Exercises

Example

A vendor submits lots of fabric to a textile manufacturer. The manufacturer wants to know if the lot average breaking strength exceeds 200 psi. If so, she wants to accept the lot. Past experience indicates that a reasonable value for the variance of breaking strength is 100(psi-sq).

Exercises

Example

Average breaking strength **exceeds** 200 psi.

variance of breaking strength is 100(psi-sq) , $\sigma = 10$

$$H_0: \mu = 200$$
$$H_1: \mu > 200$$

Table 2-3 Tests on Means with Variance Known

Hypothesis	Test Statistic	Criteria for Rejection
$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$		$\lvert Z_0 \rvert > Z_{\alpha/2}$
$H_0: \mu = \mu_0$ $H_1: \mu < \mu_0$	$Z_0 = \dfrac{\bar{y} - \mu_0}{\sigma/\sqrt{n}}$	$Z_0 < -Z_\alpha$
$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$		$Z_0 > Z_\alpha$
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$		$\lvert Z_0 \rvert > Z_{\alpha/2}$
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 < \mu_2$	$Z_0 = \dfrac{\bar{y}_1 - \bar{y}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	$Z_0 < -Z_\alpha$
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 > \mu_2$		$Z_0 > Z_\alpha$

Exercises

Example

Four specimens are randomly selected, and the average breaking strength observed is $\bar{y} = 214$ psi. The value of the test statistic is

$$Z_0 = \frac{\bar{y} - \mu_0}{\sigma/\sqrt{n}} = \frac{214 - 200}{10/\sqrt{4}} = 2.80$$

If a type I error of $\alpha = 0.05$ is specified, we find $Z_\alpha = Z_{0.05} = 1.645$ from Appendix Table I. Thus H_0 is rejected, and we conclude that the lot average breaking strength exceeds 200 psi.

z	.00	.01	.02	.03	.04	z
.0	.50000	.50399	.50798	.51197	.51595	0
.1	.53983	.54379	.54776	.55172	.55567	.1
.2	.57926	.58317	.58706	.59095	.59483	.2
.3	.61791	.62172	.62551	.62930	.63307	.3
.4	.65542	.65910	.66276	.66640	.67003	.4
.5	.69146	.69497	.69847	.70194	.70540	.5
.6	.72575	.72907	.73237	.73565	.73891	.6
.7	.75803	.76115	.76424	.76730	.77035	.7
.8	.78814	.79103	.79389	.79673	.79954	.8
.9	.81594	.81859	.82121	.82381	.82639	.9
1.0	.84134	.84375	.84613	.84849	.85083	1.0
1.1	.86433	.86650	.86864	.87076	.87285	1.1
1.2	.88493	.88686	.88877	.89065	.89251	1.2
1.3	.90320	.90490	.90658	.90824	.90988	1.3
1.4	.91924	.92073	.92219	.92364	.92506	1.4
1.5	.93319	.93448	.93574	.93699	.93822	1.5
1.6	.94520	.94630	.94738	.94845	.94950	1.6
1.7	.95543	.95637	.95728	.95818	.95907	1.7
1.8	.96407	.96485	.96562	.96637	.96711	1.8
1.9	.97128	.97193	.97257	.97320	.97381	1.9
[illegible]	[illegible]	.97778	.97831	.97882	.97932	2.0

z	.05	.06	.07
.0	.51994	.52392	.52790
.1	.55962	.56356	.56749
.2	.59871	.60257	.60642
.3	.63683	.64058	.64431
.4	.67364	.67724	.68082
.5	.70884	.71226	.71566
.6	.74215	.74537	.74857
.7	.77337	.77637	.77935
.8	.80234	.80510	.80785
.9	.82894	.83147	.83397
1.0	.85314	.85543	.85769
1.1	.87493	.87697	.87900
1.2	.89435	.89616	.89796
1.3	.91149	.91308	.91465
1.4	.92647	.92785	.92922
1.5	.93943	.90462	.94179
1.6	.95053	.95154	.95254
1.7	.95994	.96080	.96164
1.8	.96784	.96856	.96926
1.9	.97441	.97500	.97558
2.0	.97982	.98030	.98077

Exercises

Example

A chemical engineer is investigating the inherent variability of two types of test equipment that can be used to monitor the output of a production process. He suspects that the old equipment, type 1, has a larger variance than the new one.

Exercises

Example

two types of test equipment . **He suspects** that the old equipment, type 1, **has a larger variance** than the new one.

$$H_0: \sigma_1^2 = \sigma_2^2$$
$$H_1: \sigma_1^2 > \sigma_2^2$$

Two random samples of $n_1 = 12$ and $n_2 = 10$ observations are taken, and the sample variances are $S_1^2 = 14.5$ and $S_2^2 = 10.8$. The test statistic is

Table 2-7 Tests on Variances of Normal Distributions

Hypothesis	Test Statistic	Criteria for Rejection
$H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 \neq \sigma_2^2$	$F_0 = \dfrac{S_1^2}{S_2^2}$	$F_0 > F_{\alpha/2, n_1-1, n_2-1}$ or $F_0 < F_{1-\alpha/2, n_1-1, n_2-1}$
$H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 < \sigma_2^2$	$F_0 = \dfrac{S_2^2}{S_1^2}$	$F_0 > F_{\alpha, n_2-1, n_1-1}$
$H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 > \sigma_2^2$	$F_0 = \dfrac{S_1^2}{S_2^2}$	$F_0 > F_{\alpha, n_1-1, n_2-1}$

Exercises

Example

$$F_0 = \frac{S_1^2}{S_2^2} = \frac{14.5}{10.8} = 1.34$$

From Appendix Table IV we find that $F_{0.05,11,9} = 3.10$, so the null hypothesis cannot be rejected. That is, we have found insufficient statistical evidence to conclude that the variance of the old equipment is greater than the variance of the new equipment.

IV. Percentage Points of the F Distribution (*continued*)

$F_{0.05,\nu_1,\nu_2}$

ν_2 \ ν_1	Degrees of Freedom for the Numerator (ν_1) 1	2	3	4	5	6	7	8	9	10	12	15	20	24
	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35

inator (ν_2)

Exercises

Example

. Twenty observations on etch uniformity on silicon wafers are taken during a qualification experiment for a plasma etcher. The data are as follows:

5.34	6.65	4.76	5.98	7.25
6.00	7.55	5.54	5.62	6.21
5.97	7.35	5.44	4.39	4.98
5.25	6.35	4.61	6.00	5.32

(a) Construct a 95 percent confidence interval estimate of σ^2.
(b) Test the hypothesis that $\sigma^2 = 1.0$. Use $\alpha = 0.05$. What are your conclusions?
(c) Discuss the normality assumption and its role in this problem.
(d) Check normality by constructing a normal probability plot. What are your conclusions?

Exercises

Example

. Twenty observations on etch uniformity on silicon wafers are taken during a qualification experiment for a plasma etcher. The data are as follows:

5.34	6.65	4.76	5.98	7.25
6.00	7.55	5.54	5.62	6.21
5.97	7.35	5.44	4.39	4.98
5.25	6.35	4.61	6.00	5.32

(a) Construct a 95 percent confidence interval estimate of σ^2.
(b) Test the hypothesis that $\sigma^2 = 1.0$. Use $\alpha = 0.05$. What are your conclusions?
(c) Discuss the normality assumption and its role in this problem.
(d) Check normality by constructing a normal probability plot. What are your conclusions?

n=20

statistics of the sample

n	Mean	StDev
20	5.829	0.889

Exercises

Example

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} \leqslant \sigma^2 \leqslant \frac{(n-1)S^2}{\chi^2_{1-(\alpha/2),n-1}} \qquad (2\text{-}46)$$

$$\frac{(20-1)(0.88907)^2}{32.852} \le \sigma^2 \le \frac{(20-1)(0.88907)^2}{8.907}$$

$$0.457 \le \sigma^2 \le 1.686$$

III. Percentage Points of the χ^2 Distribution[a]

ν	.995	.990	.975	.950	.500	.050	.025	.010	.005
				α					
1	0.00 +	0.00 +	0.00 +	0.00 +	0.45	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	1.39	5.99	7.38	9.21	10.60
3	0.07	0.11	0.22	0.35	2.37	7.81	9.35	11.34	12.84
4	0.21	0.30	0.48	0.71	3.36	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	4.35	11.07	12.38	15.09	16.75
6	0.68	0.87	1.24	1.64	5.35	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	6.35	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	7.34	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	8.34	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	9.34	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	10.34	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	11.34	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	12.34	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	13.34	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	14.34	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	15.34	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	16.34	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	17.34	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	18.34	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	19.34	31.41	34.17	37.57	40.00
25	10.52	11.52	13.12	14.61	24.34	37.65	40.65	44.31	46.93

Exercises

Example

Test the hypothesis that $\sigma^2 = 1.0$. Use $\alpha = 0.05$. What are your conclusions?

$$H_0 : \sigma^2 = 1$$

$$H_1 : \sigma^2 \neq 1$$

Table 2-7 Tests on Variances of Normal Distributions

Hypothesis	Test Statistic	Criteria for Rejection
$H_0: \sigma^2 = \sigma_0^2$ $H_1: \sigma^2 \neq \sigma_0^2$		$\chi_0^2 > \chi^2_{\alpha/2,n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$
$H_0: \sigma^2 = \sigma_0^2$ $H_1: \sigma^2 < \sigma_0^2$	$\chi_0^2 = \dfrac{(n-1)S^2}{\sigma_0^2}$	$\chi_0^2 < \chi^2_{1-\alpha,n-1}$
$H_0: \sigma^2 = \sigma_0^2$ $H_1: \sigma^2 > \sigma_0^2$		$\chi_0^2 > \chi^2_{\alpha,n-1}$

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$$

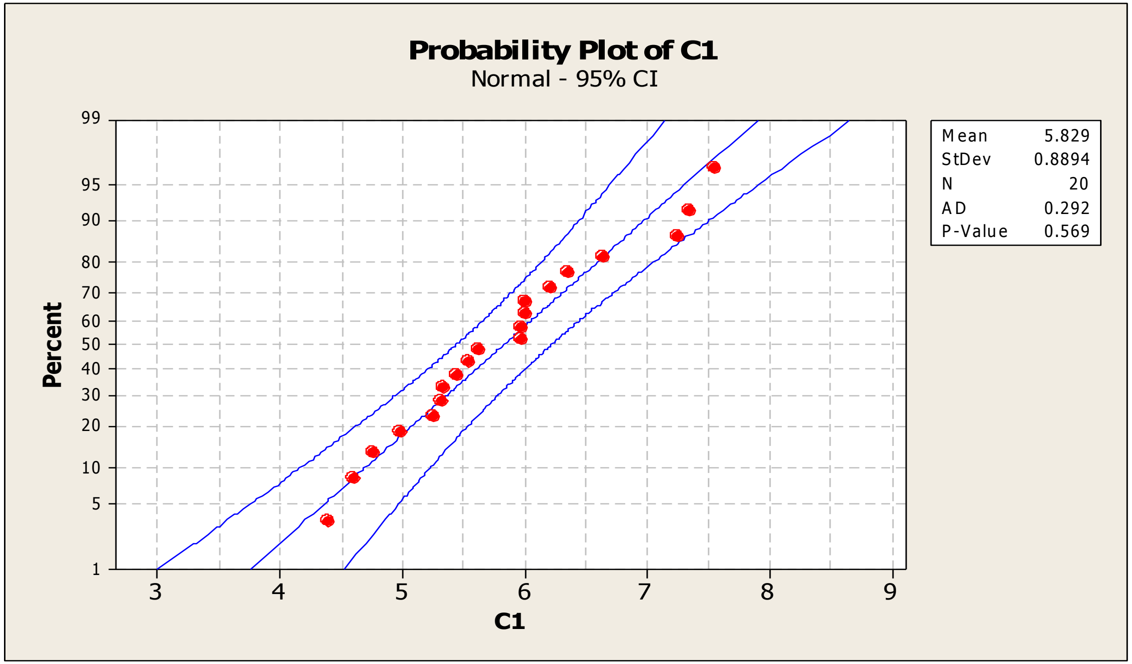
Exercises

Example

$$\chi^2_{0.025,19} = 32.852 \qquad \chi^2_{0.975,19} = 8.907$$

Do not reject. There is no evidence to indicate that $\sigma^2 \neq 1$

The normality assumption is much more important when analyzing variances then when analyzing means. A moderate departure from normality could cause problems with both statistical tests and confidence intervals. Specifically, it will cause the reported significance levels to be incorrect.= 15.016



Normality Plot

Exercises

Example

An article in the Journal of Strain Analysis (vol. 18, no. 2, 1983) compares several procedures for predicting the shear strength for steel plate girders. Data for nine girders in the form of the ratio of predicted to observed load for two of these procedures, the Karlsruhe and Lehigh methods, are as follows:

Girder	Karlsruhe Method	Lehigh Method
S1/1	1.186	1.061
S2/1	1.151	0.992
S3/1	1.322	1.063
S4/1	1.339	1.062
S5/1	1.200	1.065
S2/1	1.402	1.178
S2/2	1.365	1.037
S2/3	1.537	1.086
S2/4	1.559	1.052

(a) Is there any evidence to support a claim that there is a difference in mean performance between the two methods? Use $\alpha = 0.05$.

(c) Construct a 95 percent confidence interval for the difference in mean predicted to observed load.

(d) Investigate the normality assumption for both samples.

Exercises

Example

difference in mean performance between the two methods? Use a = 0.05

Girder	Karlsruhe Method	Lehigh Method	Difference	Difference^2
S1/1	1.186	1.061	0.125	0.015625
S2/1	1.151	0.992	0.159	0.025281
S3/1	1.322	1.063	0.259	0.067081
S4/1	1.339	1.062	0.277	0.076729
S5/1	1.200	1.065	0.135	0.018225
S2/1	1.402	1.178	0.224	0.050176
S2/2	1.365	1.037	0.328	0.107584
S2/3	1.537	1.086	0.451	0.203401
S2/4	1.559	1.052	0.507	0.257049
		Sum =	2.465	0.821151
		Average =	$\boxed{0.274}$	

$$H_0: \mu_1 = \mu_2 \qquad\qquad H_0: \mu_d = 0$$
or equivalently
$$H_1: \mu_1 \neq \mu_2 \qquad\qquad H_1: \mu_d \neq 0$$

The test statistic for this hypothesis is

$$t_0 = \frac{\bar{d}}{S_d/\sqrt{n}} \tag{2-41}$$

where

$$\bar{d} = \frac{1}{n}\sum_{j=1}^{n} d_j \tag{2-42}$$

is the sample mean of the differences and

$$S_d = \left[\frac{\sum_{j=1}^{n}(d_j - \bar{d})^2}{n-1}\right]^{1/2} = \left[\frac{\sum_{j=1}^{n} d_j^2 - \frac{1}{n}\left(\sum_{j=1}^{n} d_j\right)^2}{n-1}\right]^{1/2} \tag{2-43}$$

Exercises

Example

$$\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i = \frac{1}{9}(2.465) = 0.274$$

$$s_d = \left[\frac{\sum_{i=1}^{n} d_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} d_i\right)^2}{n-1}\right]^{1/2} = \left[\frac{0.821151 - \frac{1}{9}(2.465)^2}{9-1}\right]^{1/2} = 0.135$$

$$t_0 = \frac{\bar{d}}{\frac{S_d}{\sqrt{n}}} = \frac{0.274}{\frac{0.135}{\sqrt{9}}} = 6.08$$

$t_{\alpha/2, n-1} = t_{0.025,9} = 2.306$, reject the null hypothesis.

95 percent confidence interval for the difference in mean predicted to observed load

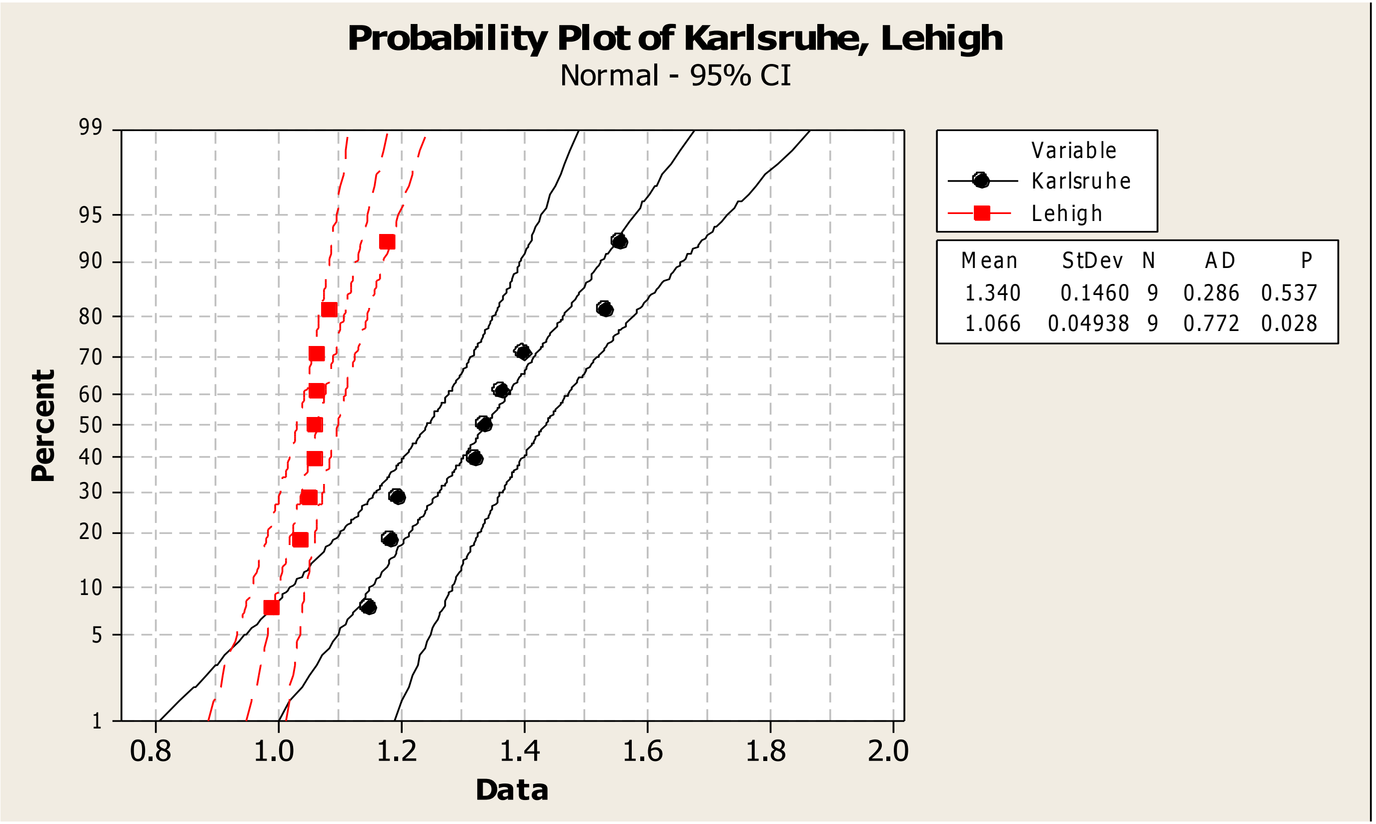
$$\bar{d} - t_{\alpha/2, n-1}\frac{S_d}{\sqrt{n}} \le \mu_d \le \bar{d} + t_{\alpha/2, n-1}\frac{S_d}{\sqrt{n}}$$

$$0.274 - 2.306\frac{0.135}{\sqrt{9}} \le \mu_d \le 0.274 + 2.306\frac{0.135}{\sqrt{9}}$$

$$0.17023 \le \mu_d \le 0.37777$$

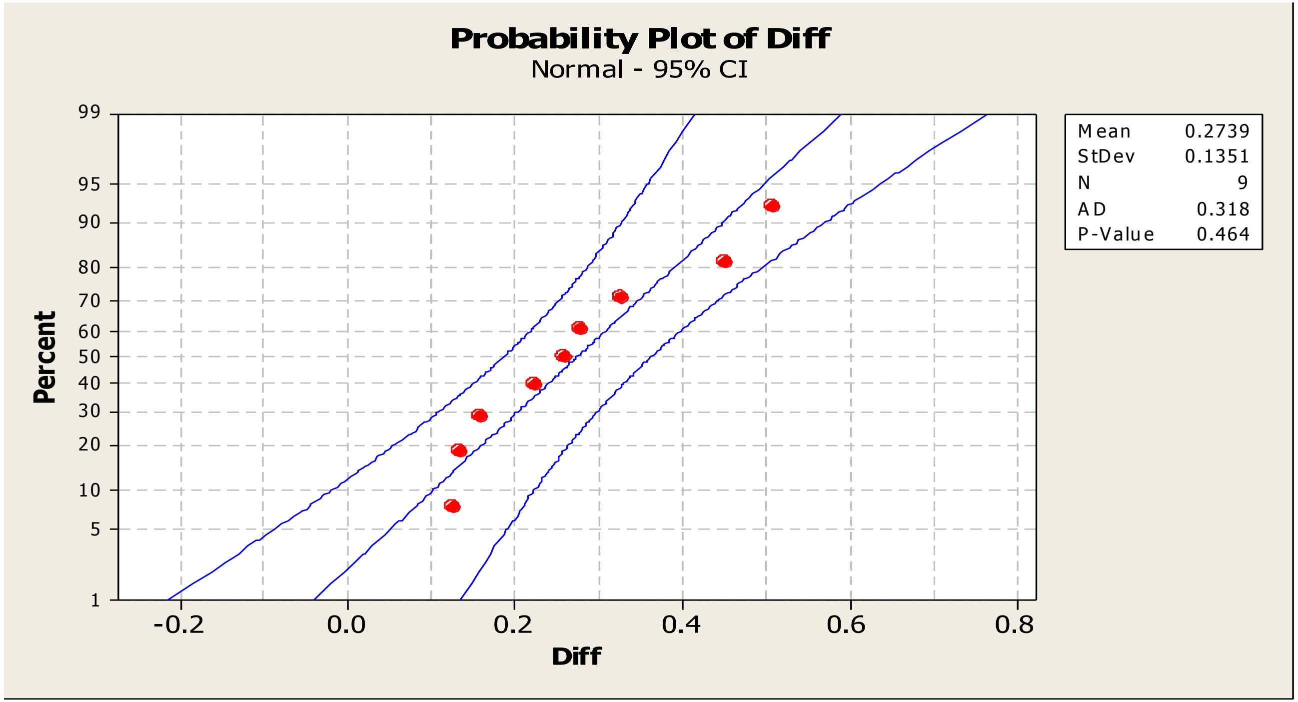
Exercises

Example



Exercises

Example



In the paired t-test, the assumption of normality applies to the distribution of the differences. That is, the individual sample measurements do not have to be normally distributed, only their difference