d) Analyze the residuals. Are there any obvious problem?

From Minitab the residuals are normally distributed with a mean of zero and constant variance



e) On the basis of an analysis of main effect and interaction plots, what coded factor levels of A, B, and C would you recommend using?



From the main effects diagram we can recommend the use of the high level of factor A (Cutting speed), the high level of factor B (Tool geometry) and the high level C (Cutting angle).

Question 6.2: An experiment was performed to improve the yield of a chemical process. Four factors were selected, and two replicates of a completely randomized experiment were run. The results are shown in the following table:

Treatment Combinatio	Replicat I	Replicat II	Treatment Combinatio	Replicat I	Replicat II
(1)	90	93	*d*	98	95
a	74	78	*ad*	72	76
b	81	85	*bd*	87	83
ab	83	80	*abd*	85	86
c	77	78	*cd*	99	90
ac	81	80	*acd*	79	75
bc	88	82	*bcd*	87	84
abc	73	70	*abcd*	80	80

(a) Estimate the factor effects.
(b) Prepare an analysis of variance table, and determine which factors are important in explaining yield.
(c) Write down a regression model for predicting yield, assuming that all four factors were varied over the range from -1 to +1 (in coded units).
(d) Plot the residuals versus the predicted yield and on a normal probability scale. Does the residual analysis appear satisfactory?
(e) Two three-factor interactions, *ABC* and *ABD*, apparently have large effects. Draw a cube plot in the factors *A*, *B*, and *C* with the average yields shown at each corner. Repeat using the factors *A*, *B*, and *D*. Do these two plots aid in data interpretation? Where would you recommend that the process be run with respect to the four variables?

Answer: By Minitab:

(a) Estimate the factor effects.

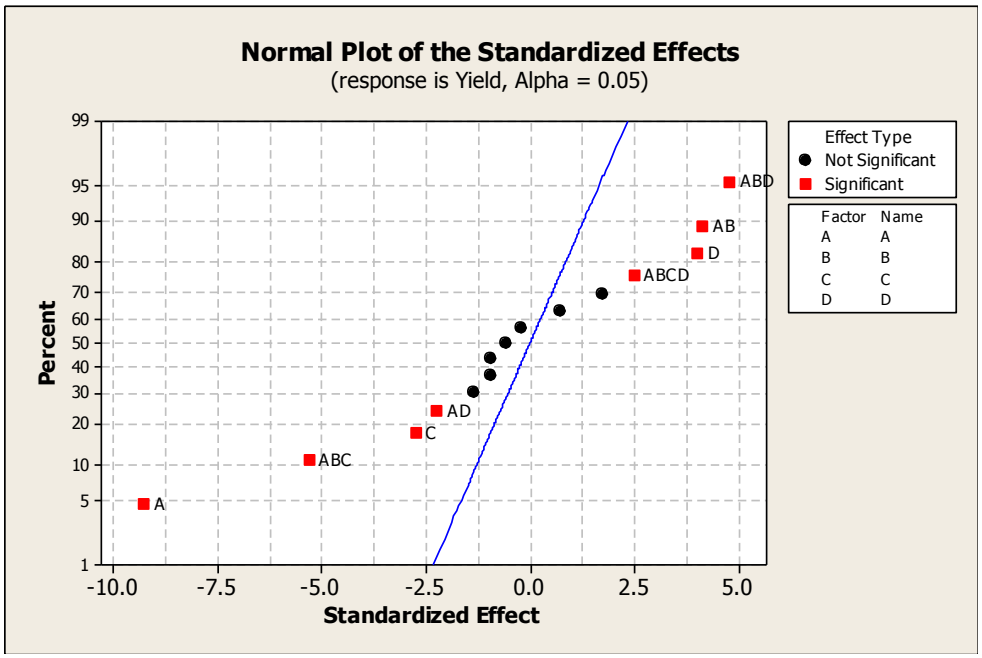
Minitab Output

Factorial Fit: Yield versus A, B, C, D

Estimated Effects and Coefficients for Yield (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		82.781	0.4891	169.24	0.000
A	-9.062	-4.531	0.4891	-9.26	0.000
B	-1.312	-0.656	0.4891	-1.34	0.198
C	-2.688	-1.344	0.4891	-2.75	0.014
D	3.937	1.969	0.4891	4.02	0.001
A*B	4.063	2.031	0.4891	4.15	0.001
A*C	0.687	0.344	0.4891	0.70	0.492
A*D	-2.188	-1.094	0.4891	-2.24	0.040
B*C	-0.563	-0.281	0.4891	-0.57	0.573
B*D	-0.187	-0.094	0.4891	-0.19	0.850
C*D	1.687	0.844	0.4891	1.72	0.104
A*B*C	-5.187	-2.594	0.4891	-5.30	0.000
A*B*D	4.687	2.344	0.4891	4.79	0.000
A*C*D	-0.938	-0.469	0.4891	-0.96	0.352
B*C*D	-0.938	-0.469	0.4891	-0.96	0.352
A*B*C*D	2.437	1.219	0.4891	2.49	0.024

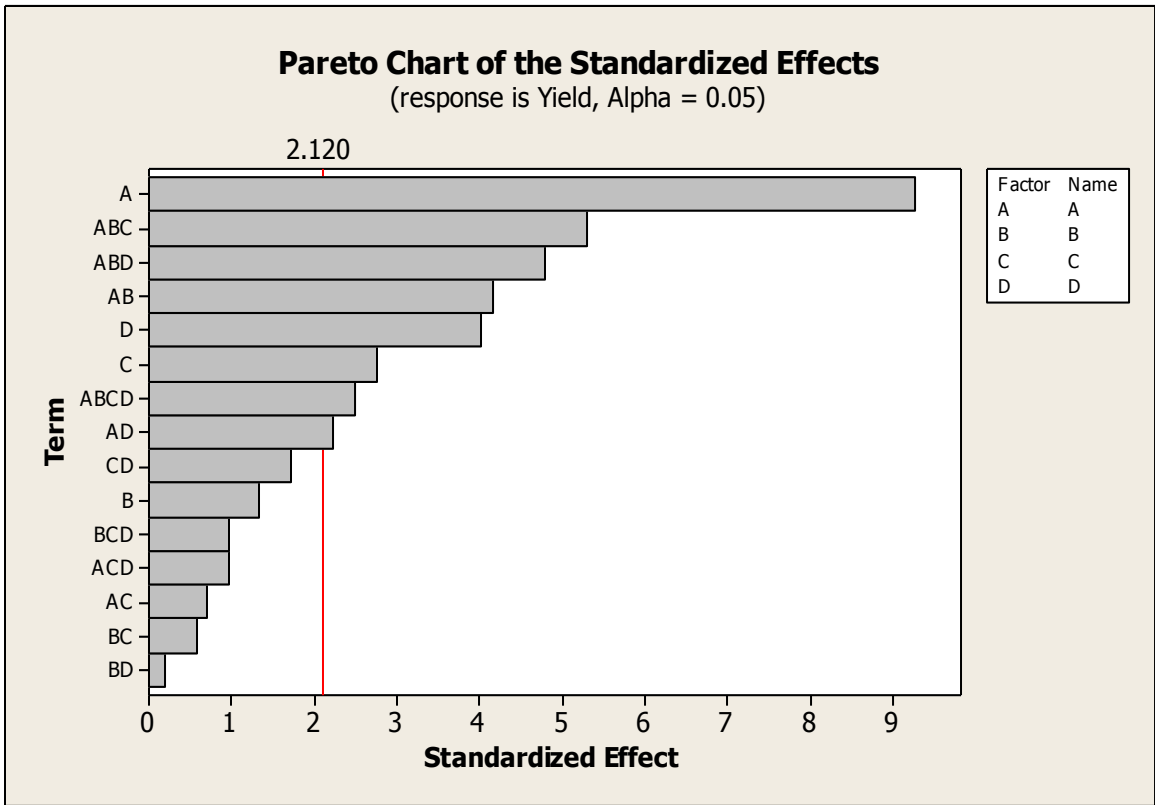
S = 2.76699 PRESS = 490
R-Sq = 92.47% R-Sq(pred) = 69.89% R-Sq(adj) = 85.42%



b) Prepare an analysis of variance table, and determine which factors are important in explaining yield.

Analysis of Variance for Yield (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	852.62	852.625	213.156	27.84	0.000
A	1	657.03	657.031	657.031	85.82	0.000
B	1	13.78	13.781	13.781	1.80	0.198
C	1	57.78	57.781	57.781	7.55	0.014
D	1	124.03	124.031	124.031	16.20	0.001
2-Way Interactions	6	199.69	199.688	33.281	4.35	0.009
A*B	1	132.03	132.031	132.031	17.24	0.001
A*C	1	3.78	3.781	3.781	0.49	0.492
A*D	1	38.28	38.281	38.281	5.00	0.040
B*C	1	2.53	2.531	2.531	0.33	0.573
B*D	1	0.28	0.281	0.281	0.04	0.850
C*D	1	22.78	22.781	22.781	2.98	0.104
3-Way Interactions	4	405.12	405.125	101.281	13.23	0.000
A*B*C	1	215.28	215.281	215.281	28.12	0.000
A*B*D	1	175.78	175.781	175.781	22.96	0.000
A*C*D	1	7.03	7.031	7.031	0.92	0.352
B*C*D	1	7.03	7.031	7.031	0.92	0.352
4-Way Interactions	1	47.53	47.531	47.531	6.21	0.024
A*B*C*D	1	47.53	47.531	47.531	6.21	0.024
Residual Error	16	122.50	122.500	7.656		
Pure Error	16	122.50	122.500	7.656		
Total	31	1627.47				



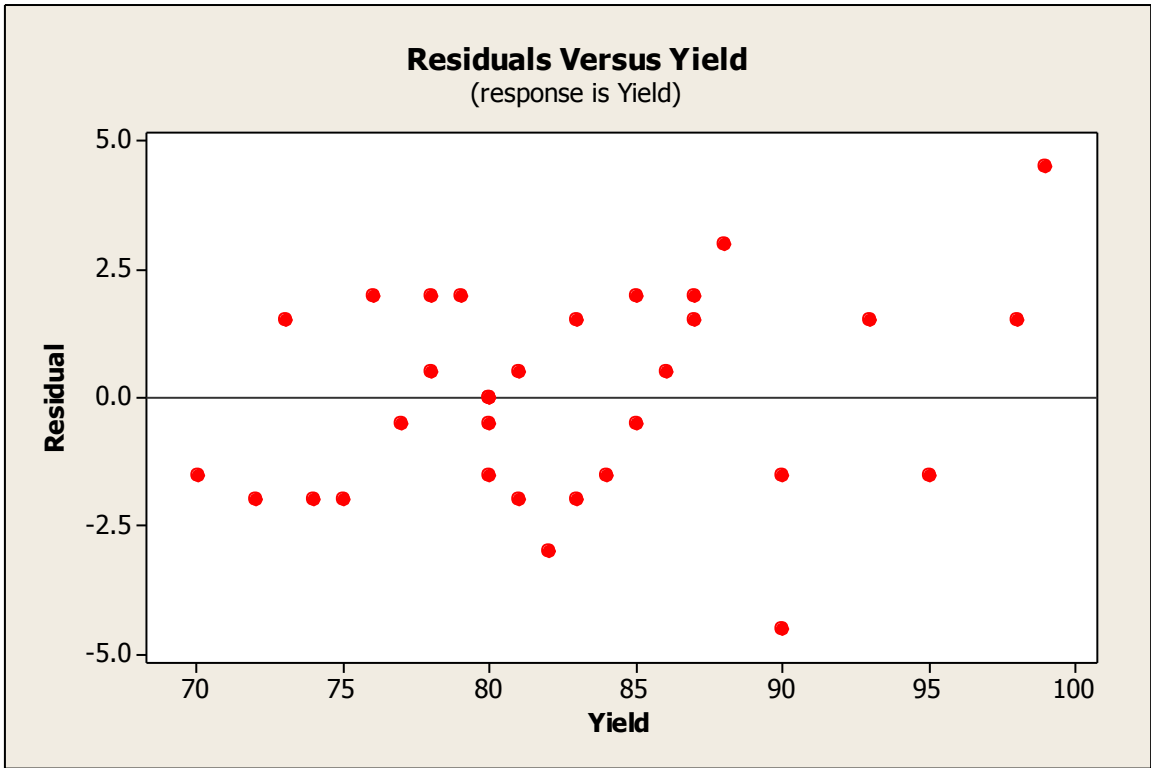
From the P-values we can find that the factors which are important in explaining yield are A, C, D, AB, AD, ABC, ABD and ABCD. The Pareto chart and Normal plot of the standardize effect explain that.

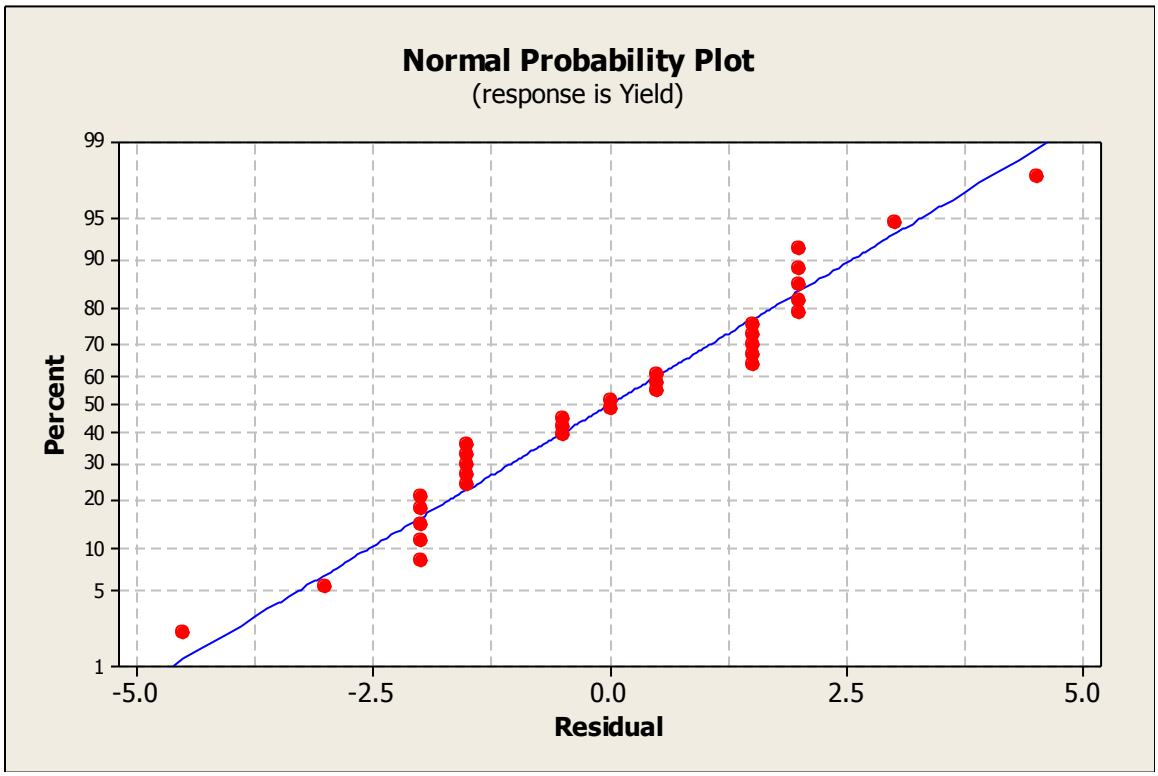
c) Write down a regression model for predicting yield …

$$Y\ (yield) = 82.781 - 4.531\ A - 0.656\ B - 1.344\ C + 1.969\ D + 2.031\ AB + 0.344\ AC - 1.094\ AD - 0.281\ BC - 0.094\ BD + 0.844\ CD - 2.594\ ABC + 2.344\ ABD - 0.469\ ACD - 0.469\ BCD + 1.219\ ABCD$$

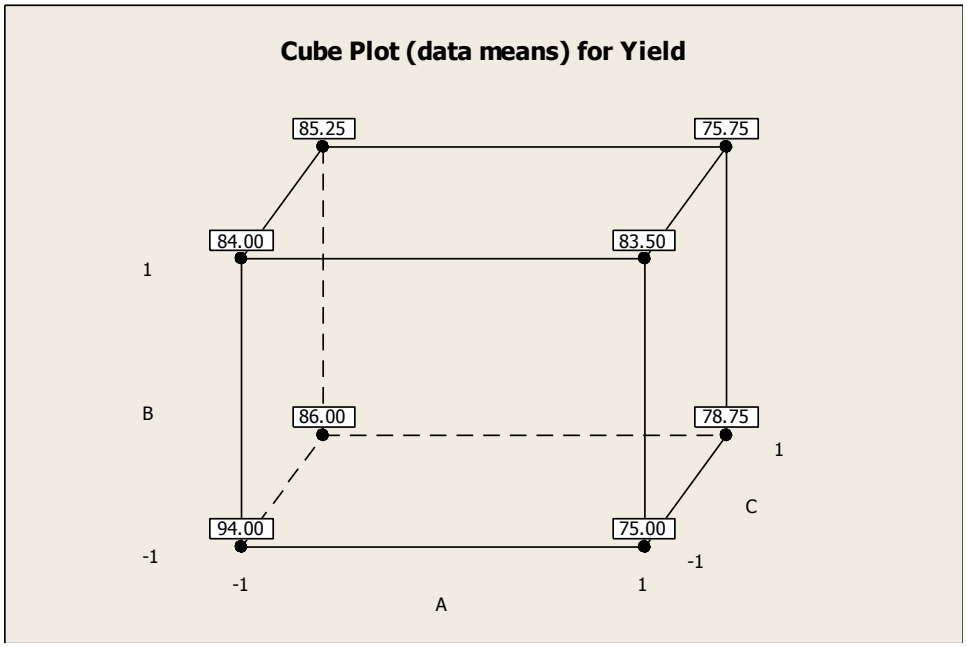
d) Plot the residuals versus the predicted yield and on a normal probability scale. Does the residual analysis appear satisfactory?

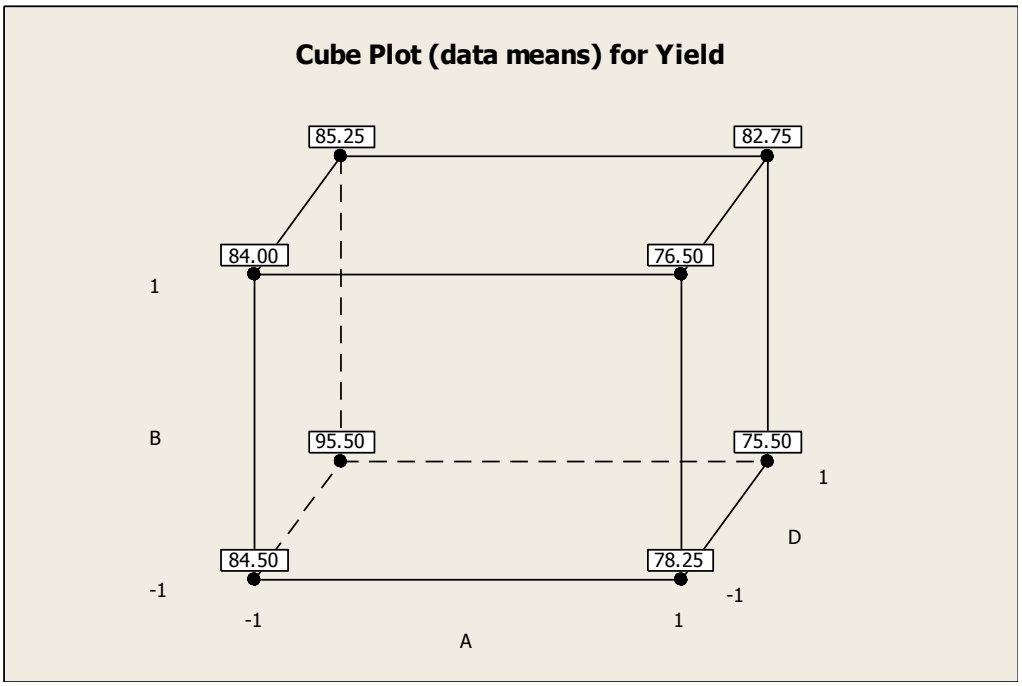
The residuals versus the predicted yield and the normal probability plot are shown below and from those graphs we can find the residual analysis appear satisfactory.




Normal Probability Plot
(response is Yield)
Percent
99
95
90
80
70
60
50
40
30
20
10
5
1
-5.0
-2.5
0.0
2.5
5.0
Residual

e) Draw a cube plot for (A, B and C) and (A, B and D)? Do the two plots aid in data interpretation? Where would you recommend that the process be run with respect to the four variables?





Yes, the two plots aid in data interpretation. And we recommend to run the process at (A) low (B) low, (C) low and (D) high.

Question6.18: Consider a variation of the bottle filling experiment from Example 5.3. Suppose that only two levels of carbonation are used so that the experiment is a 2^3 factorial design with two replicates. The data are shown below.

Run	Coded Factors			Fill Height Deviation	
	A	*B*	*C*	Replicate 1	Replicate 2
1	-	-	-	-3	-1
2	+	-	-	0	1
3	-	+	-	-1	0
4	+	+	-	2	3
5	-	-	+	-1	0
6	+	-	+	2	1
7	-	+	+	1	1
8	+	+	+	6	5

	Factor Levels	
	Low (-1)	High (+1)
A (%)	10	12
B (psi)	25	30
C (b/m)	200	250

(a) Analyze the data from this experiment. Which factors significantly affect fill height deviation?

(b) Analyze the residual from this experiment. Are there any indications of model inadequacy?

(c) Obtain a model for predicting fill height deviation in terms of the important process variables. Use this model to construct contour plots to assist in interpreting the results of the experiment.

(d) In part (a), you probably noticed that there was an interaction term that was borderline significant. If you did not include the interaction term in your model, include it now and repeat the analysis. What difference did this make? If you elected to include the interaction term in part (a), remove it and repeat the analysis. What difference does this make?

Answer: By Minitab:

a) Analyze the data from this experiment. Which factors significantly affect fill height deviation?

Minitab Output:

Factorial Fit: Fill Height Deviation versus Carbonation, Pressure, Speed

Estimated Effects and Coefficients for Fill Height Deviation (coded units)

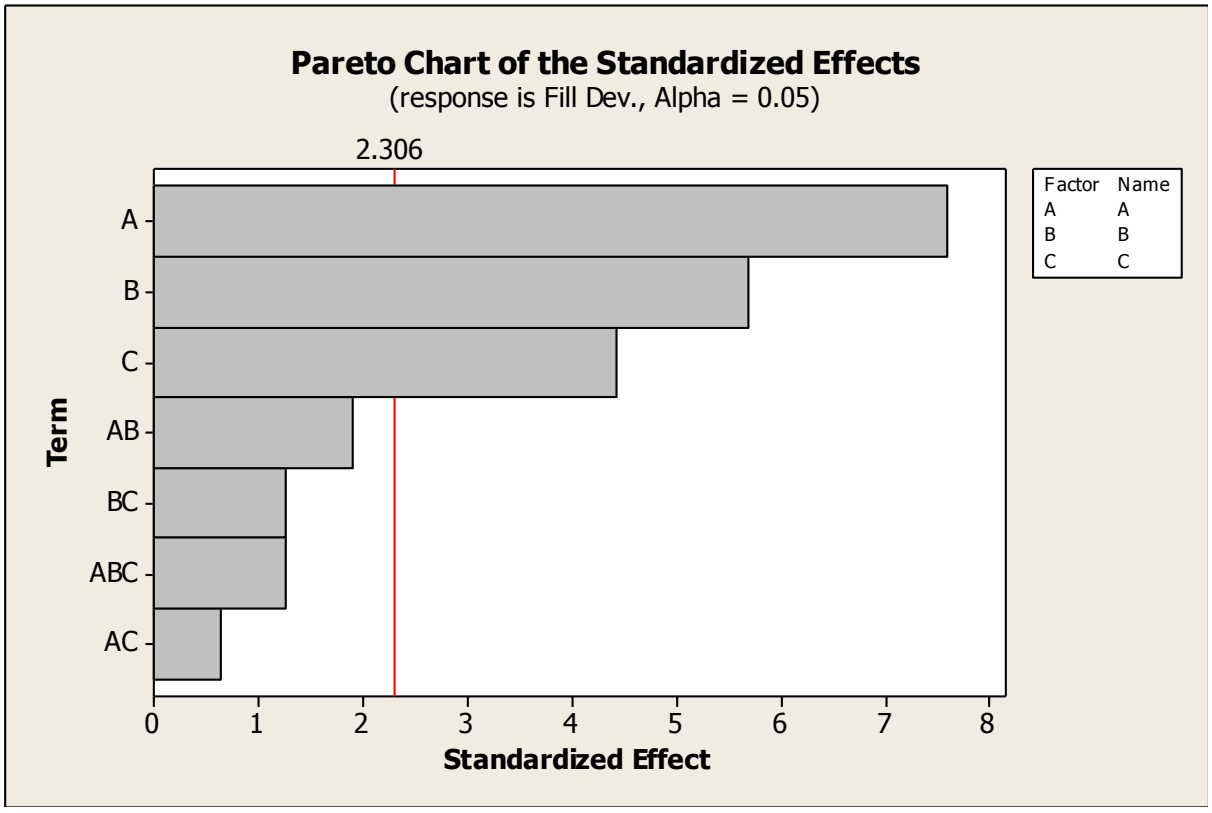
Term	Effect	Coef	SE Coef	T	P
Constant		1.0000	0.1976	5.06	0.001
Carbonation	3.0000	1.5000	0.1976	7.59	0.000
Pressure	2.2500	1.1250	0.1976	5.69	0.000
Speed	1.7500	0.8750	0.1976	4.43	0.002
Carbonation*Pressure	0.7500	0.3750	0.1976	1.90	0.094
Carbonation*Speed	0.2500	0.1250	0.1976	0.63	0.545
Pressure*Speed	0.5000	0.2500	0.1976	1.26	0.242
Carbonation*Pressure*Speed	0.5000	0.2500	0.1976	1.26	0.242

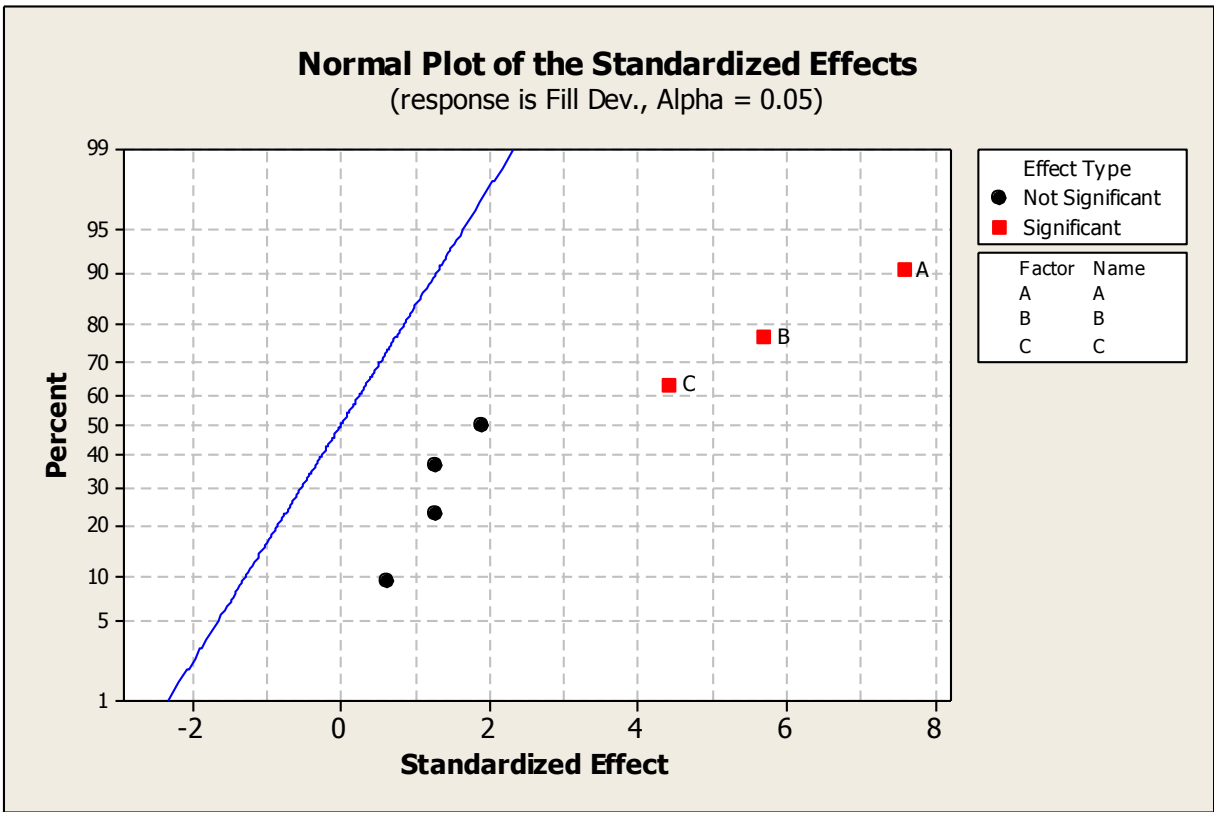
S = 0.790569 PRESS = 20
R-Sq = 93.59% R-Sq(pred) = 74.36% R-Sq(adj) = 87.98%

Analysis of Variance for Fill Height Deviation (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	68.5000	68.5000	22.8333	36.53	0.000
Carbonation	1	36.0000	36.0000	36.0000	57.60	0.000
Pressure	1	20.2500	20.2500	20.2500	32.40	0.000
Speed	1	12.2500	12.2500	12.2500	19.60	0.002
2-Way Interactions	3	3.5000	3.5000	1.1667	1.87	0.214
Carbonation*Pressure	1	2.2500	2.2500	2.2500	3.60	0.094
Carbonation*Speed	1	0.2500	0.2500	0.2500	0.40	0.545
Pressure*Speed	1	1.0000	1.0000	1.0000	1.60	0.242
3-Way Interactions	1	1.0000	1.0000	1.0000	1.60	0.242
Carbonation*Pressure*Speed	1	1.0000	1.0000	1.0000	1.60	0.242
Residual Error	8	5.0000	5.0000	0.6250		
Pure Error	8	5.0000	5.0000	0.6250		
Total	15	78.0000				

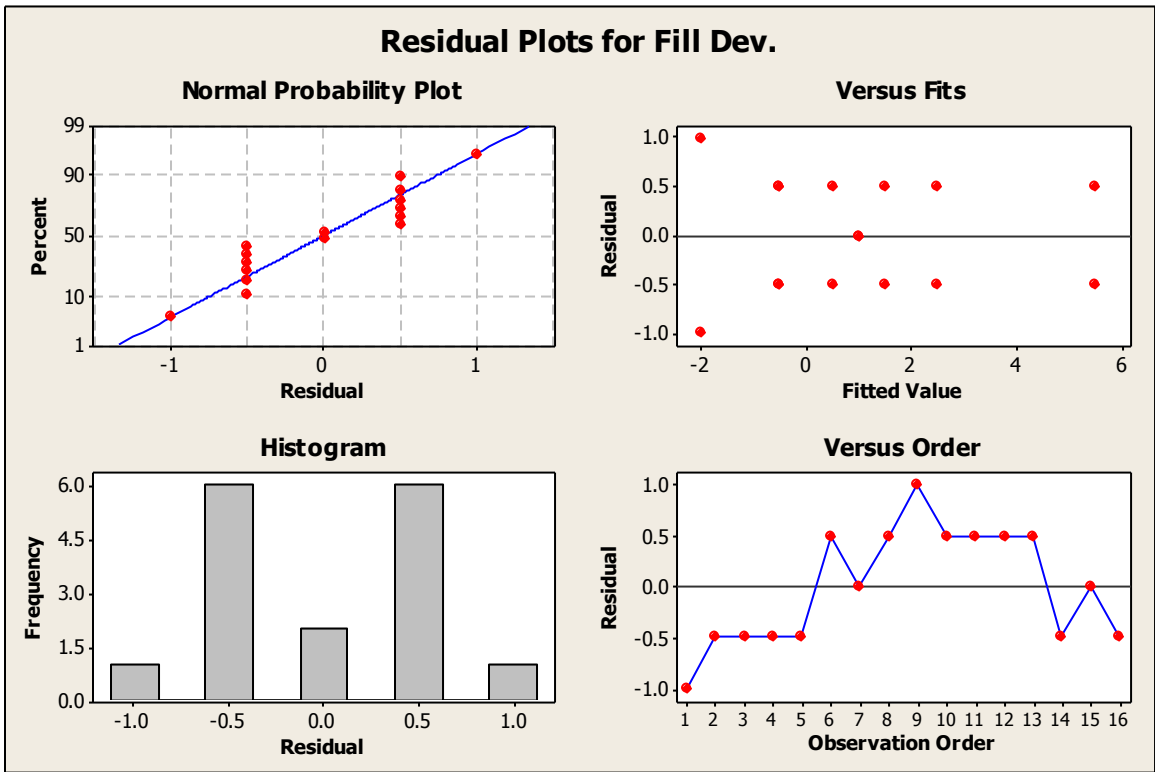
From the P-value we can find that the factors A, B and C have a significant effect on the response (fill height deviation) and the interactions have no significant effect. Pareto Chart and Normal Plot of the Standardized Effects shown below.




Normal Plot of the Standardized Effects
(response is Fill Dev., Alpha = 0.05)
Percent
99
95
90
80
70
60
50
40
30
20
10
5
1
-2
0
2
4
6
8
Standardized Effect
A
B
C
Effect Type
Not Significant
Significant
Factor Name
A A
B B
C C

(b) Analyze the residual from this experiment. Are there any indications of model inadequacy?

Using Minitab we analyze the residual and the results is as follows:



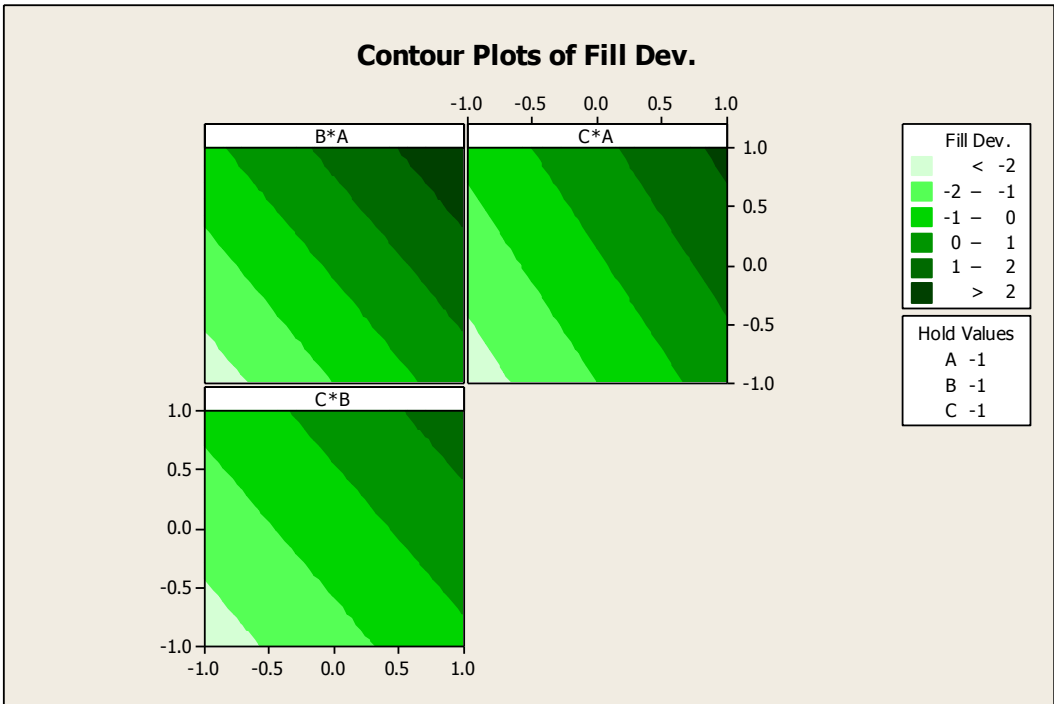
No, there is no indications of model inadequacy.

(c) Obtain a model for predicting fill height deviation in terms of the important process variables. Use this model to construct contour plots to assist in interpreting the results of the experiment.

$$Y \text{ (fill height deviation)} = 1 + 1.5\,A + 1.125\,B + 0.875\,C + 0.375\,AB + 0.125\,AC + 0.25\,BC + 0.25\,ABC$$

By using Minitab we construct the contour plots and from these plots we can assist our interpretation which was "No effect for the interactions on the response"

because the contour plots represent straight lines which means that the interactions effect isn't significant (the same result we get in part (a)).



(d) In part (a) we include the interaction term, here we remove the interaction term and repeat the analysis and the results was as follow:

Minitab Output:

Factorial Fit: Fill Height Deviation versus Carbonation, Pressure, Speed

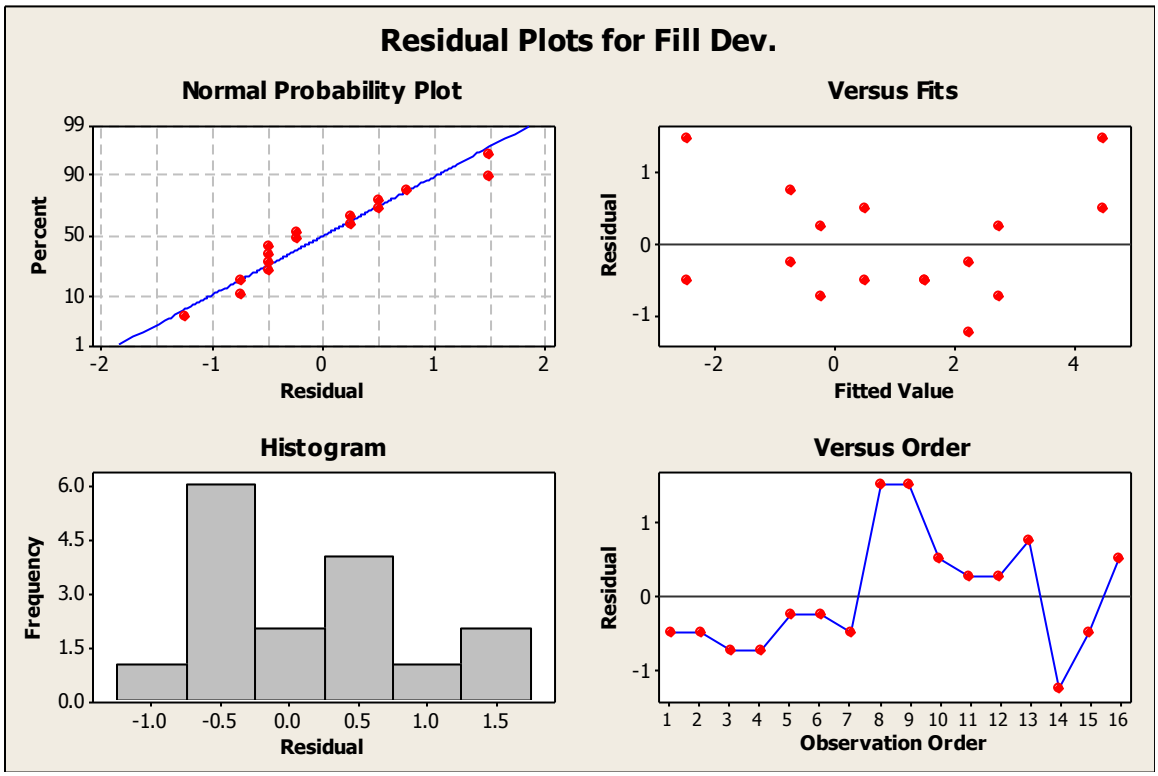
Estimated Effects and Coefficients for Fill Height Deviation (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		1.0000	0.2224	4.50	0.001
Carbonation	3.0000	1.5000	0.2224	6.74	0.000
Pressure	2.2500	1.1250	0.2224	5.06	0.000
Speed	1.7500	0.8750	0.2224	3.93	0.002

S = 0.889757 PRESS = 16.8889
R-Sq = 87.82% R-Sq(pred) = 78.35% R-Sq(adj) = 84.78%

Analysis of Variance for Fill Height Deviation (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	68.500	68.500	22.8333	28.84	0.000
Carbonation	1	36.000	36.000	36.0000	45.47	0.000
Pressure	1	20.250	20.250	20.2500	25.58	0.000
Speed	1	12.250	12.250	12.2500	15.47	0.002
Residual Error	12	9.500	9.500	0.7917		
Lack of Fit	4	4.500	4.500	1.1250	1.80	0.222
Pure Error	8	5.000	5.000	0.6250		
Total	15	78.000				



The residual analysis appears satisfactory but the R-sq(adj) in part (a) larger than the R-sq(adj) in this part which means that with including the interaction in our analysis we are closer to the actual experiment than analyzing without including the interactions.